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## The Fate of Inventions. What can we learn from Bayesian learning in strategic options model of adoption ?

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## The Fate of Inventions

What can we learn from Bayesian learning in strategic options model of adoption ?

*(Working Paper)*

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### Abstract

We develop a game where heterogeneous agents have the option of adopting an invention of uncertain quality or postponing their decision to benefit from others' experience through Bayesian learning. Messages produced on the invention nature are noisy, representing the "*teething troubles*" of innovation. Our model gives microeconomic foundations to the S-shaped innovation diffusion curves, informational externality inducing strategic delay in agents' behavior. Moreover, noise could nip in the bud the diffusion of inventions: numerical simulations underline a bi-modal distribution of steady states for innovation diffusion, stillborn or fully developed, bringing to light a reputational valley of death for inventions.

*Keywords:* Innovation diffusion ; Invention adoption ; Information ; Strategic options ; Bayesian learning.

*JEL classification:* O33 ; L15 ; D83 ; C73.

# 1 Introduction

Schumpeter (1911) defined innovation as "*the market introduction of a technical or organizational novelty, not just its invention*", shedding light on the gap existing between a well-functioning invention, which can provide benefits to its adopters, and the actual market diffusion of this invention. In his book "*The Fate of the Edsel*", Brooks (1963) describes for instance how the eponym Ford car, launched in the late fifties, failed to bridge this gap, despite the strong financial commitment of the company.

The challenge of an invention entering a market is to prove its value : when facing an invention, economic agents (households, firms or even governments) are in a situation of uncertainty. If the invention is effective and well-working, its adoption will generate profits or well-being ; but if the invention turns out being only a gizmo which benefits do not cover adoption costs, agents will lose their investment. Economic agents are then constantly on the lookout for information on these new products. As the most reliable information on invention quality is the experience of its users, agents rely on informational hubs, such as consumer associations, professional unions, private networks or one of the numerous rating websites, to exchange information and learn from others' feedback.

Information production by agents is then key to the full development of an invention. In this setting, information has the characteristics of a public good, and, consequently, the free rider problem arises. Each agent has an incentive to postpone her adoption of the invention to benefit from information generated by others' adoption. Thus, free riding delays adoption decisions and spreads over time the production of information. This strategic behavior leads only a few agents to adopt the invention at the time of its entry on the market, and then only a limited number of messages on the performance of innovation are generated.

However, when an invention enters a market, it often meets start-up problems. Even an effective invention can lead to failures in its first stages of development, by early mishaps, misuses or misunderstandings. These failures are commonly known as "*teething troubles*" and generate noise in the information produced by early adopters. This noise can put shade on the invention true quality and nip in the bud the development of a socially good invention. A recent article from *The Economist*

(2015)<sup>1</sup> presents the example of frugal innovation, and attributes its delayed development to early mishaps which created people's mistrust.

The present paper gives a rational framework to analyze this phenomena. We focus on agents' behaviors on the demand side, i.e. adopters' behaviors as a prerequisite to the analysis of inventors' behaviors. The research develops a microeconomic model enabling the analysis of an informational externality threatening the diffusion of an innovation. In the second section we expose some of the market-based examples which motivate our research question and modeling choices. The third section reviews the academic literature on innovation diffusion models, from holistic models of technological diffusion to informational cascade and social learning models. The fourth section describes the framework of our model, which consists in a strategic option representation of the invention adoption decision, in a context of Bayesian learning. We firstly study the interaction between two agents and then extend the model to  $n$  agents. Results from numerical simulations using this model are described in the fifth section, underlining the bimodal distribution of the steady state caused by the informational externality. The last section evidences that this result is robust to an endogenous determination of invention's price.

## 2 Stylized Facts

We present in this section two empirical economic facts which evidence both the importance of early reputation in an invention diffusion with the case of the Edsel car, and the variability of diffusion paths for similar inventions with the case of wind turbines.

### 2.1 The Fate of the Edsel

In September 1957, on the "E Day", the Ford company launched its Edsel model which was one of the first large sedan car commercialized at an affordable price for most American households. With this new model, the Ford company was pursuing a vertical differentiation in the car market. But

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<sup>1</sup>Schumpeter - Cheap and cheerful - After some teething troubles, frugal innovation is on the rise. <http://www.economist.com/news/business/21640330-after-some-teething-troubles-frugal-innovation-rise-cheap-and-cheerful>

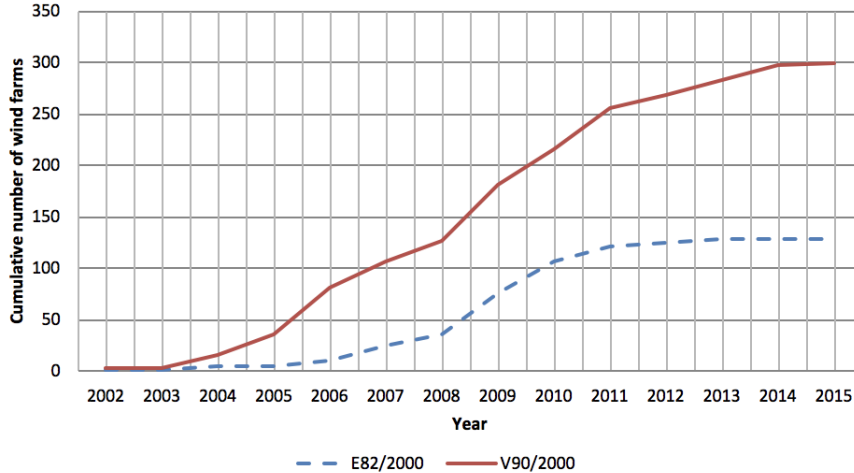
whereas the Ford Company had invested \$250 Million on Edsel development, manufacturing and marketing, the car is today a symbol of commercial failure (Bonsall (2002)). While various reasons are used to explain it, like the controversial design of its front grille, an interesting reading is the national survey conducted by *Popular Mechanics* when the car entered the market: this survey of Railton (1958) was published only six months after the "E Day". In this survey, 1,000 Edsel owners throughout the U.S. have been asked about their thoughts regarding the car. If owners enjoyed performance and ease of handling, surprisingly this survey emphasized one frequent complaint, about poor workmanship in assembly. More than 16% of the owners surveyed listed that default. In the report, Arthur R. Railton, the magazine's journalist in charge, underlines aptly that this kind of defaults does not show up in usual road test, and was unusual coming from a well-known brand. This poor workmanship in the first models which came out of the factory is explained by the Edsel industrial production management: the Edsel did not get its own assembly lines in Ford factories. It was assembled alternatively on lines of other Ford company cars, such as the Mercury, and then often unfinished (see Brooks (1963)).

This illustrates why, when the Edsel entered the market, a joke on its name quickly spread: "Edsel stands for Every Day Something Else Leaks". Despite a powerful launching campaign, information produced and shared by consumers about poor workmanship plagued the reputation of the Edsel, and contributed to its historical failure: the Ford Motor Company lost about \$300 million and stopped the production less than two years after Edsel's launch.

## **2.2 Turbines in the wind**

More recently, the importance of new products early reputation has also been exhibited in the sector of low-carbon innovations, as illustrated by the peer-effect and social spillovers in solar panels adoption by Rode and Weber (2016). We chose to investigate the case of wind turbines, using *TheWindPower* database: it gathers technical information about 1,580 wind turbines from 219 different makers. The database also lists wind farms installed across the world, counting 26,869 farms. In Germany, this database covers about three quarters of the total wind power installed in 2017. We chose to study the diffusion of two specific wind turbines on the German market: the

E-82 from Enercon and the V-90 from Vestas. We picked those two turbines for several reasons enabling their comparison. Firstly, they share almost exactly the same power performance (see power curves compared in annex A.1) and rated power. Secondly, they were both introduced on the market in 2002. Thirdly their respective constructors, Enercon and Vestas, are comparable, as shown by Hau and von Renouard (2003): they both develop and produce all their turbines' components, both of them are major players in the wind turbines market with close market shares for Enercon and Vestas in Germany (respectively about 30% and 24% in 2017), and review from Hansen et al. (2004) shows that those companies were the two leaders in the world in 2002, when E82 and V90 turbines were launched. Last, diffusions of the two turbines are observed in the same country, Germany, a mature market, in order to control for legislation and economic conditions. We do not have the price for each commissioning contract, nevertheless we can reasonably make the assumption that the two turbines' prices are not strongly different: as underlined in the European Commission report on Wind Energy (Lacal-Arántegui (2014)), "*wind turbines are viewed as a kind of commodity, it is likely that non-technological factors will have a stronger influence in the onshore turbine price*", such as demand and public subsidies.



