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EQUITY RISK PREMIUM AND TIME HORIZON: WHAT DO THE FRENCH SECULAR DATA SAY ?

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1 – Introduction

Investors in the stock market have different decision-making time horizons. They can be intra-day or daily arbitrageurs, individual non-professional portfolio managers, long-term institutional investors such as pension funds, etc. Over the last twenty years, heterogeneity models have been developed especially by distinguishing fundamentalists and chartists, the first one characterising the behaviour of long-term stockholders and the second one the behaviour of short-term stockholders.[1] Because many empirical studies have suggested that returns are somewhat forecastable[2], this implies that the efficient market hypothesis does not prevail and then that equity risk premia (ERP) are horizon-dependent, which of course does not prevent the market clearing condition leading to a single market price.[3] The analyses of term structure of (ERP) are rather recent since they have developed for only ten years[4] (Lemke and Werner (2009), Lettau and Wachter (2011), Binsberger et al. (2012), Binsbergen et al. (2013), Boguth et al. (2013), Muir (2013), Prat (2013), Croce et al. (2014), Damodaran (2015), Le Bris et al. (2019)). All these studies show that ERPs are both time-varying and horizon-dependent.

Surveys on ERP determination by Prat (2013), Damodaran (2015) and Duarte & Rosa (2015) have highlighted that, due to cognitive limits and informational costs[5], if the rational

[1] Among others, see Brock and Hommes (1998), Boswijk et al. (2007).

[2] For recent studies, see Rapach et al. (2016) and Harvey et al. (2016), the latter giving a very interesting overview of the literature about the degree of significance of variables that were found to be stock returns predictors. Paper by Golez and Koudijs (2018) use long run stock market indices data for Netherlands, UK and US and shows that predictability of returns holds for annual and multi-annual horizons works both in- and out-of-sample, hence providing evidence that expected returns - and consequently *ex-ante* ERP - are time-varying and horizon dependent.

[3] See Prat (2013), footnote (3).

[4] Apart from Prat (2001) who used survey data to measure 6- and 12 month ahead expected stock returns, and found that the corresponding ex-ante ERPs are horizon-dependant.

[5] For example, the "mental accounting" defined by Thaler (1999) as "cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities" is incompatible with REH since

expectation hypothesis (REH) does not hold, the risk premium modelling must respect three main features of this concept : its *ex-ante* character on which the decision-making is based, its time-varying character and its horizon-dependent character. In particular, Prat (2013) built a model where *ex-ante* risk premia are both time-varying and horizon-dependent. The author considers a representative investor whose wealth is made of a combination of an equity market portfolio and of a riskless asset, and who determine the weight of these two assets to maximize the expected utility of their future wealth for a given horizon. The solution of this program is that, for a given horizon, the required value of the premium at time *t* – that can be regarded as the required premium - equals the price of risk times the expected variance of returns. According to this modelling, the expected variance depends on the past values of variance while the price of risk is determined as an unobservable variable assessed using the Kalman filter methodology, which is assumed to capture the influence of hidden factors. In this framework and using annual US secular data from 1871 to 2008, the author proposes an *ex-ante* equity risk premium modelling for the one-year horizon (the so-called 'short-term' premium) and the infinite horizon (the so-called 'long-term' premium). Representing expected returns by mixing the three traditional extrapolative, adaptive and regressive processes, Prat (2013) evidenced large disparities in the dynamics of the two *observed* premia. Moreover, possibly due to risky arbitrage and transaction costs, an error correction model describes the adjustment of observed premia towards their required values. According to this approach, the question of equity risk premia *measurement* – which involves hypotheses about the representation of expected stock returns – and the question of their *explanation* - which involves the portfolio choice theory - are solved within the same econometric model. Overall, this modelling offered a valuable representation of the U.S. short-term and long-term premia over the period 1881-2008, and gives an explanation of the large disparities in their dynamics.

In line with Prat's approach, the present paper aims to modelling one-year horizon and infinite horizon ERPs for the French stock market using the secular data base established by Le Bris and Hautcoeur (2010). It is worth noting that France has experienced very strong shocks (war, inflation, nationalizations, political changes,…), so that the French stock market has had a much more turbulent history than the US stock market, which likely explains why

individuals make their decision in a piecemeal fashion, creating different categories for spending, each category corresponding to a separate mental account. The economically rational expectation theory introduced by Feige and Pearce (1976) is in accordance with this, since it states that the optimal amount of information used by an agent is such that the unit cost of information they face relative to their aversion to making forecast errors equals the marginal gain achieved by a decrease in the forecast error due to additional information, which suggests that it may be rational to do not expect rationally.

only a few authors have ventured to study the French market in the long run (Aburlu (1998), Friggit (2007), Le Bris (2018), Le Bris et al. (2019) for an individual equity). Apart from the recent contributions dealing with the term structure of ERPs mentioned above - and from contributions mentioned hereafter dealing with the effects of the term spread of interest rates and of the US stock market on French ERPs - the reader can refers to Prat (2013) both for a survey of the literature on equity premium and to position the approach proposed by the author to this literature. Accordingly, the rest of the paper is organized as follows. **Part 2** displays the theoretical framework allowing to express the one-year- and infinite horizon required ERP. **Part 3** presents auxiliary assumptions relative to the determination of expectations (returns and variances) and the prices of risk, while adding two tentative factors that are the term spread of interest rates and the US stock market effect. Using French secular data from 1872 to 2018, **Part 4** presents estimations of one-year- and infinite horizon ERPs using the Kalman filter methodology, and shows that, due to risky arbitrage and transaction costs, of observed premia tend to adjust gradually towards their required values. **Part 5** concludes that the secular French data tend to validate our two-horizon modelling, which, despite some differences, confirm those of Prat (2013) on US data.

2 - Theoretical framework

Let us consider a risk averse representative investor at time *t*, who holds their wealth W_t made by a combination of the risk-free asset and a replica of the French stock market portfolio (i.e. the risky asset). Their investment horizon is of duration *n*, and the proportions of the risk-free and risky assets held are θ_{nt}^o and θ_{nt}, respectively, with $\theta_{nt}^o + \theta_{nt} = 1$ and $-1 \leq \theta_{nt} \leq 1$.[6] Conditional to a given set of information Ω_t, the investor determines the value of θ_{nt} in order to maximise at time *t* the expected utility of their wealth in *t*+*n* (*n*>0), where *n* is of any duration (a day, a month, a year, etc.) At time *t*, the utility function is concave ($U'(W_t) > 0$ and $U''(W_t) < 0$) with an absolute risk aversion coefficient $\lambda_{nt} = \frac{-U''(W_t)}{U'(W_t)} > 0$.

Let r_{ot}^n be the *n* maturity risk-free rate and R_{t+n} the market portfolio stock return from *t* to

[6] The possibility of short selling may lead to a negative value of θ_{nt} (e.g. anticipating a reduction in the stock's price the investor decides today to sale equities that they do not own, knowing that they will have to deliver these equities at the maturity of the contact). The case $\theta_{nt} < 0$ would happen if the amount of short sales exceeded the market capitalization of shares held by the representative agent.

$t+n$; the *ex-ante* risk premium defined as the difference between risk-free rate and expected stock return is then $\Phi_{nt} = E[R_{t+n})\big|\Omega_t] - r_{ot}^n$. Put in expectation-variance form, the program of the investor leads to the following classical solution (see Prat (2013)) :

$$\Phi_{nt}^* = \gamma_{nt} V[R_{t+n}\big|\Omega_t] \qquad (1)$$

where Φ_{nt}^* is the required risk premium for the time horizon *n*, $E[R_{t+n})\big|\Omega_t]$ the expected stock return for the time span from *t* to *t+n*, $\gamma_{nt} = \lambda_{nt}\theta_{nt}$ the price of risk and $V[R_{t+n}\big|\Omega_t]$ the expected variance of stock returns at *t* for horizon *n*.

It is worth noting that relation (1) says nothing about the set of information Ω_t used by the investor to make expectations. If stock price contains all the relevant information (i.e. the price is expected rationally), then the efficient market hypothesis holds, so that the return is a white noise with constant mean and variance and then is unpredictable. In addition, if the price of risk is independent of the state of the nature, the expected return, the variance and thus the risk premium are constant, so that any attempt to model the expected time-varying return and risk premium is not a good challenge. On the other hand, if stock prices are not expected rationally, returns are predictable so that the expected return and risk premium are time-varying and horizon-dependant.[7] In this latter context, any empirical work must consider the *ex-ante* premium since the *ex-post* premium is not a straightforward decision-making concept, but this arises the question of the representation of expectations (see Part 3). Another important issue in relation (1) lies in the fact that it leaves unspecified the duration of the 'next period' corresponding to the time horizon *n* of the investor. As in Prat (2013), two traditional representative horizons are considered: the *one-year horizon* (since we consider annual data) that will be called the 'short-term' horizon (*n=1*) and the *infinite time horizon* that will be called the 'long-term' horizon ($n = \infty$). In the literature, these two horizons are analysed most often by convenience, but another reason must be considered here. Because this choice extends to the maximum the time span between the short- and long-term horizons, it allows to support the implicit assumption of independence between the behaviour of short-term and long-term investors, which is necessary to consider the determination of short-term

[7] See in particular Prat (2013), Appendix A.

and long-term premia separately. From now, subscripts 1 and 2 stand for the one-year and infinite time horizon, respectively.

