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# Dependent failure behavior modeling for risk and reliability: A systematic and critical literature review

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## Abstract

This paper presents a systematic and critical literature review on dependent failure behavior modeling in risk and reliability. A literature search is conducted systematically based on pre-defined protocols. The resulting papers are first analyzed through a meta-data and bibliometric analysis, in which the trend of publication growth, important publication sources and authors, collaboration among the authors are identified. The evolution of research topics in three time periods (prior to mid-1990s, mid-1990s - 2010 and 2010 - present) is also discussed through a semantic clustering: Early research (prior to mid-1990s) mainly came from nuclear risk assessment or statistical modeling. Due to the limitation of computational power, the models in this period are limited to small-scale and simplified systems. More computationally-demanding models, *e.g.*, copula, frailty, dynamic fault trees and Bayesian network, emerged since mid-1990s. Another important research topic that appeared since mid-1990s is maintenance optimization considering dependencies. More recent research trends (since 2010) include the dependent failure models considering degradation processes and random shocks, and large-scale, complex engineering systems like critical infrastructure and cyber-physical systems.

Then, the most important papers from the literature search are chosen for a content analysis and critical review. The main results of the critical literature review include: First, we summarize the dependent failure behavior in different system hierarchies, *i.e.*, failure mechanism level, component level, system level and systems-of-systems level. In each level, the main dependent failure behavior from literature is discussed with examples. Second, we develop a classification framework for the dependent failure behavior models. Depending on whether the dependency mechanism is explicitly considered, we broadly classified the existing models into statistical dependency models and mechanistic dependency models. Statistical dependency models do not explicitly consider the dependency mechanisms but model them in terms of statistical association among the variables, and can be further divided into lifetime distribution models, system state models and degradation process models. Mechanistic dependency models consider the dependency mechanisms explicitly, and can be further divided into failure interaction models and failure propagation models. The most frequently-used models in each category are critically reviewed, based on which we identify five challenging problems the current dependency models face, give our perspectives on their possible solutions, and discuss future research opportunities.

## Index Terms

Dependent failure behavior, risk, reliability.

# 1 Dependent failure behavior modeling for risk 2 and reliability: A systematic and critical 3 literature review

## 4 I. INTRODUCTION

5 On the evening of 31 May 2009, flight AF447 took off as usual in the beautiful sunset of Rio de Janeiro,  
6 Brazil, intended to provide its 216 passengers with a nice and comfortable journey to a city of love and romance,  
7 Paris – But the love and romance never arrive. Shortly after taking off, the plane crashed into the Atlantic ocean,  
8 killing all the 228 people (including 12 crew members) onboard [1]. The tragedy shocked the entire world, as  
9 modern aircrafts like AF447, and modern complex engineering systems in general, have been designed with a  
10 large number of redundant safety systems connected in a “defensive-in-depth” manner. Such accidents could  
11 occur only if all these safety systems fail, which, according to classical reliability theory, is highly unlikely,  
12 given the assumption that the failures of the safety systems are independent from one another.

13 What went wrong, then? The AF447 accident was initiated by a wrong air speed measurement. To have  
14 such a failure event, all the three pitot tubes in the air speed measurement system have to fail simultaneously,  
15 which is considered highly unlikely if the three pitot tubes are independent. In the AF447 accident, however,  
16 the three pitot tubes do fail at the same time, since the plane flew into a thunderstorm and the low temperature  
17 froze the three pitot tubes simultaneously [1]. In other words, the failures of the three pitot tubes are dependent  
18 due to a shared common event. Similar dependent failure behavior appears frequently in practice. For example,  
19 various experiment observations discover that the presence of erosion accelerates the degradation process due to  
20 corrosion [2]. Simultaneous failure of multiple safety barriers due to common cause events like environmental  
21 shocks or human errors has been acknowledged as one of the significant threat to nuclear and aerospace safety  
22 [3]. Failing to consider the dependent failure behavior in reliability analysis could severely overestimate of the  
23 reliability and significantly mislead decision makers.

24 Due to its extreme importance, dependent failure behavior has received great interest in the reliability  
25 community as early as 1960s. For example, Gumbel [4] presented a bivariate distribution for two marginal  
26 exponential lifetime distribution that are correlated. Krohn [5] modeled the statistical dependence among the  
27 seven failure modes in a electrical circuit. Today, the research on dependent failure behavior modeling has  
28 grown in substantial quantity, quality and diversity. A large number of models have been developed in the  
29 literature to describe dependent failure behavior in different hierarchies of a complex system. A good review  
30 of these works would be beneficial to the researchers and practitioners in the risk and reliability community  
31 for better understanding the state-of-the-art and identifying potential future research directions. Table I lists  
32 the related review articles and discusses their focuses and limitations. From Table I, we can conclude that the  
33 existing review papers can be improved from the following aspects:

- 34 • most of the existing reviews focus on maintenance optimization, but not failure behavior and reliability

- 1 modeling;
- 2 • the existing reviews only partially cover some particular types of dependencies: no holistic review exists
- 3 that covers dependent failure behavior in different hierarchies of a complex system.

TABLE I: Related review papers.

Paper	Focuses	Limitations
Dekker <i>et al.</i> [6]	Economic dependency on multi-component maintenance models.	Only maintenance grouping models under economic dependency are reviewed. No discussions on failure behavior.
Mkrtchyan <i>et al.</i> [7]	Use of Bayesian Belief Networks (BBNs) in human reliability analysis, and in particular modeling dependency assessment among human failure events.	Only BBN is reviewed. No discussions on the problems other than human reliability.
Keizer <i>et al.</i> [8]	Condition-based maintenance for multi-component systems considering economic, structural, stochastic, resource and performance dependence.	The focus is on maintenance modeling, but not on failure behavior and reliability modeling.
Kabir <i>et al.</i> [9]	Applications of Bayesian networks and Petri nets in safety, reliability, and risk assessments.	Only Bayesian network and Petri net are reviewed. Only the dependencies among basic events are discussed.
Meango and Ouali [10]	Reliability models that consider stochastic dependence between failure modes, including degradation processes. Three categories are considered: reliability indexes' interaction, state-based interaction and copula-based interaction.	Not a systematic literature review. Therefore, some important works are not included, <i>e.g.</i> , Bayesian network, cascading failure models, dynamic fault tree, frailty models.
Xing [11]	Cascading failure models.	Only discuss cascading failure but not the other dependent failure behavior.
Zhao <i>et al.</i> [12]	Maintenance Strategies considering economic, structural, stochastic and resource dependence.	The focus is on maintenance modeling, but not on failure behavior and reliability modeling.

4 Apart from the two points listed above, it should also be noted that all of the existing reviews rely on

5 subjective categorization and qualitative analysis to summarize previous research, which are highly dependent

6 on the experience of the authors and might lead to inaccurate interpretation and unreliable results [13]. Different

7 from standard literature reviews, the systematic literature review employs a replicable, scientific and intelligible

8 structured process to weaken the bias of studies selection and uses rigorous bibliometric tools to analyze the

9 selected literature objectively [13]. Hence, we use a systematic literature review to collect important papers

10 on dependent failure behavior modeling, and then conduct a critical literature review on the resulted papers.

11 Compared to the existing works, the contributions of this paper can be summarized as follows:

- 12 • it is the first known systematic and critical literature review on dependent failure behavior modeling;
- 13 • the dependent failure behavior in different hierarchies of a complex system are systematically summarized;
- 14 • a classification framework is presented for failure behavior and reliability models considering dependencies.

15 It should be noted that we limit the scope of this paper to dependent failure behavior modeling. Therefore, the

16 dependencies mainly arise in maintenance planning, *e.g.*, economical dependency, are not considered in this

17 review. For dependencies in maintenance planning, interested readers can refer to the review articles [6, 14].

18 The rest of this paper is summarized as follows. In Section II, we present the research questions we aim

19 to answer through the review and the methodology designed to answer these questions. Section III presents

20 the results of meta-data and bibliometric analysis. Section IV discusses typical dependent failure behavior in

21 different hierarchies of a complex system. Section V critically reviews the most important dependent failure

1 behavior models and presents a classification framework for them. In Section VI we discuss the challenges  
 2 faced by the current models and possible future opportunities. Finally, we conclude this paper in Section VII.

## 3 II. RESEARCH METHODOLOGY

4 Our research methodology starts by identifying the research questions we intend to answer through the  
 5 literature review:

- 6 1) What are the main research topics in the area of dependent failure behavior modeling in risk and reliability?
- 7 2) How do the research topics evolve over time?
- 8 3) What are the important sources to follow (journal, conference, key researchers)?
- 9 4) What are the challenges and opportunities for future research?

10 To answer these four questions, we conduct a systematic literature review following the methodology pre-  
 11 sented in Figure 1. The relevant papers are collected by a carefully-designed systematic protocol (see Sections  
 12 II-A for detail). The collected papers are then analyzed statistically based on their meta-data and bibliometrics  
 13 (see Section II-B for detail). Then, the important papers in the initial data set are selected for a content analysis  
 14 and critically reviewed, in order to identify the challenges and opportunities for the current research (see Section  
 15 II-C for detail).

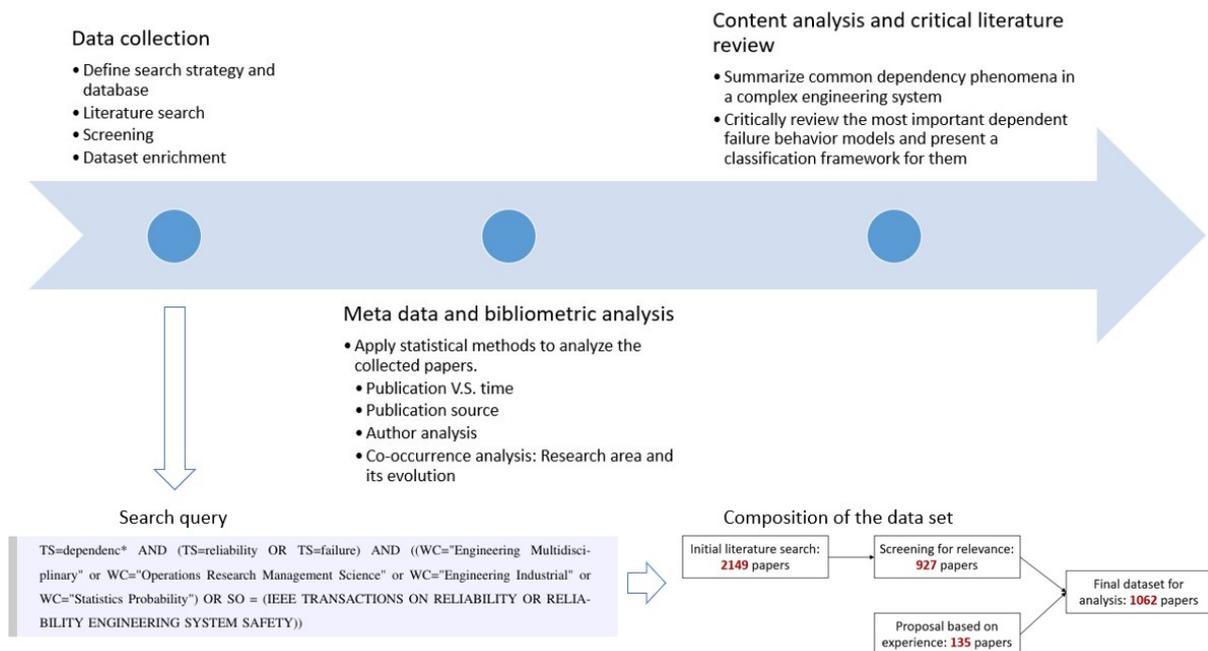


Fig. 1: Research methodology and results.

16 The research methodology in Fig. 1 is a combination a systematic literature review and a critical literature  
 17 review. A systematic literature review is designed to collect and identify important papers, and have a broad  
 18 perspective of the state-of-the-art of this field through bibliometric analysis. We intend to be as objective as  
 19 possible by presenting clearly the protocol of data collection and the method of analysis (Sect. III). Then, the  
 20 most important papers identified from the systematic literature review are critically reviewed by the authors. The

1 aims of the critical review are to provide a classification framework for dependency phenomena to be modeled  
2 (Sect. IV), critically review the existing dependent failure behavior models used to model the dependency  
3 phenomena (Sect. V), and propose the authors vision on challenges and opportunities in this domain (Sect. VI).

#### 4 A. Data collection

5 An initial literature search is conducted in the Web of Science Core Collection database based on predefined  
6 search query. We chose Web of Science (WoS) Core Collection because it is one of the most widely-accepted  
7 database for scientific research and it contains data from both journals and conferences. The search strategy is  
8 designed to limit the resulted papers only to the areas of dependencies in reliability and failure modeling. To  
9 do this, we require that the field "topic" in WoS has to include both "dependenc\*" and either "reliability" or  
10 "failure". The symbol "\*" means that any word starting with "dependenc", *e.g.*, "dependency", "dependencies",  
11 "dependence" all match the search query. As the keyword "dependency" has been used very widely in other  
12 scientific domains like physics or microelectronics, we add a few constraints to further exclude unrelated papers:  
13 A relevant paper must:

- 14 • contain keyword "reliability" or "failure";
- 15 • come from a limited range of research areas or published in a reliability-related journal.

16 The query we finally used for the initial literature search is given Figure 1, where TS, WC, SO are fields  
17 in the WoS database, representing Topic, Web of Science Categories and Publication Titles (*i.e.*, journal  
18 names), respectively.