**Figure 2.2:** Diffusion of the E82 and V90 turbines in Germany.

On figure 2.2 we represent the cumulative number of german farms equipped with the two models of turbines. Representing installed turbines number gives a similar graphic, nevertheless we

chose to represent the number of farms. Indeed, as most of them are equipped with a unique model, the relevant economic decision to study is then which turbine the agent chooses to equip its farm. Figure 2.2 evidences the well-known S-shaped diffusion curves of new product, but steady states for the two turbines are not similar. The number of farms equipped with Vestas' turbine stabilizes at 300 in 2014, whereas the number of farms equipped with Enercon's turbine already stopped its growth in 2011, with about 130 farms. The earlier and lower steady state of the E82 turbine is interesting, as its power curve was similar to the V90 turbine, and happened on the same market. Moreover, whereas Germany is the home market of Enercon and is not for the Danish firm Vestas, Enercon did not take advantage of a home market effect.

Sarja and Halonen (2013) investigated the determinants of new turbines adoption in Finland: their findings underline that reputation was the key driver leading to the choice of a wind turbine over other ones, ahead of other classical factors such as turbine's technical performance or costs. The second driver identified is the volume of electricity generated, which is similar for E82 and V90 as demonstrated on the power curves (annex A.1). Costs are only listed as the third factor, and in these costs the turbine's price is poorly cited by the interviewees, strengthening our hypothesis that turbines with similar power have converging prices, at least in Europe, and then that prices do not strongly affect commissioning decision. In their article, Sarja and Halonen underline that the reputation was not referring to production statistics, but "*interviewee's own past experiences and by sharing information with other companies*", which emphasizes the role of public information that accrues from private decisions made by farm owners.

### **3 Literature review: information and innovation diffusion**

Whereas the S-shaped diffusion curves of innovation have been identified by economists in the fifties, and while this stylized-fact has for long been attributed to an information effect, we evidence in this section that the economic literature still lacks models based on rational agents decisions to explain this phenomenon.



### 3.1 Holistic models of innovation diffusion

Since Griliches (1957), many empirical analysis have highlighted the S-shaped curve of innovations diffusion. The Bass diffusion model, firstly exposed in Bass (1969), provides a good description of this dynamics phenomena, and has been widely applied to the diffusion of innovations in the last fifty years, including works on the diffusion of renewables, such as the one by Rao and Kishore (2010) and by Jenner et al. (2013). Nevertheless, these analyzes are holistic and lack microeconomic foundations, whereas social sciences widely recognize the key-role of information in this logistic diffusion of innovation, by word of mouth processes for instance as shown by Rogers (2010). Indeed microeconomic approaches on innovation diffusion focus either on network externalities (where frameworks similar to the one of Cabral (1990) have numerous applications for ICT) and on learning curves, as in the study by Beck et al. (2018). Our framework of social learning with an informational externality is linked to the concept of learning curve: indeed the invention adoption by some agents reduces the adoption costs (in terms of uncertainty) for the following adopters; this link was underlined by Baudry and Bonnet (2017). Those studies all suppose that in the end, after the S-shaped diffusion, an innovation always reaches its full potential development. By contrast, the aim of the present work is to build a microeconomic model eliciting the role of information in technology adoption and innovation diffusion. One of the main lessons from this model is that imperfection of information can randomly cap the diffusion of an innovation below its optimal level.

A first theoretic microeconomic model, taking into account the role of information and the agents' trade-off between adopting an innovation with uncertain outcomes and waiting for more information, was proposed by Jensen (1982). This model evidences that information could be a driver of the S-shaped curve of innovation diffusion, but still relies on an exogenous arrival of information. As shown in Hall (2004) and Peres et al. (2010), the economic literature still lacks models representing the rational choice of agents in interaction with their environment. As the individual decision rooting innovation diffusion is either to adopt immediately the invention or to postpone the adoption in the aim to obtain information arising from others' adoption, it is relevant to look at this decision as a real option problem. Moreover, as the action of an agent has an impact

on the choice of others, and reciprocally, it requires a game theoretic approach, leading to our choice of a strategic option model.

### 3.2 Real options and game theory

Irreversible discrete decisions of investment in a situation of uncertainty have been firstly analyzed by Henry (1974). This analysis was then extended to the precautionary principle in a continuous choice framework by Gollier et al. (2000). Simultaneously to Henry's contribution, another seminal paper on irreversible decisions in a context of uncertainty was published by Arrow and Fisher (1974), where information arrival stochastic process left room for various interpretations, mainly Markov process and Bayesian learning. The first interpretation is privileged by the literature on real options theory, mainly known through the textbook by Dixit and Pindyck (1994). The Bayesian interpretation has been applied to climate economics by Kelly and Kolstad (1999) for instance, and the synthesis of real options theory with Bayesian learning applied to the precautionary principle is realized by Baudry (2008). However those works focus on the precautionary principle for policy-makers at a global level, avoiding the strategic dimension which has to be taken into account when focusing at the level of states, and *a fortiori* at the level of individuals.

Indeed, beyond the use of options to analyze choices under uncertainty, the invention adoption problem we described in the previous section requires to introduce strategic interactions between agents. Investment decision in an innovation becomes subject to a waiting game: by delaying adoption, agents can learn from others' experience. A more recent literature has focused on strategic options: in the wake of Lambrecht and Perraudin (2003) and Smit and Trigeorgis (2006), research works such as the ones of Thijssen (2010), Mason and Weeds (2010) and Thijssen et al. (2012) have started modeling strategic behavior of firms facing an investment with uncertain outcomes, especially in the context of R&D. But, by focusing on the decision of inventors to develop and market their invention, their works exhibit situations where preemption strategies become dominant, whereas our model is interested in situation where potential adopters' waiting strategies are reinforced: agents are interested in others' experience. Such behaviors are usually described in models of herding and informational cascades.

### 3.3 Informational cascades, herding and social learning

Two seminal papers, independantly published the same year, exposed the fundamentals of informational cascades: Banerjee (1992) and Bikhchandani et al. (1992) describe a sequential decision model where each decision maker looks at the decisions taken previously. Herd behavior derives from the fact that some individuals have private information on the good decision to make, but can decide rationally to ignore their private information to mimic others' behavior. But these models rely on an exogenously determined order of arrival, which are made one by one. These restrictions were exposed by Shiller (1995), who outlines the limits of the sequentiality and the first movers' issue, namely the removal of strategic interplay. Gale (1996) underlines the issue of endogenous sequencing as one of the main limits of informational cascades models, but also outlines an important feature of informational cascades: the first best could be unreached due to the informational externality. In their article, Chamley and Gale (1994) implement an endogenous timing of decisions, but still rely on agents differing by their private information, whereas our problem relies on the public nature of information.

There is a wide brand of literature dealing with the social aspects of innovation diffusion. They usually acknowledge the key role to information sharing. Review made by Young (2009) evidences three types of models in the economic literature: contagion ones, where innovation spreads like epidemics, social influence ones, where innovation spreads thanks to a conformity motive (also called peer effects) and lastly social learning. The two first categories of informational effects are dug into by Xiong et al. (2016), but this research does not investigate the strategic aspect of delaying adoption. The last category, social learning, is underlined as the most relevant for economic analysis, as decisions made by actors are rational, people waiting for empirical evidence before adopting a new product. However, Young (2009) does not introduce the notion of "teething troubles" which is key to us to explain some less evident diffusion patterns, and does not either explicitly model the waiting game in which agents could engage in. Our aim is then to fill this gap by modeling rational agents having the option of adopting the innovation immediately or postponing their decisions to benefit from social learning. As information transmission is not perfect, noise is introduced in our

model, evidencing an alternative aborted diffusion path for new products.