The *ex-ante* risk premium characterizing the short-term investor's behaviour is defined by Eq.(1) for *n=1* :

$$\Phi_{1t} = E[R_{t+1} \mid \Omega_t] - r^1_{o,t} \qquad (2)$$

where $R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$ is the one-year stock return and $r^1_{o,t}$ the one-year-to-maturity risk-free rate, both expressed in percent per annum. The *ex-ante* risk premium characterizing the long-term investor's behaviour is deduced from the dividend discount model (DDM) with an infinite horizon. As a rule of thumb due to limited cognitive capacity and to information costs, stockholders are assumed to consider the infinite horizon as a whole and then make expectations uniformly between *t* and all the future successive periods, this heuristic being reconsidered at *t + 1, ... t+k , etc ...* Hence, supposing at any time *t* that the expected rate of growth in dividends and the actualization rate are uniform between *t* and all the successive periods *t+k*, we obtain the well-known "Gordon-Shapiro" stock valuation formula from which we can deduce the expression of the risk premium Φ_{2t} for a long-term investor (expressed in percent per annum):[8]

$$\Phi_{2t} = E[R^\infty_t \mid \Omega_t] - r^\infty_{ot} \qquad \text{with} \qquad E[R^\infty_t \mid \Omega_t] = \frac{D_t}{P_t} + \tilde{g}^\infty_t \qquad (3)$$

where $E[R^\infty_t \mid \Omega_t]$, $\tilde{g}^\infty_t$ and r^∞_{ot} stand for the long-term expected stock return, the long-term expected rate of growth in dividends and the long-term riskless rate of interest, respectively. Note that, according to (3), the implicit observed stock return for the long-term investor is $R^\infty_t = \frac{D_t}{P_t} + g_t$ with $g_t = \frac{D_t - D_{t-1}}{D_{t-1}}$.

While the short- and long-term premia Φ_{1t} and Φ_{2t} are *defined* by (2) and (3), these two premia are *explained* in accordance with the general relation (1). The market equilibrium condition is when risk premia offered by the market Φ_{1t} and Φ_{2t} equal risk premia Φ^*_{1t} and

[8] Jawadi and Prat (2016) compare this traditional formula with its so-called "dynamic version" proposed by Campbell and Shiller (1988) and explain why they used the first one rather than the second one.

Φ_{2t}^{*} required by investors. Accordingly, we can write the following market equilibrium conditions :

$$\Phi_{1t} = E[R_{t+1} \mid \Omega_t] - r_{o,t}^{1} = \gamma_{1t} V[R_{t+1} \mid \Omega_t] \qquad (4)$$

$$\Phi_{2t} = \frac{D_t}{P_t} + \tilde{g}_t^{\infty} - r_{ot}^{\infty} = \gamma_{2t} V[R_t^{\infty} \mid \Omega_t] \qquad (5)$$

3 - Expected returns, expected variances and prices of risk representations

To check empirically the validity of Eqs. (4) and (5), assumptions about the representation of the expected stock returns for the one-period horizon and the expected dividends growth rate for the infinite horizon are needed on the left-hand sides, while assumptions about the expected variances and the prices of risk are needed on the right-hand sides. It is worth noting that, according to our approach, *measurement* (on the left of Eqs.) and *explanation* (on the right of Eqs.) of risk premia are modelled simultaneously.

Expected stock returns

Many empirical studies suggest that returns are somewhat forecastable, which suggest that markets are not informationally efficient, and then that REH does not holds.[9] Otherwise, expected returns revealed by survey data conducted with experts show that mixing the three traditional extrapolative, adaptive and regressive processes can be a way to represent expectations.[10] Accordingly, as supposed by Prat (2013) for US data, we assume that expectations are determined by mixing these three traditional processes. For the one year horizon, the following expectation processes was found to be in accordance with our data :

$$E[R_{t+1} \mid \Omega_t] = R_{t,1}^{m} + a_{1,1}(\overline{R} - R_t) + a_{1,2}(R_t - R_{t-1}) + a_{1,3}(R_{t-1} - R_{t-2}) \quad a_{1,i} > 0 \qquad (6)$$

where $R_{t,1}^{m} = \beta_1 R_t + (1-\beta_1) R_{t-1}^{m}$ $(0 \le \beta_1 \le 1)$ is the adaptive component, $(\overline{R} - R_t)$ the mean-reverting component ($\overline{R} = 6.18\%$ per year is the average of R_t), while the two last terms are

[9] See Prat (2013), note (12).
[10] See especially Prat (1994) and Abou & Prat (2000).

extrapolative components. Considering the expected long term stock return $E[R_t^{\infty} \mid \Omega_t] = \frac{D_t}{P_t} + \tilde{g}_t^{\infty}$, its assessment needs the determination of $\tilde{g}_t^{\infty}$, which is given by

$$\tilde{g}_t^{\infty} = g_{t,\infty}^{m} + a_{2,1}(\bar{g} - g_t) + a_{2,2}(g_t - g_{t-1}) + a_{2,3}(g_{t-1} - g_{t-2}) \qquad a_{2,i} > 0 \qquad (7)$$

where $g_{t,\infty}^{m} = \beta_2\, g_t + (1-\beta_2)\, g_{t-1,\infty}^{m}$ $(0 \le \beta_2 \le 1)$ is the adaptive component, $(\bar{g} - g_t)$ the mean-reverting component ($\bar{g} = 2.17\%$ per year is the average of g_t), while the two last terms are extrapolative components. Note that, in Eqs. (6) and (7), the coefficients β_1 , β_2, $a_{1,i}$ and $a_{2,i}$ are determined simultaneously for the two horizons in the course of the estimation of the equilibrium conditions (4) and (5).

Expected variances

Insofar as only the variance of the unpredictable component of returns intervenes in determining the required risk premia, the expected variances of returns was estimated using GARCH processes. For each horizon, we considered alternately two measures of the stock return : (i) the observed returns, that are $R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$ and $R_t^{\infty} = \frac{D_t}{P_t} + g_t$ for the short and the long term horizons respectively, and (ii) the return considered as endogenous variables in the structural equations of the risk premia, that are $R_{t,1}^{m}$ and $R_{t,\infty}^{m} = \frac{D_t}{P_t} + g_{t,\infty}^{m}$ (see hereafter Eq. (12) and (16)). For both horizons, GARCH processes based on the return measure (ii) led to best results for our structural model, so that we retained it. These findings seem rather intuitive since in principle only the variance of the unpredictable component of returns intervenes in determining the risk premia because the expected variance of the predictable component is null. On the other hand, because $R_{t,1}^{m}$ and $R_{t,\infty}^{m}$ can be viewed as "memorized" values of returns, it can make sense that the expected volatilities are based on these values rather than on observed gross returns; as shown by Eqs. (6) and (7), $R_{t,1}^{m}$ and $R_{t,\infty}^{m}$ introduce a prior smoothing effect on returns depending on the optimal values of coefficients β_1 and β_2, which are determined in the course of the estimation of the structural model using a grid search.

By anticipating a bit the presentation of the results of our structural model, we found for the one year horizon that the expected variance $\tilde{V}_{1t} = V[R^m_{t,1} \mid \Omega_t]$ is a GARCH(1,1) process augmented by the lagged value of the squared inflation rate $\pi_t = 20 \log(CPI_t / CPI_{t-1})$ (% per year) [11]:

$$\tilde{V}_{1t} = \underset{(0.9)}{0.001} + \underset{(3.0)}{0.20}\, \hat{u}^2_{1t-1} + \underset{(16.6)}{0.77}\, \tilde{V}_{1t-1} + \underset{(3.3)}{0.02}\, \pi^2_{t-1} \qquad (8)$$

with the associated following mean equation obtained for optimal value $\beta_1 = 0.07$ deduced from the estimation of our structural model:

$$R^m_{t,1} = \underset{(16.1)}{0.99}\, R^m_{t-1,1} - \underset{(-3.0)}{0.20}\, R^m_{t-2,1} - \underset{(-2.5)}{0.16}(r^1_{o,t} - r^1_{o,t-1}) + \underset{(5.2)}{0.38}(r^L_{o,t-1} - r^1_{o,t-1}) + \underset{(3.3)}{0.62} + \hat{u}_{1t} \quad (\bar{R}^2 = 0.787)$$

where $r^1_{o,t}$ is the short term interest rate and $r^L_{o,t}$ the French government bonds yield .

For the infinite horizon, we found that $\tilde{V}_{2t} = V[R^m_{t,\infty} \mid \Omega_t]$ is represented by a GARCH-M(1,1) process, where[12]

$$\tilde{V}_{2t} = \underset{(1.4)}{0.18} + \underset{(2.8)}{0.30}\, \hat{u}^2_{2t-1} + \underset{(4.7)}{0.65}\, \tilde{V}_{1t-1} \qquad (9)$$

with the associated mean equation obtained for optimal value $\beta_2 = 0.03$ deduced from the estimation of our structural model (the intercept is removed since insignificant) :

$$R^m_{t,\infty} = \underset{(2.1)}{0.68}\, \tilde{V}_{2t} + \underset{(14.7)}{0.85}\, R^m_{t-1,\infty} + \underset{(3.7)}{0.36}(r^L_{o,t-1} - r^L_{o,t-2}) - \underset{(-3.06)}{0.14}\,(\pi_{5t-1} - \pi_{5t-2}) + \hat{u}_{2t} \quad (\bar{R}^2 = 0.70)$$

where $\pi_{5t} = 20 \log(CPI_t / CPI_{t-5})$ (% per year).

Prices of risk

The prices of risk for one-year and infinite time horizons are unobservable variables while the theory does not specify *a priori* their factors. To circumvent this problem, we implement a state-space model in which, for each horizon, a signal equation describes the risk premium while a stochastic state equation describes the unobservable price of risk. Accordingly, γ_{1t} and γ_{2t} are generated by state equations that are supposed to capture the

[11] The null of normality was rejected in testing the residual distribution in prior estimations and a general error distribution (GED) was then assumed. No asymmetry was found using EGARCH and TARCH variants of the conditional variance equation and no GARCH-in-mean effect was identified.

[12] The null of normality was still rejected in testing the residual distribution so that a general error distribution (GED) was assumed. No asymmetry was found using EGARCH and TARCH variants of the conditional variance equation but a GARCH-in-mean effect was significant. The intercept was not found to be significant in the conditional mean equation, where no residual autocorrelation was detected

influence on the price of risk of hidden factors comprising psychological effects. These two state equations are AR(1) processes, possibly augmented by a constant drift, by an AR(2) term and by macroeconomic variables (such as the rate of inflation and its volatility, change in interest rates), but none of them was found to be statistically significant. As a result, the prices of risk γ_{1t} and γ_{2t} are determined according to the following relations:

$$\gamma_{1t} = \rho_1 \gamma_{1t-1} + \eta_{1t} \qquad 0 \leq \rho_1 \leq 1 \qquad (10)$$

$$\gamma_{2t} = \rho_2 \gamma_{2t-1} + \eta_{2t} \qquad 0 \leq \rho_2 \leq 1 \qquad (11)$$

where errors η_{1t} and η_{2t} are Gaussian white noises. The time variability of γ_{1t} and γ_{2t} join to the conditional expected returns and variances are key features of our modelling that allow us to hope capturing both structural changes and strong shocks suffered by the French stock market during the secular period analysed.