19 The initial literature search returns a total number of 2,149 papers (As of December, 2022). However, a quick  
20 examination of the results reveals that although we already considered a number of constraints in the query,  
21 the results still contain a large number of unrelated publications. This is because, the keywords "dependency"  
22 and "reliability" all have diverse meanings. For example, papers discussing "temporal-dependency" of some  
23 events and the reliability of its statistical estimation match the defined query but are unrelated to our topic. To  
24 further remove these unrelated results, a screening is performed by performing a cursory review of the titles  
25 and abstracts of the initial results. After the screening, only 927 relevant papers are kept for further analysis.

26 Finally, we enrich the search results by manually adding another 135 papers that we find, based on our  
27 experience, crucial in this domain. The final data set for further analysis comprise of 1062 papers, as shown  
28 in Figure 1. One might wonder why the 135 manually added important papers are not captured by the search  
29 query defined in Figure 1. This is because, when designing the query, we add a few constraints to filter out  
30 irrelevant papers in order to improve the precision score of search query. While improving the precision, these  
31 constraints will inevitably reduce the recall score. For example, we could have some papers that are important  
32 but not published in the required field or journals. These papers are not captured by the designed search query.  
33 We already tried our best to improve the search query so that a balance could be obtained between its precision  
34 and recall. Another important reason is that, the WoS database does not cover all the papers, especially the  
35 papers published early (*e.g.*, before the 1990s) when the publications are not digitalized.

## 1 *B. Meta-data and bibliometric analysis*

2 Before thoroughly investigating their contents, the collected papers are preliminary analyzed based on their  
3 meta-data (title, abstract, author, publication source, publication year, *etc.*) and bibliometric like citations [15].

4 As shown in Figure 1, we mainly consider four analyses:

- 5 • Publication time analysis, which examines how does the number of publications evolves over time. This  
6 analysis will help us answer the research question 2 of this paper.
- 7 • Co-occurrence analysis, which aims at investigating the research areas and their evolution over time. This  
8 is done through a semantic clustering of the keywords from the title and abstract of each paper [16]. The  
9 results help us understand research questions 1 and 2 of this paper.
- 10 • Publication source analysis, which aims at identifying the important publication source. The results will  
11 help us answer research question 3 of this paper.
- 12 • Author analysis, which aims at identifying the important authors. This is done by checking the number of  
13 publications from each author. The results help us answer research question 3 of this paper.

14 The analyses are performed using open-source software VOSViewer [16] and Bibliometrix [17].

## 15 *C. Content analysis and critical literature review*

16 We then select a few important and representative papers for a deeper content analysis and critical review.  
17 The papers are selected following a two-stage strategy: in the first stage, the top 10% most cited papers in  
18 every 10 years in the dataset are analyzed. We choose 10 years as an evaluation horizon as citations need time  
19 to accumulate. Then, in the second stage, based on the initial analysis, we define a few keywords to search for  
20 other highly relevant papers to enrich the content analysis and critical literature review.

21 Through the analyses, we intend to give more insights to research questions 1, 2 and 4 of this paper. In  
22 particular, we:

- 23 • summarize the commonly encountered dependent failure behavior in different hierarchies (failure mecha-  
24 nism, component, system and system-of-system levels) of a complex system (see Section IV).
- 25 • critically review important dependent failure behavior models and develop a classification framework for  
26 these models. (See Section V).

## 27 III. META-DATA AND BIBLIOMETRIC ANALYSIS

28 This section presents the results from meta-data and bibliometric analysis. The publication numbers over  
29 time are analyzed in Section III-A. Then, in Section III-B, we discuss how does the research topics in this area  
30 evolve over time through semantic clustering. Important publication sources and key researchers in this area  
31 are discussed in Sections III-C and III-D, respectively.

### 32 *A. Publication growth*

33 Figure 2 shows the number of publications in each year (blue bars). A clear growth trend can be discovered  
34 for the numbers of publications in this area. Looking at the number of papers each year, however, could be  
35 biased, as in general, people publish more papers in recent years than several decades ago. To adjust for the

1 the potential bias, we collect the total number of papers published in two flagship reliability journals, *i.e.*, IEEE  
 2 Transactions on Reliability and Reliability Engineering and System Safety, in the same period of time, as a  
 3 measure of total number of publications related to risk and reliability in a given period of time. Then, we  
 4 calculate the ratio between the blue bars in Figure 2 and the number of publications in these two journals. The  
 5 result is presented as the red curve in Figure 2.

6 From Figure 2, there are two initial observations. First, both the absolute (blue bar) and relative (red curve)  
 7 number of publications in this area grows overtime, indicating that dependency has become an increasingly  
 8 popular topic in risk and reliability. Second, based on the slope of the growth curve, we discover two areas  
 9 where research interests on dependency grows dramatically: one is in the mid-1990s and the other is around  
 10 2010. This observation indicates that some new research topics might arise in these two periods of time, which  
 11 deserves further detailed analysis (see Section III-B).

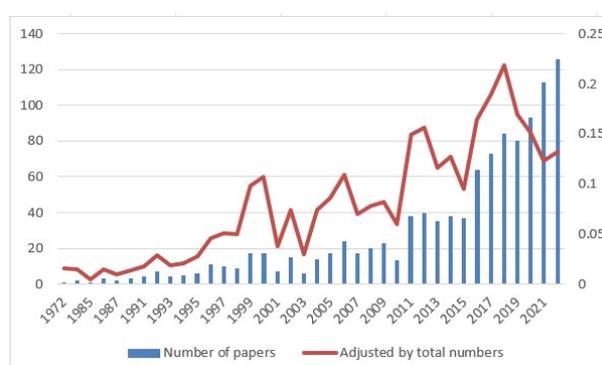


Fig. 2: Publications by year.

## 12 B. Co-occurrence analysis

13 In this section, a co-occurrence analysis is conducted based on the title and abstract of the collected papers  
 14 [16]. The result of such an analysis is a semantic network, where the nodes are words or phrases that appear in  
 15 the titles or abstracts. The presence of links between nodes indicate that the two words/phrases tend to appear  
 16 together in the same document. The size of the nodes indicates their appearance frequency, while the width of  
 17 the link indicates the degree of correlation between two items. From the constructed network, one can identify  
 18 different research topics by identifying clusters in the network. This is done by examining the network structure  
 19 and the semantic meanings of the nodes. The analysis is conducted using VOSViewer [16].

20 As discussed in Section III-A, we can discover from Figure 2 that significant increments on research activities  
 21 occurred around the mid-1990s (1995) and 2010. We intend to explore the cause of such growth by identifying  
 22 the main research topics in each period. Therefore, we consider the three ranges for the co-occurrence analysis:  
 23 before 1994, 1995-2009 and 2010 to present. The results are presented in Figs. 3 - 5. These results can help  
 24 us formulate a broad picture of the research field and understand the fundamental research topics in this field,  
 25 which laid down foundations for the critical literature reviews in Sects. IV - VI.

26 1) *Prior to 1994*: Only 36 papers (less than 2 papers per year) in our data set are published before 1995.  
 27 The co-occurrence network for these papers are presented in Figure 3. From the figure, we can identify two

1 key semantic clusters that represent different research topics, as shown in Table II. In general, the research  
 2 considering dependency modeling in this period is relatively scarce, with a strong emphasize on developing  
 3 statistical models for joint probability distribution from marginal distributions. Models that consider the detailed  
 4 description of dependency mechanisms, especially on a system level, are not discussed extensively. The very  
 5 few existing probabilistic models are developed only for relatively simple systems with exact (or approximate)  
 6 solutions for system reliability (*e.g.*, see [18]). Part of the reason might be, in this period of time, the  
 7 computational and simulation capability is not well-developed, which prevents the researchers from developing  
 8 very sophisticated (but realistic) models for dependency that cannot be solved analytically.

TABLE II: Representative keywords for the clusters (before 1994).

Cluster	Representative keywords	Representative papers
cluster 1	marginal distribution, multivariate failure time data, covariate	Liang <i>et al.</i> [19], Liu and Kiureghian [20]
cluster 2	probabilistic safety assessment, common cause failure, fault tree	Apostolakis and Moieni [21], Acosta and Siu [22]

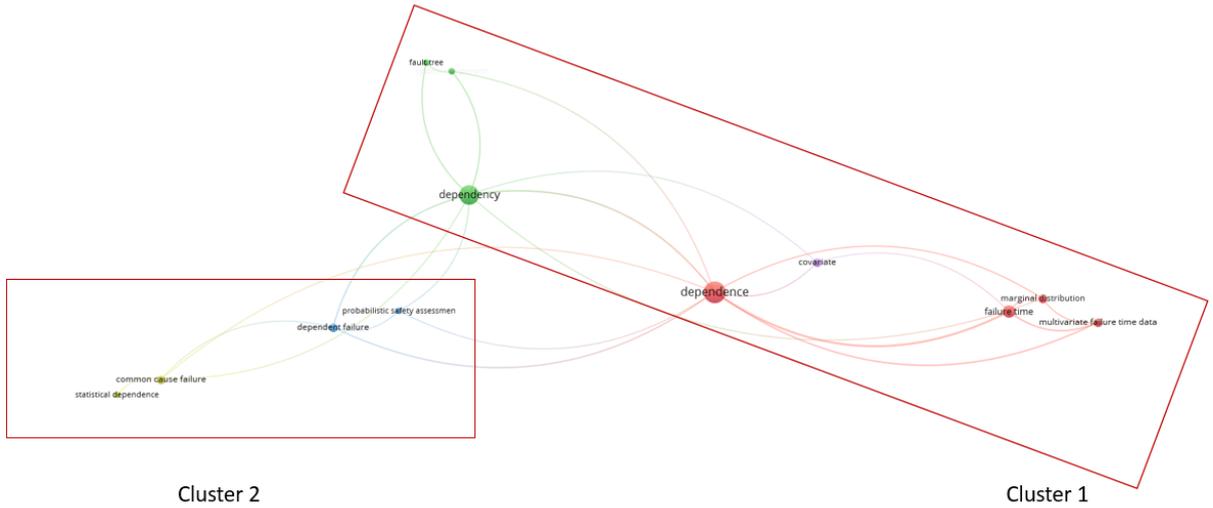


Fig. 3: Co-occurrence network for papers prior to 1994.

9 2) *From 1995 to 2009*: A total number of 261 papers ( $\approx 18$  per year) in our literature search results was  
 10 published in this period of time. The co-occurrence network for this period of time is presented in Figure 4.  
 11 From Figure 4, we discover three clusters. The representative keywords for each cluster are listed in Table III.

TABLE III: Representative keywords for the clusters (1995-2009).

Cluster	Representative keywords	Representative papers
cluster 1	multivariate failure time data, frailty, copula, covariate	Huard <i>et al.</i> [23], Lebrun and Dutfoy [24]
cluster 2	system reliability, component failure, dynamic fault tree, Markov chain, petri net, Bayesian network, maintenance	Andrews and Dunnett [25], Malhotra and Trivedi [26], Kang <i>et al.</i> [27]
cluster 3	probabilistic safety assessment, basic event, human reliability analysis, common cause failure	Boudali and Dugan [28], Cepen [29]

1 Among the three clusters, cluster 1 and 3 exist also in an earlier time period (see the previous subsection).  
 2 However, compared to the previous period, copula [23] and frailty models [30] have emerged in cluster 1 while  
 3 common cause failure has become an important topic in cluster 3. The second cluster, which considers system  
 4 reliability modeling considering dependent components, is a new research cluster that emerges in this period.  
 5 Some new models, *e.g.*, dynamic fault tree and Bayesian network, also appear in this period. Also, the study  
 6 about maintenance planning given dependencies emerges in cluster 2. The new models and problems in cluster  
 7 2 partly explain the first significant growth trend in Figure 2.

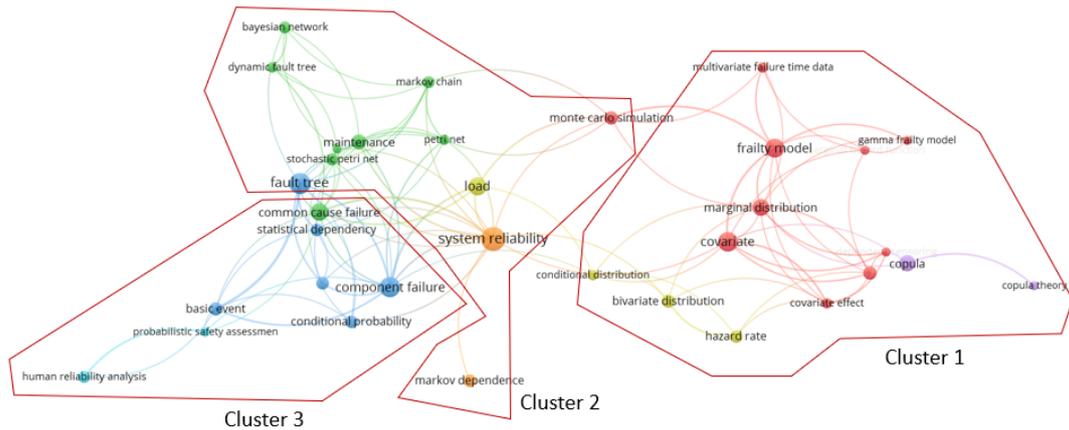


Fig. 4: Co-occurrence network for the papers published between 1995 to 2009.