## 4 Model Description

In the lights of the models reviewed above, we innovate on several aspects. We consider a framework where  $N$  agents face an invention of uncertain quality, and share information about this invention.

Agents are free to adopt the invention immediately or to postpone the decision to the following period. They differ by their preference for quality  $\theta$ , and are in asymmetric information on others' preferences, that they treat as uniformly distributed between the minimal preference  $\theta_m$  and the maximum one  $\theta_M$ . Economic agents are rational, risk-neutral and in strategic interaction; moreover, they are one-period forward looking to capture inter-temporal choices. At each period, each agent decides to invest if and only if her expected gains from immediate adoption are greater than expected gains of postponing the decision to the next period.

Invention is a durable good of uncertain quality: effective  $Q_{sup}$ , or counter-productive  $Q_{inf}$  (with  $Q_{sup} > Q_{inf}$ ), with a price  $P$  fixed by a firm that is assumed to have a monopoly power on it due to patenting or secrecy. Adoption is irreversible: the irreversibility arises, for instance, from a "market for lemons" in the case of product innovation, or from the specificity of assets mobilized in the case of other types of innovations.

Belief ( $X_t$ ) in the invention nature is common and shared among agents, i.e. information about invention quality is public. This belief is revised by Bayes' rule according to messages produced by adopters: each time an agent adopts the invention, she produces a message on its nature, but information transmission is not perfect (e.g., adoption of an invention of effective quality can give birth to a negative message, and conversely for an invention of counter-productive quality). Noise is multi-sourced (teething troubles, measurement issues, Chinese whispers...), and captures the phenomena exposed in introduction.

Expected gains then incorporate the possibility of receiving messages of both types, and the possibility of postponing adoption if expected gains are negative. As they are in an information asymmetry, agents give a common probability  $p$  to other agents to invest, and they adopt a strategic

behavior. Our game theoretic framework is solved recursively in pure strategies.

## 4.1 A basic two agents - two periods model: proof of concept

### 4.1.1 Framework of the game

We use the basic model of vertical differentiation developed by Shaked and Sutton (1982). The utility flow  $u_i$  of agent  $i$  is written  $u_i = \theta_i * Q + Y$ ; her budget constraint is  $Y + P = R_i$ , where  $\theta_i$  is the marginal rate of substitution of the agent  $i$  between the aggregate good and the differentiated good,  $Q$  is the quality of the differentiated good,  $Y$  is the quantity of the aggregate good,  $P$  the price of the differentiated good and  $R_i$  the revenue of the agent  $i$  at each period. Each agent is supposed to buy only one unit of the differentiated good, and we consider our good as a durable one<sup>2</sup>. By substitution, we obtain the following expression for agent  $i$ 's utility flow:

$$u_i = \theta_i * Q - P + R_i \quad (1)$$

We consider 2 agents  $A$  and  $B$  who are facing the decision to adopt a same product. Quality preferences of the two agents are respectively  $\theta_A > 0$  and  $\theta_B > 0$ ; each agent knows her preference, but it is private information. Agents do not know the preference of their partner, they only know that quality preferences are in the range  $[\theta_m, \theta_M]$ . Prior adopting the invention, quality is normalized to  $Q = 0$  obtained at price  $P = 0$ . Invention quality is  $Q_{sup}$  if the invention is effective, and reciprocally  $Q_{inf}$  if the the invention is counter-productive, with  $Q_{sup} > Q_{inf} > 0$ . Then, for agent  $i \in \{A, B\}$ , willingness to pay for the invention is  $Q_{sup} * \theta_i$  in the good quality scenario, or  $Q_{inf} * \theta_i$  in the bad quality scenario. Initial common belief that the invention is counter-productive is  $X_0$  and, thus, belief in the good scenario is  $1 - X_0$ , with  $X_0 \in [0, 1]$ . Subsequent belief at  $t = 1$  is denoted  $X_1$ . We define the expected quality:

$$Q_{exp}(X_t) = X_t * Q_{inf} + (1 - X_t) * Q_{sup} \text{ for } t \in \{0, 1\} \quad (2)$$

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<sup>2</sup>The model could alternatively be presented as a decision to adopt a process, managerial or marketing invention by two firms.  $P$  would then denote the sunk cost of investing in the invention, whereas  $Q$  would be the multiplicative impact on gross profit  $\theta_i$  of the resulting change on total factor productivity of the firm. Accordingly, the net profit in case of adoption would be:  $\Pi_i = \theta_i * Q - P + R_i$ , where  $R_i$  is the unaffected source of profit of firm  $i$ .

The discount rate used by all agents is fixed at  $r \geq 0$ .

When an agent decides to invest at the first period, she produces a message which will re-evaluate  $X_0$  into  $X_1$ . Reliabilities of messages created are defined as follows:  $p^{pos} > 0.5$  is the probability to receive a message compliant with a positive scenario (probability that the message is positive when the invention is effective);  $p^{neg} > 0.5$  is the probability to receive a message compliant with a negative scenario (probability that the message is negative when the invention is counter-productive).

As  $p^{pos}$  and  $p^{neg}$  are common knowledge, from above we can define rational expectations of agents on the receipt of positive messages from  $t = 0$  to  $t = 1$ , and respectively on the receipt of negative messages:  $Prob_{pos,0}$  is the probability of receiving a positive message if the other player adopts the invention at  $t = 0$ , respectively  $Prob_{neg,0}$  the probability of receiving a negative message if the other player adopts the invention at  $t = 0$ .

$$\begin{cases} Prob_{pos,0} = p^{pos} * (1 - X_0) + (1 - p^{neg}) * X_0 \\ Prob_{neg,0} = (1 - p^{pos}) * (1 - X_0) + p^{neg} * X_0 \\ Prob_{pos,0} + Prob_{neg,0} = 1 \end{cases} \quad (3)$$

As a message will be incorporated in the common belief on the nature of the innovation, Bayesian re-evaluation will give the following  $X_{1,+}$  (respectively  $X_{1,-}$ ) if the message is positive (respectively negative). If no message is received between  $t = 0$  and  $t = 1$ , then  $X_{1,\emptyset} = X_0$ .

$$\begin{cases} X_{1,+} = \frac{1 - p^{neg}}{Prob_{pos,0}} * X_0 \\ X_{1,-} = \frac{p^{neg}}{Prob_{neg,0}} * X_0 \\ X_{1,\emptyset} = X_0. \end{cases} \quad (4)$$

As we have  $p^{pos} + p^{neg} > 1$ , then  $X_{1,-} > X_0 > X_{1,+}$  and then  $Q_{exp}(X_{1,+}) > Q_{exp}(X_0) > Q_{exp}(X_{1,-})$  where  $Q_{exp}(X_t)$  is defined in (0).

With  $\lambda$  the probability each agent gives to the other one to invest at the first period, we analyze agents' strategic choices. As in this first framework only two agents are interacting, the analysis of

two periods is sufficient. Indeed, reasoning recursively, there is three possible states of nature at the second period:

- Both agents have adopted the invention at the first period. Then further analysis is not needed.
- None of the agents has adopted the invention during the first period: as information is endogenously produced in this game, it means that beliefs on invention nature (effective or counter-productive) have not been revised, and then option problem is similar to the one of the first period, and rationally, each agent will keep her strategy of postponing. By recurrence, in this case no agent will ever adopt the invention, and the two periods game is sufficient to analyze strategies.
- A third possible state of nature is that only one of the two agents has adopted the invention. Then, for the remaining agent with the option of adopting, belief on invention nature has been revised according to the message produced by the agent who has adopted the invention. Remaining agent can then make her decision to adopt or postpone the adoption, but if she postpones, she is in reality giving up definitely as no more information can be revealed about invention nature. Again, a two periods game captures all possible strategies of the two agents.

Thus there are only two relevant periods of analysis for our game. The option problem in the first period is to decide between adopting immediately the invention or postponing the decision to the second period. If the expected utility derived from immediate adoption is easy to calculate with the initial belief on invention nature, expected utility of postponing is more complex as it embodies both the possibility that the decision to adopt or not will be enlightened by the adoption of the other agent during the first period (with probability  $\lambda$ ) and the alternative state of nature where no more information will be disclosed (with probability  $1 - \lambda$ ). If she receives a message, agent  $i$  can rationally anticipate the evolution of expected utility if this message is positive or negative.