4 - Empirical evidence

We used French annual secular data established from 1854 to 2007 by Le Bris and Hautcoeur (2010), which was completed the data until 2018. The authors reconstituted the CAC 40 stock price index P_t before its creation in 1988 (for each year, at the beginning of January), the dividends per share corresponding D_t for the last year (corresponding to the Le Bris-Hautcoeur's data at t+1).[13] Le Bris also established over this period time series of the short term interest rates r_{ot}^1 (money market rate, TMM "Taux Moyen du Marché Monétaire", Bank of France), of the long-term interest rate r_{ot}^L (French government bonds yield)[14], and of the Consumer Price Index CPI index that are used in the present paper. The historical dynamics of these variables are represented on **Figure A** for P_t and D_t and on **Figure B** for r_{ot}^1 and r_{ot}^L (see **Appendix 1**).[15]

[13] To avoid any survival bias, the components of the stock index as selected at the beginning of each year as the 40 main market capitalizations. The performances of those stocks is observed until the next January before to select the components of the new year.

[14] Annual average of r_{ot}^1 and r_{ot}^L of the preceding year are considered, which is consistent with the time span of stock returns and dividends growth rate.

[15] Note that, as for all other studies in the literature on ERP, the difference in taxation of stocks returns and debt securities yields is assumed to impact negligibly risk premia values.

Risk-free rates representations

The risk-free discount rate is represented by r_{ot}^{1} for the short term premium. Concerning the risk-free rate for the long-term premium, we followed Jawadi and Prat (2016), who considered the government bonds yield French government bonds yield r_{ot}^{L} with two assumptions. First, the Hodrick-Prescott filter trend $\bar{r}_{ot}^{L}$ of r_{ot}^{L} was considered. Indeed, although there is *a priori* no default risk in this guaranteed yield, consideration of trend allows to underplay the market risk component included in short-term fluctuations. It is thus not surprising that considering trend rather than the observed yield improved the empirical results. Second, we estimated the value of a constant liquidity premium in r_{ot}^{L} by regressing the term spread $TMS_t = r_{ot}^{L} - r_{ot}^{1}$ on actual and past values of change in the short rate r_{ot}^{1} (we found 4 significant lags that represent short interest rate expectations), where the intercept of 0.95% corresponds to the value of the premium. Accordingly, the risk-free discount rate representing the time preference of long term investors is assessed as $r_{ot}^{\infty} = \bar{r}_{ot}^{L} - 0.95$.

Adding the term spread and US stock market as tentative factors of required ERPs

Eqs. (4) and (5) give the theoretical values of the required equity risk premia when there are no exogenous disturbing effects. But such effects may exist due to liquidity constraints or/and to the contagion from international stock markets. That is why Eqs. (4) and (5) was completed using two tentative factors that are the term spread of interest rates and the US equity risk premia. Concerning the first one, studies in the literature had repeatedly shown during the 1980s that the slope of the term structure of interest rates is a reliable variable to predict ERP (Campbell (1987), Fama and French (1989))[16], and recent studies have confirmed this result (Goyal and Welch (2008), Rapach et al.(2016), Harvey et al. (2016), Jawadi and Prat (2017), Faria and Verona (2018a, 2018b)), the relation being positive. This helps to understand why the term spread (TMS_t) is continuously monitored by market participants while straightforward to know from publicly available data. The underlying supposed mechanism is that a decrease in TMS_t reflects an increase of in the liquidity constraints -

[16] Concerning the link between the term spread and equity risk premium through the APT, see among others Chen, Roll and Ross (1986), Elton, Gruber and Mei (1994)) and Kryzanowski et al. (1997).

especially due to a restrictive monetary policy - leading some stockholders to sales equities for compensating these constraints, ending with a fall in ERP because the induced fall in equity prices triggers an increased equity risk exposure. Accordingly, it seems a priori all the more relevant to add $TMS_t = r_{ot}^L - r_{ot}^1$ in the two structural equations of required ERPs since we have available data to do it. However, this variable was not found to be insignificant for the long term horizon, which joins Damodaran (2015) who found no significance when the ERP is deduced from the DDM.

Regarding now the second tentative variable, many observers have shown that the US market exerts a significant positive influence in mean and in volatility on other international stock markets and especially on European stock markets, whereas the reciprocal hypothesis is rather weakly and rarely supported (among other, see Bollerslev et al., Avouyi-Dovi and Neto (2004), Rapach et al. (2013), Siliverstovs (2016)). Such spillover effects may be due to pure contagion but also to structural links because a proportion of the market capitalization of French CAC 40 companies were held by non-resident investors.[17] To capture such influences in our modelling, we build a composite variable characterised by three specific features. First, because our endogenous variables in the structural equations of risk premia are in the form of $R_{t,1}^m - r_{ot}^1$ and $\frac{D_t}{P_t} + g_{t,\infty}^m - \bar{r}_{ot}^{\infty}$ for the one- and infinite horizons (see hereafter Eqs. (12) and (16)), we consider the same variables for US, that are ${}_{US}R_t^m - {}_{US}r_{ot}^1$ and $\frac{{}_{US}D_t}{{}_{US}P_t} + {}_{US}g_{t,\infty}^m - {}_{US}r_{ot}^{\infty}$, respectively.[18] Second, to compare US variables to the French ones, we correct the first ones for exchange rate between USD and Franc (euro since 2000). For that, based on the PPP, we subtracted from the US variables the Hodrick-Prescott trend of the inflation differential dif_t between France and US;[19] this somewhat improved empirical results compared to a no

[17] For example, in December 2009, 42% of the market capitalization of French CAC 40 companies was held by non-resident investors including 16% of US investors.

[18] These variables represent the risk premia under the hypothesis that only the adaptive behaviour would work in the expected return formation. US series are those of Shiller's data base : ${}_{US}P_t$ and ${}_{US}D_t$ stand for de Standard and Poors' 500 stock price Index at January and dividends per share over last 12 months, ${}_{US}r_{ot}^1$ for the one-year interest rate and ${}_{US}\bar{r}_{ot}^L$ the 10-year government bonds yield HP trend. The variables ${}_{US}R_{t,1}^m$ and ${}_{US}g_{t,\infty}^m$ are calculated with the optimal values $\beta_1 = 7\%$ and $\beta_2 = 3\%$ per year, respectively.

[19] According to the PPP, when the inflation differential between France and US increases, this means a devaluation of the French Franc (Euro from 2000). Accordingly, the US stock return related to USD becomes more interesting for French, hence compensating the magnitude of the US equity risk premium. Trend of the

correction specification. Third, it is rather unlikely that the influence of the US market exerts with the same intensity over the whole period. That is why we introduce a dummy variable dum_t set to 1 until a given year T (an alternative assumption was to set the dummy to 0 until T), then increasing according to a linear trend. The best results were obtained for setting dum_t to 1 until T=1950, which seems in accordance with the development of the internationalization of stock markets. The US variables then are $US_{1t} = dum_t\,({}_{US}R^m_{t,1} - {}_{US}r^1_{ot} - dif_t)$ and $US_{2t} = dum_t\,(\frac{{}_{US}D_t}{{}_{US}P_t} + {}_{US}g^m_{t,\infty} - {}_{US}r^\infty_{ot} - dif_t)$ for the one-year and infinite horizon, respectively, where $dum_t = 1$ until 1950 (including) and following a linear positive trend after[20], $dif_t = (\pi_t - {}_{US}\pi_t)_{HP}$ with $\pi_t = 100\log(CPI_t / CPI_{t-1})$ and ${}_{US}\pi_t = 100\log({}_{US}CPI_t / {}_{US}CPI_{t-1})$, CPI_t and ${}_{US}CPI_t$ Consumer Price Indices for France and US respectively, where the subscript *HP* stands for Hodrick-Prescott filter trend.

Reporting the expressions of the expected return (6), of the price of risk (8) in the structural relation of the short-term premium (4), adding the TMS_t, US_{1t} and a Gaussian noise υ_{1t} independent of the auxiliary residuals η_{1t} in the state equation, we get the signal equation (12) for the short-term premium, which is estimated with the state equation for given values of β_1 and of the initial value of $R^m_{t,1}$:

$$R^m_{t,1} - r^1_{ot} = \gamma_{1t}\,\tilde{V}_{1t} - a_{1,1}(\bar{R} - R_t) - a_{1,2}(R_t - R_{t-1}) - a_{1,3}(R_{t-1} - R_{t-2}) + \kappa_1 US_{1t} + \delta TMS_t + \upsilon_{1t} \qquad (12)$$
$$a_{1,i},\ \kappa_1, \delta > 0$$

where γ_{1t} is given by Eq. (10). Eq.(12) allows us to identify the values of short-term *"observed"* one-year premium $\Phi_{1t} = E[R_{t+1} \,|\, \Omega_t] - r^1_{ot}$ as

$$\Phi_{1t} = R^m_{t,1} + a_{1,1}(\bar{R} - R_t) + a_{1,2}(R_t - R_{t-1}) + a_{1,3}(R_{t-1} - R_{t-2}) - r^1_{ot} \qquad (13)$$

where

$$E[R_{t+1} \,|\, \Omega_t] = R^m_{t,1} + a_{1,1}(\bar{R} - R_t) + a_{1,2}(R_t - R_{t-1}) + a_{1,3}(R_{t-1} - R_{t-2}) \qquad (14)$$

---

inflation differential is considered to blur the impact of the strong French inflation that occurred during the second world war.

