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**Broadband negative reflection of underwater acoustic waves from
a simple metagrating : modeling and experiment**

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Abstract

Metagratings are periodic arrays of subwavelength scatterers, or atoms, engineered to refract or reflect waves towards anomalous directions with unit efficiency. Here we design and build a metagrating to control the reflection direction of water-borne ultrasound waves impinging on rigid or free surface. The grating and its atoms are designed to cancel the specular reflection and to redirect acoustic power towards a negative reflection direction, through the first negative order Floquet mode. Despite a simple design, based on c-shaped brass particles acting as Helmholtz resonators, the grating is efficient ($> 90\%$) over a relatively broad range of frequencies (74-103 kHz) for a broad range of incident angles (14 to 54°). This good performance is obtained by tuning the distance between the atoms and the reflective surface. A multiple scattering analytical model is presented to explain the phenomenon, and a finite element model is developed to further investigate the performance of the proposed design. Predictions from the model are confirmed experimentally in a water tank. The simplicity, reconfigurability and scalability of the design, as well as its high efficiency, broadband behavior and robustness to incident angle are all features that make the grating potentially useful for various applications in underwater acoustic such as telemetry, communication, or noise mitigation.

I. INTRODUCTION

Bulk 3D metamaterials and thin 2D metasurfaces are designed to offer extraordinary wave propagation properties, usually from an assembly of multiple subwavelength elements. Many metasurfaces have been developed in the past years, both for electromagnetic waves [1–5] and acoustic waves [6–9]. One application of metasurfaces is to redirect waves toward anomalous directions not classically allowed by the reflection and refraction laws [10–13]. This can be achieved using a phase-gradient metasurface : a local phase shift is applied by the surface to modify the wave’s momentum and reroute it toward the desired direction, according to the so-called generalized Snell-Descartes’s laws [14]. The phase gradient can be discretized by engineering multiple small elements, or atoms, to offer the desired local phase shift. In acoustics, space-coiling structures or Helmholtz-like resonators are often employed [7], but require tedious design and precise manufacturing. In air acoustics, 3D printing can provide complex structures quickly and at low cost, but most 3D printing materials are inappropriate for underwater applications because of a low

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contrast in acoustic impedance. Furthermore, such surfaces usually work optimally in a narrow frequency band, as complex meta-atoms will offer the desired phase response only at their design wavelength. Recent works have highlighted another limitation : even with a perfect realization of the phase gradient, the efficiency is intrinsically limited for large steering angles [15, 16]. This can be overcome by designing locally active or non-local metasurfaces, relying on near field coupling between the atoms [16, 17]. However, this comes at the cost of further increased design and fabrication challenges.

The concept of metagrating relies on a different physical principle and on a simpler design based on a periodic cell containing down to one single atom [18–25]. The possible outgoing wave directions are governed by the periodicity (Fig. 1). By engineering the atoms, the unwanted diffraction channels can be suppressed and all the impinging energy rerouted to a desired direction with unitary efficiency [18, 20]. Such gratings have been recently demonstrated experimentally for acoustics in air [25] and numerically for underwater acoustics[24]. Another benefit over phase-gradient metasurfaces is the existence of configurations effective over a large bandwidth and a large range of incident angles, as recently demonstrated in optics [26–28] and in this study for acoustics.

Resonant atoms can provide the desired scattering properties [18]. Here we use c-shaped atoms, which behave as Helmholtz resonators with a low frequency resonance mainly governed by the size of the cavity and neck opening. Such atoms have recently attracted interest in acoustic wave manipulation, as they are easy to fabricate and offer interesting scattering properties [25, 29]. In this study, we design and build a metagrating for anomalous reflection of underwater ultrasound waves around 90 kHz, from small (radius $\approx \lambda/6$) brass atoms, with a focus on negative reflection : the goal is to redirect the incoming energy toward the direction of the incident wave. We demonstrate how, by adequately choosing the distance between the grating and the reflective surface, it is possible to obtain near unit efficiency over a large frequency band and range of incident angles. The broadband and broad-angle effects are linked to the multiple scattering of the wave by the surface and the atoms, and an analytical model is provided, as well as Finite Element Modeling and an experimental demonstration.

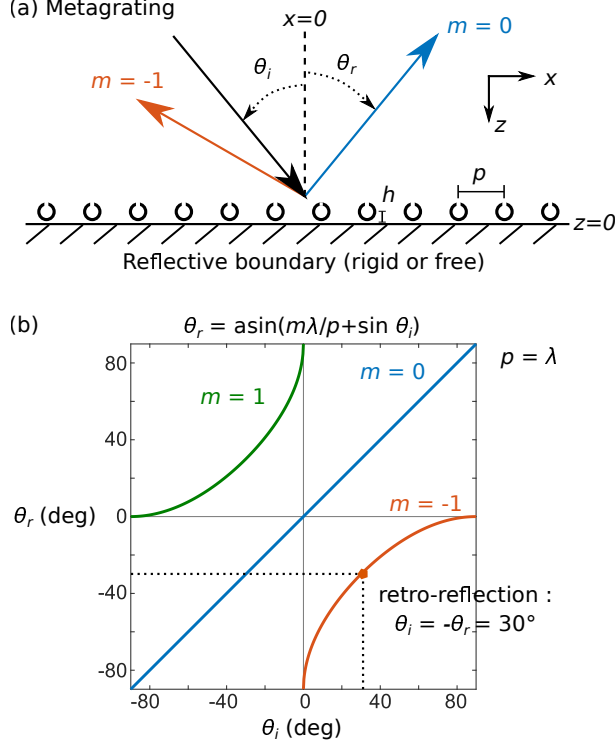


FIG. 1. (a) To achieve anomalous reflection, metagratings rely on periodicity, which allows only a finite number of reflected modes to exist, given by the grating equation, illustrated in (b) for $p = \lambda$. By tailoring the scattering properties of the atoms, the $m = 0$ mode and the specular reflection from the surface behind the grating can be tuned to cancel each other so that all the energy is rerouted toward the desired anomalous direction.