8 3) 2010 - present: A total number of 799 papers ( $\approx 61$  papers per year) in our literature search results  
 9 were published in this period. We can discover six clusters in Figure 5. Table IV lists most representative  
 10 keywords and papers for each cluster. Among them, clusters 1 - 3 are inherent from the previous time periods,  
 11 while clusters 4 - 6 represent some new research trends that emerge in this period. The first emerging cluster  
 12 (cluster 4) represents research on dependent competing failure processes, where the dependencies between  
 13 hard failure caused by random shocks and soft failure caused by performance degradation are considered. We  
 14 will discuss this cluster in detail in Section V-A3. The second new research cluster (cluster 5) focuses on  
 15 applying combinatorial methods to model dependencies. For details, readers can refer to Section V-A2. Another  
 16 important cluster that emerges in this period is cluster 6 that focuses mainly on dependent failure behavior  
 17 related to vulnerability and resilience of critical infrastructures. For details, readers can refer to Section IV-D.

### 18 C. Publication source

19 Figure 6 listed the top 35 publication sources on which the papers we collected are published. It can be seen  
 20 that most of the collected papers are coming from journals. More specifically, the journals of interests might  
 21 involve several categories:

- 22 • Risk and reliability, *e.g.*, Reliability Engineering and System Safety, IEEE Transactions on Reliability,  
 23 Journal of Risk and Reliability;
- 24 • Industrial engineering, *e.g.*, IISE Transactions, EJOR, Computer and Industrial Engineering, IJPR;
- 25 • Statistics, *e.g.*, Technometrics, Lifetime Data Analysis, Communication in Statistics;

TABLE IV: Representative keywords for the clusters (2010-present).

Cluster	Representative keywords	Representative papers
cluster 1	copula, frailty model, vine copula, bivariate copula	Li <i>et al.</i> [31], Navarro and Durante [32]
cluster 2	system reliability, component dependency, maintenance optimization, condition-based maintenance	Guo <i>et al.</i> [33], Meango and Ouali [10]
cluster 3	probabilistic safety assessment, human error, human reliability analysis, Bayesian network	Mkrtychyan <i>et al.</i> [7], Kabir and Papadopoulos [9]
cluster 4	dependent competing risk, degradation, shock, hard failure, soft failure	Peng <i>et al.</i> [34], Jiang <i>et al.</i> [35]
cluster 5	combinatorial method, decision diagram, stochastic dependency, failure isolation, propagated failure	Xing and Leviten [36], Mo <i>et al.</i> [37]
cluster 6	critical infrastructure, resilience, vulnerability	Cagno <i>et al.</i> [38], Hosseini <i>et al.</i> [39]

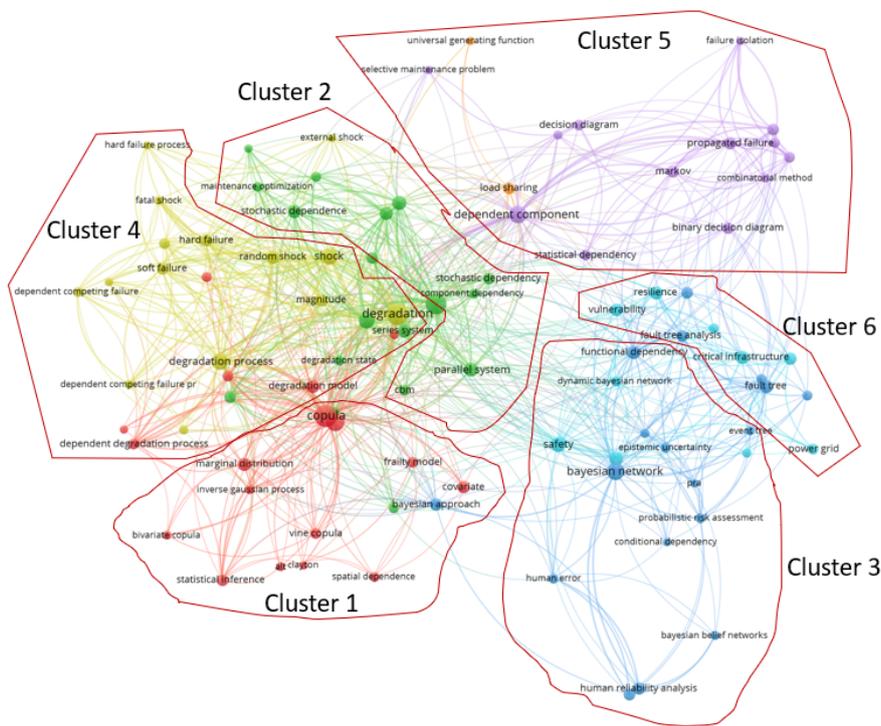


Fig. 5: Co-occurrence network for papers published from 2010 to present.

- 1 • Mechanical and civil engineering, *e.g.*, Probabilistic Engineering Mechanics, International Journal of
- 2 Critical Infrastructure Protection.
- 3 Besides, there are two conferences that also contribute large amounts of papers in our search result:
- 4 • European Conference on Safety and Reliability (ESREL) and
- 5 • Annual Reliability and Maintainability Symposium (RAMS).
- 6 These publication sources could be watched carefully for future research updates.

#### 7 D. Author analysis

8 To identify important researchers in this field, we analyze the total number of papers by each author in our  
9 collected database. The result is given in Figure 7. Another interesting analysis is to identify the collaboration

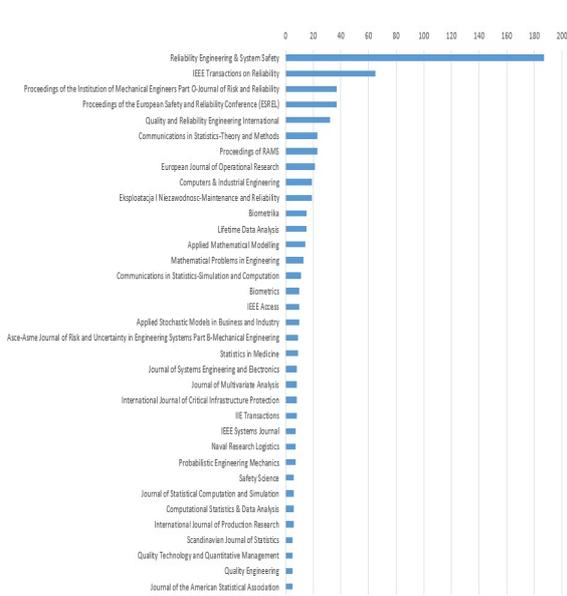


Fig. 6: Publication sources.

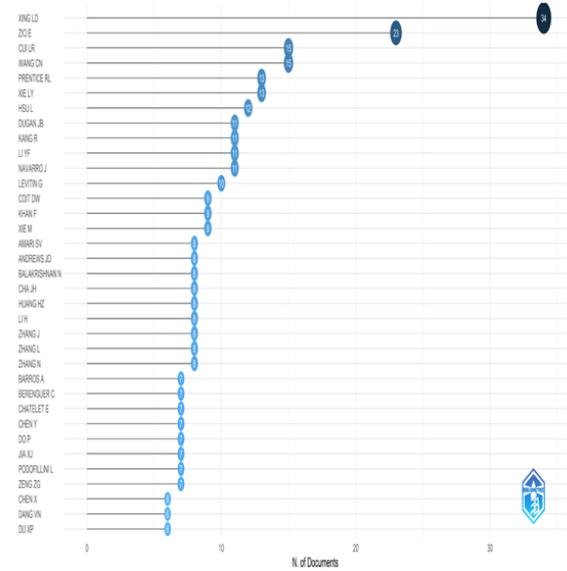


Fig. 7: Important authors based on number of related publications.

1 among the researchers. A collaboration network is constructed for the important authors listed in Figures 7 by  
 2 checking if a pair of authors have co-authored papers together [16]. The result is given in Figure 8. From this  
 3 figure, it is easy to see some researchers are the focal point of collaborations. We should take special care on  
 4 the updates of these active researchers.



Fig. 8: Collaborations among the relevant authors.

5 IV. DEPENDENT FAILURE BEHAVIOR IN DIFFERENT SYSTEM HIERARCHIES

6 In this section, we present the first result of the content analysis: a summary of frequently encountered  
 7 dependent failure behavior in the literature. The important papers identified from the systematic literature review  
 8 and bibliometric analysis are analyzed manually by the authors to identify the frequently encountered dependent  
 9 failure behavior. A classification framework for the dependent failure behavior is proposed by the authors from  
 10 a systems engineering perspective: the frequently encountered dependent failure behavior is summarized based  
 11 on the hierarchies of a complex engineering system in which the dependent failure behavior occurs, as shown  
 12 in Figure 9. Four levels of system hierarchies are considered:

- 1 • The bottom level aims at modeling failure mechanisms, *i.e.*, the physical, chemical, thermodynamic or  
 2 other processes that result in failure [40]. The impact of dependencies on failure mechanisms is discussed  
 3 in Section IV-A.
- 4 • The component level modeling focuses on the failure behavior of components in a system. Usually, a  
 5 component might be subject to one or several failure mechanisms. Section IV-B, then, discusses the  
 6 dependencies among the failure mechanisms in a component.
- 7 • On the next modeling level, the interactions and dependencies among components are considered to develop  
 8 failure behavior models for the system (see Section IV-C).
- 9 • System-of-system level mainly concerns large-scale interconnected complex systems that, by themselves,  
 10 are comprised of multiple heterogeneous complex systems. The major difference between a system-of-  
 11 system and a "system", according to the authors, is that, the scale and degree of complexity of a system-of-  
 12 system is much larger than a "system", which requires a different modeling approach than the top-down  
 13 bottom-up approaches in traditional systems engineering literature. Section IV-D discusses the dependent  
 14 failure behavior that could occur on the system-of-system level.

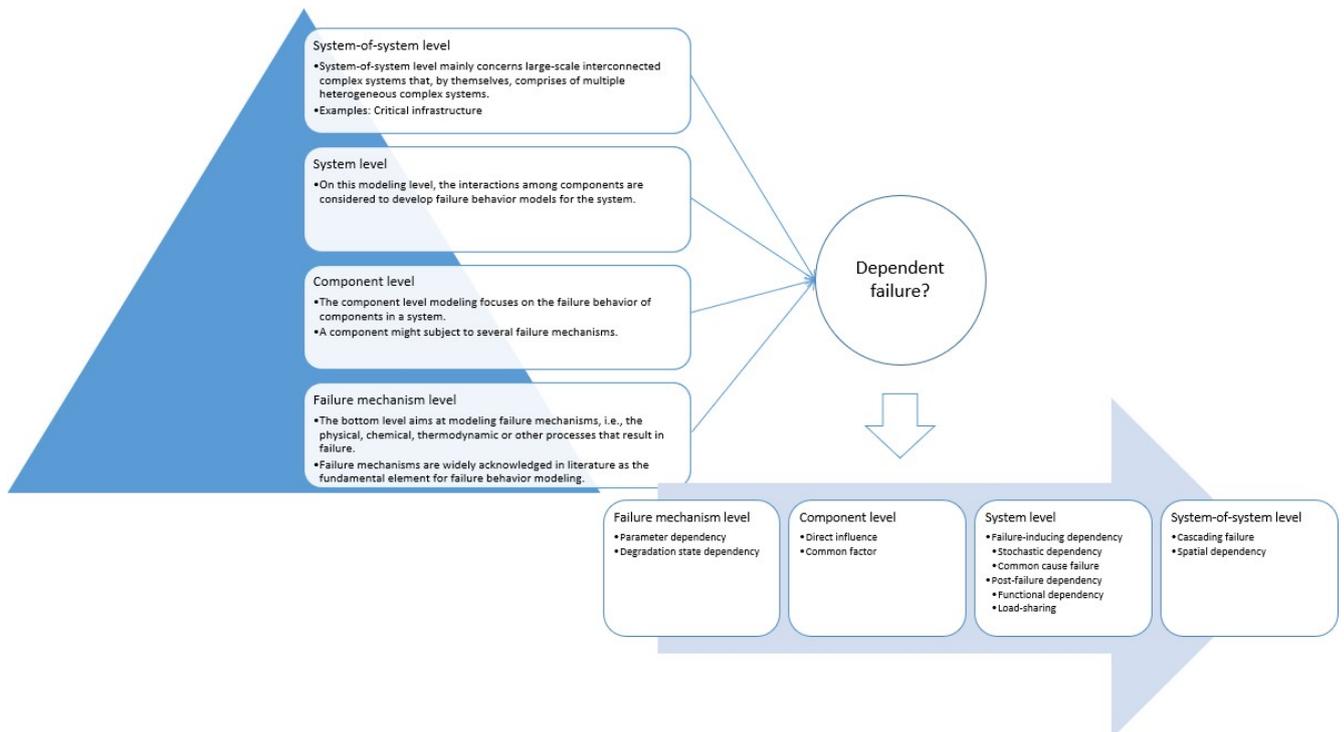


Fig. 9: Dependent failure behavior in different system hierarchies.

#### 15 A. Failure mechanism level

16 The physics-based failure mechanism models are often used as the smallest building blocks to construct more  
 17 complex system reliability models following a bottom-up approach [40]. As shown in Figure 9, dependent failure  
 18 behavior can be observed in a single failure mechanism model. In the failure mechanism level, typical dependent  
 19 failure behavior includes parameter dependency and degradation state dependency.

1    1) *Parameter dependency*: Parameter dependency arises naturally when the model parameters in the failure  
 2 mechanism model are statistically associated/correlated. For example, correlations are reported between wind  
 3 speed and wind direction, and between Young’s modulus and Poisson’s ratio, both of which are model parameters  
 4 in a failure mechanism model [41]. The exact cause of such a statistical association could be one parameter is  
 5 related to another in some ways, or because the correlated parameters are influenced by some common factors  
 6 like operating environment.