Accordingly, the option problem writes as follows:

$$F_i = \text{Max} \left\{ \begin{array}{l} \text{Utility of immediate exercise of the option:} \\ U_{i,adoption} = \theta_i * Q_{exp}(X_0) + R_i - P + \frac{\theta_i * Q_{exp}(X_0) + R_i}{1+r} \\ \\ \text{Utility of postponing:} \\ U_{i,delay} = R_i + (1 - \lambda) * \frac{\text{Max}\{\theta_i * Q_{exp}(X_0) + R_i - P; R_i\}}{1+r} \\ + \lambda * \frac{\text{Prob}_{pos,0} * \text{Max}\{\theta_i * Q_{exp}(X_{1,+}) + R_i - P; R_i\} + \text{Prob}_{neg,0} * \text{Max}\{\theta_i * Q_{exp}(X_{1,-}) + R_i - P; R_i\}}{1+r} \end{array} \right. \quad (5)$$

The linearity of  $u_i$  implies that  $R_i$  has no influence on the exercise rule of the option. Depending on parameters initial values, three cases have to be considered:

- If  $0 > U_{i,adoption} \Leftrightarrow P > \theta_i * Q_{exp}(X_0) * \frac{2+r}{1+r}$  : agent  $i$  systematically delays, no matter how the other agent behaves. Indeed, the value of immediate execution is negative, whereas the value of report is always superior or equal to 0, because the agent is never forced to adopt the invention. The strategic interaction has no influence on the agent's decision in this case.
- If  $P < \theta_i * Q_{exp}(X_{1,-})$  : agent  $i$  always decides to exercise her option immediately and to adopt the invention. Indeed, even with a negative message, expected net gains resulting from the adoption will be positive. Then the information hypothetically earned through waiting will not change agent's decision, whereas waiting has a cost for the agent, through the discount rate. The strategic interaction never influence the agent's decision in this case.
- If  $\theta_i * Q_{exp}(X_0) * \frac{2+r}{1+r} > P > \theta_i * Q_{exp}(X_{1,-})$  : expected net gains from immediate exercise is positive for the agent, but if a negative message is received between the first and the second period, expected net gains become negative. Then in the second period the agent will not choose to adopt. The optimal decision relies on the probability  $\lambda$  conferred to the other agent to invest. Finding  $\lambda$  is a prerequisite to solve the option problem.



### 4.1.2 Solving the 2 agents model with strategic delay

We consider the third case presented above:  $\theta_i * Q_{exp}(X_0) * \frac{2+r}{1+r} > P > \theta_i * Q_{exp}(X_{1,-})$ . We can rewrite the option problem as follows:

$$F_i = Max \left\{ \begin{array}{l} \text{Utility of immediate exercise of the option:} \\ U_{i,adoption} = (\theta_i * Q_{exp}(X_0)) * \frac{2+r}{1+r} - P \\ \text{Utility of postponing:} \\ U_{i,delay} = (1 - \lambda) * \frac{\theta_i * Q_{exp}(X_0) + R_i - P}{1+r} \\ + \lambda * \frac{Prob_{pos,0} * (\theta_i * Q_{exp}(X_{1,+}) + R_i - P)}{1+r} + \lambda * \frac{Prob_{neg,0} * R_i}{1+r} \end{array} \right. \quad (6)$$

Where  $\lambda$  belongs to the interval  $[0, 1]$ . By definition,  $\lambda$  is the probability that the value of immediate exercise is superior to the value of postponement for the other player: it is an anticipation made by agents on the probability that others adopt. As agents are rational, share of adopters observed at the end of the period has to be consistent with the adoption probability used by agents in their economic rationale.

$$\lambda = Pr\{U_{i,adoption} > U_{i,delay}\} \quad (7)$$

Substituting in equation (7) the expression of  $U_{i,adoption}$  and  $U_{i,delay}$  given in (6), this is equivalent to:

$$\lambda = Pr\left\{\theta_i > P * \frac{r + \lambda * Prob_{neg,0}}{(1 + \lambda + r) * Q_{exp}(X_0) - \lambda * Prob_{pos,0} * Q_{exp}(X_{1,+})}\right\} \quad (8)$$

Solving this inequation requires to specify the belief agents have on the marginal rate of substitution of others. For computational convenience, we use a uniform distribution of  $\theta$  on the interval  $[\theta_m, \theta_M]$ . Equation (8) then becomes:

$$\lambda = \frac{\theta_M - P * \frac{r + \lambda * Prob_{neg,0}}{(1 + \lambda + r) * Q_{exp}(X_0) - \lambda * Prob_{pos,0} * Q_{exp}(X_{1,+})}}{\theta_M - \theta_m} \quad (9)$$

This equation in  $\lambda$  can be conveyed into the second-order polynomial stated in equation 10:

$$0 = \lambda^2 * (\theta_m - \theta_M) * (Q_{exp}(X_0) - Prob_{pos,0} * Q_{exp}(X_{1,+})) + \lambda * (\theta_M * (Q_{exp}(X_0) - Prob_{pos,0} * Q_{exp}(X_{1,+})) + (\theta_m - \theta_M) * (1 + r) * Q_{exp}(X_0) - Prob_{neg,0}) - P * r \quad (10)$$

As shown in Appendix A.2, we can easily prove that this polynomial admits a unique positive solution in  $\lambda$ , noted  $\lambda_{sol}$ , and that this solution is strictly positive. The solution of the option problem is denoted by  $\lambda^*$  and may depart from  $\lambda_{sol}$ . If  $\lambda_{sol} > 1$  then the corner solution  $\lambda^* = 1$  is obtained and agents will both adopt the invention at the first period. In this case, the option value of waiting is not high enough in comparison to the expected loss due to discounting. But if  $\lambda_{sol} < 1$ , then  $\lambda^* = \lambda_{sol}$ , the adoption at the first period is not systematic anymore. A sufficient condition ensuring  $\lambda^* < 1$  is  $\theta_m < \frac{P}{Q_{sup}} * \frac{1 - p^{pos}}{r + p^{pos}}$ . This condition is not limiting for our analysis: it simply means that an agent might have a quality preference low enough to prevent him from ever investing in the invention.

**Proposition 1.** *In a two agents game, the probability that an agent affects to the other exercising her option to adopt immediately the invention has a unique solution  $\lambda^* \in ]0; 1]$  which depends on  $p^{pos}$ ,  $p^{neg}$ ,  $\theta_m$ ,  $\theta_M$ ,  $Q_{inf}$ ,  $Q_{sup}$ ,  $X_0$ ,  $P$  and  $r$ .*

## 4.2 Generalization: $N + 1$ agents and up to $N + 1$ periods

### 4.2.1 Framework of the game

We now consider  $N + 1$  agents,  $A_1, A_2, \dots, A_{N+1}$  who can adopt the same invention. As in the previous section, their quality preferences are respectively  $\{\theta_1, \theta_2, \dots, \theta_{N+1}\} \in [\theta_m; \theta_M]^{N+1}$  and are private information of each agent. Willingness to pay for the invention of agent  $i$  is  $\theta_i * Q_{sup}$  if the invention is effective, or  $\theta_i * Q_{inf}$  if the invention is counter-productive. Initial common belief in the

bad scenario is  $X_0$ , and respectively initial belief in the good scenario is  $1 - X_0$ , with  $X_0 \in [0, 1]$ . We define expected quality at period  $t$  as  $Q_{exp}(X_t) = X_t * Q_{inf} + (1 - X_t) * Q_{sup}$ . The discount rate used by all agents is fixed at  $r \geq 0$ .

