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A comprehensive study on the instabilities arising in the wake of a rotating cylinder

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Summary A complete description of the bifurcation scenario of a uniform flow past a rotating circular cylinder has been recently provided by [1]. Linear stability theory is used to depict neutral curves and analyse the arising unstable global modes. Special emphasis is paid to the identification of bifurcations with codimension higher than one. Three codimension-two bifurcation points are detected, namely a Takens-Bogdanov, a cusp and generalised Hopf, which are closely related to qualitative changes in orbit dynamics. The occurrence of the cusp and Takens-Bogdanov bifurcations for very close parameters (corresponding to an imperfect codimension-3 bifurcation) is shown to be responsible for the emergence of multiple steady states. Direct numerical simulations confirmed the presence of homoclinic and heteroclinic orbits which are classical in the presence of Takens-Bogdanov bifurcations. Finally, a weakly nonlinear analysis is accomplished in the neighborhood of the Generalised Hopf bifurcation to determine the fold of two limit cycles of opposing stability in the vicinity the subcritical Hopf bifurcation curve.

INTRODUCTION

The flow past a circular cylinder is one of the classical flow configurations which has long received attention from the fluid dynamics community. This kind of flow has direct implications for flow control by using wall motion and has recently received particular attention [2, 3, 4, 5].

Linear stability approaches [6, 7, 8], see i.e. [9, 10], have precised the picture, showing the existence of two separated regions of instability in the (Re, α) plane, where α is the nondimensional rotation rate and Re the Reynolds number. At large rotation rates $\alpha \geq 4.5$, there is a region in the plane of parameters (Re, α) where multiple steady states coexist [9, 11, 12]. It has been conjectured by [11] the fact that multiple steady states collapse into a *cusp* bifurcation point. In the vicinity of *cusp* bifurcation, three coexisting steady states exist. The counting of steady states supports the hypothesis of a *cusp* bifurcation. However, near a *cusp* bifurcation one should not find an unsteady transition as it is the case of Mode II. The presence of a connection between a Hopf bifurcation and a saddle-node is an indicator of the other type of codimension-two bifurcation, a 0^2 or *Takens-Bogdanov* bifurcation. Nevertheless, the generic type of this bifurcation does not explain the existence of three steady states. Actually, in the neighbourhood of the onset at which multiple steady states arise, dynamics are driven by the third-order normal form of the unfolding of the TB bifurcation.

Furthermore, it has been identified a third codimension-two point, namely a Generalized Hopf (GH) bifurcation.

PRELIMINARY RESULTS

Cusp-Takens Bogdanov normal form The Cusp-Takens Bogdanov normal form reads:

$$\frac{dy_1}{dt} = y_2, \quad \frac{dy_2}{dt} = \beta_1 + \beta_2 y_1 + \beta_3 y_2 + \epsilon y_1^3 + c_1 y_1 y_2 - y_1^2 y_2 \quad (1)$$

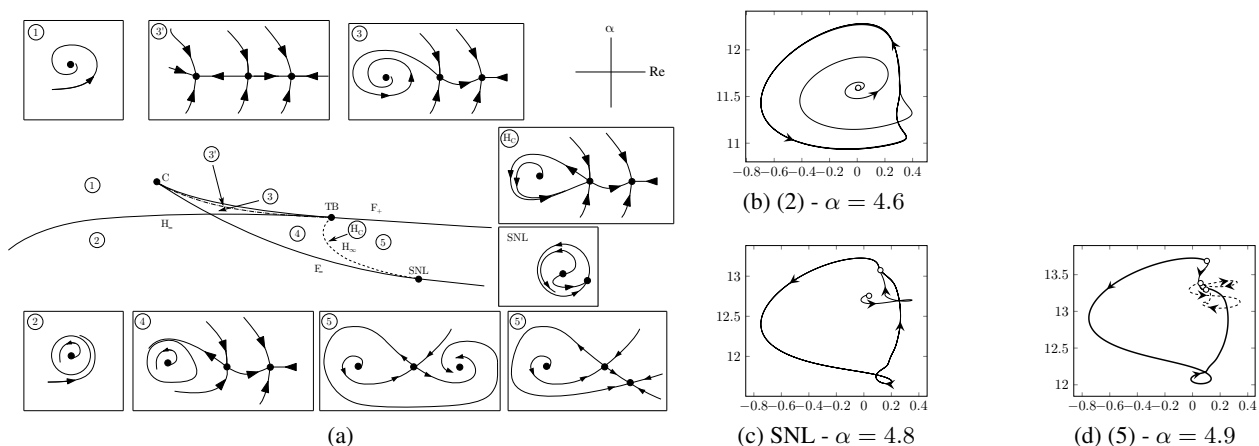


Figure 1: (a) Bifurcation diagram predicted using the normal form 1 in the stable focus case (adapted from [13]), and qualitative phase portrait in regions (1), (2), (3), (3'), (4), (5), (5') and along curve H_∞ . (b-d) Phase portrait (F_x, F_y) of the dynamics of the rotating cylinder at $Re = 170$, where empty dots denote steady state solutions.