7    An early but classical example of parametric dependencies can be found in the works of Kiureghian and Liu  
 8 [20, 42], where they considered a generic limit state equation as a physics-based failure mechanism model with  
 9 dependent parameters  $\mathbf{X}$ . Huang and An [43] considered a simple physics-based model  $g_m(\mathbf{X}) = C - S$ , where  
 10  $\mathbf{X} = [S, C]$  and  $S, C$  represent the stress on a material and its associated strength, respectively. The dependency  
 11 arises because the conditional probability density function of the strength varies as the stress changes. Aghatise  
 12 *et al.* [44] developed a physics-based failure mechanism model based on finite element simulation for a off-  
 13 shore structure where the dependency among the input parameters was captured using vine copulas. Zheng *et*  
 14 *al.* [45] considered the dependency between wind speed and direction through Bayesian copulas and used them  
 15 to evaluate the deformation- and comfort-based total damage probability.

16    2) *Degradation state dependency*: For degradation failure mechanisms, another type of dependency could  
 17 arise when the new degradation state depends not only on the current state, but also on the historical states [46].  
 18 In literature, this kind of dependency is also referred to as long-term/long-range dependency, memory effect  
 19 or history-dependenct degradation [46]. This phenomenon can be explained by the fact that the two adjacent  
 20 moments might be subject to similar environment or use conditions [46]. Xi *et al.* [46] presented a numerical  
 21 index called the Hurst exponent to quantitatively measure the degree of the degradation state dependency, and  
 22 use it to investigate the degradation state dependencies in two real datasets (see Section II of [46] for details).  
 23 Degradation state dependency challenges the Markovian degradation models (*e.g.*, the models based on Wiener  
 24 or gamma process [47]), as most of them are based on the assumption that the future degradation state only  
 25 depends on the current state but not the history [47]. A perturbed Markovian process model is developed by  
 26 Oumouni and Schoefs [48] where degradation state dependency is considered by assuming that the degradation  
 27 increments are state-dependent but perturbed by a Gaussian white process with degradation-dependent variances.

## 28 B. Component level

29    A component is likely to experience multiple failure mechanisms that are mutually dependent. For example,  
 30 experimental data showed that corrosion and erosion of materials can accelerate each other, resulting in faster  
 31 degradation [2]. Another example is that when test specimens are susceptible to high temperatures and heavy  
 32 loads, fatigue interacts with creep so that the specimens’ times-to-failure are severely reduced [49]. As shown  
 33 in Figure 9, we can distinguish two types of dependent failure behavior: direct influence and common factors.

34    1) *Direct influence*: A schematic illustration of direct influence is given in Figure 10, where the dependency  
 35 between the two failure mechanisms ( $FM_1$  and  $FM_2$ ) are caused by the direct influence from  $FM_2$  to some  
 36 parameters ( $x_{dep}$ ) of  $FM_1$ . To model the direct influence among failure mechanisms explicitly, a graphical  
 37 method called an interaction graph is developed in [50]. Based on the graph, the component failure behavior  
 38 model considering the dependencies among failure mechanisms can be derived.

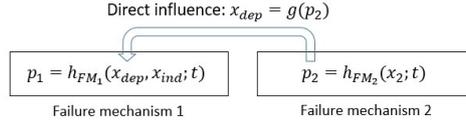
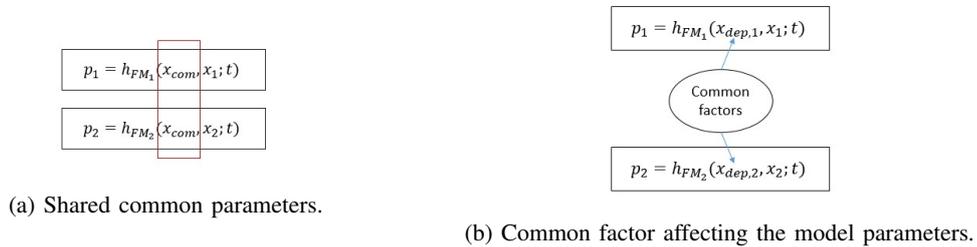


Fig. 10: A schematic of direct influence.

1 A good example of the direct influence can be found in the works of Peng *et al.* [34] on failure of  
 2 Micro-Electro-Mechanical Systems (MEMS) device. According to an earlier test performed at Sandia National  
 3 Laboratory [51], an MEMS can fail due to either a degradation failure caused by wear, or a catastrophic failure  
 4 caused by a random shock process. The two failure mechanisms are dependent as the random shock process not  
 5 only causes the catastrophic failure, but also brings additional increments to the degradation process. Another  
 6 example of the direct influence can be found in [50], where dependency between adhesive and abrasive wear in  
 7 a sliding spool is considered. The adhesive wear depends on the surface roughness, which could be impacted  
 8 by the abrasive wear.

9 2) *Common factor*: Dependencies in the component-level can also be due to common factors affecting the  
 10 failure mechanism models, as illustrated in Figure 11. Common factors could be the parameters shared by  
 11 several failure mechanism models (Figure 11(a)), or some common external factors that affect different failure  
 12 mechanism models simultaneously (Figure 11(b)).



(a) Shared common parameters.

(b) Common factor affecting the model parameters.

Fig. 11: A schematic of common factor.

13 After examining the degradation data, Fang *et al.* [52] found out that the degradation processes of the  
 14 benzene ring mass loss and the stretching of aryl ether are statistically correlated. They identified cause of such  
 15 correlation as shared common factors (temperature, humidity, as well as UV spectrum and intensity) between  
 16 the two degradation processes [52]. In [53], the results of an accelerated degradation test on a bearing under  
 17 different rotational speeds reveal that both the degradation rate and variation in the failure mechanism model  
 18 are associated with the rotational speed. Therefore, the rotational speed is a common factor affecting the two  
 19 degradation parameters.

### 20 C. System level

21 Systems are made up of components. Therefore, when considering the system's failure behavior, the depen-  
 22 dencies among the components need to be taken into account. In the results of our literature search, four types  
 23 of dependencies among the components can be found. These four dependencies can be further classified into

1 two categories, based on whether the dependency leads to the initial failure (failure-inducing dependency), or  
2 appears after the initial failure occurs (post-failure dependency), as shown in Figure 9.

3 1) *Failure-inducing dependency*: As can be seen in Figure 9, failure-inducing dependency includes stochastic  
4 dependency and common cause failure. Stochastic dependency arises when the degradation or failure processes  
5 of components are stochastically dependent or correlated [8]. As in the component level, the root cause of  
6 the stochastic dependency could be direct influence or common factors (See Section IV-B). An example of  
7 stochastic dependency in the system level can be found in [54], where stochastic dependency appears among  
8 the four stations in a manufacturing system as the times-to-failure of the four stations are statistically correlated.  
9 Stochastic dependency could also involve degradation processes in different components. For example, in [55],  
10 a system comprised of two interacted gears is tested. The results show that the degradation rate of one gear  
11 is dependent on the degradation state of the other. In [56], the component degradation paths are grouped as  
12 different clusters, where in each cluster, the degradation paths from the different components are dependent. For  
13 example, in a sliding spool, the spool and sleeve are physically in touch with the same hydraulic oil and share  
14 the same operational conditions. Therefore, they form a dependency cluster and the degradation path models  
15 share a common factor. The degradation process of one component could also impact the lifetime distribution  
16 of another component. For example, in [57], the failure rate of a motor is modeled as a baseline failure rate  
17 times an adjustment factor that depends on the vibration magnitude of a pump in the same system.

18 According to the definition from IEC, Common-Cause Failures (CCFs) are simultaneous failures of multiple  
19 components of a system due to a shared root cause [58]. According to [59], common cause failures could occur  
20 due to undetected design errors, operator and maintenance errors, equipment failure caused by the failure of  
21 a different equipment, and equipment failure resulting from some unforeseen external event. For example, in  
22 [58], a protection system comprised of four safety barriers is considered. However, two common cause failure  
23 events could affect the system: one is caused by tornado and affects three safety barriers while the other is  
24 caused by an earthquake and affects all four safety barriers [58]. Another example of common cause failure  
25 can be found in [60] on an oil supply system to an aero-engine. Two common cause failures might affect the  
26 system: (1) The differential pressure switch and the oil catheter can fail at the same time due to the physical  
27 interactions between them; (2) The warning subsystem and the oil filter fail simultaneously when an extreme  
28 work load appears due to an external shock from the environment.

29 2) *Post-failure dependency*: As shown in Figure 9, post-failure dependency includes load-sharing and func-  
30 tional dependency. Load-sharing often occurs when several components share the same load. Then, the failure of  
31 one component increases the load on the remaining ones and affects their failure behavior. A detailed real-world  
32 example of load-sharing is presented in [61], where they considered a simplified model for an aircraft hydraulic  
33 system of four identical active pumps and one backup pump working in warm-standby mode. The four active  
34 pumps jointly share the system working load until all the active pumps fail. If any working pump fails, the  
35 degradation of the remaining pumps accelerates as the the loads on the remaining pumps are increased. Apart  
36 from affecting the degradation of the remaining components, load-sharing could affect directly the failure rate of  
37 the remaining components. For example, in [62], a load-sharing 3-out-of-7 CNC machine system is considered.  
38 Due to load-sharing, if any machine fails, the failure rate of the remaining machine is increased, which is  
39 modeled as a baseline failure rate multiplied by an adjustment factor related to the load on the remaining

1 machine.

2 Functional dependency is defined as the functions of some components (referred to as functional dependent  
3 components) rely on the function of other components (referred to as functional trigger components) [63].  
4 The failure of one trigger component causes other dependent components within the same system to become  
5 unusable or isolated from the system. An example of functional dependency is given in Wang *et al.* [64], where  
6 sensors are functionally dependent on relays as the former requires the latter to provide communication supports.  
7 If the relay in a functional group fails, the sensors in the same functional group are isolated. In [65], a smart  
8 home is considered where the sensors in the rooftop solar electric system is functionally dependent on a relay  
9 of the energy storage system, which is further functionally dependent on a relay in the energy management  
10 system. The reliability of this smart home considering the cascading of functional dependencies is analyzed  
11 based on a new combinatorial method in [65].

#### 12 *D. System-of-system level*

13 In the literature, there are a lot of complex engineering systems that can be regarded as a system-of-system,  
14 *e.g.*, critical infrastructure, cyber-physical system, industrial Internet-of-things, smart grid. Luijff and Klaver  
15 [66] analyzed a database of critical infrastructure failures reports from 2004 until the mid-2018, and concluded  
16 that, for a complex system-of-system like critical infrastructure, two types of dependent failure behavior can be  
17 identified: cascading failure and spatial dependency (in fact, in [66], they use the term common cause failure to  
18 describe spatial dependency). Cascading failure is caused by the propagation of failures among the individual  
19 systems comprising the system-of-system, while spatial dependency refers to the fact that the components in a  
20 geographical region tends to fail together when impacted by the same event like natural hazards.

21 *1) Cascading failure:* A good example of cascading failure in systems-of-systems can be found in [67],  
22 where the initial failures can propagate in an interconnected power-communication network: A failure in a  
23 node in the power network can propagate to the interconnected nodes in the communication network, as the  
24 latter loses electricity supports from the power network. Bellè *et al.* [68] discussed an interconnected rail-power  
25 network. The failure in the power network can propagate within itself due to load shedding, and can propagate  
26 to the connected railway network, as the traction stations in the railway network cannot function without the  
27 electricity supported by the power network. In [69], an example is presented where an initial failure in one line  
28 could spread and cause congestion in the whole network, since the passengers affected by the failed line have  
29 to redirect themselves and increase the congestion on the other parts of the network.

30 *2) Spatial dependency:* Electrical power systems subject to seismic risks are good examples of spatial  
31 dependency, as the fragility of the power system components depends on the peak ground acceleration caused  
32 by the earthquake [70]. An electricity distribution and metro networks in London are examined in [71], where  
33 spatial dependency arises when the network is subject to the risk of flooding: if some assets are located in  
34 the same flood zones, they fail simultaneously due to the damages from the flood. In [72], examples of spatial  
35 dependency in a deteriorating water infrastructure and transportation infrastructure are given, including both  
36 physical dependency from road to pipe, and operational dependency from pipe to road. He and Cha [73]  
37 discussed an infrastructure network, consisting of power, water and telecommunication systems under the threat  
38 of hurricanes. The arrival of a hurricane can destroy several items in a geographical area at the same time,

1 creating spatial dependency in the systems-of-systems.

## 2 V. MODELS FOR DEPENDENT FAILURE BEHAVIOR

3 In this section, we aim to critically review important dependent failure models and propose a classification  
 4 framework based on their inherent natures. The developed classification framework is presented in Figure 12.  
 5 We can first distinguish broadly between statistical and mechanistic dependency models. Statistical dependency  
 6 models do not explicitly consider the mechanisms from which the dependencies arise. Rather, the focus is to  
 7 develop stochastic models to capture the dependencies in terms of statistical association among variables in the  
 8 model. In contrast, mechanistic dependency models first look at the mechanisms that generate the dependencies,  
 9 and then develop the dependency models based on the dependency mechanisms. It should be noted that the  
 10 mechanistic models can be either stochastic or deterministic. The key difference between the mechanistic and  
 11 statistical dependency model is whether the dependency mechanism is considered explicitly, not whether the  
 12 model is stochastic or deterministic.

