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TILINGS AND SYMMETRY USING THE LABOSAÏQUE

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We present here our original scientific mediation device, called Labosaïque designed to offer attractive activities in schools for discovering mathematical concepts (forms, symmetry, multiplication, infinite repetition) by means of concrete objects: geometric puzzle pieces, “turn-and-slide table”, ordinary and articulated mirrors, mirror chambers. The mathematical context is the study of symmetry and periodic tilings of the plane. We describe here this material and its use in primary school activities.

INTRODUCTION

Labosaïque (<https://www.lmno.cnrs.fr/equipes/dcm/labosaique>) is a project aimed at discovering the symmetries of the plane and space through the manipulation and observation of concrete and attractive objects. This material focuses on the study of tilings and symmetries, allowing to explore at the same time the links between mathematics and physics of materials, art, architecture, biology ... Since its first public use in 2011, Labosaïque has been widely used for interventions in schools, from primary school to high school (more than 5,000 students).

Partially inspired by an Italian project on symmetries and mirrors (Bellingeri et al. 2002), it exists mainly in 2 different versions, one of large size and excellent aesthetic quality, the other portable for easy distribution and use in schools. It is important to make our materials directly available to teachers so that they can take ownership of our workshops and make them “consistent” with the curriculum and their groups of students. We have already organized teacher training days, for primary or high schools.

THE DEVICE : CONTEXT AND PUBLICS

The Labosaïque offers several activities to discover mathematics related to the tilings, friezes and rosettes in the plane, theme currently highlighted in the French school curriculum. This theme is a preparation for the study of central symmetries, translations and rotations by installing mental images that will later facilitate the analysis of geometric figures as well as the punctual definition of the transformations studied.

The Labosaïque consists of several mostly innovative objects. At the primary school level we provide a turn-and-slide table, a toolset of puzzle pieces allowing the study of periodic tilings as well as a Penrose tiling, mirror chambers, articulated mirror, set of cards and mirrors to play with mirror symmetry.

The educational activities that we propose around these objects have been created and improved by feedback as we have carried out our interventions in schools. These usually take

the form of one to two hour workshops with a class. During a workshop we encourage students to engage a group research process. Our concrete and playful material is a support to tackle different problems according to the school levels.

The first specific aspect of our material is the ability for students to manipulate given geometric shapes, rather than cut them out or draw them themselves. Compared to other devices of this type, we have a wide variety of shapes (more than twenty) mixing regular and non regular figures. In addition, the Labosaïque pieces are magnetized, which facilitates the arrangement and manipulation and allows sharing on a board in front of the whole class. Having a wide variety of play sets allows us to modulate activities according to the different levels and quickly lead most students to ask questions and cooperate with each other.

The second specific aspect is the use of several types of mirrors, of different sizes, articulated, assembled to form mirror chambers. This set of mirrors allows us to approach axial symmetries, their compositions, but also multiplication and fractions, the notion of angle, the concept of infinite plane (see next section for a brief description of activities based on our material). The choice to work on symmetries with mirrors is not insignificant. In our device the different sizes of mirrors, the chambers, but also the presence of slots (in the square chamber) allow us to understand or at least perceive symmetries as a transformation of the whole plane.

Perhaps these two aspects are precisely the fundamental point of our approach: the wide variety of mirrors, parts and tools for visualizing symmetries allow us to show that the world of shapes and their symmetries is surprisingly rich.

We do not yet consider our workshops as educational material for school curricula (this work is a longer-term objective), but as activities to discover mirrors, shapes and the underlying mathematics, which teachers can then return to in class. Discussions with teachers and several training sessions motivated us to adapt our workshops for training (future) teachers in the primary school, with three objectives in relation with the proposition of Mathematics laboratories in (Torossian & Villani, 2018). The first one is to help future teachers providing objects and tools « realizing » different mathematical notions: symmetries, multiplication properties, fractions... (see next section for some descriptions). The second one is to show teachers several activities that they can return in class. Finally, we hope in this way to obtain a larger feedback on the impact and the pertinence of our devices in students performances and discover possible improvements through exchanges with teachers. An experience in this direction will start this year in collaboration with INSPE, the french institute for education and training of teachers.