[20] A quadratic trend did not do better.

represents the one-year expected return. According to Eqs (10) and (12) the theoretical *required short-term premium* writes

$$\Phi_{1t}^{*} = \gamma_{1t}\,\widetilde{V}_{1t} + \delta\, TMS_t + \kappa_1\, US_{1t} \tag{15}$$

In a similar manner, reporting the expressions of the expected rate of dividends growth (7) and of the long-term price of risk (11) in the structural relation of the long-term premium (5), adding the US_{2t} factor in the structural relation of the long-term premium (5), and adding a Gaussian noise υ_{2t} independent of the auxiliary residuals η_{2t} in the state equation, we get the signal equation (16) for the long-term premium, which is estimated simultaneously with the state equation

$$\frac{D_t}{P_t} + g_{t,\infty}^{m} - \bar{r}_{ot}^{\infty} = \gamma_{2t}\,\widetilde{V}_{2t} - a_{2,1}(\bar{g} - g_t) - a_{2,2}(g_t - g_{t-1}) - a_{2,3}(g_{t-1} - g_{t-2}) + \kappa_{2t} US_{2t} + \upsilon_{2t} \tag{16}$$

$a_{2,i}$, $\kappa_2 > 0$

where γ_{2t} is determined by the state equation (11).

For given values of β_1 and of the initial value of $g_{t,\infty}^{m}$, Eqs. (16) and (11) allow us to identify the value of the *infinite horizon "observed" premium*

$$\Phi_{2t} = \frac{D_t}{P_t} + g_{t,\infty}^{m} + a_{2,1}(\bar{g} - g_t) + a_{2,2}(g_t - g_{t-1}) + a_{2,3}(g_{t-1} - g_{t-2}) - \bar{r}_{ot}^{\infty} \tag{17}$$

where

$$\widetilde{g}_{t,\infty} = g_{t,\infty}^{m} + a_{21}(\bar{g} - g_t) + a_{2,2}(g_t - g_{t-1}) + a_{2,3}(g_{t-1} - g_{t-2}) \tag{18}$$

gives the long-term expected rate of dividends growth. Equations (16) and (11) allow us to determine the theoretical *required long-term premium* as

$$\Phi_{2t}^{*} = \gamma_{2t}\,\widetilde{V}_{2t} + \kappa_{2t} US_{2t} \tag{19}$$

For given values of the pair $[\beta_1, \beta_2]$ and of initial values of $R_{t,1}^{m}$ and $g_{t,\infty}^{m}$, the 4-equations-system (12), (10), (16) and (11) is estimated using the Kalman filter methodology, where (12) and (16) are the two *signal equations*, while (10) and (11) are the two *state equations*. All parameters (including initial values of state variables) are those minimizing the

Akaike, Schwarz and Hannan-Quinn information criteria. The above 4-equation system was estimated over the period 1872-2018 (147 years) using the maximum likelihood method (Harvey [1992], Hamilton [1994]). The initial values of the state variables γ_{1t} and γ_{2t} are determined using a grid search to minimize the information criteria (Akaike (AIC), Schwarz (SC) and Hannan-Quinn (HQ)).[21] The successive values of the state variables are revised each year on the basis of new information assumed to reflect a positive, negative or null variation in the price of risk. This new information takes the form of additive normally distributed white noises, whose variances belong to the vector of the hyperparameters to be estimated. Since the effects of these informational shocks are supposed to cumulate gradually over time, the dynamics of these two state variables are characterized by AR(1) processes with drift. As it is frequent with nonlinear models, the Kalman filter gives often rise to problems of convergence especially in estimating the variances of the residuals ("innovations") of the signal equations and the variances of the residuals ("noises") of the state equations. It is possible to minimize this difficulty by initializing these variances with sufficiently high values (Stock and Watson, 1998; Durbin and Koopman, 2001), as we done. When such problems persist, as was the case here, econometricians solve it by setting the value of the ratio between innovations and noises named "Signal-to-Noise Ratio".[22] This constraint was used in our estimations (see Table 1). Finally, because no significant correlation was found between innovations υ_{1t} and υ_{2t}, it was not relevant to estimate their covariance among the hyperparameters to avoid estimation bias that could have been induced.[23] **Table 1** gives the Kalman filter estimates of our structural model made of a system composed of two signal equations (12) and (16) and two state equations (10) and (11). **Table 2** gives descriptive statistics of the two "observed" premia issued from equation (13) for the one-year premium and equation (17) for the infinite horizon premium, with estimates in Table 1. Expected returns that intervene in short- and long term ERPs are issued from Eqs. (6) and (7), with

[21] Estimates were made using Eviews10. Since we considered a structural model, "smoothed" state variables are considered.

[22] Note that, analysing unemployment, Gordon (1997) advised to use an "aesthetic criterion", which refers to the time pattern of an estimated state variable : in his analysis, it made sense that the equilibrium unemployment rate must not be constant or erratic but smoothed compared to the observed rate. It is indeed possible to obtain very different dynamics of the state variable with rather close values for the information criteria. When a system is estimated with several signal and state equations, it also can happen that estimates focus on one of the equation at the expense of the other, leading to a quality of fit exaggeratedly good for ones while exaggeratedly bad for others.

[23] The covariance between the two noises η_{1t} and η_{2t} was also found to be insignificantly different from zero. We also verified the absence of correlation between the signal residuals and the state residuals for each horizon, which is a condition for updating state equations.

optimal values of $\beta_1 = 7\%$ and $\beta_2 = 3\%$, respectively.[24] These coefficients show a relatively long delay of influence for the two horizons, the longer delay found for the long-run horizon being somewhat intuitive. The short-tem premium appears to be higher in mean than the long-term one, which correlates with the fact that short-term returns are more volatile than long-term returns. It can be noted that the rather low average values of the two ERPs seem not subject to the famous "equity risk premium puzzle" arising from the Lucas' consumption-based model using US data (Mehra and Prescott (1985)), which confirms that this puzzle was not clearly evidenced in France.

Table 1 - Estimating the one-year and infinite time horizon equity risk premia using the Kalman filter methodology

	one-year time horizon (τ = 1)	**infinite time horizon (τ = 2)**
	State equations (10) and (11)	
ρ_τ	0.983*** (78.9)	0.980*** (65.6)
c_τ	-1.87*** (-12.1)	-2.10*** (-18.6)
	Signal equations (12) and (16)	
$a_{\tau,1}$	0.14*** (7.2)	0.07*** (6.7)
$a_{\tau,2}$	0.08*** (6.0)	0.04*** (3.8)
$a_{\tau,3}$	0.03*** (3.4)	0.02*** (2.9)
$\delta_\tau\ (TMS_t)$	1.56*** (12.0)	-
$\kappa_\tau\ (US_{\tau t})$	0.01*** (5.1)	0.01*** (5.0)
Signal-to-noise	9.05	3.95

[24] Because the likelihood was flat around the values of the adaptive coefficients β_1 and β_2 obtained by Prat (2013) for US, we set the same values in France as those found by the author for the USA. The initial values of the adaptive components of expected return (Eq. (18)) and of the expected growth rate in dividends (Eq.(22)) are 4.79% and 1.77% per year, respectively.

ratio

R^2	0.950		0.968
R_D^2	0.796		0.836
JB p-value	0.52		0.01
LM p-value	0.00		0.01
AIC		6.88	
SC		7.15	
HQC		6.99	

Notes - Estimation cover the period 1872-2018 (147 years). The two signal equations (12) and (16) and the two state equations (10) and (11) are estimated as a system of four equations by using the Kalman filter methodology. Estimates are obtained for $\beta_1 = 7\%$ and $\beta_2 = 3\%$. To ensure positive values, the variances of noises $\eta_{\tau t}$ (i=1,2) are estimated as $\exp(c_\tau)$, while the variances of $\upsilon_{\tau t}$ (i=1,2) are determined by fixing the Signal-to-Noise ratios. AIC, SC and HQC stand for Akaike, Schwarz and Hannan and Quinn information criteria for the estimated system. The initial values of $\gamma_{\tau t}$ have been optimized as 3 both for $\tau = 1$ and $\tau = 2$. Numbers in brackets are the *t*-values. ***, ** and * indicate that estimates are significant at the 1%, 5% or 10% levels, respectively. R^2 and R_D^2 are two measures of the goodness of fit of the signal equations (see footnote 31). *JB* and *LM* stand for the Jarque-Bera normality test and the Breusch Godfrey serial correlation LM test (4 lags), respectively.

Table 2 – Short-term and long-term observed risk premia : descriptive statistics

	Mean Median	Maximum Minimum	Std deviation	% of positive values
One-year horizon premium (τ = 1)	1.74 2.62	8.34 -11.7	4.10	68%
Infinite horizon premium (τ = 2)	1.37 1.14	6.87 -5.34	2.64	73%

Note - The two risk premia are expressed in percent per year. The sample period is 1872-2018 (147 observations).