II. ANALYTICAL MODEL AND PARAMETER SELECTION

Consider a periodic grating of small atoms located at a height h from a perfectly reflecting surface. The surface can be either an acoustic free (acoustic pressure = 0) or a rigid (normal velocity = 0) boundary condition. When the grating is illuminated by a plane acoustic wave with incidence angle θ_i , only a finite number of reflected plane waves, or Floquet modes, can exist. Their reflection angles θ_r are related to the periodicity p and wavelength λ by the grating equation [30, 31]:

$$\sin(\theta_r) - \sin(\theta_i) = m \frac{\lambda}{p}, \quad (1)$$

where m is an integer. Solutions of Eq. (1) are plotted on Fig. 1(b) for $p = \lambda$. In this case, three reflection channels exist : $m = 0$, for which $\theta_r = \theta_i$ for all θ_i , and the $m = -1$ and $m = 1$ modes, existing respectively for $\theta_i > 0$ and $\theta_i < 0$. With these parameters, $\theta_i = 30^\circ$ corresponds to the

case of retro-reflection, where the $m = -1$ mode is reflected exactly toward the direction of the incoming wave. When two or more channels co-exist at a given incidence angle, their relative weight depends on the scattering properties of the atoms [18]. The goal is to tune the scattering such that the weight of one selected mode is made as close as possible to 1, while other unwanted modes are close to 0. In this study we focus on negative reflection : in the range of parameters for which the $m = 0$ and $m = -1$ modes exists, we design the grating such that the $m = 0$ mode cancels the specular reflection from the surface behind the grating, so that all energy is reflected by the $m = -1$ channel, in a negative direction as shown in Fig. 1(a).

Under a monopolar scattering approximation, valid for small atoms, the $m = 0$ mode exactly cancels the specular reflection if, for the rigid boundary condition, (see the Appendix)

$$\frac{1}{\alpha} = \frac{2i}{p} \frac{\omega c}{\cos \theta_i} \cos^2(kh \cos \theta_i), \quad (2)$$

and for the free boundary condition

$$\frac{1}{\alpha} = \frac{-2i}{p} \frac{\omega c}{\cos \theta_i} \sin^2(kh \cos \theta_i), \quad (3)$$

where α is the effective monopolar polarizability of an atom and $k = 2\pi/\lambda = 2\pi f/c$. We see in both cases that α must be purely imaginary, which happens at the metagrating resonance [18, 32]. Note that α is the *effective* polarizability of an atom, defined in the presence of all other atoms and reflective surface. Futhermore, in order to perfectly match the power of the incident wave to power radiated through the $m = -1$ channel we need to satisfy [18] (see also the Appendix) :

$$\frac{1}{\cos \theta_i} \cos^2(kh \cos \theta_i) = \frac{1}{\cos \theta_r} \cos^2(kh \cos \theta_r), \quad (4)$$

for the rigid boundary condition, and

$$\frac{1}{\cos \theta_i} \sin^2(kh \cos \theta_i) = \frac{1}{\cos \theta_r} \sin^2(kh \cos \theta_r), \quad (5)$$

for the free boundary condition. We see, as noted by Ra'di *et al.* [18], that retro-reflection ($\theta_r = -\theta_i$) is a trivial solution of these equations, meaning that it can be obtained whatever the distance h . However, as we will show now, the solution is more robust to a change in wavelength or angle for some values of h , providing a broadband and broad-angle behavior.

We define ε as the ratio of left hand to right hand term in eq. (4) or (5). By considering wave scattering by the grating and its *virtual* counterpart on the other side of the reflective surface, we can arrive at (see the appendix)

$$\varepsilon = \frac{\cos \theta_r \cos^2(kh \cos \theta_i)}{\cos \theta_i \cos^2(kh \cos \theta_r)}, \quad (6)$$