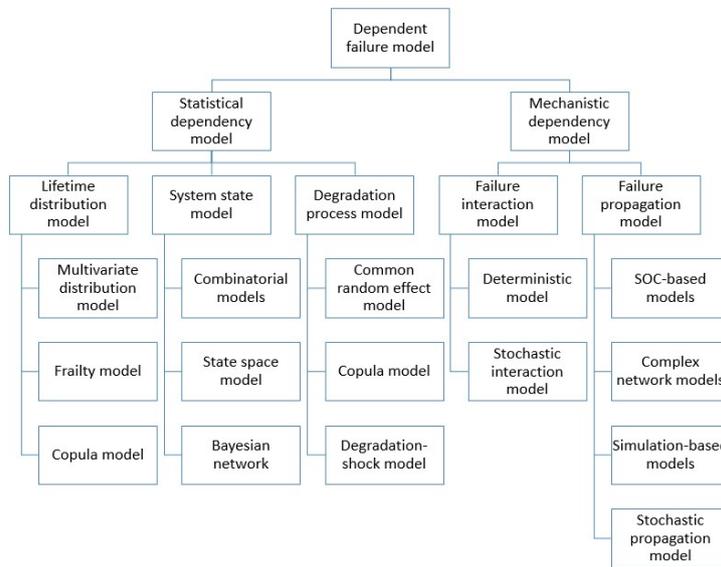


Fig. 12: A classification framework for dependent failure behavior models.

13 The statistical dependency models can be further divided into lifetime distribution models, system state  
 14 and degradation process models, as shown in Figure 12 (See Section V-A for details). Lifetime distribution and  
 15 system state models both belong to a broader class of random variable models, as they model the statistical  
 16 dependencies among random variables. In the lifetime distribution models, the considered random variables are  
 17 time-to-failure, while in system state models, the random variables considered are state variables that often take  
 18 discrete state values (binary or multi-state) that represent the state of components and systems. Unlike the lifetime  
 19 distribution and system state model, which only models the result of the failure evolution process, degradation  
 20 process models focuses on the failure evolution process itself. In particular, the statistical dependencies among  
 21 degradation processes are taken into consideration in these models.

22 Based on whether the dependency mechanism depends on an initial failure, the mechanistic dependency  
 23 models can be further divided into failure interaction models and failure propagation models. The failure

1 interaction models consider the interaction among the individual failure processes explicitly; while the failure  
 2 propagation models focus on the mechanisms that the initial failures propagate and cause subsequent failures.  
 3 The different types of mechanistic dependency models are discussed in detail in Section V-B.

4 It should be noted that the focus of this section is to review the models frequently used for dependency  
 5 modeling, while in Sect. IV, what we reviewed is the possible dependent failure behavior in different system  
 6 hierarchies. Different models might be suitable to be applied on only certain system hierarchies. Table V  
 7 summarizes the application levels of different models.

TABLE V: Application levels of dependency models.

	Multivariate distribution model	Frailty model	Copula model	Combinatorial models	State space model	Bayesian network	Common random effect model	Degradation-shock model	Deterministic model	Stochastic interaction model	SOC-based models	Complex network models	Simulation-based models	Stochastic propagation models
Failure mechanism level		×	×				×	×	×	×				
Component level	×	×	×	×			×	×	×	×				
System level		×	×	×	×	×		×						
System-of-system level											×	×	×	×

#### 8 A. Statistical dependency models

9 1) *Lifetime distribution models*: Lifetime distribution models focus on modeling the statistical dependency  
 10 among lifetime distributions. As shown in Figure 12, lifetime distribution models can be further divided into  
 11 multivariate distribution model, frailty model and copula model.

##### 12 Multivariate distribution model

13 In the multivariate distribution-based models, the joint distribution of dependent marginal lifetime distributions  
 14 is modeled directly by multivariate distributions. Research on multivariate distribution-based models can be dated  
 15 back to as early as the 1960s. A good example of the multivariate distribution model is the Marshall-Orkin  
 16 model developed in [74]:

$$P(X > s, Y > t) = e^{-\lambda_1 s - \lambda_2 t - \lambda_{12} \max(s, t)}, s, t > 0. \quad (1)$$

17 This model and its multivariate extensions have been widely applied, not only because it has exponential  
 18 marginals, but also its physical basis. For example, one way to look at this model is to view it as the lifetime  
 19 of a two-component series system subject to two fatal shocks affecting the two components individually (with  
 20 arrival rates  $\lambda_1$  and  $\lambda_2$ , respectively), and a common-cause shock that fails the two components simultaneously  
 21 (with arrival rate  $\lambda_{12}$ ). The parameters in the multivariate distribution model can be estimated based on lifetime  
 22 data, collected from in-lab testing or the field. For example, the parameter of a generalized Marshall-Orkin  
 23 distribution can be estimated using the EM algorithm [75] and Bayesian methods [76].

1 Multivariate distribution models have known and explicit form of the joint distribution, which is useful for  
 2 some theoretical analyses. However, to estimate the distribution parameters, we need a large number of high-  
 3 dimensional time-to-failure data, which might be difficult to obtain in practice. Another potential drawback is  
 4 that, deriving the system reliability from the joint distribution of component time-to-failures is not a trivial task,  
 5 except for some simple cases like series and parallel systems [77].

### 6 Frailty model

7 Frailty models attempt to derive the joint distribution of lifetimes from the marginal distributions. The basic  
 8 assumptions in frailty models are that (1) the marginal lifetime distributions are conditional independent given  
 9 some unobserved frailty terms, and (2) the hazard functions of the different marginal lifetime distributions  
 10 share a common frailty term, which describes the dependency arising from common factors affecting different  
 11 samples. For example, in the commonly used gamma frailty model, the joint cumulative distribution function  
 12 of the lifetimes  $T_1, T_2, \dots, T_k$  given a common frailty term  $Z$  is assumed to be [78]

$$\begin{aligned} P(T_1 < t^{(1)}, T_2 < t^{(2)}, \dots, T_k < t^{(k)} | Z) &= G(t^{(1)}, t^{(2)}, \dots, t^{(k)} | Z) \\ &= e^{-Z \cdot \sum_{j=1}^k \Lambda(t^{(j)})}. \end{aligned} \quad (2)$$

13 where  $t^{(1)}, t^{(2)}, \dots, t^{(k)}$  are realizations of the corresponding random variables,  $Z$  is a shared frailty term  
 14 that is assumed to follow a gamma distribution  $Z \sim \text{Gamma}(w, \tau)$ , and  $\Lambda(t^{(j)})$  is the cumulative hazard  
 15 function of the marginal distribution  $T_j$ . The individual hazard function is often further assumed to follow  
 16 Cox's proportional hazard model [79]. The frailty term can follow other distributions as well, resulting in a  
 17 variety of frailty models. Hougaard [80] summarized different frailty distributions used in the frailty model and  
 18 discussed their effects.

19 Frailty models are often used in reliability engineering for lifetime data analysis and test planning. For  
 20 example, Liu [78] used a gamma frailty model to describe the dependencies among different failure modes,  
 21 whose lifetimes follow a generic log-location-scale distribution, and derived the optimal accelerated test plans  
 22 by minimizing the large-sample approximate variance of the maximum likelihood estimator of a certain life  
 23 quantile at use condition. Hajiha *et al.* [81] proposed a model for multiple degradation processes under dynamic  
 24 operating conditions, where the multiple degradation paths follow Wiener processes and a frailty model is used  
 25 to consider the statistical dependence among the latent remaining lifetimes of the multiple degradation processes  
 26 due to unobserved future environmental factors. Moustafa *et al.* [82] developed a Bayesian framework to estimate  
 27 the system lifetime based on both system and component-level accelerated life testing data, in which shared  
 28 frailty models are used to model the dependence between failure time distributions of the components of a  
 29 system.

30 Frailty models allow us to construct the joint distribution based on the marginal distribution. Besides, the  
 31 dependency structure in the frailty model have clear physical explanations (shared common factors affecting  
 32 multiple components). However, the assumptions of shared frailty might not always hold in practice [78]. Also,  
 33 as compared to the copula models to be discussed in the next sub-section, the frailty model works well with  
 34 dependencies among the latent failure times, but cannot be applied directly to model the dependencies among  
 35 degradation processes.

### 36 Copula model

1 Copula models are another popular model that constructs the joint probability distribution of correlated random  
 2 variables based on their marginal distributions [83]. A copula function  $C(\cdot)$  is defined as a joint distribution  
 3 function of standard uniform random variables  $U_i \sim U(0, 1)$ ,  $i = 1, 2, \dots, p$ :

$$C(u_1, u_2, \dots, u_p; \boldsymbol{\theta}_c) = P(U_1 < u_1, U_2 < u_2, \dots, U_p < u_p), \quad (3)$$

4 where  $\boldsymbol{\theta}_c$  are the parameters of the copula function. According to Sklar's Theorem (see [83]), for a joint  
 5 distribution of  $p$  random variables with continuous marginals  $F_i(\cdot)$ ,  $i = 1, 2, \dots, p$ , one can always find a  
 6 copula function  $C(\cdot)$  so that [83]

$$F(x_1, x_2, \dots, x_p) = C(F_1(x_1), F_2(x_2), \dots, F_p(x_p); \boldsymbol{\theta}_c). \quad (4)$$

7 Various copula functions have been proposed with different functional form of  $C(\cdot)$  and statistical properties.  
 8 For a review of the commonly used copula function, readers can refer to [83].

9 Copulas have been widely used to consider the dependencies among time-to-failures. Jia *et al.* [77] used  
 10 copula functions to model the joint distribution of component lifetimes, and then derived the expressions of  
 11 commonly-used reliability indexes for series, parallel and  $k$ -out-of- $n$  systems. Eryilmaz [84] applied Clayton  
 12 and Gumbel copulas to model the dependent lifetimes of the components in a weighted- $k$ -out-of- $n$  system and  
 13 calculated the system reliability. Navarro and Durante [85] considered a general coherent system and derived  
 14 the residual reliability based on the copula representation for the component lifetimes. Eryilmaz [86] considered  
 15 a coherent system with dependent components modeled by an Archimedean copula and developed a moment  
 16 estimation method for the component lifetime distribution based on system-level testing data. Zhang *et al.* [87]  
 17 derived an maximum likelihood estimation for accelerated life testing data comprised of multiple dependent  
 18 failure modes that are described using copulas. Gu *et al.* [88] developed a maximum likelihood estimation for a  
 19 copula function used to model the multiple correlated failure modes in a mechanical system. Copula functions  
 20 have also been applied for modeling degradation processes, which will be discussed in Section V-A3.

21 A significant strength of copula models, as shown in Eq. (4), is that it separates the marginal behavior, as  
 22 represented by the  $F_i(\cdot)$ s, from the dependence structure described in  $C(\cdot)$ . However, the methods used for  
 23 copula-based dependency models have significant mathematical complexity in comparison to other methods. It  
 24 is also difficult to associate a physical explanation to its parameters [10]. Another practical problem when trying  
 25 to apply the copula models is how to select the most appropriate copula functions [23].

26 2) *System state models*: System state models focus on the statistical dependencies among random variables  
 27 that represent the system/component states. As shown in Figure 12, system state-based models mainly include  
 28 combinatorial models, state space models and Bayesian network.

### 29 Combinatorial models

30 Combinatorial models are system reliability models that intend to derive the system state variable as a  
 31 function (Boolean function for binary systems and multi-value functions for multistate systems) of component  
 32 state variables by considering all combinations of component states that lead to a system failure [89]. Typical  
 33 combinatorial models include reliability block diagrams, fault tree/event trees, decision diagrams, universal  
 34 generating function, *etc.* [89]. Traditionally, combinatorial models assume that the basic events are independent.  
 35 In recent years, however, a large number of extended combinatorial models have been proposed to consider  
 36 dependencies.

1 A large variety of the extended combinatorial models aim at considering the dependencies among basic  
 2 events by their joint failure probability. An intuitive approach, called the implicit approach [90, 91], calculates  
 3 the system reliability by simply replacing the multiplication of failure probabilities of the dependent basic  
 4 events with their joint probability ( $P(X_1) \cdot P(X_2) \rightarrow P(X_1 \cap X_2)$ ). The joint failure probability is assumed  
 5 to be known or can be derived based on conditional probabilities [91]. Vaurio [91] presented a general rule  
 6 for transforming any groups of  $n$  dependent events into  $2^n - 1$   $s$ -independent virtual events whose occurrence  
 7 probability can be obtained based on the joint probabilities of the original dependent events. Bobbio *et al.* [92]  
 8 transformed fault trees into a Bayesian network, in which the dependencies among basic events can be captured  
 9 naturally. Tolo and Andrews [93] introduced a novel methodology based on binary decision diagrams for the  
 10 analysis of fault trees that allows considering component dependencies and dynamic features in the fault trees.

11 Combinatorial models have been extended to consider other types of dependencies. For example, the implicit  
 12 approach was applied in [90] to model the dependency caused by common cause failure in fault trees. Andrews  
 13 and Dunnett [25] proposed a binary decision diagram-based model to consider the dependencies in event  
 14 trees, which are caused by common cause failures due to shared basic events. Post-failure dependencies (see  
 15 Section IV-C) can also be considered in combinatorial models. For example, Sun and Andrews [94] developed  
 16 an efficient algorithm to identify the sections in a fault tree that are subject to post-failure dependencies  
 17 and applied Markov modeling only for the identified sections. Dynamic fault trees have been developed as  
 18 an extension of fault trees to consider post-failure dependencies by introducing new gates, like functional  
 19 dependency gates [95]. Multi-valued decision diagrams were used in [37] to model the stochastic dependencies  
 20 among the component states in different phases of a phased-mission system.