Figures 1 and 2 exhibit the dynamics of the components in the observed premia for the one-year (expected stock return and risk-free rate) and for the infinite time horizon (dividends yield, expected dividends growth, risk-free rate), respectively. It can be seen that, in both cases, the components exhibit significant fluctuations compared to each other. **Figure 3** compares the values of the "observed" premia according to horizons (Eqs. (13) and (17)), hence confirming that horizon is a discriminant parameter conditioning the dynamics of ERP. At a first glance, an equity risk premium should be expected to be positive. However, as in all empirical studies of the literature using data over a long period, Table 2 indicates that,

although the average values of the premia are positive, negative values often occur (between a quarter and a third of observations). In fact, notwithstanding errors measurement, a negative risk premium can make sense from several ways. Considering the security market line of the CAPM, when the beta of an individual share is negative, a rational investor will hold it even though it yields less than the risk-free rate, since it makes a "recession insurance" in their diversified portfolio.[25] According to the consumption-based model of Lucas (1978) from which the CCAPM is deduced, a negative risk premium may also occurs for a well-diversified stock portfolio if returns are negatively correlated with the aggregate consumption growth, which may encourage risk-adverse investors to pay a premium to hedge the consumption risk.[26] More generally, this could also be the case if, due imperfect correlations with other assets, held the well-diversified stock portfolio reduces the risk of the whole wealth of the investor. In an international portfolios context, because the currency risk premium can be positive or negative, this may lead to negative equity risk premia for any national stock market (among others, see Arouri et al.(2012)). Considering our modelling, required risk premia depend formally on the product of price of risk by the expected variance and on exogenous disturbing effects represented by the tentative variables US_{1t} and US_{2t} for short and long term horizons respectively, and by TMS_t for the short horizon. About that, the negative values of TMS_t over the period are found to be weakly in line with those of the short-term ERP. Contrariwise, the negative and lasting values of the short premium during the sub-period 1967-90 - and to a lesser extent of the long-term premium - are found to be in line with the negative values of US_{1t} and US_{2t}, hence suggesting international factors. Concerning the product of the price of risk by the expected variance, our results show that the prices of risk γ_{1t} and γ_{2t} that are estimated by state variables can take negative values that are rather in line with the observed risk premia, and this will be discussed hereafter. Anyway, it may be reassuring to see that, during the 2000's, the values issued from our model are close to those estimated by circles of financial experts. Indeed, over the sub-period 2000-2018 in France, the values of the equity risk premium from Fenebris' (Frankfurt) oscillate around an

[25] In the same vein, because put options on a stock price index (e.g. S&P500) can hedge systematic risk when the market portfolio is held, they carry a negative risk premium (i.e. their expected returns are below the risk-free rate). This means that stockholders pay a positive insurance premium to get this hedging advantage. Note that negative short term interest rate at the end of our period also illustrate the fact that agents can accept to pay to secure their funds (see Appendix 1, Figure A).

[26] Moreover, in the CCAPM framework, Liu et al (2016) suggest that ERP is determined both by this traditional consumption risk and by a "liquidity risk" they define as the covariance between transaction costs and consumption growth. Consequently, if for any reason at time *t* this covariance takes negative value, this "liquidity risk" would also contribute in explaining a negative ERP.

average level of about 5%[27] while we find 6.5% and 4.6% for the short and long term premia, respectively. In the same way, Statista provides an estimation averaging of 6% over the sub-period 2011-18 while we find 5.98% and 6.08% for the short and long premia, respectively.[28]

Figure 1 - the two components of the one-year equity risk premium, 1872-2018

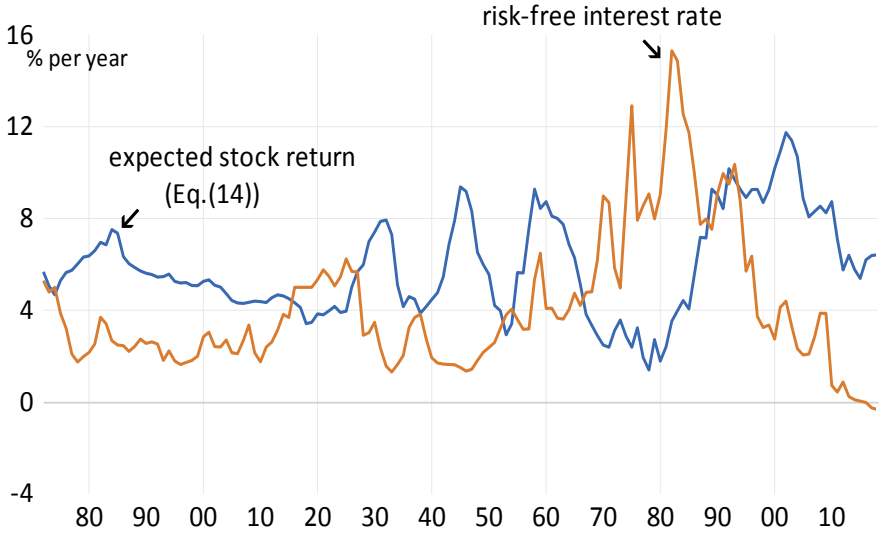
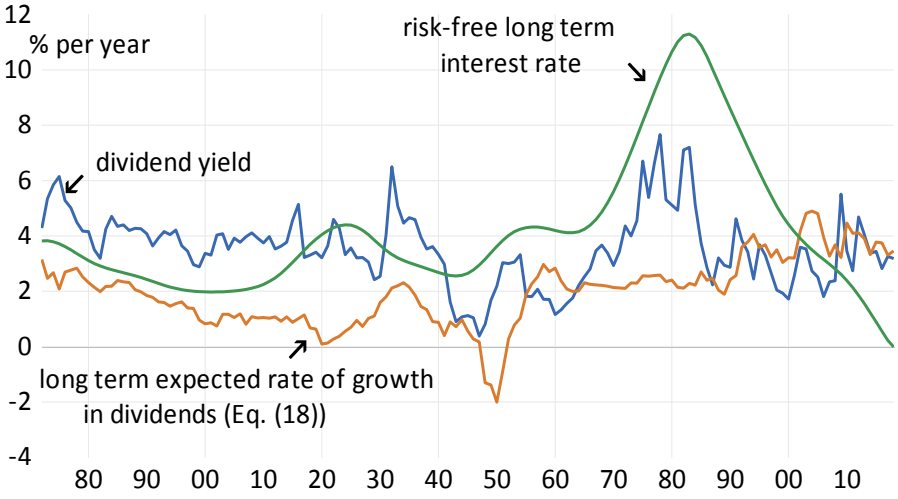


Figure 2 - The three components of the infinite horizon equity risk premium, 1872-2018



[27] See http://www.market-risk-premia.com/ fr.html
[28] See https://www.statista.com/statistics/664857/average-market-risk-premium-france-europe/

Figure 3 - The one year horizon and the infinite horizon equity risk premia, 1872-2018

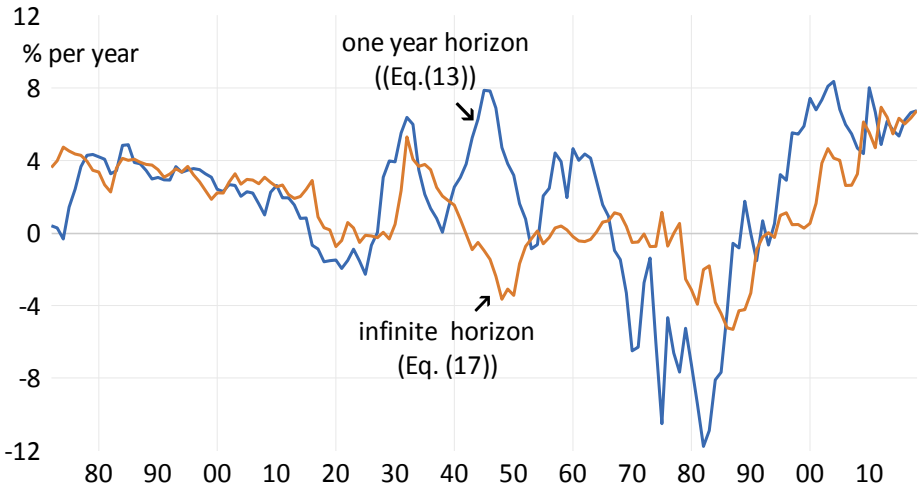
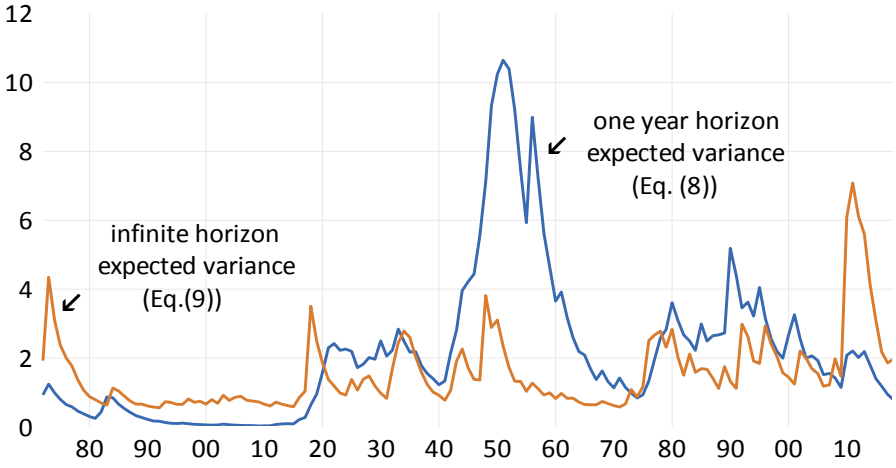


Figure 4 - Short term and long term expected variances of stock returns, 1872-2018



We now turn to the expected variances and the prices of risk that are the main arguments of the required premia. **Figure 4** shows that the short term and long term expected variances exhibit different dynamics ($R^2 = 0.06$), the first one being more volatile than the second one.[29] Concerning the prices of risk, since we consider a structural model, the state variables are estimated conditionally at each point in time on the whole sample data (*smoothed inference*) rather than only using past observations (*predicted inference*). At any time, the Kalman filter yields the standard deviations of each state variable. For an accepted

[29] Considering the short term horizon expected variance, the sharp rise that occurred between 1915 and 1951 and the strong subsequent decline are in line with the time pattern of the 10-year rolling variance of stock returns estimated by Le Bris (2018). However, probably because a GARCH process only considers the unpredictable component of returns to determine the expected variance of returns, it does not produce persistence of the expected variance level found by Le Bris.

5% level of significance, the standard deviation allows us to determine confidence intervals defined at each date by the line ranging between the estimated value plus or minus 1.96 times the standard deviation, which leads to associate upper and lower bounds to the estimated values of the price of risk γ_{1t} and γ_{2t}, as shown by **Figures 5 and 6**, respectively. Prices of risk are significantly time-varying since a horizontal line can not be located inside the confidence intervals, while **Figure 7** shows that γ_{1t} and γ_{2t} are clearly correlated ($R^2 = 0.59$) although they exhibit large own fluctuations. Observed risk premia Φ_{1t} and Φ_{2t} are of course clearly correlated with prices of risk γ_{1t} and γ_{2t}, but much less than with estimated required premia : we found $R^2(\Phi_{1t}, \gamma_{1t}) = 0.33$ compared to $R^2(\Phi_{1t}, \hat{\Phi}^*_{1t}) = 0.94$, and $R^2(\Phi_{2t}, \gamma_{2t}) = 0.38$ compared to $R^2(\Phi_{2t}, \hat{\Phi}^*_{2t}) = 0.96$, which shows that the determinist factors in the model play a crucial role in the determination of risk premia that go beyond the stochastic factors (i.e. the state variables).