The expected utility gain of the agent  $i$  when adopting at period  $t$  is then written:

$$u_{i,t} = \theta_i * Q_{exp}(X_t) - P + R_i \quad (11)$$

Agents' rational expectations on positive and negative messages at period  $t$  are written following the same lines than in the previous section (two players game):

$$\begin{cases} Prob_{pos,t} = p^{pos} * (1 - X_t) + (1 - p^{neg}) * X_t \\ Prob_{neg,t} = (1 - p^{pos}) * (1 - X_t) + p^{neg} * X_t \\ Prob_{pos,t} + Prob_{neg,t} = 1 \end{cases} \quad (12)$$

Unlike the previous model, multiple messages can now be incorporated in the revision of common belief from date to date. Indeed we do not limit the number of agents who can choose to adopt the invention at each period - contrary to most informational cascades models. Bayesian re-evaluation will give the following  $X_t = Rev_{\alpha,\beta}(X_{t-1})$  common belief <sup>3</sup> on the nature of invention at a given period  $t$ , given that  $\alpha$  positive messages and  $\beta$  negative messages have been received since the previous date  $t - 1$ . The function  $Rev_{\alpha,\beta}(\cdot)$  gives the belief on invention nature revised bayesianly with those  $\alpha$  positive and  $\beta$  negative messages.

$$Rev_{\alpha,\beta}(X_{t-1}) = \frac{\left(\frac{1 - p^{neg}}{p^{pos}}\right)^\alpha * \left(\frac{p^{neg}}{1 - p^{pos}}\right)^\beta * X_{t-1}}{1 - X_{t-1} * \left(1 - \left(\frac{1 - p^{neg}}{p^{pos}}\right)^\alpha * \left(\frac{p^{neg}}{1 - p^{pos}}\right)^\beta\right)} \quad (13)$$

Demonstration of equation (13) is given in Appendix A.3. We consider an agent  $A_i$  facing the decision of investing immediately or postponing for one period to gather information from other agents on the invention's nature. We generalize the option problem firstly presented in equation

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<sup>3</sup>We operate a change from previous section's notations: here  $X_{1,+}$  becomes  $Rev_{1,0}(X_t)$ .

(5) at a period  $t \geq 0$ , when there are still  $n$  other agents who have not adopted the invention yet ( $n \leq N+1$ ). As the information produced about the invention is public, all agents share with  $A_i$  the same belief on invention nature. Moreover, as preferences are private information, all agents affect the same probability  $\lambda_t$  of investing at the period  $t$  to other agents. For computational convenience, agents are only one-period forward looking. Therefore, the value function of their decision is:

$$F_{A_i,t} = \text{Max} \left\{ \begin{array}{l} \text{Utility of immediate exercise:} \\ U_{i,t,adoption} = \theta_i * Q_{exp}(X_t) + R_i - P + \frac{\theta_i * Q_{exp}(X_t) + R_i}{1+r} \\ \\ \text{Utility of postponing:} \\ U_{i,t,delay} = R_i \\ + \frac{\sum_{k=0}^n \binom{n}{k} * \lambda_t^k * (1-\lambda_t)^{n-k} * \sum_{j=0}^k \binom{k}{j} * Prob_{pos,t}^j * Prob_{neg,t}^{k-j} * \text{Max}\{\theta_i * Q_{exp}(Rev_{j,k-j}(X_t) - P + R_i; R_i)\}}{1+r} \end{array} \right. \quad (14)$$

With  $Rev_{j,k-j}(X_t)$  common belief on the nature of invention when  $j$  positive messages and  $k-j$  negative messages have altered the belief  $X_t$ .

#### 4.2.2 Solving the $N+1$ agents game

Like in the two agents-two periods model, we assume that belief each agent has about other agents' preferences for quality can be represented by a uniform distribution of  $\theta_i$  on the interval  $[\theta_m, \theta_M]$ . Hence, by the following rationale, we deduce the  $(n+1)$ -order polynomial representing the option: ( $P^{n+1}$ ). To obtain this polynomial, we calculate  $\theta_{agent,t}$ , threshold of  $\theta$  separating agents who choose to invest at period  $t$  and those who choose to postpone at period  $t+1$ . But, unlike the 2 agents-2 periods model, in this  $n+1$  agents framework we have to take into account that, at each period, agents who have already adopted the invention quit the game. Over periods and adoptions, there are fewer and fewer agents in the game, and the remaining rational agents take this demographic effect into account in their expectations of new messages. More precisely, as the first agents to invest are the ones with the highest preferences for quality,  $\theta_M$  decreases with the number of adopters. We thus switch to the notation  $\theta_{M,t}$ , with  $\theta_{M,0} = \theta_M$  and  $\theta_{M,t+1} = \theta_{agent,t}$ .

By definition,  $\lambda_t$  is the probability that an agent  $j$  invests at the period  $t$ :

$$\begin{aligned}\lambda_t &= Pr\{\theta_j > \theta_{agent,t}\} \\ &= \frac{\theta_{M,t} - \theta_{agent,t}}{\theta_{M,t} - \theta_m}\end{aligned}$$

$$\Leftrightarrow \theta_{agent,t} = \theta_{agent,t-1} - \lambda_t * (\theta_{agent,t-1} - \theta_m) \quad (15)$$

By proceeding along the same method than in section 2, and using jointly the value function (14) and the equation (15), we write ( $P_t^{n+1}$ ):

$$\begin{aligned}(P_t^{n+1}) &= (1+r) * (Q_{exp}(X_t) * (\theta_{M,t} + \lambda_t * (\theta_m - \theta_{M,t})) - P) \\ &\quad + Q_{exp}(X_t) * (\theta_{M,t} + \lambda_t * (\theta_m - \theta_{M,t})) \\ &\quad - \sum_{k=0}^n \left[ \binom{n}{k} * \lambda_t^k * (1 - \lambda_t)^{n-k} \right. \\ &\quad * \sum_{j=0}^k \left[ \binom{k}{j} * Prob_{pos,t}^j * Prob_{neg,t}^{k-j} \right. \\ &\quad \left. \left. * Max\{(\theta_{M,t} + \lambda_t * (\theta_m - \theta_{M,t})) * Q_{exp}(Rev_{j,k-j}(X_t)) - P; 0\} \right] \right] \quad (16)\end{aligned}$$

According to the sign of the polynomial ( $P_t^{n+1}$ ) for  $\lambda_t \in [0; 1]$ , three cases have to be envisioned:

- If  $(P_t^{n+1})(\lambda_t) < 0$  on  $[0; 1]$ : all the  $(n+1)$ -agents delay the adoption at period  $t$ , there is no adoption of the new product. The solution to the option problem is then  $\lambda_t^* = 0$  and the diffusion stops.
- If  $(P_t^{n+1})(\lambda_t) > 0$  on  $[0; 1]$ : all the  $(n+1)$ -agents decide to exercise their option at period  $t$  and to adopt the invention. The solution to the option problem is then  $\lambda_t^* = 1$  and the diffusion over all the population is completed.
- If  $(P_t^{n+1})(\lambda_t)$  switches its sign on  $[0; 1]$ : only a fraction of agents will adopt the innovation at period  $t$ . This is the most interesting case. The fix point value of  $\lambda_t^*$  associated with the

option problem is then the polynomial root between 0 and 1. Proposition 2 states the unicity of the solution.

**Proposition 2.** *In a  $N + 1$ -agents game, the probability that an agent who has not yet adopted the invention at period  $t$  optimally decides to adopt immediately is unique.*

*Proof. Proposition 2*

**Existence:** Immediate from the discussion on the sign of polynomial  $(P_t^{n+1})$  (see equation (16) above).

**Uniqueness:** Immediate if  $(P_t^{n+1})(\lambda_t)$  does not switch sign on  $[0; 1]$ ; if it does, we make proof by contradiction. Assume that there is more than one solution to  $(P_t^{n+1})(\lambda_t) = 0$  between 0 and 1.  $(P_t^{n+1})(\lambda_t)$  is a  $n + 1$  degree polynomial, it has a finite number of solutions. Let consider two successive different solutions  $(\lambda_a, \lambda_b) \in [0; 1]^2$ . Being distinct,  $\lambda_a$  and  $\lambda_b$  admit an order relation. We arbitrarily posit that  $\lambda_a < \lambda_b$ . According to equation (15), their associated quality preference frontiers admit the opposite order relation  $\theta_a > \theta_b$ . We can then choose a quality preference verifying  $\theta_a > \theta_x > \theta_b$ . As  $\theta$ 's distribution is continuous and uniform among agents, we can find the corresponding agent  $x$ .

As  $\theta_a > \theta_x$ , the optimal decision of agent  $x$  is to postpone rather than exercising immediately.

As  $\theta_x > \theta_b$ , the optimal decision of agent  $x$  is to exercise immediately rather than postponing.