21 Extending combinatorial models to capture dependencies can be beneficial as combinatorial models are  
 22 widely used in risk and reliability and one could easily make use of existing code/simulation tools to assist the  
 23 reliability analysis. However, a common limitation of these approaches is that, the derivation of the extended  
 24 model needs to be done in a case-by-case manner, and for complex system structure, one cannot guarantee  
 25 obtaining the required system reliability function. Also, the complexity of deriving and calculating the model  
 26 grows significantly when the system size increases, which limits the application of such methods on a large-scale  
 27 system with complex structures.

### 28 State space models

29 According to Trivedi and Bobbio [89], apart from the combinatorial models, system reliability can also be  
 30 modeled by state space models like Markov models and Stochastic Petri net models. These models get their  
 31 names because these models enumerate all the possible system states to form a finite state space, and calculate  
 32 the system reliability and availability by modeling the transitions among the system states. State space models  
 33 can naturally capture the dependencies arising from the conditional dependence of some component states on the  
 34 state of the other components. Thus, they can be naturally applied to model post-failure dependencies like load  
 35 sharing, functional dependencies and failure propagation. For example, a Markov model is used as a benchmark  
 36 model for an imperfect coverage system subject to functional dependency [96]. A semi-Markov model is used in  
 37 [97] to model bitcoin nodes under eclipse attacks, where the dependency exists as different state has a different  
 38 migration behavior. Petri net is also a widely-used state space model for describing dependencies. For example,  
 39 Whiteley *et al.* [98] developed a comprehensive Petri-Net model integrated with a 0-D fuel cell performance

1 model of the fuel cell system to develop a more accurate degradation model for a polymer electrolyte membrane  
 2 fuel cell considering harbouring dependencies between multiple failure modes.

3 State space models can directly describe various dependencies of some component states on the states of  
 4 the other components. Therefore, they are commonly used as a benchmark for validating the newly developed  
 5 models. However, they are required to be defined over the global state space of the system, thus incurring into the  
 6 well-known state-space explosion problem for large-scale systems [36]. Also, state space models have analytical  
 7 solutions only for some special cases (e.g., Markov models). For a general system without analytical solutions,  
 8 evaluating the time-dependent reliability through numerical methods is also very computationally intensive [99].

### 9 Bayesian network

10 Bayesian networks are widely used to represent uncertain knowledge and derive the joint probability distri-  
 11 bution between random variables in a compact way. A Bayesian network is comprised of a tuple  $\langle V, E, P \rangle$ ,  
 12 where  $V$  and  $E$  represent the nodes and the edges of a directed acyclic graph, and  $P$  is a probability distribution  
 13 over  $V$  [92]. Discrete random variables  $V = [X_1, X_2, \dots, X_n]$  are assigned to the nodes, while the edges  $E$   
 14 represent the causal probabilistic relationship among the nodes. The joint probability distribution of the nodes  
 15 can be calculated in a compact way:

$$P[X_1, X_2, \dots, X_n] = \prod_{i=1}^n P[X_i \mid \text{Parents}(X_i)] \quad (5)$$

16 Equation (5) provides a flexible way of calculating the joint probability distribution of dependent basic events.  
 17 For example, Bobbio *et al.* [92] presented a general protocol of using Bayesian network for system reliability  
 18 analysis, where component and system states become nodes in the network and the failure logic is represented  
 19 by the conditional probability tables attached to the nodes. The dependencies can be captured by adding edges  
 20 between the dependent nodes and deriving the joint probability distribution through the calculation rules in Eq.  
 21 (5). Khakzad *et al.* [100] applied a Bayesian network to calculate the blowout risk in the process industry and  
 22 showed that, compared to conventional probabilistic risk assessment models like event tree and bow-tie models,  
 23 the benefits of a Bayesian network is its flexibility to capture various dependencies between basic events and  
 24 due to common cause failures. To consider the time-dependent behavior, especially the degradation of dependent  
 25 components, Rebello *et al.* [101] proposed a system functional reliability assessment model based on a dynamic  
 26 Bayesian network and hidden Markov model. Morato *et al.* [102] used a dynamic Bayesian network to capture  
 27 the probabilistic dependencies in a deteriorating system, based on which a deep decentralized multi-agent  
 28 actor-critic reinforcement learning approach is developed for inspection and maintenance planning.

29 Bayesian networks allows using modeler's knowledge on the causal relationships between the dependent  
 30 events to derive the joint probability distribution, which make modeling of dependent events more intuitive  
 31 [92]. However, to apply the model, it is required to construct the network structure based on the knowledge of  
 32 the modeler, which is often not an easy task, especially for a large scale system. Also, Bayesian networks are  
 33 more mature with respect to discrete nodes. When the state of the nodes are continuous, although theoretically  
 34 possible, evaluating the network in practice is not a trivial job. Another difficulty of applying Bayesian networks  
 35 is one needs a large amount of fully observed failure data to accurately estimate the conditional probability  
 36 tables, which is not often achievable in practice.

1 3) *Degradation process models*: Degradation process models are statistical dependency models that involves  
 2 degradation processes. As shown in Figure 12, the degradation-based models can be further divided into common  
 3 random effect models, copula models and degradation-shock models. It should be noted that the degradation  
 4 process could also be modeled by a discrete state random process like Markov/semi-Markov process. In this  
 5 case, the state space model reviewed in Section V-A2 can be applied. Shen *et al.* [103] used Markov processes  
 6 to describe the degradation of two dependent components, where the transition rate matrix of one component  
 7 depends on the degradation state of the other component. To reduce repetition, we do not include the discrete  
 8 state degradation models in this section.

### 9 Common random effect models

10 Common random effect models consider the dependencies among multiple degradation processes. Each  
 11 degradation process is modeled separately by statistical degradation models, and the dependencies among the  
 12 degradation processes are captured by some common parameters shared by different degradation models (the idea  
 13 is similar to the frailty model discussed in Section V-A1). Common random effect models are named because  
 14 the shared parameters are often used to describe random effect in the degradation processes. As reviewed by  
 15 Ye and Xie [104], frequently used degradation process models in the literature include general path model  
 16 [105] and stochastic process model (*e.g.*, Wiener process [106], gamma process [107], inverse Guassian process  
 17 [108]). Each degradation model can be used as a basic model to derive the common random effect model for  
 18 multiple dependent degradation processes.

19 Si *et al.* [109] generalized the famous general path model from Lu and Meeker [105] into a multivariate  
 20 general path model:

$$\begin{cases} y_{ijk} = x_{ijk} + \epsilon_{ijk} = \eta(t_k; \Psi, \psi_{ij}) + \epsilon_{ijk} \\ \Phi_i = (\psi_{i1}, \psi_{i2}, \dots, \psi_{ip}) \sim MVN(\mu_{\Phi}, \Sigma_{\Phi}). \end{cases} \quad (6)$$

21 In their developed model (Eq. (6)),  $y_{ijk}$  represents the degradation level of the  $j$ th performance characteristics  
 22 of the  $i$ th sample being tested, measured at time  $t_k$ ;  $x_{ijk}$  is the corresponding true value of the performance  
 23 characteristics;  $\epsilon_{ijk}$  is the measurement noise which are assumed to be independently and identically distributed  
 24 with a Gaussian distribution;  $\eta(\cdot)$  is the degradation path function with fixed-effect parameter vector  $\Psi$ , which  
 25 is shared by all the samples, and a random-effect parameter vector  $\psi_{ij}$  that is unique for the  $i$ th sample  
 26 and  $j$ th performance characteristics. The stochastic dependencies among the  $p$  performance characteristics are  
 27 modeled by assuming that the random effect vectors of different performance characteristics in the same sample,  
 28  $\Phi_i = (\psi_{i1}, \psi_{i2}, \dots, \psi_{ip})$  follows a multivariate normal distribution. A similar model was developed by Lu *et*  
 29 *al.* [110] to quantify the multidimensional degradation data considering the dependencies among the different  
 30 performance characteristics. An expectation-maximization algorithm combined with the Markov chain Monte  
 31 Carlo simulation is developed for estimating the model parameters and predicting system reliability.

32 Common random effect models can also be derived based on the stochastic process-based degradation models.  
 33 In Sun *et al.* [111], a multivariate Wiener process is used to model the correlation among different dimensions  
 34 of degradation characteristics through a correlation matrix. Fang *et al.* [108] developed a common random effect  
 35 model based on inverse Gaussian process where the drift parameters of each degradation model are assumed  
 36 to share a common effect, which is modeled by a multivariate normal distribution.

1 Common random effect models are attractive in modeling dependent multivariate degradation processes  
 2 because of their tractability and flexibility in handling both heterogeneity among the units and the dependencies  
 3 among the multiple degradation processes [111]. However, some limitations still exist for such models. For  
 4 example, parameter estimation is in general complex due to the shared parameters in the model [108]. Besides,  
 5 calculating multivariate integrals is required for the reliability assessment since multiple degradation processes  
 6 are involved, which could be very demanding in terms of computations, especially when a large number of  
 7 degradation processes are considered [108].

### 8 Copula models

9 Similar to in the lifetime-based models discussed in Section V-A1, copula models can also be used to describe  
 10 the dependencies among the multiple dependent degradation processes. In the literature, there are mainly two  
 11 ways to use copula models for multiple dependent degradation processes. The first way is to estimate the  
 12 parameters in the marginal degradation processes as if they were independent, and then use a copula function  
 13 to model the joint distribution of the first passage times (latent failure times) of the marginal degradation  
 14 processes. A typical example can be found in Sari *et al.* [112] where they considered the dependencies between  
 15 two degradation processes of LEDs. The marginal degradation process is assumed to follow the general path  
 16 model [105] and fit independently. Then, a copula function is used to represent the joint distribution of the  
 17 predicted time-to-failures from the two marginal degradation paths:

$$\begin{aligned} P(TTF^{(1)} > t, TTF^{(2)} > t) &= P(Y_t^{(1)} < c^{(1)}, Y_t^{(2)} < c^{(2)}) \\ &= C(F^{(1)}(t; y^{(1)}), F^{(2)}(t; y^{(2)})), \end{aligned} \quad (7)$$

18 where  $TTF^{(1)}$  and  $TTF^{(2)}$  are the predicted time-to-failure of the marginal degradation paths 1 and 2,  
 19 respectively;  $Y_t^{(1)}$  and  $Y_t^{(2)}$  represent the degradation values of the two degradation paths at  $t$ ;  $c^{(1)}$  and  $c^{(2)}$   
 20 are the degradation thresholds;  $F^{(1)}(t; y^{(1)})$  and  $F^{(2)}(t; y^{(2)})$  are predicted marginal failure probability at  $t$ ,  
 21 respectively, and  $C(\cdot)$  is the copula function. A similar model was developed by Ruiz *et al.* [113] where  
 22  $n$  dependent degradation paths are considered where the marginal degradation paths follow inverse Gaussian  
 23 processes. Ye *et al.* [114] used random-effect nonlinear Wiener-based model with measurement errors to model  
 24 the marginal degradation processes and copula functions to consider the dependencies among the degradation  
 25 paths when estimating the reliability. Frank, Gumbel and Clayton copulas are compared and selected based on  
 26 the value of Akaike information criterion.

27 The second way of applying copulas for dependent degradation processes is to model the dependencies  
 28 among the degradation increments. Fang *et al.* [52] presented a general framework of using copulas to model  
 29 the dependent degradation processes that have independent increments:

$$\begin{aligned} (\Delta Y_{i1}(t_k), \Delta Y_{i2}(t_k), \dots, \Delta Y_{iM}(t_k)) &\sim C(F_1(\Delta y_{i1}(t_k)), F_2(\Delta y_{i2}(t_k)), \dots, F_M(\Delta y_{iM}(t_k))); \theta^{Cop}) \\ \Delta Y_{ij}(t_k) &\sim MDP(\mathbf{Y}_j, \theta_j^{Mar}) \end{aligned} \quad (8)$$

30 where  $C(\cdot)$  is a copula function with parameter  $\theta^{Cop}$ ;  $F_j(\Delta y_{ij}(t_k)), j = 1, 2, \dots, M$  are the marginal CDF  
 31 of individual degradation indicators, respectively;  $\Delta Y_{ij}(t_k)$  is the degradation increments of the marginal  
 32 degradation process, which is assumed to follow a given Marginal Degradation Process (MDP) with parameter  
 33  $\theta_j^{Mar}$ . The MDPs considered in this model need to have independent increments, *e.g.*, Wiener process, gamma  
 34 process and inverse Gaussian process. Rodríguez-Picón *et al.* [115] developed a similar model for two fatigue

1 crack processes, where the marginal degradation processes are assumed to follow the inverse Gaussian process,  
 2 and the dependency between the degradation increments is modeled by copulas. A systematic framework for  
 3 applying copulas to model the dependencies among the degradation increments and predict the reliability and  
 4 remaining useful life is presented in Fang and Pan [116], where they compare the performances of different  
 5 marginal degradation models and copula functions through a numerical and a real-world case study.

6 Compared to the other methods that consider dependencies among multiple degradation processes, a signifi-  
 7 cant strength of the copula models is that they allow separating the marginal distribution with the dependence  
 8 structure. In this way, one could be more flexible in describing the dependencies [52]. The drawbacks of  
 9 the copula approach include (1) there are usually no analytic forms for calculating the reliability indices for  
 10 high-dimensional problems [108]; (2) it is hard to incorporate physical knowledge of the correlation structure  
 11 described by copula functions [111] and (3) estimating parameters in copula models with three or more  
 12 degradation processes can be numerically challenging [110].