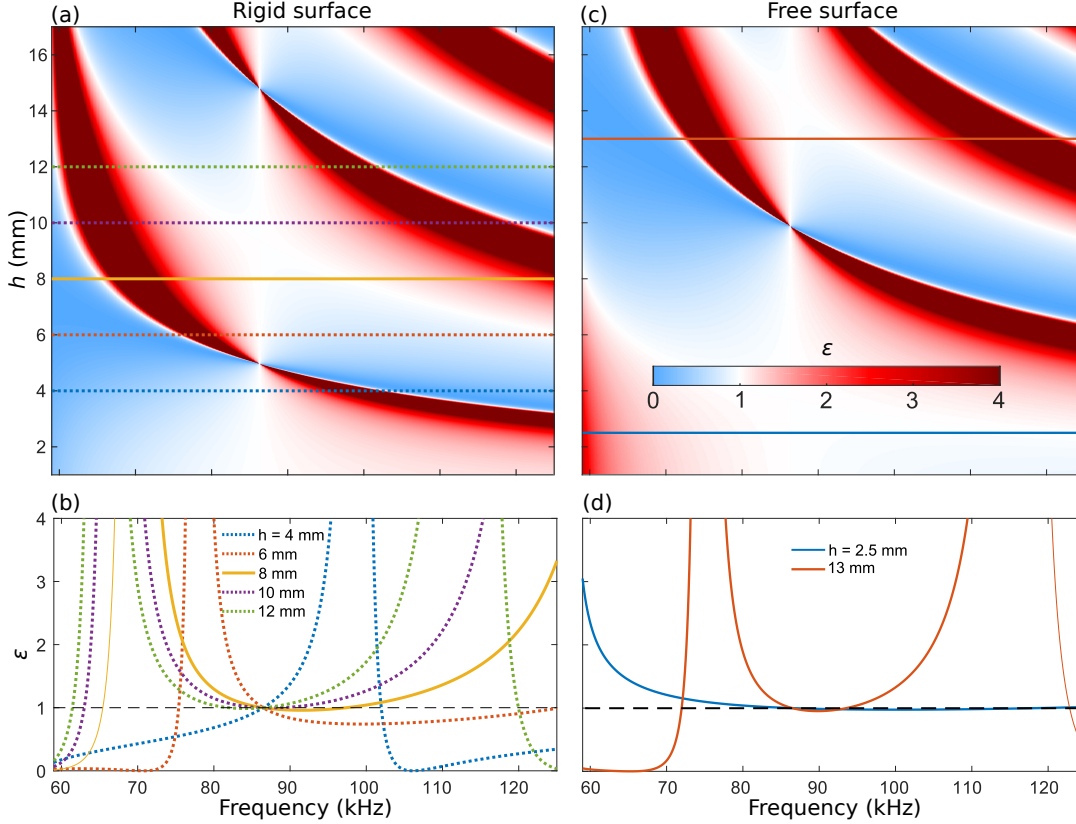


FIG. 2. Tuning the distance h between the grating and the rigid (a)-(b) or free (c)-(d) reflective surface. ε is the ratio of left hand to right hand term in eq. (4) or (5). A value of 1 (white color) satisfies the conditions for anomalous reflection with unitary efficiency. The vertical white line at 87 kHz is the trivial solution $\theta_r = -\theta_i$. For $h = 8$ mm for the rigid surface, ε is close to one over a broad range of frequencies, particularly in the range [80-105] kHz. For the free surface, two interesting solutions exist : $h = 13$ mm and h close to 0 (the smallest possible value in practice being equal to the atom's radius).

for the rigid boundary condition, and

$$\varepsilon = \frac{\cos \theta_r \sin^2(kh \cos \theta_i)}{\cos \theta_i \sin^2(kh \cos \theta_r)}, \quad (7)$$

for the acoustic free boundary. We plot ε as a function of h and incident wave frequency f in Fig. 2 for both boundary conditions, for $\theta_i = 30^\circ$, $c = 1470$ m/s, and $p = \lambda = 17$ mm (and using the grating equation Eq. (1) to obtain θ_r as a function of f). In addition to the trivial solution ($\theta_r = -\theta_i$) corresponding to the vertical white lines at about 87 kHz visible in Fig. 2(a)-(b), other solution branches exist. In the rigid case, for $h = 8$ mm we observe a large valley of near unity ε , due to a solution branch crossing the $\theta_r = -\theta_i$ solution. In this region, the metagrating is able to efficiently couple the incoming power to the $m = -1$ channel over a broad range of frequencies,

through complex wave interactions between the grating and the reflective surface. In the free boundary case, two interesting values of h exist : h very small, with the minimal values in practice being equal to the radius of the atoms (2.5 mm here, see below), or $h = 13$ mm.

The dispersion relation between the wavelength and reflective angle (Eq. 1) implies that, if the specular reflection is canceled over a broad frequency range, the energy is reflected toward a varying angle. Consequently, a broadband wave cannot be redirected to a given direction. This is a limitation of the metagrating. On the other hand, we can also observe that reflective and incident angle have a symmetric role in Eqs. (4)-(5), implying that the broadband behavior is associated to robustness with respect to the incident angle.

III. FINITE ELEMENT MODELING OF THE PERIODIC GRATING

A Finite Element Model (FEM) of the periodic metagrating was built using Comsol Multiphysics. The acoustic structure interaction module was used to take into account the elastic nature of the atoms. Geometry and material properties were set according to the actual grating (see section IV below). Wave velocity and mass density of water are 1470 m/s and 1000 kg/m³, respectively. Modeling was done in the frequency domain. A single periodic cell of the grating was modeled as a rectangular box, using Floquet periodic boundary conditions to obtain the behavior of an infinite grating. A rigid (normal velocity = 0) or free (acoustic pressure = 0) boundary condition was set behind the atoms and a matched boundary condition at the opposite boundary was set to perfectly absorb the $m = 0$ and $m = -1$ reflected modes. The incident wave was a planar wave background pressure field with unit amplitude. The power radiated by each of the possible propagation mode was evaluated by integration over the opposite boundary, and expressed as (quadratic) reflection coefficient $|R_m|^2$.

Results of the FEM modeling of the periodic grating for the rigid boundary case for $h = 8$ mm are presented on Fig. 3. Panel (a) displays the reflection coefficient of the anomalous reflection $m = -1$ for all incidence angles from 0 to 90°. Efficiency remains excellent (> 0.9) for a broad range of incident angles (14° to 54°) around the design angle of 30°. The bandwidth is shrinking for extreme angles but is still reasonably large angles from about 20° to 40°. The reflection angle here depends on both incidence angle and frequency, according to Eq. (1). Retro-reflection ($\theta_r = -\theta_i$) is achieved along the purple line. More than 50% of the incoming power can be retro-reflected for frequencies ranging from 65 to 111 kHz, and angles from 23° to 43°. The sharp bright lines in

panel (a) correspond to the appearance of new possible reflection modes among $m = -2$, $m = -1$, and $m = 1$. The reflection coefficients of each channel are plotted on panel (b) for $\theta_i = 30^\circ$. Until 115 kHz only the $m = 0$ and $m = -1$ modes exist, in agreement with Eq. 1. Above 70 kHz, the $m = -1$ reflection mode becomes dominant, up to 108 kHz. Between 74 kHz and 103 kHz, more than 90% power is directed toward the anomalous reflection direction, demonstrating near unit efficiency over a large bandwidth ($\approx 1/3$ of the central frequency). Panels (c) and (d) show respectively the incident and reflected wave field (real part of the acoustic pressure) at 87 kHz ($p = \lambda$) for $\theta_i = 30^\circ$. In this case retro-reflection is perfect, with efficiency > 0.998 . At 75 kHz and 100 kHz (Fig. 3(e)-(f)) anomalous reflection is less than perfect, as visible from the slightly distorted wave fronts, but remains efficient.

Similar results are displayed on Fig. 4 for the free boundary case, for height $h = 13$ mm and $h = 2.5$ mm respectively on panels (a-b) and (c-d). For $h = 13$ mm, the observed effect is qualitatively similar to the effect observed for $h = 8$ mm for the rigid boundary. This is not surprising, as in both case the broadband effect is due to a crossing of two solution branches of the equation $\varepsilon = 0$ in the frequency - distance h space (Fig. 2). For $h = 2.5$ mm the effect is qualitatively different : the range of incidence angle over which the efficiency is > 0.9 is even larger, ranging from 10 to 78° . The grating is therefore highly efficient from near normal to very grazing incidence. Frequency band of high efficiency is somewhat narrower in this case, but the decrease in efficiency when moving away from the design frequency is less sharp than in the previous cases and the grating still has > 0.5 efficiency on a reasonably broad band (67 to 107 kHz at 30° incidence).