From the two previous statements we deduce that the option value of agent  $x$  is the same if she immediately exercises or if she postpones: then  $\lambda_x$  associated to  $\theta_x$  is also a solution of  $(P^{n+1})(\lambda) = 0$ . Yet  $\lambda_a < \lambda_x < \lambda_b$ , which is impossible as we have taken two successive solutions of the polynomial.

Thus the solution  $\lambda_t^*$  of the equation  $(P_t^{n+1})(\lambda_t) = 0$  is unique. □

Proposition 2 is the theoretic foundation of the invention progressive diffusion:  $\lambda_t^*$  is not necessarily equal to 0 or 1, it lies in this interval and diffusion is progressive.

## 5 Bimodal distribution of steady state

If an analytical solution is computationally complex to establish, numerical simulations enable an insightful illustration of the model. Indeed, the objective is to evidence some effects on diffusion paths unprecedented in the economic literature. Especially, numerical simulations highlight that steady states exhibit special characteristics.

## 5.1 Calibration

In order to further analyze the micro-founded model of adoption diffusion, and more specifically its properties, we parametrize the model as follows. There are  $N = 101$  agents in strategic interaction. These agents can all either adopt a new brand product with uncertain quality, or postpone their decision at the following period. If the invention is effective, its quality will be  $Q_{sup} = 1$ . If it is counter-productive, its quality will be  $Q_{inf} = 0$ . Quality preferences of agents are uniformly distributed between  $\theta_m = 10$  and  $\theta_M(t = 0) = 110$ . The common initial belief on the invention is  $X_0 = 0.9$ . Noise parameters are fixed at  $p^{pos} = 0.6$  and  $p^{neg} = 0.65$ . The price of the product is constant over time and fixed at  $P = 19$ ; this price is the one maximizing the firm's profit, as further discussed in subsection 5.4. The discount rate of agents is  $r = 0.05$ . We fix the time limit of the game at 101 periods, since the maximum number of learning periods equals the number of agents.

The model is solved recursively, subtracting to  $n$  at period  $t + 1$  the number of agents having adopted the product at period  $t$ . We compute  $\lambda^*(t)$  given the current belief  $X_t$  obtained with Bayes rule. As agents are rational, they anticipate that those adopting first are the ones with the highest preferences for quality. Then, on the basis of how many agents have adopted the invention, they are able to revise the maximal preference for quality of agents still playing as follows:

$$\theta_{M,t+1} = \theta_{M,t} - \lambda^*(t) * (\theta_{M,t} - \theta_m). \quad (17)$$

At each period  $t$ , once the number of agents  $a(t)$  who choose to invest in the invention, whereas they have not already, is determined, we make a random draw from the binomial distribution defined either by  $p^{pos}$  or  $p^{neg}$  (depending on which scenario we exogenously impose) to determine how many positive and negative messages are emitted. The shared common belief on the nature of the invention among the agents still in the game is revised on the basis of these messages.

## 5.2 S-shape of the diffusion curve and steady state

In this subsection, we present our simulations results in the case of an *effective* invention, with the aim to highlight how an "intrinsically good" invention can be doomed due to informational externalities. Given the calibration of parameters detailed in the previous subsection, if the invention is effective, its optimal diffusion is 99% among our population, which corresponds to 100 agents<sup>4</sup>.

Figure 1 displays the result of one of our one-shot simulations, when the real nature of the invention is *effective*. We can observe in this case that after 5 periods, the full development of the invention is reached in the population of 100 agents. Besides, the diffusion path follows the S-shaped curve generally observed empirically, as detailed in Section 2.

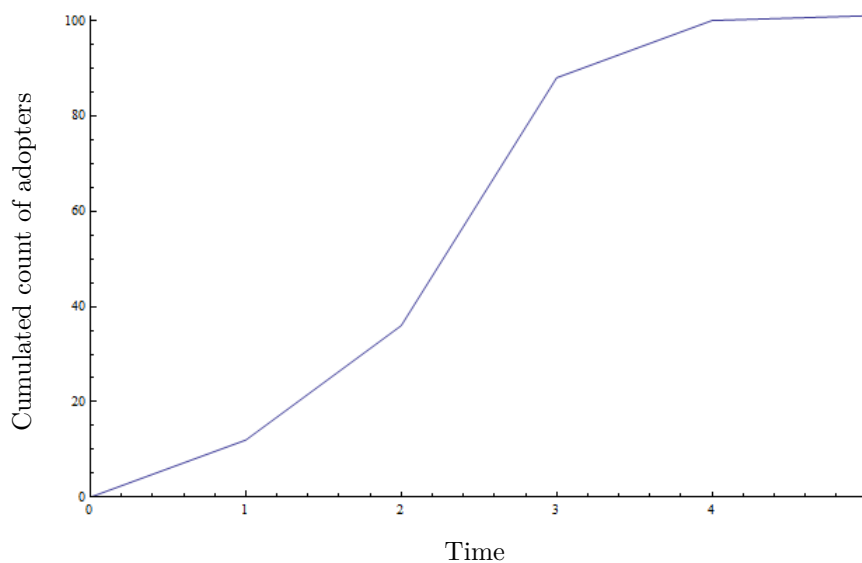


Figure 1: S-shaped diffusion ("Good invention" scenario)

However, each simulation yields a different diffusion path of the invention, as shown in Figure 2. These graphs deserve two main observations: firstly the steady state is always reached after 5 periods, sometimes quicker. This might come from a peculiarity of our model : agents are short-sighted and only implement in their decision at period  $t$  the possibility of postponing the decision

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<sup>4</sup>Indeed, population is made up of 101 agents, but given the quality of invention and their preference quality, only 100 agents would derive a positive utility from invention adoption.



at period  $t + 1$ , but do not take into account the possibility of postponing it in further periods. Secondly, the steady state does not always match with the full development of the innovation. Sometimes the adoption process is stopped after a few periods. This result stems from the random nature of messages. Indeed, in the case studied here, the innovation is effective, but its adoption can still generate negative messages. As in the first periods only a few agents adopt the innovation, a limited number of messages on the invention nature is produced. If noise is loud enough and negative messages are received first, the shared belief on the invention's nature leans towards the "bad" scenario. Therefore agents stop adopting the invention, nipping in the bud the invention's diffusion.

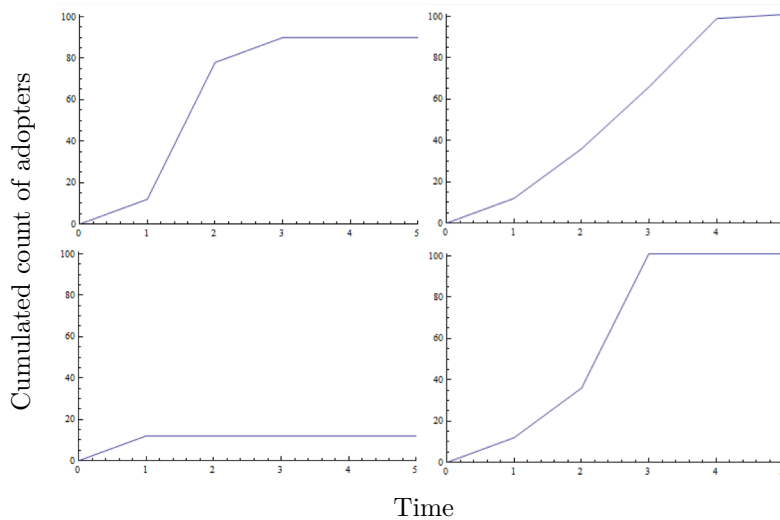


Figure 2: Numerical illustrations of varying diffusion paths ("Good invention" scenario)

### 5.3 Bimodal distribution of the steady state

In order to have a look at the distribution of steady states generated with numerical simulations, we have drawn 10,000 simulations under the "good invention" scenario, and computed the resulting mean diffusion path, with its standard deviation interval. Result is shown in Figure 3.