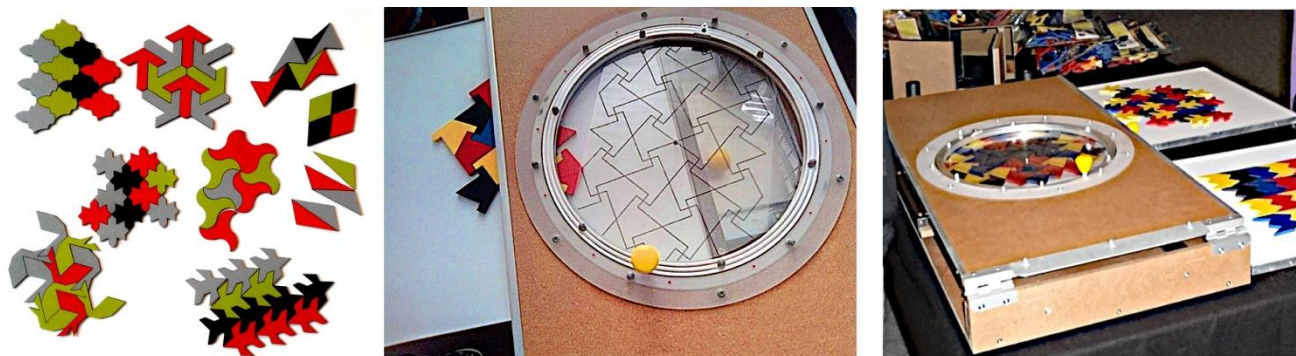
DETAILED DESCRIPTION OF THE EQUIPMENT AND ACTIVITIES WITH PUPILS

We give in this section a description of the material that makes up the Labosaïque, and examples of activities to be carried out with pupils. In practice, almost every one of our objects is used in several activities in different contexts and at different school levels.

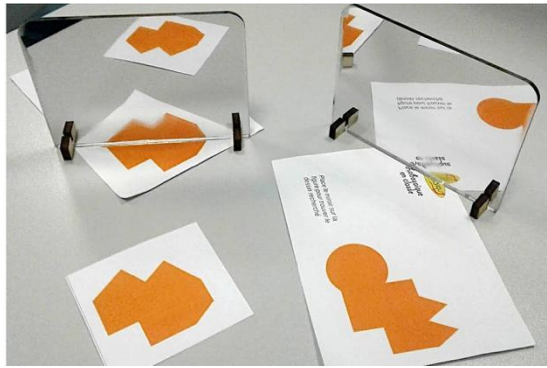
Very simple sets of puzzle pieces (triangles, quadrilaterals, regular polygons) allow to study which shapes may tile the plane. The most important here is not really to answer the question of tiling the plane or not with basis shapes. This investigation is rather a pretext to work with the pupils on definition and properties for the polygons concerned. For instance, if pupils suggest to use diamonds for the tiling, we will question them about what is a diamond? is a square a diamond? Those simple sets of pieces allow also to check, by arrangement or superposition, certain geometric properties such as the equality of angles or the notion of similarity.

A special set of one thousand pieces is devoted to Penrose tilings and several activities have been designed to allow people to explore surprising aspects of the so-called “quasi-periodic” tilings. At the primary school level, we use it essentially to construct tilings respecting given rules.

A thousand magnetized puzzle pieces permit to reproduce most of the 17 types of periodic tiling of the plane (some shapes allow to construct two or more different tilings). A turn-and-slide table is then used to visualize the effect of rotations and translations over the constructed tiling. At the primary school level this serves just as an introduction to the notion of plane transformations. The aim is that pupils can describe these basic transformations by usual words like “slide in this direction”, “make a third of a turn”. Free-standing mirrors are also used to identify mirror symmetries of a given tiling.

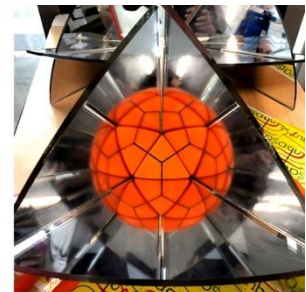


These small mirrors are also used in a shape research activity related to symmetry. Students are presented with a geometric drawing on which they must place a mirror to reveal another symmetrical, imposed shape. Used as a challenge between teams, this work is particularly suitable for working on oral communication between pupils.



An articulated mirror (two hinged vertical mirrors), allows to explore a surprisingly unintuitive notion: the link between the number of images and the mirror's angle. What about an odd sub-multiple of a full turn? The images of a left hand? The counting of objects placed between mirrors provides an aesthetic and playful pretext for discussions on algebraic concepts avoiding any difficult vocabulary (commutativity of multiplication, distributivity, factorization of integers) ; just look and explain! Different photos may be placed to visualize the “fundamental domain” of a rosette or the impossibility of reproducing certain “cyclic” drawings without mirror symmetry (e.g. some car wheel covers). A third vertical mirror on an independent stand can be added to make you imagine the passage of rosette motives to the tiling of the plane. The transition from finite to infinite is very rarely anticipated by primary (even high) school students, and is a source of wonder and thinking.

The triangular and square mirror chambers (open boxes of 3 or 4 vertical mirrors) allow to build infinite tilings by mirror symmetries. They are first used to visualize infinity: reproduce infinitely any object you put inside. This experiment also makes it possible to study the compositions of axial symmetries, each reflection corresponding to a composition of symmetries given by the mirrors constituting the walls of the chamber. Different activities developed with this material then allow us to work on the notion of symmetry. For example, a drawing (square or triangular) can be proposed for insertion in the room and pupils can be asked to understand what paving will be generated by this image. Conversely, students can be provided with a paving image and asked to understand which drawing to insert in the room to reproduce the paving.



Three types of “sphere tiling mirror chamber”, each constructed with three concurrent mirrors, allow people to view and understand spatial geometry by displaying regular object like polyhedra or spherical tilings. At the primary school level it is used to discover spatial reflections (for instance symmetries in the soccer ball).

More details can be found in (Bellingeri et al.,2018).

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