IV. EXPERIMENTAL DEMONSTRATION WITH A RIGID SURFACE

Experimentally, the atoms making up the metagrating were made from 300 mm long brass hollow cylinders, with external radius $r_{ext} = 2.50$ mm and internal radius $r_{in} = 2.05$ mm. They were opened on one side using a milling cutter to form c-shaped atoms (see Fig. 5). The opening was $\delta = 2$ mm large, forming a Helmholtz resonator with a broad resonance at about 90 kHz. Brass was selected because 1) it could be found easily in the desired shape at low cost, 2) it is resistant to corrosion, even when immersed in water for several consecutive days, 3) it can be easily milled (as compared to stainless steel), and 4) it is stiff ($E = 100$ GPa) and heavy ($\rho = 8500\text{kg/m}^3$), providing a large acoustic impedance contrast with water. A pairs of PVC blocks were drilled to the desired periodicity $p = 17$ mm ($\approx \lambda$ at 87 kHz) to assemble the grating from 17 atoms.

the structure was then attached in front of a 10 mm thick stainless steel plate ($L = 300$ by 300 mm) acting as an almost perfect reflector for ultrasound waves in the frequency range of interest. The distance between the center of the tubes and the plate was set to $h = 8$ mm, according to the analysis above.

Experiments were conducted in a water tank (3 m diameter, 1.5 m height). Two identical ultrasonic transducers were used (Panametrics V3052, 100 kHz central frequency, 38 mm diameter) for transmission and reception. Both transducers were located at about 0.9 m from the bottom of the tank. Distance from the center, where the target (grating + stainless steel plate) was suspended to a rotating stage, was about 1.4 m for the emitter and 0.4 m for the receiver, respectively. The setup

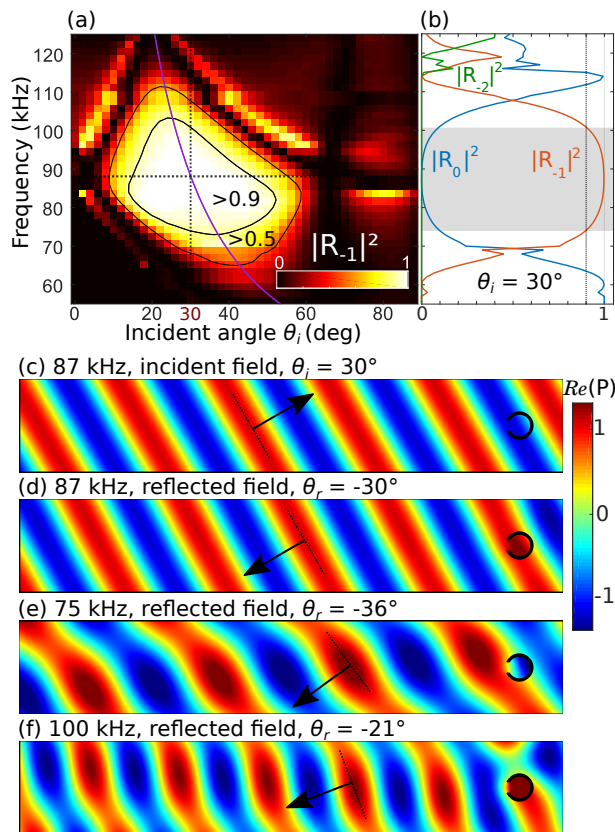


FIG. 3. FEM results for the proposed metagrating over a rigid surface with height $h = 8$ mm. (a) Reflection coefficient of the anomalous reflection ($m = -1$) channel as a function of incident angle and frequency. The purple line indicates retro-reflection ($\theta_r = -\theta_i$). (b) Reflection coefficient of the channels $m = 0$ (specular), $m = -1$, and $m = -2$ for $\theta_i = 30^\circ$. The bandwidth over which $|R_{-1}| > 0.9$ is highlighted in grey. (c-f) Maps of the incident (c) or reflected (d-f) acoustic pressure field. At 87 kHz, retro-reflection is perfect. At 75 kHz and 100 kHz anomalous reflection is less than perfect but still efficient.

is presented on Fig. 5. A pulser-receiver (5052R, Panametrics) was used to generate a broadband input signal. The received signal was recorded on a digital oscilloscope (HDO6034, Teledyne Lecroy). The receiving transducer was rotated over 180° in front of the plate, with 2° steps. For each receiving position, 100 recordings were averaged to improve signal-to-noise ratio. Time windowing was applied to isolate the reflection on the grating from reflections on the tank boundary and water surface. The spectrum was obtained from Fast Fourier Transform and frequency band correction was applied using the spectrum of the specular reflection on the steel plate alone. The experiment was repeated for three incidence angles (15° , 30° , 45°).

The experimentally recorded reflected pressure is displayed as a function of the receiving angle and frequency on Fig. 6(a)-(d). From Fig. 6(a) (no grating, plate only) we can see that the incoming beam contains energy on relatively broad range of angles, from about $\theta_i - 15^\circ$ to $\theta_i + 15^\circ$, due to the finite size of the source. After source signal spectrum correction, the pressure level is homo-