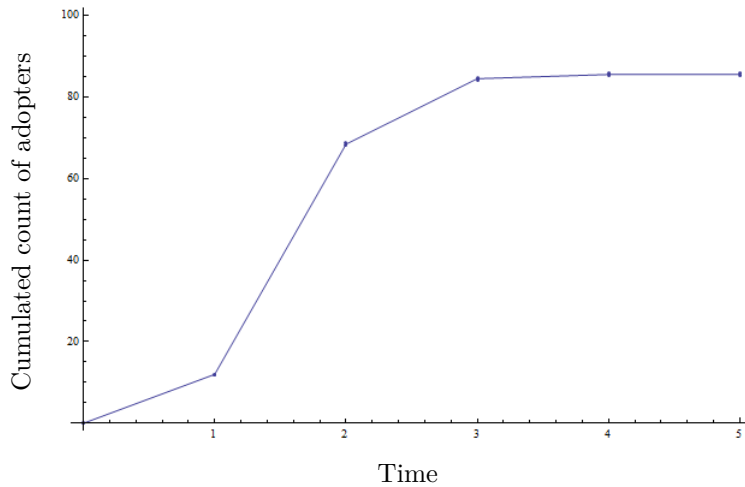


Figure 3: Mean diffusion path over 10,000 simulations ("Good invention" scenario)

Figure 3 highlights that even if the invention is effective for all agents, the mean steady state does not correspond to full development, but is capped (in this configuration, the mean diffusion is limited to around 85% of the population).

Besides the mean diffusion path, we can observe the distribution of steady states over 10,000 simulations. Figure 4 presents the histogram of the distribution of the maximum number of adopters across steady states. We can observe a bi-modal distribution of these steady states. The first mode is a full diffusion of the invention, with 100 adopters in the steady state. This mode gathers about 80% of simulation results. The second mode is a stillborn diffusion, with less than 15 adopters, gathering about 15% of simulations in this setting.

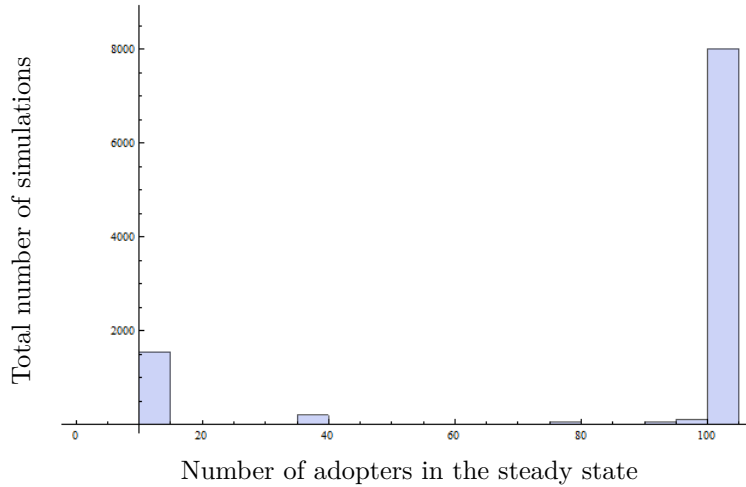


Figure 4: Distribution of steady states for 10,000 simulations ("Good invention" scenario)

This bi-modal distribution is explained by the imperfection of information transmission: at early stages, messages are crucial to sustain the dynamics of innovation diffusion. But, notably because of the strategic interactions and because of the heterogeneity of agents' preferences, at this early stage only a few agents adopt the invention. Among this limited number of messages produced, if negative messages are received first, then noise about the real nature of the invention shades its true nature, and the diffusion is capped. When there are enough positive messages among early messages about the invention's quality, the probability of stopping the diffusion process is lower. Figure 4 typically illustrates that a "good" invention may have a "bad" fate.

By contrast, under the "bad" scenario, if the invention is counter-productive, we do not have a bi-modal distribution. The rationale for this result is that the more messages you get, the more likely it is agents discover that the invention nature is "bad" and the diffusion process stops. The histogram of steady states distribution in a counter-productive invention scenario is presented in Figure 5.

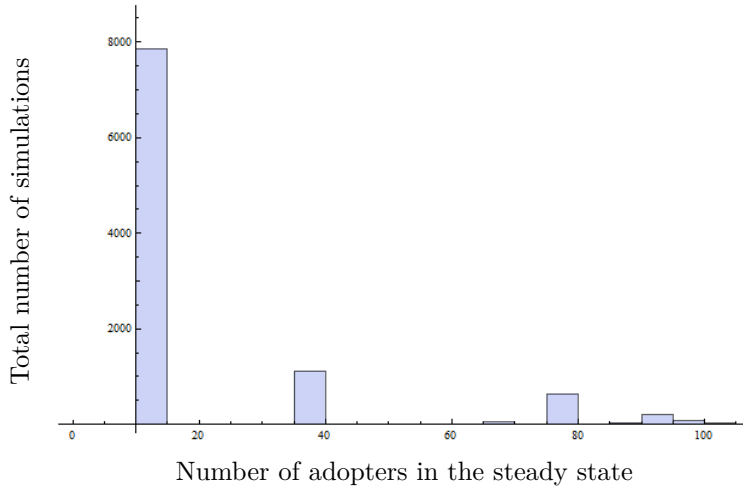


Figure 5: Distribution of steady states for 10,000 simulations ("Bad invention" scenario)

#### 5.4 Firm's optimal pricing strategy

One could argue that firms producing inventions are aware of the reputational risk, and could cope with the fate of inventions exhibited in the previous numerical simulations by adjusting their pricing of the invention. Indeed, as the price decreases, there are more adopters in the first periods, and messages multiplies. With more messages, the probability that noise shades the invention's true nature gets smaller, and the fate phenomena softens.

In a second step of our analysis, we thus consider the possibility for the invention's producer to choose the selling price maximizing its expected profit. A rational producer will choose the price maximizing its expected profit. We determine the associated optimal price by firstly computing the inter-temporal profit for each set of simulations conditional on the price level, as described in the previous section, with a discount rate of 5%. We iterate this calculus with steps of 0.25 for the invention's price. We draw the expected profit curve conditional on the price in Figure 6, all others parameters being kept as specified earlier.

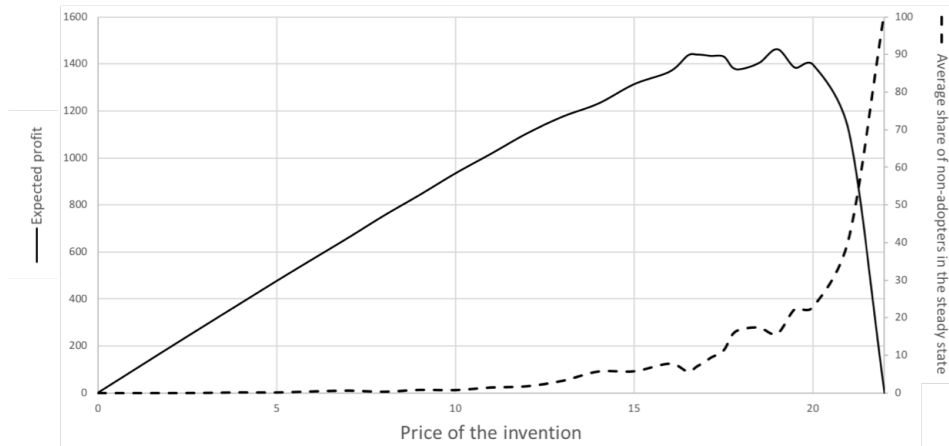


Figure 6: Maximization of firm's profits with the fate of inventions ("Good invention" scenario)

The curve brings out that expected profits become strictly superior to 0 only when the price belongs to the range  $(0, 22)$ . Below  $P = 0$  the firm will obviously earn no positive profit. The upper limit,  $P = 22$ , is due to information imperfection. As invention quality is uncertain, when  $P = 22$  no agent will derive a positive expected utility from adoption, then none will adopt and expected profits are zero from that point. For a price level between 0 and 16.5, the discounted sum of expected profits increases quasi linearly with the price, and are almost stable from 16.5 to 20. Then, they fall down to zero when  $P = 22$ . The maximum expected profit is reached when the price amounts to 19. The two factors influencing this profit pattern are the classical price and volume effects. The volume effect is linked to the expected diffusion of the invention, which depends on the price as the fate of inventions becomes stronger when the invention's price increases. We can observe that the plateau where profits are the highest is tied in with a strong reputational risk. As seen in section 4.3, when the price is optimal,  $P = 19$ , about 15% of inventions will fail to conquer their market with this price. In annex C, we draw profit maximization when information is perfect: optimal price is then much higher, about 105. Together, Figure 6 and Appendix A.4 underline that information imperfection induces a much lower pricing by the firm compared to the full information case, but the firm's optimal pricing strategy does not obliterate the fate of inventions.