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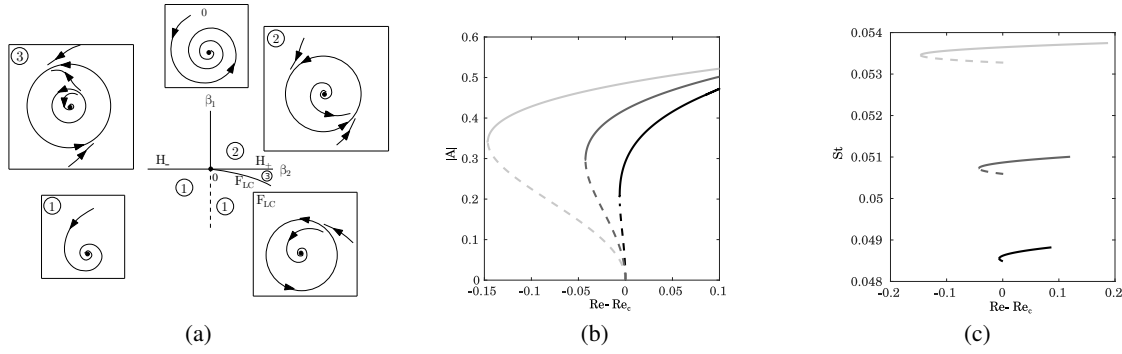


Figure 2: (a) Qualitative bifurcation scenario in the vicinity of the GH bifurcation. (b) Comparison of the amplitude of stable (solid line) and unstable (dashed line) limit cycles for three Re_c , where Re_c denotes the Reynolds number at the subcritical Hopf bifurcation. Light grey curve corresponds to $Re_c = 250$, dark grey to $Re_c = 200$ and black to $Re_c = 170$. (c) Strouhal number for each limit cycle is shown.

where β_1, β_2 and β_3 are unfolding parameters (mapped from the physical parameters (Re, α)), c_1 and ϵ (which can be rescaled to ± 1) are fixed coefficients which depend on the nonlinear terms of the underlying system. Note that this normal form Eq. 1 generalizes both the normal form of the standard TB bifurcation (which is recovered for $\beta_1(Re, \alpha) = 0$) and the one of the fold bifurcation (which is recovered for $\beta_3(Re, \alpha) = 0$). The occurrence of both these codimension-2 conditions for very close values of the parameters is characteristic of an imperfect codimension-3 bifurcation and justifies the relevance of the associated normal form.

The dynamics of the normal form has been explored by [13, 14] who classified the possible phase portraits and the associated bifurcation diagrams as a function of the unfolding parameters $(\beta_1, \beta_2, \beta_3)$ along a spherical surface. They showed that all possible bifurcation diagrams fall into three possible categories, called *focus*, *saddle* and *node* according to the values of the coefficients c_1 and ϵ . The situation $0 < c_1 < 2\sqrt{2}$ and $\epsilon = -1$ corresponds to the *stable focus case* and is found to lead to a bifurcation diagram Fig. 1. In order to confirm the global stability results, we performed also a parametric study by using time-stepping simulations [15] for several rotation rates at a fixed Re , see Fig. 1 (b-d).

Generalised Hopf bifurcation The governing equation is a Stuart Landau equation, depending on a small parameter $\epsilon^2 = Re_c^{-1} - Re^{-1}$:

$$\frac{dA}{dt} = (i\omega_0 + \epsilon^2\lambda_0 + \epsilon^4\lambda_1)A + (\nu_{1,0} + \epsilon^2\nu_{1,1})|A|^2A + \nu_{2,0}|A|^4A \quad (2)$$

From Eq. 2, a GH bifurcation occurs on the Hopf bifurcation curve and whenever $Re(\nu_{1,0}) = 0$. When $Re(\nu_{1,0}) < 0$ (resp. $Re(\nu_{1,0}) > 0$) the Hopf bifurcation is subcritical (resp. supercritical). The amplitude of the limit cycle $|A|_{\pm} = \sqrt{-\frac{Re(\nu_1)}{2Re(\nu_2)} \pm \sqrt{\frac{Re(\nu_1)^2}{4Re(\nu_2)^2} - \frac{Re(\lambda)}{Re(\nu_2)}}}$ and its frequency $\omega_{\pm} = \omega_0 + Im(\nu_1)|A|_{\pm} + Im(\nu_2)|A|_{\pm}^2$ where $\nu_1 = \nu_{1,0} + \epsilon^2\nu_{1,1}$, $\lambda = \epsilon^2\lambda_0 + \epsilon^4\lambda_1$ are directly computed from Eq. 2 via Weakly NonLinear analysis, see Fig. 2 (b-c). When the Hopf bifurcation is subcritical ($Re(\nu_{1,0}) < 0$) there is a range in ϵ^2 where both solutions are positive real solutions, $|A|_{\pm} \in \mathbb{R}_+$. The inner unstable (resp. outer stable) limit cycle increases (resp. decreases) its amplitude as the distance from the Hopf bifurcation increases until both periodic orbits collide and disappear, that occurs when $\frac{Re(\nu_1)^2}{4Re(\nu_2)^2} = \frac{Re(\lambda)}{Re(\nu_2)}$.

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