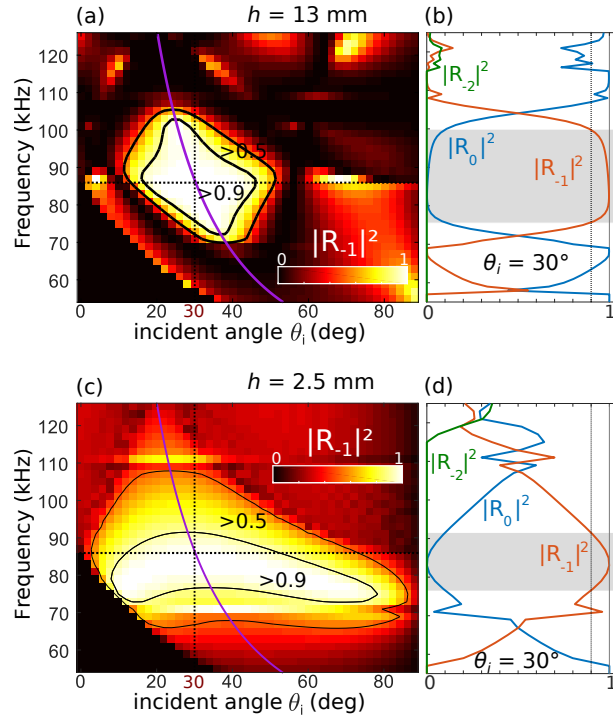


FIG. 4. FEM results for an acoustic free surface, with height $h = 13$ mm (a-b) and $h = 2.5$ mm (c-d). Maps (a) and (c) show the reflection coefficient of the anomalous reflection ($m = -1$) channel as a function of incident angle and frequency. The purple line indicates retro-reflection ($\theta_r = -\theta_i$). Plots (b) and (d) show the reflection coefficient of the channels $m = 0$ (specular), $m = -1$, and $m = -2$ for $\theta_i = 30^\circ$. The bandwidth over which $|R_{-1}| > 0.9$ is highlighted in grey.

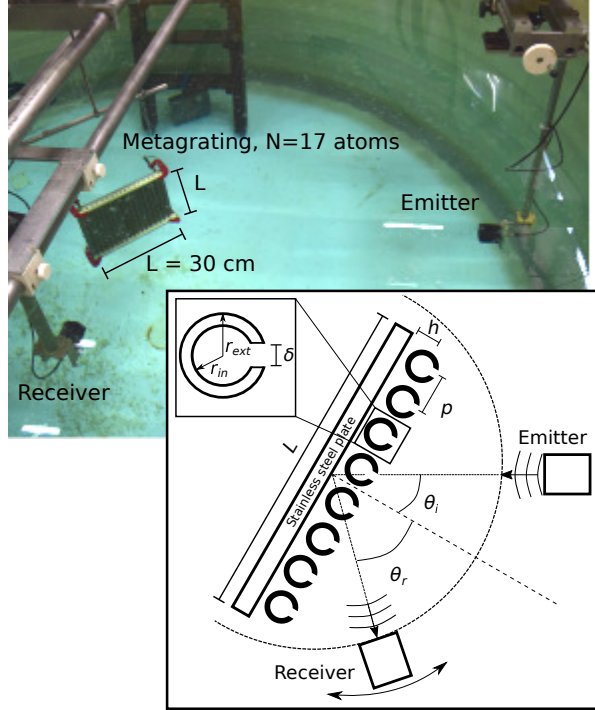


FIG. 5. Photograph and scheme of the experimental setup. The target can be rotated to vary the incidence angle, while the receiving angle can also be modified. The design parameters of the atom and grating are given in the text.

geneous from 55 kHz to 165 kHz. On Fig. 6(b)-(d) we can see how acoustic power is redirected toward the various modes allowed by the grating equation (eq. 1). The reflected amplitude in the specular ($m = 0$) direction is dramatically reduced in the operating bandwidth of the metagrating (Fig. 6(e)-(g)), particularly for $\theta_i = 30^\circ$. The experiments are also compared to the FEM prediction on panels (e-g). At $\theta_i = 30^\circ$, a reduction down to 30 dB (power divided by 1000) can be observed, and the effective bandwidth at -10 dB (power divided by 10, or 90% efficiency) is about 30 kHz, experimentally confirming the broad bandwidth of the designed grating. The broad-angle behavior is confirmed on Fig. 6(c) by the fact that for $\theta_i = 30^\circ$, the broad acoustic beam is totally canceled in the specular direction, and by the good efficiency observed for $\theta_i = 15^\circ$ and $\theta_i = 45^\circ$ in Fig. 6(b)-(g). We can also observe narrow gaps in the reflected mode spectra, at frequencies corresponding to the appearance of an additional mode, which are the acoustic analog of the Wood's anomaly [33, 34], whose discussion is outside the scope of the present work.

Fig. 7 provides an alternative representation of the experimentally recorded pressure at 87 kHz, at which the metagrating is the most effective, for the same four cases : (a) no metagrating, (b)-(d)