### 13 Degradation-shock models

14 Degradation-shock models are those models that consider the dependencies between degradation processes  
 15 and random shock processes. The research on such models can be dated back to the 1960s, when Mercer  
 16 [117] presented a partial differential equation to consider failures due to wear caused by the arrival of random  
 17 shocks. The failure rate is assumed to be dependent on the wear level  $x$ :  $\lambda(t, x) = \lambda_1(t) + \lambda_2 x$ , where  $\lambda_1(t)$  is  
 18 a baseline failure rate function. Since then, a large number of degradation-shock models have been developed  
 19 in the literature.

20 Singpurwalla [118] summarized three typical ways of considering degradation-shock dependencies (although  
 21 in his original paper, the focus is to introduce stochastic processes in degradation modeling, not discussing  
 22 dependencies). The first way is to model the occurrences of shocks by a Poisson process whose arrival rate is  
 23 dependent on the degradation state [118]. We refer to these models *dependent intensity models* in this paper. A  
 24 good example of the dependent intensity models is presented in Lemoine and Wenocur [119], in which a system  
 25 is subject to a degradation process  $X = \{X(t), t \geq 0\}$  and a fatal shock process. The system fails whenever the  
 26 degradation exceeds a threshold value or a fatal shock occurs. The arrival rate of the shock process is assumed  
 27 to be a function of  $X(t)$ . Therefore, the system reliability can be calculated as:

$$P(T > t) = E_X \left[ e^{-\int_0^t k(s, X(s)) ds} \cdot I_{\{\tau > t\}} \right], \quad (9)$$

28 where  $T$  is the time to failure of the system;  $E_X[\cdot]$  represents taking expectation over the random process  $X(t)$ ;  
 29  $I_{\{\cdot\}}$  is an indicator function, and  $\tau$  is the instant when  $X(t)$  exceeds the failure threshold, as predicted by the  
 30 degradation process model. The dependence structure  $k(s, X(s))$  simply means the arrival rate is a function of  
 31 current age and the degradation levels [120]. Parameter estimation under different specific degradation processes  
 32 and dependency structure have been discussed by Lehmann [120] and Bagdonavicius and Nikulin [121]. Such  
 33 a formulation has been widely applied in the literature. For example, Huynh *et al.* [122] assumed that the  
 34 arrival rate of the shock processes is a piecewise function of  $X(t)$  and presented a periodic inspection and  
 35 replacement policy considering the degradation and traumatic events due to shocks. Hu *et al.* [123] considered  
 36 condition-based maintenance planning for systems subject to dependent soft and hard failure, where the failure  
 37 rate of hard failure increases as the degradation level becomes higher.

1 The second type of approach does not model the degradation process explicitly. Rather, the time-to-failure  $T$   
 2 is considered as a random variable and its failure rate function is assumed to be dependent on the random shock  
 3 processes [118]. In this paper, we refer this type of models the *dependent failure rate models*. For example,  
 4 in Lemoine and Wenocur [119], it is assumed that the failure rate function  $\lambda(t)$  of the lifetime distribution is  
 5 dependent on the accumulative effect of a random shock process:

$$\lambda(t) = \sum_{i=1}^{\infty} D_i h(t - S_i) \quad (10)$$

6 where  $D_i$  is the magnitude of the  $i$ th shock that arrives at instant  $S_i$ , and  $h(t)$  is a decreasing function that  
 7 models the attenuation of the shock effect overtime. The arrival of the random shocks  $S_i, i = 1, 2, \dots$  is, by  
 8 itself, a random process (e.g., a Poisson process). Then, the reliability function can be derived as:

$$R(t) = P(T > t) = E \left[ e^{-\int_0^t \lambda(s) ds} \right], \quad (11)$$

9 where  $\lambda(s)$  is defined in Eq. (10) and the expectation is taken over  $\lambda(s)$  as it is a random variable given  $S$ .  
 10 Note that the formulation in Eq. (10) is also known in literature as a shot-noise process, which is defined  
 11 in [124] as a stochastic process that models the accumulative effects of a counting process. The failure rate  
 12 dependency models have been widely applied in literature. For example, Qiu *et al.* [125] investigated the  
 13 preventive maintenance of a system whose failure rate is influenced by a shot-noise process. Wang *et al.* [126]  
 14 extended the dependent failure rate models to a case where the failure rate increments caused by shocks need  
 15 not be identical or identically distributed.

16 The third type of approach focuses on modeling the impact from the random shock processes to the  
 17 degradation process. The shock processes might further depend on some covariates that models the influence  
 18 from the environment [118]. In this paper, we call this type of model *dependent shock-degradation model*, as  
 19 the dependency is caused by the impact of the shock process on the degradation process. A typical example of  
 20 such models was presented in Figure 13, where a system switches between three operation modes due to an  
 21 environment covariate process  $Z(t)$ , and in mode 1, the system is subject to degradation described by a Wiener  
 22 process, while in the other two modes, the system is subject to random shock processes where the arrivals of  
 23 shocks introduce an additional increment to the degradation measurements [118].

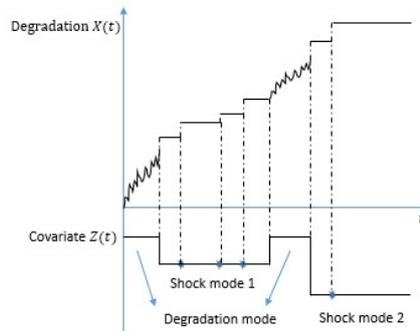


Fig. 13: An an illustration of the sample path from a dependent degradation model [118].

24 A typical example of such models can be found in Peng, Feng and Coit [34], where they modeled the  
 25 competing failure processes of soft failures caused jointly by continuous smooth degradation with additional

1 abrupt degradation damage from a shock process, and catastrophic failures caused by an abrupt and sudden  
2 stress from the same shock process. Following their work, many dependent shock-degradation models have been  
3 proposed to consider different shock patterns and dependence structure between the shock and degradation  
4 processes. For example, Rafiee *et al.* [127] considered a dependent competing failure process where the  
5 degradation rate could be impacted by the arrival of random shocks. Four different shock patterns are considered  
6 in their model, *i.e.*, generalized extreme shock, when the first shock above a critical value is recorded; generalized  
7  $\delta$ -shock model, when the inter-arrival time of two sequential shocks is less than a threshold  $\delta$ ; generalized  $m$ -  
8 shock model: when  $m$  shocks greater than a critical level are recorded; and generalized run shock model, when  
9 there is a run of  $n$  consecutive shocks that are greater than a critical value. Jiang *et al.* [35] developed a model  
10 in which the arrival of the shocks shifts the failure threshold of the degradation process. The considered shock  
11 patterns include the generalized extreme shock model, generalized  $\delta$ -shock model and generalized  $m$ -shock  
12 model. Lyu *et al.* [128] considered a  $\delta$ -shock model, where the  $\delta$ -shock causes failure of the system directly,  
13 while the other shocks affect the degradation process by changing the degradation rate and causing additional  
14 degradation increments.

15 The three typical models, *i.e.*, the dependent intensity, failure rate and shock-degradation models, can be  
16 combined to develop more complicated degradation-shock models. For example, Fan *et al.* [129] developed a  
17 dependent failure behavior model for a spool valve that integrates the dependent intensity and shock-degradation  
18 model, where the arrival of shocks leads to additional increments of the natural degradation process, while the  
19 intensity of the random shock process also increases as the degradation gets worse. Yang *et al.* [130] developed  
20 models where the arrivals of shocks have two impacts: (1) increase the failure rate of sudden failure; (2) cause  
21 abrupt degradation increments. Recently, there is a growing trend in the literature that aims at providing a  
22 unified framework for different degradation-shock dependencies. For example, Fan *et al.* [58, 131] developed  
23 a unified modeling framework based on stochastic hybrid systems, which is a special case of a more general  
24 model called stochastic hybrid automaton [132], to model different types of degradation-shock dependencies.  
25 Similar attempts have been made in [133] based on piece-wise deterministic Markov processes, which are  
26 another special case of stochastic hybrid automaton.

27 A significant strength of the degradation-shock models is that it allows modeling dependencies from a physics-  
28 based perspective, *i.e.*, the interaction between the shock and degradation processes are explicitly considered  
29 [34]. When covariates are used, the models also allows analysts to consider the dependencies that are related  
30 to some external factors from the environmental [118]. There are a few limitations of the degradation-shock  
31 models as well. First, the evaluation of the reliability function often involves high-dimensional convolution  
32 or summation over infinite terms [131]. When the number of the involved processes increases, it becomes  
33 computationally difficult for the evaluation. Besides, the developed degradation-shock models often contain  
34 parameters, especially those related to the shock processes, which do not have sufficient historical data to  
35 support a good estimation [134]. Finally, the validation of the model, especially the assumptions made on the  
36 dependence structure involving the shock processes, is a difficult task due to lack of available validation data.

## 1 *B. Mechanistic dependency models*

2 The other category of the dependent failure models, as shown in Figure 12, is the mechanistic dependency  
 3 models. In mechanistic dependency models, the dependency mechanisms, *i.e.*, the physical cause of the depen-  
 4 dency, is considered explicitly in the developed dependency models. This fact makes the mechanistic models  
 5 different from the stochastic models, as the latter considers the dependencies as statistical correlation without  
 6 looking into the exact cause of the correlation. Based on the dependency mechanisms being modeled, the  
 7 mechanistic dependency models can be further divided into failure interaction models and failure propagation  
 8 models.

9 *1) Failure interaction models:* Failure interaction models focus on modeling the interactions among different  
 10 failure mechanisms. As shown in Figure 12, failure interaction models can be further divided into deterministic  
 11 interaction and stochastic interaction models.

### 12 *Deterministic interaction models*

13 As shown in its name, the deterministic interaction models consider the interactions among different failure  
 14 processes from a physics-based perspective and modeled in a deterministic way. Zeng *et al.*, [135] considers  
 15 three basic interactions, *i.e.*, competition, superposition, and coupling, and proposed a physics-based framework  
 16 to develop failure behavior models considering the dependencies among failure mechanisms. In their framework,  
 17 the performance margin of each failure mechanism is modeled based on physics-of-failure, and the interactions  
 18 among the failure mechanisms are considered through an interaction graph as a combination of the three  
 19 basic interactions. Chen *et al.* [136] proposes another three types of interaction, *i.e.*, trigger, where one failure  
 20 processes causes another, acceleration, where the presence of one process accelerate the evolution of another,  
 21 inhibition, where the occurrence of one process will prohibit another process from occurring, and discussed  
 22 how to consider each basic interaction in a dependent failure behavior model. Li *et al.* [137] considered three  
 23 common failure mechanisms, including hot-carrier injection, time-dependent dielectric breakdown and negative  
 24 bias temperature instability, and simulate the evolution of the three failure mechanisms and its impact on the  
 25 performance of MOSFET using a electrical circuit simulation tool SPICE. The dependencies among the failure  
 26 mechanisms are considered as the failure mechanism models share some common parameters.

27 The deterministic interaction model has the significant strength of being able to explicitly describe how do  
 28 the dependencies arise. The major limitation of such models is that, the model development requires deep  
 29 understanding of the failure mechanisms and the interactions among them, which is not always achievable in  
 30 practice [50]. Another potential drawback is that the model parameters are often estimated from data generated  
 31 from controlled in-lab experiments. Whether the parameters can be safely generalized to a practical case needs  
 32 to be carefully examined [135].

### 33 *Stochastic interaction models*

34 In stochastic interaction models, the interactions among the failure processes involve some random events.  
 35 Most commonly-used stochastic interaction models are expressed in terms of the degradation-shock models  
 36 as reviewed in Section V-A3. However, not all degradation-shock models are used to describe the interactions  
 37 explicitly. That is why we chose to put the degradation-shock model in Section V-A3 with other stochastic  
 38 models. Examples of the stochastic interaction models include the works by Peng *et al.* [34] and Fan *et al.*  
 39 [129], which have been discussed in Section V-A3. To reduce repetition, we directly refer the readers to Section

1 V-A3 for further details. The stochastic interaction can be caused by not only degradation, but also failure of  
 2 other components. For example, Zhang *et al.* [138] presented a stochastic interaction model where the failure  
 3 rate of surviving components in a multi-component load sharing system increases after initial failure of some  
 4 components occurred. A similar model, called a tempered failure rate model is used in [139] to consider the  
 5 fact that the impact of load-sharing to the surviving components in a  $k$ -out-of- $n$ :G system.

6 2) *Failure propagation models*: Failure propagation models focus on describing the dependencies in which  
 7 some initial failures trigger the failure of the other components in a system (also referred to as cascading  
 8 failure). A large fraction of failure propagation models in the literature originated from the study of cascading  
 9 failures in electrical power grids but have demonstrated wide applicability in other domains as well (see [140]  
 10 and [141] for a good review). According to Xing [11], the main failure propagation models include, but are not  
 11 limited to, the self-organized critical (SOC) theory-based, complex network, and simulation models. The three  
 12 models do not consider randomness when evaluating the failure propagation. To be more complete, we add a  
 13 fourth category of the propagation model, the stochastic propagation model, as shown in Figure 12.

#### 14 SOC models

15 SOC models include a variety of models that are dedicated to modeling the self-evolution process of a  
 16 complex dynamical system until it reaches a critical steady state, known as the SOC state [11]. One of the  
 17 most well-known example of the SOC-based failure propagation model is the ORNL-PSerc-Alask (OPA) model,  
 18 which combines DC load flow and optimal power flow models to simulate the cascading failure on transmission  
 19 lines in a power grid after some trips due to over-loading caused by load redistribution [68]. CASCADE model  
 20 was another widely used SOC-based models for cascading failures in the power grid [142]. In the CASCADE  
 21 model, the propagation process is modeled by adding a fixed load increment on all the remaining functioning  
 22 components, and the propagation continues until all the components fail or loads on the remaining functioning  
 23 components are all within their acceptable limits [142].