As like for risk premia, although the estimated values of the prices of risk γ_{1t} and γ_{2t} are mostly positive with positive averages over the whole period, negative values are numerous, notably since 1965 and 1980 for the short- and the long term prices of risk, respectively. This contrasts with the results in Prat (2013) according to which few negative values were found for the US stock market (about 5% of observations). Considering the upper bound values, we find 24% of negative values for γ_{1t} while this percentage drops to 12% for γ_{2t}, which confirms the existence of a significant number of negative values. The presence of negative values of prices of risk was persistent with our modelling by using different specifications for expected returns and expected variances, with or without the two tentative added variables in required premia equations, and whatever the values of signal-to-noise ratios or initial values of state variables. As part of our approach, this suggests a "price of risk puzzle" for the French stock market, which evidences a limit of our modelling to identify and separate different phenomena. Recall that price of risk γ_{nt} (n =1,2) is the product of the share θ_{nt} of stock market portfolio held by the representative investor in their wealth by the risk aversion coefficient λ_{nt}, where θ_{nt} can take a negative value if short sales

exceed the amount of equities held, hence leading to a negative values of γ_{nt}.[30] But existence of short sales as an explanation of the negative values is far from to be a valuable explanation; to give an example, in 2004, the amount of open positions on the CAC 40 Index Future was roughly only 2% of the CAC 40 market capitalization. In fact, although at the theoretical level the price of risk is formally identified to the product $\lambda_{nt}\theta_{nt}$, the state variable used to represent this product is likely to capture also other unidentified effects. Generally speaking, for a variety of motives that do not directly depend on stock returns, stockholders may accept to pay services attached to equities. In the simplest way, γ_{nt} could represent $(\lambda_{nt}-\alpha_{nt})\theta_{nt}$ rather than $\lambda_{nt}\theta_{nt}$, where $\alpha_{nt}>0$ is a preference parameter representing advantages of shares holding. In this context, when at time *t* stockholders believe that advantages overtake risk aversion (i.e. $\lambda_{nt}<\alpha_{nt}$), the state variable takes a negative value. This suggests that our "price of risk" must be viewed as net of all unquantifiable advantages related to the share holding. For example, voting rights attached to the shares also may motivate stockholders to pay for preserving their right to participate in the control of companies. Also, stock holding can constitute a comparative advantage when risk of loss in real capital is felt to be lower in the stock market than in other secondary markets, so that investors accept to pay for keeping equities in their portfolios. Otherwise, although in times of high and widespread euphoria in the stock market, we could understand that the price of risk take negative values - since many investors may join to risk seeking speculators – it is rather unconvincing the case during most of the years in which negative values appeared during our period, even though an euphoria could be born and spread during the sub-period 1983-2000 : the CAC 40 index was multiplied by 13 while the NASDAQ was multiplied by 10, which undoubtedly encouraged the purchase of shares. However, the Rank-Dependent Expected Utility model proposed by Quiggin (1982) and the subsequent Cumulative Prospect Theory by Tversky and Kahneman (1992) can help to understand such negative values. Supposing that rationality is based both on a function describing preferences vis-à-vis risk and a function describing individuals' beliefs about the state of the market, these approaches aim to answer some criticisms addressed to the expected utility model of decision under risk (in particular the experimental Allais' paradox). More

[30] See footnote (6). On this point, note that, until the end of the twentieth century, most of transactions in Paris Stock Exchange were at the monthly settlement, i.e. settlement-delivery was at the end of the month. This settlement-delivery could be carried forward from one month to another at a rate varying according to the equities and dates (on this point, the French market was different from the US market where spot transactions dominated), and this can foster short sales. Note that the CAC 40 Index Future (FCE) created in the late 1980s, which is today one of the most traded contracts on the French market, allowed investors to make easily short selling of the market portfolio replica.

diversified behaviours are considered since probabilities of prospects are replaced by decision weights that are not necessarily equal to their respective probabilities, and this implies that the prospect value function is not always concave. As a result, these types of modelling show that the price of risk can become zero or negative in the upward phases of the cycle without the assumption that most investors are risk-preference, and may be our state variables capture such effects. About that, France has experienced several periods of recovery since the end of the 1970's: 1975-80, 1984-88, 1993-2000, 2010-11; one may also observe that interest rates have strongly and steadily declined since the early 1980s, from values above 15% to near zero at the end of our period (see Appendix 1, Figure B), which could anchor the belief in an upcoming recovery since that date. Anyway, the strong time variability of the prices of risk contributes to understand why it is not necessary to consider explicitly structural changes or shocks such as word wars in our modelling over the secular period. On the other hand, a brief preliminary analysis in **Appendix 2** shows that, for both horizons, although a decrease of deflation or an increase of inflation firstly lead to a lowering of prices of risk (this is the case for a large majority of the observations), prices of risk and inflation tend to raise together when inflation reaches a high threshold. These results seem rather intuitive and join a literature suggesting the existence of an optimal rate of inflation for the stock market (Lintner (1973), Prat (1982, chap. VII))), in the sense that a moderate inflation would be profitable for companies and then for the stock market, but that too much inflation would be harmful. Anyway, these results suggest that a more in-depth economic analysis of the price of risk should be conducted later.

Figure 5 - Price of risk for the one year horizon, 1872-2018

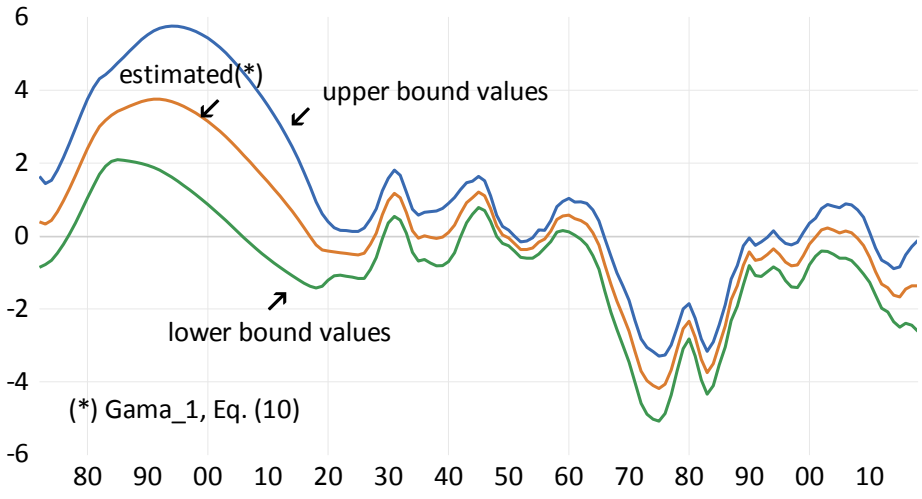


Figure 6 - Price of risk for the infinite horizon, 1872-2018

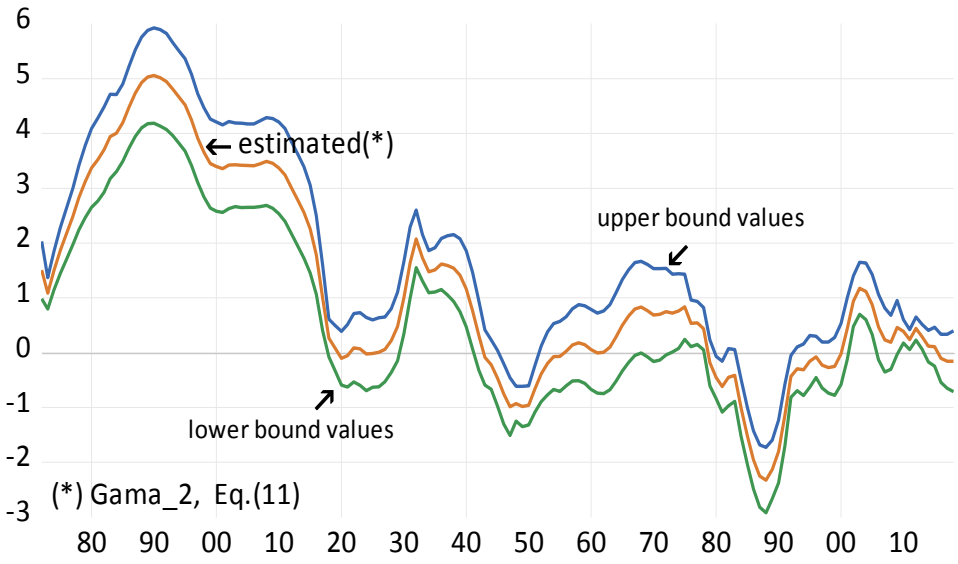
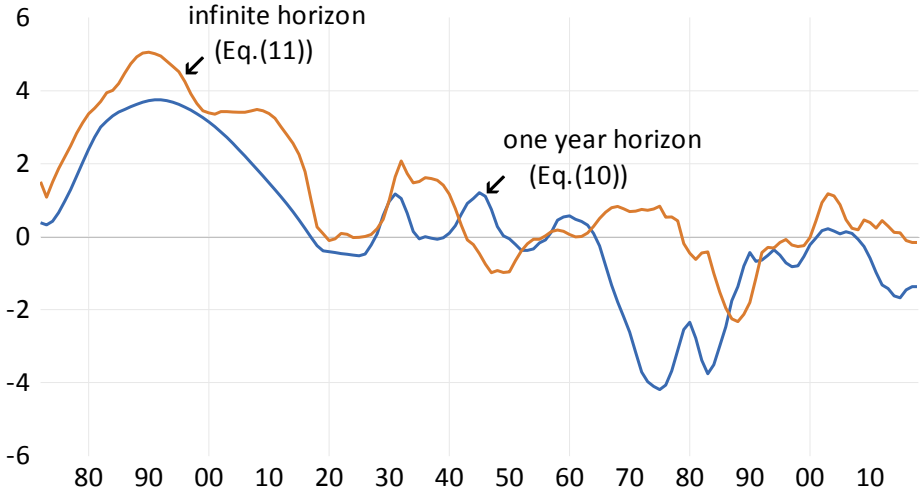