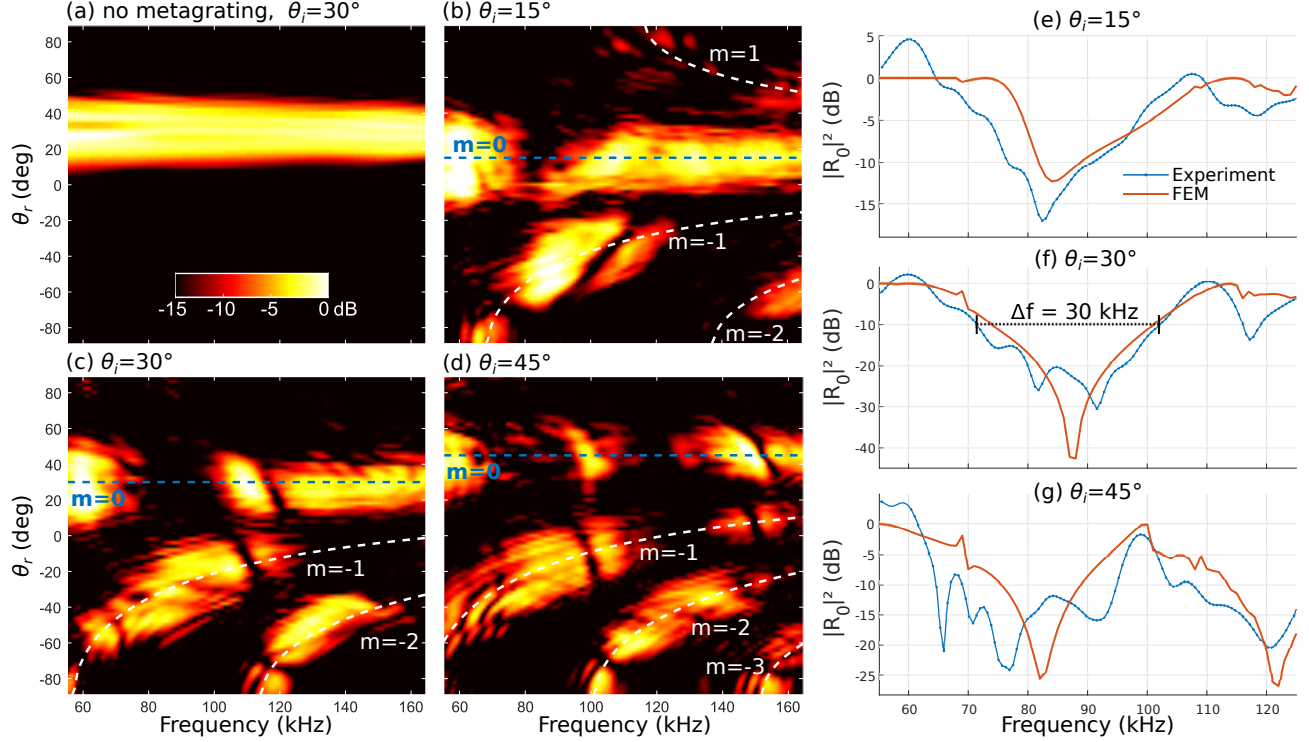


FIG. 6. Experimental results for the constructed metagrating. (a) Amplitude of the reflected pressure wave without grating (calibration). (b-d) Acoustic pressure reflected by the grating for three incidence angle (15° , 30° , 45°). (e-g) Profiles along the specular direction $m = 0$, and comparison to FEM results. For $\theta_i = 30^\circ$, the $m = 0$ mode is reduced by more than 10 dB over a broad frequency range (≈ 30 kHz). The surface is still effective for $\theta_i = 15^\circ$ and 45° .

with the metagrating for incidence angles 15° , 30° , and 45° , respectively. The negative reflection effect is clearly visible, as in all three cases with the grating, most of the wave pressure is reflected backward, towards the quadrant containing the incident wave vector. The pressure amplitude increases compared to specular reflection pressure for large negative reflection angles (panel (b)), and decreases for small negative reflection angles (panel (d)). This is expected, as a consequence of energy balance: the power carried away by the anomalous reflection mode is proportional to the product of acoustic pressure and cosine of the reflection angle (see eqs. A.16 and A.17 in the Appendix). For 30° incidence angle (panel (c)), the amplitude should be expected to be equal to the specular reflection pressure (panel (a)), but is slightly lower. This is an artefact due to our experimental system : at -30° reflection angle (perfect retroreflection) the receiver is located in front of the acoustic source, actually reducing the amount of incident energy.

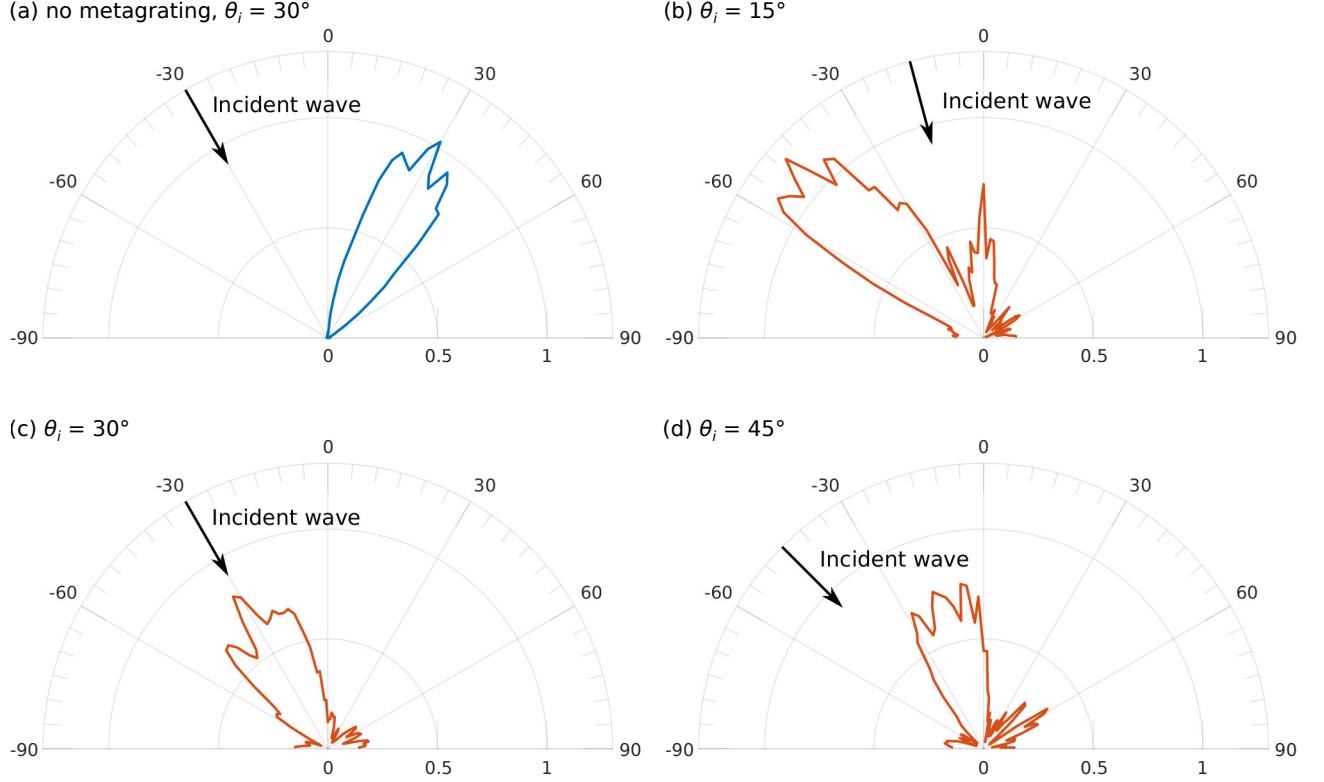


FIG. 7. Polar plots of the reflected pressure at 87 kHz, at which the grating is the most effective. (a) Without the metagrating and (b)-(d) with the metagrating for incidence angles 15° , 30° , and 45° , respectively. The direction of the incident wave is displayed as a black arrow. Amplitudes are normalized to the amplitude of the specular reflection at 30° .