24 The SOC models allow simulating the cascade of failures through some simplification of the actual cascading  
 25 mechanism. The degree of simplification lies between the complex network models and the simulation models.  
 26 Therefore, the SOC models can achieve a balance between representing reality and the computational costs  
 27 [68]. However, the SOC process has to be defined in a case-by-case manner for the system under investigation,  
 28 which limits the generalization of the SOC models. Also, when the scale of the system becomes large, the  
 29 evaluation of the model can easily becomes intractable [68].

#### 30 Complex network models

31 Another way of investigating cascading failures is through complex network theory. Scale-free and small  
 32 world networks are the two well-known and also well-studied classes of complex networks. Motter and Lai  
 33 [143] presented an early application of the complex network models for analyzing cascading failures. In their  
 34 work, it is assumed that the loads on a given node are approximated by the number of shortest paths passing  
 35 through it. A failure is modeled by removing the corresponding node and the edges passing through the failed  
 36 node. As the initial failure changes the topology of the network, the loads on the nodes are redistributed.  
 37 As a result, subsequent cascading failures can occur. The application of such models on random networks  
 38 and real-world networks of the Internet and a power grid showed that a heterogeneous network is robust to  
 39 random attacks but vulnerable to intentional attacks, while a homogeneous network appears to be more robust

1 against attacks than the heterogeneous ones [143]. Buldyrev *et al.* [144] developed a complex network model to  
 2 analyze the cascading failure behavior in an interdependent complex network. In their framework, a cascading  
 3 failure can occur due to (1) the dependent node in another network fails, and (2) the node does not belong to  
 4 any mutually connected clusters. Through their analysis, they discovered that for an interdependent complex  
 5 network, a broader degree distribution increases the vulnerability to random failure, which is opposite to how  
 6 a single complex network behaves.

7 In terms of modeling the failure cascading, the complex network models take a highly abstract approach:  
 8 cascade failures following principles that are defined based on only network structure and dynamics. The high  
 9 degree of abstraction makes the complex network model easy to generalize and evaluate, compared to the other  
 10 failure propagation models. The drawback of such an approach is that, the model cannot fully capture the actual  
 11 way of failure propagation [68]. Therefore, the assumption of failure propagation in complex network models  
 12 need to be carefully examined when applied in a practical problem.

### 13 Simulation models

14 Simulation models rely on performance simulation models to analyze dynamic response of a complex system  
 15 under certain disturbances and simulate the potential failure propagation. For example, Cadini *et al.* [145]  
 16 developed a sequential Monte Carlo scheme which combines an extreme weather model, a DC power flow  
 17 model and a proportional re-dispatch strategy to simulate the cascading failures under extreme weather events.  
 18 In Hasan *et al.*, [146] a cascade simulation model in Matlab/Simscape was proposed, which incorporates a  
 19 detailed behavioral model of protection devices to consider both cyber failures and time causality of events;  
 20 the result was validated using a different simulation platform called OpenDSS.

21 Simulation models allows us to consider in details the realistic ways of how failure propagates in practical  
 22 systems, considering different contributing factors from the system itself and also the environment. However,  
 23 the construction of simulation models requires a lot of domain knowledge and historical data. The model itself  
 24 can be very computationally expensive to evaluate as well [11].

### 25 Stochastic propagation models

26 The stochastic models, in particular the combinatorial models and state space models discussed in Section  
 27 V-A2, can also be used to model failure propagation. A widely-used combinatorial model for failure propagation  
 28 is the dynamic fault tree, in which where logic AND, OR and  $k$ -out-of- $n$  gates are used to represent the  
 29 combinations of component statuses while multiple cascading Function DEpendency (FDEP) gates are used  
 30 to model the cascading failures [11]. For example, Zhao and Xing [147] models the failure propagation in an  
 31 Internet-of-Things (IoT) system using a dynamic fault tree with probabilistic functional dependence gates. State  
 32 space models like a Markov model can also be used to model failure propagation. For example, in [148], a  
 33 CTMC-based method was developed to assess the evolution of blackout probability in time, and further the  
 34 probability mass function of the blackout size for power grids subject to cascading failures. For further details,  
 35 we refer the readers to Section V-A2, in order to avoid too much repetition in the paper.

## 36 VI. CHALLENGE AND OPPORTUNITIES

37 Through the reviews and analyses in Sects. III - V, we can see clearly the quantity and quality of the existing  
 38 research on dependent failure behavior. Although great achievements have been made, there are still a few

1 challenges that could potentially motivate future research and researchers.

## 2 *A. Identification and validation of the dependencies*

3 Dependent failure models rely on fundamental assumptions on how the dependent variables relate to one  
 4 another. In statistical dependency models, these assumptions are mainly statistical and empirical, while in  
 5 mechanistic models, these are more physics-based dependency assumptions (see Section V for details). Re-  
 6 garding these assumptions, a question arises naturally: how to identify which variables are dependent and in  
 7 which manner?

8 If one has large amount of historical data, the identification problem can be solved by checking the statistical  
 9 association among the variables directly from data [52, 114]. In a large number of practical cases, however,  
 10 we do not have enough data. Most of the existing papers rely on direct assumption or analysis based on expert  
 11 knowledge to identify potential dependencies in this case [34]. There are two shortcomings for this kind of  
 12 approach. First, the identification has to be done manually and case-by-case, which is not easy to implement  
 13 in a complex system with a large number of components. Second, the results could be subjective and highly  
 14 dependent on the knowledge level of the domain experts who make the analysis [131]. A possible improvement  
 15 could be to leverage the recent advancement on artificial intelligence, especially the research and models on  
 16 knowledge graphs and knowledge-based machine learning and reasoning [149]. For example, we can create  
 17 a knowledge graph, based on prior knowledges, describing the interlinked relationships among components,  
 18 working environment, failure mechanisms, dependencies, *etc.*, and use it as a knowledge base to design AI-  
 19 based reasoning algorithms that can automatically identify the potential dependencies in a complex system.

20 A related problem is how to validate whether the assumptions on dependence structures are valid? With  
 21 enough data, validation of the model can be done by evaluating the model performance on testing data through  
 22 metrics like Akaike information criterion, Bayesian information criterion, Kendall's  $\tau$  [116] *etc.* Validating the  
 23 dependency assumptions when few data is available is, however, a difficult task that deserves further exploration.

## 24 *B. Scalability of the model*

25 Scalability is another challenge to most of the existing models. When systems grow in size and complexity,  
 26 scalability issues can come from two aspects. First, developing the model itself becomes a complex problem,  
 27 as exploring all the possible dependencies in a large-scale system can be a time-consuming and difficult  
 28 task for the modeler. One promising idea to solve this issue is to leverage the advancements in knowledge-  
 29 based systems and AI-based reasoning, as detailed in Section VI-A. Second, evaluating the model can be  
 30 computationally intensive for a large-scale model with complex structure. A common solution is to develop  
 31 surrogate models to approximate the model output in less computational time [150]. Recently, physics-informed  
 32 deep learning has emerged as a promising approach that integrate physics-based knowledge with machine  
 33 learning models and can achieve good balance between model capability and computational complexity [151].  
 34 A possibly promising direction could be using physics-informed machine learning to improve the scalability of  
 35 the dependent failure models.

### 1 C. Parameter estimation

2 Although statistical parameter estimation approaches have been well-developed for dependent failure models  
 3 [52], applying them requires large amounts of historical data, which is not always available in practice.  
 4 Determining precise parameter values under a practical constraint of limited historical data remains a challenging  
 5 problem. There are two possible solutions. First, as sensor technology advances, it is possible to collect  
 6 condition-monitoring data during system operation through sensors. This permits us to develop a Bayesian  
 7 framework to update the parameter estimations of a dependent failure model online using the condition-  
 8 monitoring data [152]. Remaining useful life prediction and predictive maintenance can also be considered.  
 9 Currently, however, parameter updating and remaining useful life prediction are not well-explored in the research  
 10 of dependent failure models [134]. Another possible solution is to consider the epistemic uncertainty in the  
 11 parameters, and quantify the impact of such uncertainty in the model output and the decisions made based on  
 12 the models. Helton [153] summarized various frameworks for modeling and propagating epistemic uncertainty.  
 13 However, there are not much effort trying to apply them on dependent failure models.

### 14 D. Validation of the added values of considering dependencies

15 Validating the added value of considering dependencies is an important question in practice. This question,  
 16 however, is not well-addressed in the existing literature, as in most of the existing papers, the focus is on  
 17 developing new models. To show the added values of the developed models, an easy way is to apply them  
 18 on decision-making problems, and compare the results with some benchmark models that do not consider  
 19 dependencies. In Table VI, we list a few typical decision-making problems that might be useful for validating  
 20 the added value of the dependent failure models.

TABLE VI: Typical decision-making problems that could be used for validating the dependent failure models.

Problems	Descriptions
Qualification based on risk/reliability	Decisions often need to be made regarding whether to accept/reject an engineering system based on its reliability/risk. For example, in safety-critical industries like nuclear, the risks of some severe accidents are not allowed exceed given reference values [154]. The impacts on the regulatory decisions can be investigated by comparing the dependent failure models with traditional models like probabilistic risk assessment that do not consider dependencies.
Maintenance planning	Optimal maintenance policies can be decided based on the predicted lifetime and reliability from the dependent failure models. For components, a typical example is to determine optimal intervals for preventive maintenance [34], while for systems, we can consider grouping of components due to economical dependencies.
Inspection planning	For a multistate/continuous degraded system, inspection on the system can measure the current degradation level and/or reveal the actual degradation state. The optimal inspection interval has to be planned carefully by balancing the cost of inspection and the failure cost [155].
Remaining useful life prediction and predictive maintenance	For the dependent failure models that involve degradation processes, we could apply the model to make remaining useful life prediction based on condition-monitoring data [134]. The predicted remaining useful life can then be used to support predictive maintenance planning. The added value of the dependent failure models can be validated by comparing the results with a benchmark model that does not consider the dependencies.
Design optimization	Reliability can be used as a constraint or an objective function in a design optimization problem. Then, the results from a dependent failure model can be compared to that from a traditional model without dependencies. Typical use cases include redundancy allocation and reliability-based design optimization [156], etc.

### 1 E. Joint optimization of reliability, maintenance and other operational problems

2 As shown in Table VI, maintenance and reliability optimization are two important use cases of dependent  
 3 failure models with respect to decision-making. Recently, there is a growing trend in research to consider  
 4 joint optimization of reliability, maintenance and other important operational objectives. For example, joint  
 5 optimization of production scheduling and predictive maintenance of the machines is considered in [157]  
 6 by developing an optimization model to minimize the total cost comprising of production cost, preventive  
 7 maintenance cost, minimal repair cost for unexpected failures and tardiness cost. Zhang *et al.* [158] jointly  
 8 optimized the production and maintenance plans for a parallel production system, where the dependency between  
 9 production yield and the degradation process is considered. A joint optimization model is developed in [159] to  
 10 determine the optimal redundancy and maintenance strategy needed to satisfy a given reliability requirement with  
 11 lowest cost. Joint optimization of spare part supply and opportunistic condition-based maintenance for onshore  
 12 wind farms is considered in [160] where the economic and graphical dependencies among wind turbines and  
 13 the stochastic degradations of their components are considered. A promising future direction is to integrate the  
 14 dependent failure models in the joint optimization model and investigate the impact of the dependencies on the  
 15 optimal decisions.

## 16 VII. CONCLUSION

17 In this paper, we conduct a systematic and critical literature review on dependent failure behavior models  
 18 in risk and reliability. In the systematic literature review, a total number of 1062 papers are collected from  
 19 a carefully designed literature search protocol and analyzed through a meta-data and bibliometric analysis. In  
 20 particular, a semantic clustering is performed to identify the main research topics related to dependencies in  
 21 risk and reliability in three periods of time: before 1994, 1995 – 2010 and from 2011 to present. The results  
 22 of the analyses help to understand the main research topics in this domain and how they evolve over time.

23 The top 10% most-cited papers from the systematic literature review are then selected for a critical literature  
 24 reiew. First, the dependent failure behavior in different levels of a complex system, including failure mechanism,  
 25 component, system, and system-of-system levels, are summarized in a hierarchical framework. This result could  
 26 help a researcher identify the potential dependencies in a complex system that need to be considered. Then,  
 27 the most important models for dependent failure behavior are critically reviewed. A classification framework  
 28 is also proposed for models. Future researchers could benefit from this when they try to choose an appropriate  
 29 model for their dependent failure behavior modeling problem.

30 The results of our review emphasize once again that dependent failure modeling is a problem whose  
 31 importance should never be overlooked. A direct consequence of failing to capture the dependent failure  
 32 behavior is an inaccurate reliability/risk estimation, which will further impact the decisions made based on  
 33 these assessments. Although substantial works have been done on this topics, our analyses still identify five  
 34 important research gaps that deserve future investigations. We believe these research gaps, and the field of  
 35 dependent failure behavior modeling in general, should and will attract more attentions from the academia  
 36 and industry. The solution of these problems, together with introduction of promising new technologies like  
 37 knowledge graph and physics-informed machine learning, will greatly improve our capability to accurately assess  
 38 and reduce the risk, and significantly contribute to more reliable and resilient engineering systems and society.

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