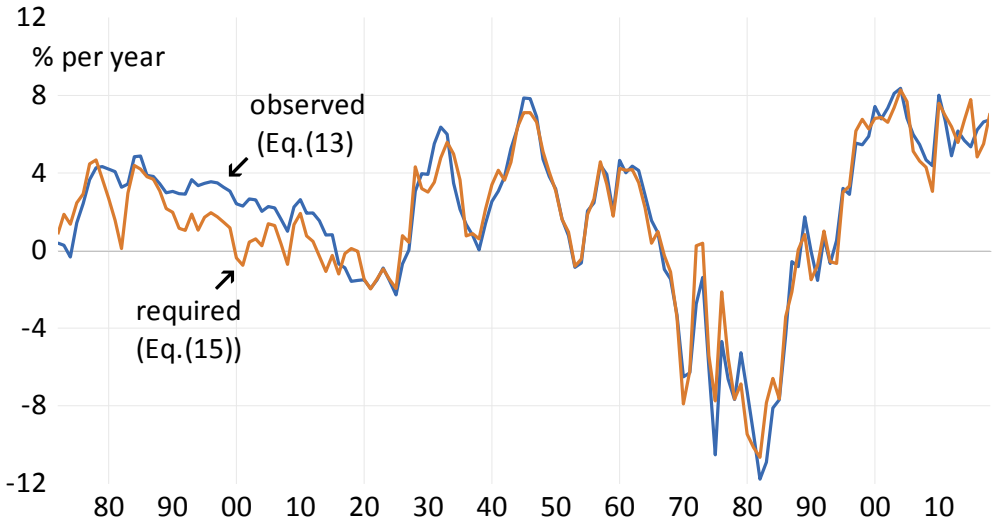
Figure 7 - Prices of risk for the one year and the infinite horizons, 1872-2018



The conventional R^2 between the observed and fitted values given by the signals equations (12) and (16) (Table 1) are high both for the short- and the long term horizons (0.95 and 0.97, respectively). However, especially when using the Kalman filter, it is suitable to check the quality of the fits by using the modified measure R_D^2 proposed by Harvey (1992), which assesses the relevance of our model with respect to a benchmark represented by a

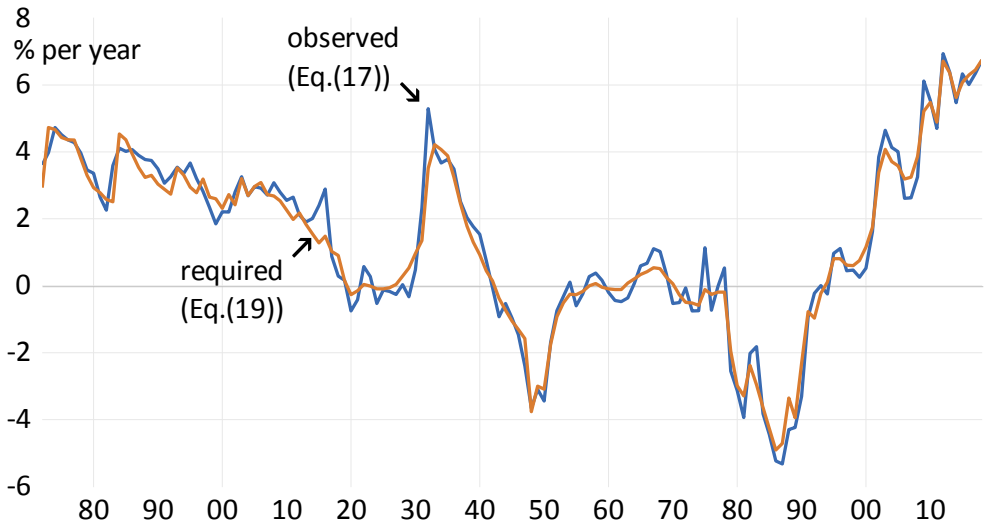
random walk plus drift.[31] The R_D^2 values found are 0.79 and 0.84 for the short- and the long term horizons respectively, indicating that our risk premia model strongly outperforms the benchmark. **Figures 8** and **9** compare the observed values Φ_{1t} and Φ_{2t} (Eqs. (13) and (17) with the estimated required values $\hat{\Phi}_{1t}^{*}$ and $\hat{\Phi}_{2t}^{*}$ of ERPs (Eqs. (15) and (19)) deduced from the kalman filter estimates (Eqs. (12) and (16)), for the short- and long-term horizons, respectively: although the required values describe the main fluctuations of the short-term and long-term premia, we can often see sharp differences (corresponding to estimated values of residuals υ_{1t} and υ_{2t}).

Figure 8 - Observed and required values of the one year equity premium 1872-2018



[31] Whereas $R^2 = 1 - SSR / \sum_{t=1}^{T} (y_t - \bar{y})^2$, we have $R_D^2 = 1 - SSR / \sum_{t=2}^{T} (\Delta y_t - \overline{\Delta y})^2$ where $y_t = \Phi_{\tau t}$ $(\tau = 1, 2)$ while *SSR* is the sum of the squared residuals υ_{1t} and υ_{2t}. A negative R_D^2 implies that the estimated model is less precise than a simple random walk plus drift.

Figure 9 - Observed and required values of the infinite horizon equity risk premium 1872-2018



We now turn to the statistical properties of the standardized estimated residuals of the signal equations υ_{1t} and υ_{2t}. In Table 1, the *p-values* of the Jarque-Bera statistics (*JB*) lead to accept the normal distribution hypothesis at the 5% level and the 1% level for the short- and long term horizons, respectively[32], but *F*-statistic *p-values* of the *LM* test show that the null of no serial correlation is rejected for the two horizons at the 5% level (the null is nevertheless accepted at the 1% level for the long term premium).[33] This suggests a possible lack in the specification of the required premia modelling, especially about the role played by inflation. Indeed, if the representative investor had no monetary illusion, they would maximize the expected utility of their real wealth. In this case, according to Eqs.(2) and (3), risk premia definitions remain unchanged since the expected rate of inflation must be subtracted from both the expected stock return and the riskless rate. However, the expected variances of returns in the structural equations (3) and (4) must be replaced by that of real returns, which equals the expected variances of returns plus the expected variance of inflation less twice the expected covariance between returns and inflation. To check for this conjecture, we analysed for each horizon the correlation between, on the one hand, the estimated values of residuals υ_{1t} or υ_{2t}, and on the other hand, actual and lagged inflation rate variances (squared inflation rates) as proxy of expected variance of inflation (10 lags) and the moving covariance over the last *k* years between observed stock returns and inflation rate (*k* = 1 to 10) as proxy of the

[32] These results show that there is not a significant number of abnormal values.

[33] Autocorrelation of residuals does not cause bias on the estimated values of parameters but the variances of parameters are undervalued. However, our finding that the *p-values* are substantially less than 1% for all parameters suggests that autocorrelation likely does not disturb conclusions about the significance of variables.

expected covariance between inflation and stock returns. For our two horizons, these attempts showed that residuals are neither correlated with variance of inflation nor with covariance between inflation and stock returns[34]; one can also add that no correlation was found between residuals and actual and lagged inflation rates themselves. As a result, these outcomes suggest that our required premia modelling does not suffer from a lack of the specification due to the role of inflation.[35] Another conjecture to explain autocorrelation of residuals is that market frictions such as transaction costs[36] and risky arbitrage[37] generate delayed adjustments of premia towards their required values. To check for this, we estimate error correction models (ECMs), where the observed premia Φ_{1t} and Φ_{2t} are given by equations (13) and (17), while targets are the required premia Φ^*_{1t} and Φ^*_{2t} estimated by equations (15) and (19), respectively. The two ECMs are estimated simultaneously as a system using the *Seemingly Unrelated Regression* method, which is robust both to the contemporaneous correlation between the two residuals and for heteroskedasticity. The results are the following :

$$\Delta\Phi_{1t} = \underset{(5.5)}{0.31}\,(\hat{\Phi}^*_{1t-1} - \Phi_{t-1}) + \underset{(26.5)}{0.85}\,\Delta\hat{\Phi}^*_{1t} + \underset{(1.9)}{0.06} + \Delta\Phi_{1t-1} + \hat{\varepsilon}_{1t} \qquad \overline{R}^2 = 0.83 \qquad (20)$$

$$\Delta\Phi_{2t} = \underset{(10.4)}{0.86}\,(\hat{\Phi}^*_{2t-1} - \Phi_{2t-1}) + \underset{(15.9)}{0.95}\,\Delta\hat{\Phi}^*_{2t} + \underset{(3.3)}{0.15}\,\Delta\Phi_{2t-1} + \hat{\varepsilon}_{2t} \qquad \overline{R}^2 = 0.72 \qquad (21)$$

A system residuals Portmanteau test for autocorrelations of $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$ leads now to accept the null of non autocorrelation at the 5% level of significance.[38] These results suggest that equity risk premia gradually adjust towards their required values, which indirectly supports the suitability of the required values estimations.

5 – Conclusion

In line with Prat's approach applied to U.S. stock market secular data and using the secular data base established by Lebris and Hautcoeur (2010), the present paper aims to modelling one-year horizon and infinite horizon ERPs for the French stock market. We consider a representative investor whose wealth is made of a replica of the equity market

[34] For each horizon, this last result is reinforced by the null value found for the unconditional covariance between stock return and inflation over the whole period.
[35] As suggested in Appendix 2, the influence of inflation is captured in our modelling by the price of risk (and also by the expected variance for the short horizon, see Eq.(8))).
[36] See Anderson (1997).
[37] See Shleifer and Summers (1990).
[38] The Q-Stat (2 lags) *p-value* found is 0.26 (this test is valid only for lags larger than the system lag order, that is 1).

portfolio and of the riskless asset, and who maximizes the expected utility of their future wealth for a given horizon. The solution of this program implies that the required risk premium equals to the price of risk times the expected variance of stock returns, which both are time-varying and horizon-dependent. Two traditional horizons are considered: the one-period-ahead horizon (i.e. the 'short-term' premium) and the infinite-time horizon (i.e. the 'long-term' premium). Representing the expected returns by mixing the three traditional adaptive, extrapolative and regressive processes, large disparities in the dynamics of the two premia are evidenced from 1872 to 2018. Considering the required premia side, the expected variances are represented by GARCH processes while the prices of risk are estimated according to the Kalman filter methodology, which allows representing unobservable variables. The term spread of interest rates and US ERPs appeared to be relevant additional factors of required premia. Due to risky arbitrage and transaction costs, the observed premia are found to gradually converge towards their required values, this process being described by an error correction model.