V. DISCUSSION AND CONCLUSION

In this work, we designed a metagrating for negative reflection of underwater acoustic waves. This metagrating can operate in front of either a rigid or an acoustic free surface. A detailed analysis of multiple wave scattering between the grating and the surface was presented. This analysis showed that the height h of the grating above the surface is a critical parameter regarding performances. Finite element modeling confirmed that, once an appropriate value for h has been selected, the negative reflection towards the $m = -1$ Floquet mode can be highly efficient (>0.9) over a relatively broad frequency range and for a broad range of incident angles. This was finally verified experimentally in a water tank for the rigid surface, using a metagrating build from c-shaped brass tubes.

The broadband efficiency and simple design are significant steps towards real-world applica-

tions in underwater acoustics, where most commonly encountered acoustic signals, from sonar to noise pollution, are broadband. Fabrication of acoustic metasurfaces is more challenging for water or water-like media than for air, since stiff and heavy materials are required to provide a large impedance contrast. We have demonstrated here that a simple brass hollow cylinder opened on the side is a viable structure for underwater applications. The broadband and broad-angle high efficiency in retro-reflection could be useful for telemetry applications based on pulse-echo methods. The relation between the reflection angle and frequency could be used as a benefit to perform real time processing operations such as e.g. filtering, frequency discrimination (i.e. spectroscopy) or multiplexing / demultiplexing in underwater acoustic communications. The use as a noise barrier can also be envisaged in shallow water, where low frequency propagation is governed by guided modes reflecting back and forth between the water surface and the ocean bottom at grazing angles. A metagrating, placed close to the water surface could have the ability to send back waves toward the source or toward the sea bottom with normal incidence, effectively blocking propagation. The design can be scaled to other frequencies, and applied to a free boundary. All analytical and numerical calculations were done under a linear acoustic propagation / linear elasticity assumption, which corresponds to the presented experimental conditions. Extremely high acoustic pressure levels may alter the behavior of the grating, by modifying the resonant behavior of the structure or through non-linear interactions of multiple scattered waves. This was not investigated here.

In naval applications, one usually seeks to reduce the acoustic power reflected back to the source by a target, which is precisely the opposite of what was achieved here. Further research is needed to develop other configurations, e.g. redirecting a normally incident wave toward a grazing direction. Using several atoms per period [20, 24] could potentially address this challenge. Finally, c-shaped atoms are known for providing strong Willis coupling [35, 36], which was not discussed here but could also allow even more challenging wave control applications by canceling multiple Floquet modes simultaneously.

Appendix: Analytical model of wave diffraction by a periodic grating close to a reflective surface

We consider a periodic array of small particles whose center is at a distance h from a perfectly reflecting surface (either a rigid or a free surface). The configuration is depicted on Fig. 1(a). The surface is aligned with the x axis and at coordinate $z = 0$. The particles are at $z = -h$ and the period is p . An acoustic plane wave of frequency f is incoming from the left, with an incidence

θ_i . The wave velocity and mass density of the propagation medium are c and ρ , respectively.

The incoming pressure wave is

$$p_{inc} = p_0 e^{i(k_x x + k_z z)} \quad (\text{A.1})$$

with $k_x = k \sin(\theta_i)$, $k_z = k \cos(\theta_i)$, and $k = \omega/c = 2\pi f/c = 2\pi/\lambda$. The pressure wave reflected at the boundary (in the absence of grating) is

$$p_{ref}^{rigid/free} = \pm p_0 e^{i(k_x x - k_z z)}, \quad (\text{A.2})$$

where the (+) sign is for the rigid case and the (-) sign for the free boundary. Under a monopolar approximation (valid for small particles), the field scattered by the infinite array of particles is

$$p_{scat}^{rigid/free} = A_0 \sum_{n=-\infty}^{+\infty} \left(H_0^1 \left[k \sqrt{(x - np)^2 + (z + h)^2} \right] \pm H_0^1 \left[k \sqrt{(x - np)^2 + (z - h)^2} \right] \right) e^{in\phi}, \quad (\text{A.3})$$

with H_0^1 the zero order Hankel function of the first kind and $\phi = 2\pi p \sin(\theta_i)/\lambda$ a phase shift term related to the oblique incidence. The first term in the infinite sum is for the actual particles and the second term is for the “virtual” particles located on the other side of the perfectly reflecting surface. The Poisson’s summation formula is then used to transform the above sum over an infinite number of particles to a sum over an infinite number of Floquet modes [32]. We obtain for the rigid boundary

$$p_{scat}^{rigid} = \frac{4A_0}{p} \sum_{m=-\infty}^{+\infty} \exp\left(i\left(\frac{2\pi m + \phi}{p}x - k_{zm}z\right)\right) \frac{\cos(k_{zm}h)}{k_{zm}}, \quad \text{for } z < -h, \quad (\text{A.4})$$

and for the free boundary

$$p_{scat}^{free} = \frac{-4iA_0}{p} \sum_{m=-\infty}^{+\infty} \exp\left(i\left(\frac{2\pi m + \phi}{p}x - k_{zm}z\right)\right) \frac{\sin(k_{zm}h)}{k_{zm}}, \quad \text{for } z < -h, \quad (\text{A.5})$$

with $k_{zm} = \sqrt{k^2 - \left(\frac{2\pi m + \phi}{p}\right)^2}$ the horizontal wave number for the m -th mode. The Floquet modes in the sums above are propagating if k_{zm} is real and are evanescent if k_{zm} is complex. Propagating modes leave the surface with an angle θ_r^m given by the relation $k \sin(\theta_r^m) = \frac{\phi + 2\pi m}{p}$, from which we can arrive at the grating equation $\sin(\theta_r^m) - \sin(\theta_i) = m \frac{\lambda}{p}$.

