

REVISITING THE $O(2)$ EQUIVARIANT STEADY–HOPF INTERACTION

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We consider the mode-interaction [1, 2] between a steady-state bifurcation and Hopf bifurcation in a system with $O(2)$ symmetry. It is somehow a natural configuration that emerges in many fluids of interest: Taylor–Couette flow (TCF), the wake flow [7, 6, 8, 4] of axisymmetric objects (WFA) and the wake of axisymmetric objects in mixed convection (WFA-MC). In here we analyse the bifurcation scenario of the WFA-MC problem for two axisymmetric bodies, a disk with variable thickness and the sphere. Other axisymmetric problems have been also recently explored, such as the hope past a circular aperture [3, 5] In these cases, the flow state $\mathbf{q} = [\mathbf{u}, p]$ composed of the velocity field and the hydrodynamic pressure (the WFA-MC also includes the temperature field T) is decomposed as:

$$\mathbf{q} = \mathbf{Q}_0 + \text{Re}[a_0(t)e^{-i\theta}\hat{\mathbf{u}}_s] + \text{Re}[a_1(t)e^{-i\theta}\hat{\mathbf{u}}_{h,-1} + a_2(t)e^{i\theta}\hat{\mathbf{u}}_{h,1}] \quad (1)$$

Here \mathbf{Q}_0 is the steady-state flow state that is invariant under the action of the whole $O(2)$ group, $\hat{\mathbf{u}}_s$ is the steady mode and $\hat{\mathbf{u}}_h$ is the Hopf (unsteady) mode. The ansatz eq. (1) takes into account the continuous symmetry by the term $e^{\pm i\theta}$, where $\theta \in S^1$ is an angle-like variable in the periodicity direction.

Using the polar representation of the complex amplitudes $a_j = r_j e^{i\phi_j}$ for $j = 0, 1, 2$, the normal form is recast to a system of four coupled equations governing the amplitudes r_0, r_1, r_2 and the mixed phase $\Psi = \phi_1 - \phi_2 - 2\phi_0$:

$$\begin{aligned} \dot{r}_0 &= [\lambda_s + l_0 r_0^2 + l_1 (r_1^2 + r_2^2)] r_0 + l_3 r_0 r_1 r_2 \cos \Psi \\ \dot{r}_1 &= [\lambda_h + B_r r_1^2 + (A_r + B_r) r_2^2 + C_r r_0^2] r_1 + r_0^2 r_2 (D_r \cos \Psi + D_i \sin \Psi) \\ \dot{r}_2 &= [\lambda_h + B_r r_2^2 + (A_r + B_r) r_1^2 + C_r r_0^2] r_2 + r_0^2 r_1 (D_r \cos \Psi - D_i \sin \Psi) \\ \dot{\Psi} &= (A_i - 2l_2)(r_2^2 - r_1^2) - 2l_3 r_1 r_2 \sin \Psi + r_0^2 D_i \cos \Psi \left[\frac{r_2}{r_1} - \frac{r_1}{r_2} \right] - r_0^2 D_r \sin \Psi \left[\frac{r_2}{r_1} + \frac{r_1}{r_2} \right] \end{aligned} \quad (2)$$

The unfolding of the normal form allows a characterization of the nonlinear states observed in the transition to complex dynamics in these flow configurations. Fig 1 displays the distinct spatio-temporal structures of the flow field in the WFA-MC (thick disk, opposing flow) problem.

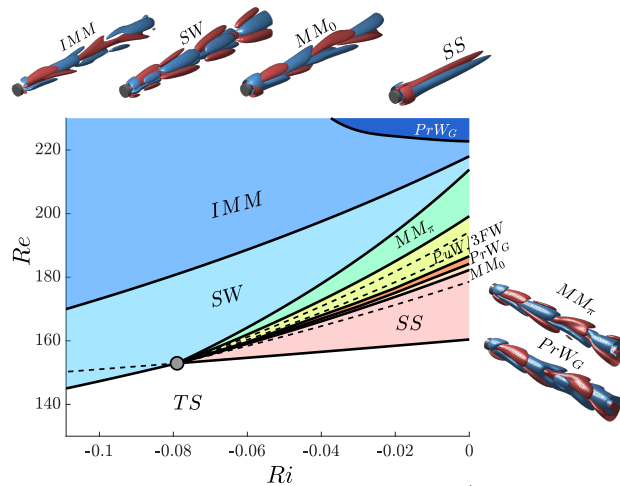


Figure 1: Parametric map of the patterns within the flow past a thick disk (aspect ratio $\chi = 3$) in mixed convection (opposing flow) conditions. Re denotes the Reynolds number and Ri the Richardson number.

References

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