This model offers a valuable representation of French short- and long-term ERPs over a long period having experienced very strong historical shocks, which shed some additional light on the existence of a time-varying term structure of ERPs. In line with Prat (2013), due to the fact that results are conditional on assumptions made to represent expected returns, expected volatilities and prices of risk, our approach is based on the simultaneous determination of the measures and of the explanations of premia. We conclude that, despite some differences in modelling[39] and in outcomes[40], our results mainly join those obtained by Prat (2013) on US secular data.

[39] The modelling we propose here presents three main differences compared to Prat (2013) : (i) the expected variances are determined using GARCH processes rather than averages of past squared returns, (ii) the term spread and the influence of the US market are introduced in the required premia equations, (iii) the state variables do not contain observable variables, but they are found *ex-post* to be inflation dependent. One can consider that these dissimilarities are rather secondary compared to what is common to both modelling.

[40] Notably, the prices of risk take more often negative values in France than in US.

Appendix 1 - Historical dynamics of basic data, France, 1854-2018

Figure A - CAC40 stock price index and Dividends per share, 1854-2018

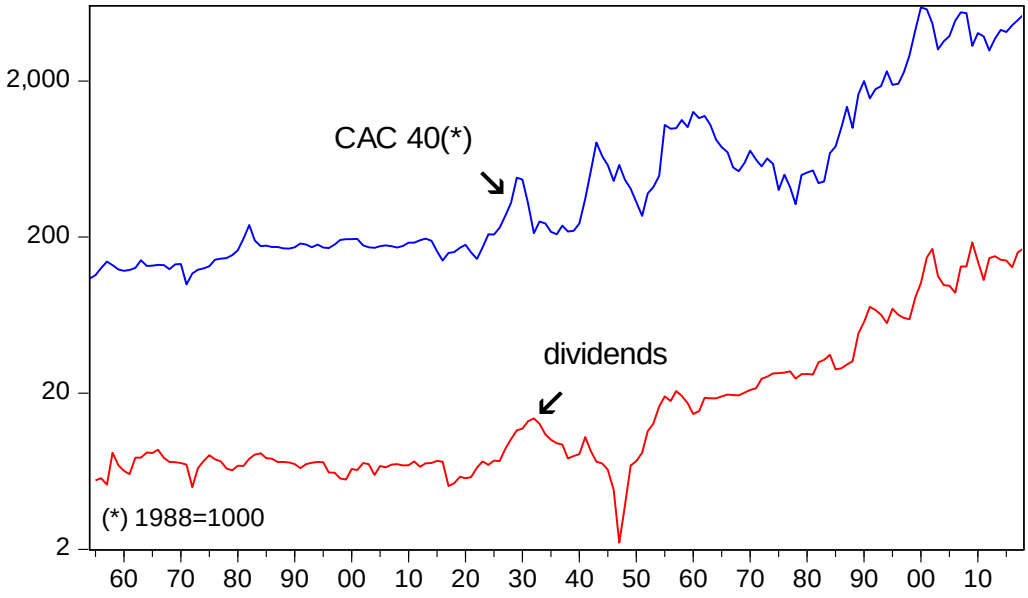
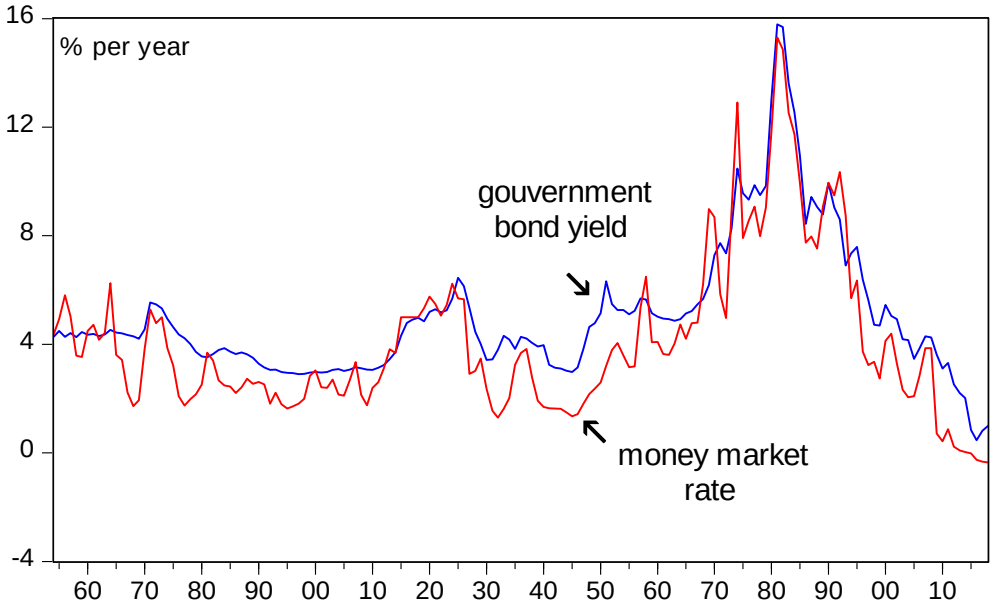


Figure B - Short term and long term interest rates, 1854-2018



Interestingly, as predicted by the simplest version of the DDM $P_t = D_t /(r_o + \phi_o - g_o)$ where r_o, ϕ_o and g_o are the constant values of the riskless rate, the risk premium and the dividends growth rates, the regression of $\log(P_t)$ on $\log(D_t)$ over the whole period 1854-2018 (Newey-West method) shows that the slope is insignificantly different from unity while the intercept indicates a capitalization rate of $r_o + \phi_o - g_o = \exp(-3.30) = 3.69\%$:

$$\log(P_t) = \underset{(16.5)}{1.025}\log(D_t) + \underset{(15.4)}{3.30} + residual \quad R^2 = 0.846 \quad DW = 0.30$$

The estimated capitalization rate value of 3.69 % seems plausible compared to the averages of 5% for government bond yield and of 2% for the dividend growth rate, which indirectly corroborate the reliability of our data. Although the ADF test (not reported) indicates stationary of residuals at the 1% level, the low *DW* statistic shows that residuals are highly auto-correlated, which suggests that the capitalization rate is time-varying as it is the case in the present paper.

Appendix 2 – Prices of risk and inflation : tentative regressions

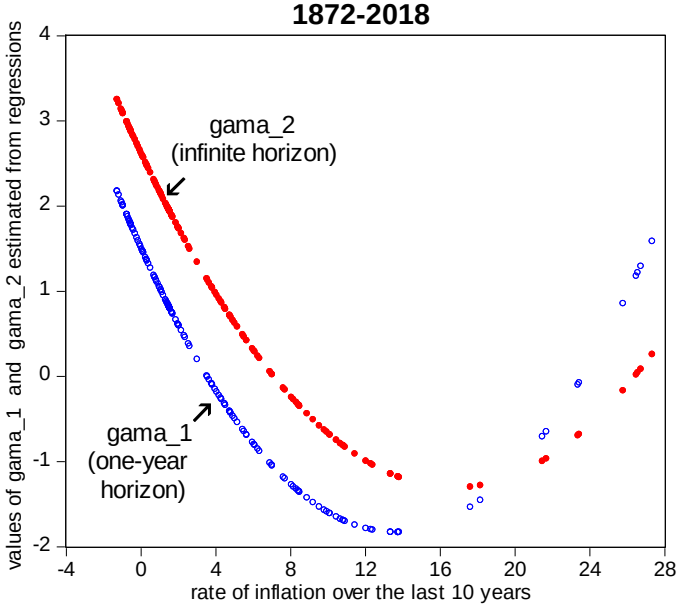
Using the *Seemingly Unrelated Regression* method that avoid estimates biases due to contemporary correlation of residuals and heteroskedasticity, we obtain the following parabolic regressions estimated as a system (1872-2018, N=147) where all variables are found to be I(1) regarding the ADF test (not reported) :

$$\gamma_{1t} = \underset{(-9.7)}{-0.49}\pi_{10,t} + \underset{(8.3)}{0.018}\pi_{10,t}^2 + \underset{(8.2)}{1.52} + res_{1t} \quad \bar{R}^2 = 0.39 \quad DW = 0.08$$

$$\gamma_{2t} = \underset{(-12.0)}{-0.47}\pi_{10,t} + \underset{(8.3)}{0.014}\pi_{10,t}^2 + \underset{(18.4)}{2.63} + res_{2t} \quad \bar{R}^2 = 0.57 \quad DW = 0.09$$

where γ_{1t} and γ_{2t} are the values of the price of risk issued from the stochastic state equations (10) and (11) for the short- and long term horizons, respectively, while $\pi_{10,t}$ stands for the average inflation rate over the last ten years, that is $\pi_{10,t} = 10\log(IPC_t / IPC_{t-10})$ (% per year). The *DW* statistics indicate clearly autocorrelation of residuals, which strongly suggests missing factors. **Figure C** represents $\pi_{10,t}$ on the x-axis while the fitted values $\hat{\gamma}_{1t}$ and $\hat{\gamma}_{2t}$ deduced from the two regressions above are on the y-axis. For both horizons, although a decrease of deflation or an increase of inflation firstly lead to a lowering of prices of risk, prices of risk and inflation raise together when inflation reaches a high threshold around 13% and 17% for $\hat{\gamma}_{1t}$ and $\hat{\gamma}_{2t}$, respectively.

Figure C - Prices of risk as parabolic function of Inflation rate
1872-2018



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