To achieve perfect anomalous reflection, i.e. having the incident wave redirected with unit efficiency into a single plane wave whose angle can be controlled, we need two propagating Floquet

modes : one being the controlled reflected wave and the other one canceling the specular reflection. One of these two modes is necessarily the mode $m = 0$, as $k_{z0} = k_z$ is real. The second can be either $m = 1$ or $m = -1$ depending on the sign of θ_i . We assume here that $\theta_i > 0$, in which case we need the modes $m = 0$ and $m = -1$ to be propagating and all other to be evanescent. A mode m is evanescent if $\sin(\theta_r^m) - \sin(\theta_i) = m \frac{\lambda}{p}$ has no solution. For $m = 1$ no solution exists if $p < \frac{\lambda}{1 - \sin(\theta_i)}$. Satisfying this condition also ensure that modes $m = 2, 3, \dots$ are evanescent. For $m = -2$ to be evanescent (and also $m = -3, -4, \dots$), we get to the condition $p < \frac{2\lambda}{1 + \sin(\theta_i)}$. For $p = \lambda$, both conditions are satisfied for all angles $\theta_i > 0$.

The total wave field is then $p_{tot} = p_{inc} + p_{ref} + p_{m=0} + p_{m=-1}$, which leads to, for the rigid boundary,

$$p_{tot} = p_0 e^{i(k_x x + k_z z)} + p_0 e^{i(k_x x - k_z z)} + \frac{4A_0}{p} e^{i(k_x x - k_z z)} \frac{\cos(k_z h)}{k_z} + \frac{4A_0}{p} e^{i(k \sin(\theta_r^{-1})x - k \cos(\theta_r^{-1})z)} \frac{\cos(k \cos(\theta_r^{-1})h)}{k \cos(\theta_r^{-1})}, \quad (\text{A.6})$$

where $k \sin(\theta_r^{-1}) = \frac{\phi - 2\pi}{p}$ and $k \cos(\theta_r^{-1}) = \sqrt{k^2 - \left(\frac{\phi - 2\pi}{p}\right)^2}$. For the free boundary we arrive at

$$p_{tot} = p_0 e^{i(k_x x + k_z z)} - p_0 e^{i(k_x x - k_z z)} - \frac{4iA_0}{p} e^{i(k_x x - k_z z)} \frac{\sin(k_z h)}{k_z} - \frac{4iA_0}{p} e^{i(k \sin(\theta_r^{-1})x - k \cos(\theta_r^{-1})z)} \frac{\sin(k \cos(\theta_r^{-1})h)}{k \cos(\theta_r^{-1})}. \quad (\text{A.7})$$

For the $m = 0$ mode to cancel the specular reflection it is therefore required that

$$p_0 = \frac{-4A_0}{p} \frac{\cos(k_z h)}{k_z}, \text{ for the rigid boundary and that} \quad (\text{A.8})$$

$$p_0 = \frac{-4iA_0}{p} \frac{\sin(k_z h)}{k_z}, \text{ for the free boundary.} \quad (\text{A.9})$$

The 0-order scattering coefficient A_0 is related to the effective monopolar polarizability α of the particle by the relation [36]

$$A_0 = \frac{-ik^2 c^2}{4} \alpha (\tilde{p}_{inc} + \tilde{p}_{ref}), \quad (\text{A.10})$$

where \tilde{p}_{inc} and \tilde{p}_{ref} denote respectively the incident and reflected pressure at the center of the particle. For the rigid boundary we have, for a particle located at $x = 0$ and $z = -h$

$$\tilde{p}_{inc} + \tilde{p}_{ref} = p_0 e^{ik_z h} + p_0 e^{-ik_z h} = 2p_0 \cos(k_z h), \quad (\text{A.11})$$

and for the free boundary

$$\tilde{p}_{inc} + \tilde{p}_{ref} = p_0 e^{ik_z h} - p_0 e^{-ik_z h} = -2ip_0 \sin(k_z h). \quad (\text{A.12})$$

Inserting these equations in eq. (A.10) and satisfying the conditions eqs. (A.8 - A.9) we find

$$\frac{1}{\alpha} = \frac{2i}{p} \frac{\omega c}{\cos \theta_i} \cos^2(kh \cos \theta_i), \text{ for the rigid boundary and} \quad (\text{A.13})$$

$$\frac{1}{\alpha} = \frac{-2i}{p} \frac{\omega c}{\cos \theta_i} \sin^2(kh \cos \theta_i), \text{ for the free boundary.} \quad (\text{A.14})$$

In both cases, we see that α must be purely imaginary for the $m = 0$ Floquet mode to cancel the specular reflection, which happens at the grating resonance [18, 32].

In addition, it is also required for perfect anomalous reflection that the power carried away from the surface by $m = -1$ mode matches the incoming power. The incoming power is equal to

$$P_i = \frac{p_0^2 p}{2\rho c} \cos \theta_i. \quad (\text{A.15})$$

The power carried away by the $m = -1$ mode is, now simplifying notation using $\theta_r^{-1} = \theta_r$, for the rigid boundary

$$P_{m=-1} = \frac{p}{2\rho c} \cos(\theta_r) \left(\frac{4A_0}{p} \right)^2 \frac{\cos^2(kh \cos \theta_r)}{k^2 \cos^2(\theta_r)}, \quad (\text{A.16})$$

and for the free boundary

$$P_{m=-1} = \frac{p}{2\rho c} \cos(\theta_r) \left(\frac{4iA_0}{p} \right)^2 \frac{\sin^2(kh \cos \theta_r)}{k^2 \cos^2(\theta_r)}. \quad (\text{A.17})$$

Using the conditions eqs. (A.8-A.9) for p_0 above and equating incident and carried away powers lead to

$$\frac{1}{\cos \theta_i} \cos^2(kh \cos \theta_i) = \frac{1}{\cos \theta_r} \cos^2(kh \cos \theta_r),$$

for the rigid boundary and (A.18)

$$\frac{1}{\cos \theta_i} \sin^2(kh \cos \theta_i) = \frac{1}{\cos \theta_r} \sin^2(kh \cos \theta_r),$$

for the free boundary. (A.19)

One of these two last conditions, depending on the nature of the reflective surface behind the grating, has to be satisfied along the grating resonance condition in order to achieve perfect negative reflection. We can see that retro-reflection, $\theta_r = -\theta_i$ is always a solution, for any distance h . However, as demonstrated in the main text, there are other non-trivial solutions and there exist values of h for which the conditions can be approximately satisfied over a broad range of frequencies and incidence angles, leading to a robust broadband negative reflection effect.

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