## 6 Conclusion

The research work presented in this article contributes to the economic literature interested in invention adoption and innovation diffusion: we model the adoption decision of economic agents as an inter-temporal and strategic choice in a situation of uncertainty, where information on the invention nature becomes a public good produced by private actions. We demonstrate that informational externality is a sufficient condition to induce an endogenous S-shaped diffusion curve. Moreover, we show that noise derived from teething troubles can nip in the bud the diffusion of an effective invention, and curse its fate. We believe that our analytical framework can be useful to explain cases of innovations developing unevenly over various markets, especially when reputational damages are identified. Firms' strategies to overcome this reputational valley of death can also be analyzed through our model. Three different strategies could be envisioned by the firm offering a new product to answer this issue: the first strategy would be to act on the price, for instance by discriminating early adopters and offering them a lower price  $P$  in order to produce enough messages on the invention quality in order to trigger the virtuous circle of information: a "launch price" strategy. The second strategy would be to set up larger informational hubs, for instance by introducing a rating website for consumers or by organizing meetings with early adopters. Both of these two first strategies aim at scaling up the number of messages gathered by potential adopters on invention's quality. The third firm's strategy could be to make information production about its invention more reliable, for instance by offering tools to estimate faithfully the benefits derived from the invention, a solution which seems especially relevant for inventions related to energy-efficiency. The "Dieselgate" enlightened recently the risk of unfair assessment of quality, and the response of the European Commission roots in this third strategy: new certifications and quality control standards are set up to provide a better information for the consumer.

# A Appendix

## A.1 Power curves

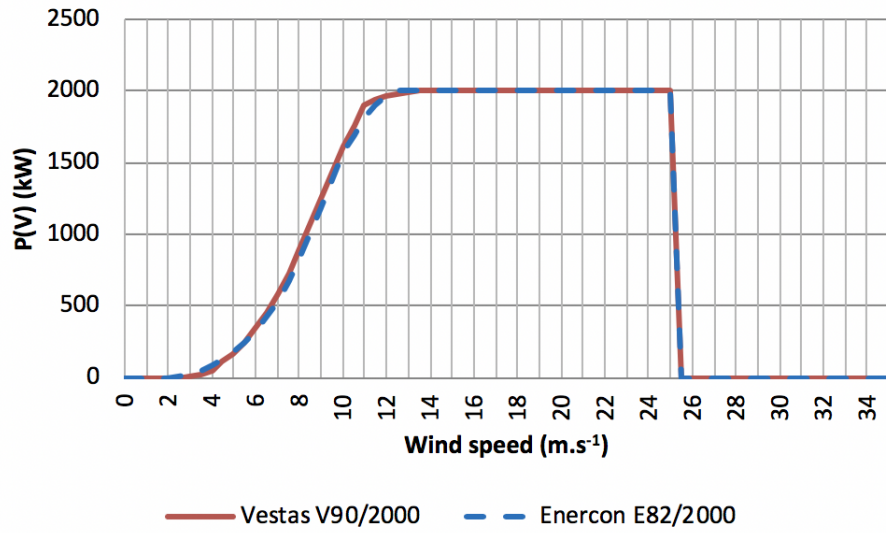


Figure 7: Power curves for the E82 Enercon turbine and the V90 Vestas turbine

## A.2 Mathematical proof: 2 agents problem

### Proof of Proposition 1

$$a * \lambda^2 + b * \lambda + c = 0, \quad (18)$$

$$\text{with } \begin{cases} a = (\theta_M - \theta_m) * (Q_{exp}(X_0) - Q_{exp}(X_{1,+}) * Prob_{pos,0}) \\ b = P * Prob_{neg,0} + \theta_M * Q_{exp}(X_{1,+}) * Prob_{pos,0} \\ \quad + (\theta_M - \theta_m) * (Q_{exp}(X_0) * (1+r) - \theta_M * Q_{exp}(X_0)) \\ c = -(1+r) * \theta_M * Q_{exp}(X_0) \end{cases} \quad (19)$$

$a$  can be rewritten as follows :

$$a = (\theta_M - \theta_m) * (Q_{sup} * (1 - X_0) * (1 - p^{pos} + Q_{inf} * X_0 * p_{neg})) \quad (20)$$

Our framework hypotheses imply that  $a > 0$  and  $c < 0$ . As we define  $\Delta = b^2 - 4 * a * c$ , then  $\Delta > 0$ . There exists then two real roots of equation (18) of opposite signs.

Let consider the positive root  $\lambda^+ = \frac{-b + \sqrt{\Delta}}{2 * a}$ . We look for a condition ensuring  $\lambda^+ \leq 1$

$$\Leftrightarrow \frac{-b + \sqrt{\Delta}}{2 * a} \leq 1$$

$$\Leftrightarrow 0 \leq 4 * a * (a + b + c)$$

$$\Leftrightarrow 0 \leq (a + b + c)$$

$$\Leftrightarrow P \geq \theta_m * \frac{Q_{sup} * (1 - X_0) * ((1+r) - (1 - p^{pos})) + Q_{inf} * X_0 * (1+r - p^{neg})}{(1 - p^{pos}) * (1 - X_0) + p^{neg} * X_0}$$

Then a sufficient condition to ensure  $\lambda^+ \leq 1$  is  $P \geq \theta_m * Q_{sup} * \left(\frac{r + p^{pos}}{1 - p^{pos}}\right)$ .



### A.3 Bayesian re-evaluation in $N + 1$ agents game

In order to compute the belief evolution after  $\alpha$  positive messages and  $\beta$  negative messages, we define the likelihood ratio:

$$Z_t = \ln \frac{X_t}{1 - X_t} \quad (21)$$

Then we define  $\Delta Z_+ = Z_{1+} - Z_0 = \ln \frac{X_{1+}}{1 - X_{1+}} * \frac{1 - X_0}{X_0}$  and we introduce expressions (3) and (4).

By computation, we obtain  $\Delta Z_+ = \ln \left( \frac{1 - p^{neg}}{p^{pos}} \right)$ .

Similarly, we can compute  $\Delta Z_- = Z_{1-} - Z_0 = \ln \left( \frac{p^{neg}}{1 - p^{pos}} \right)$ .

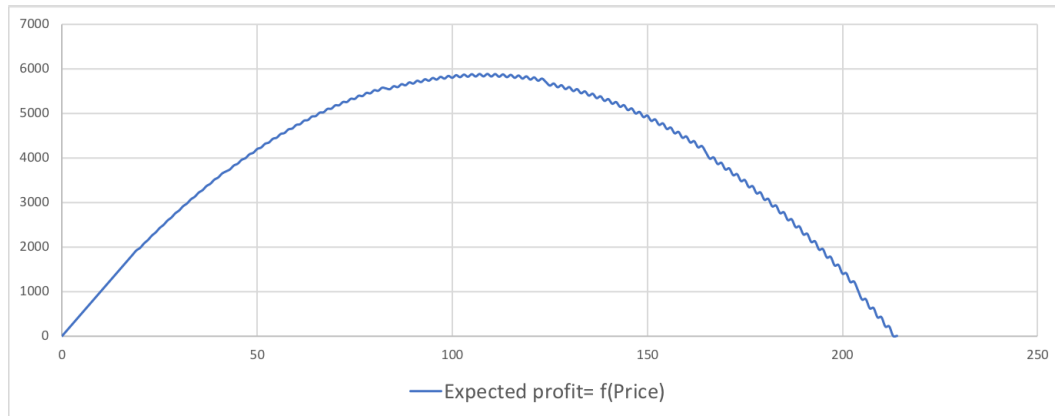
Hence, after  $\alpha$  positive messages and  $\beta$  negative messages, we have:

$$Z_t = Z_0 + \alpha * \ln \left( \frac{1 - p^{neg}}{p^{pos}} \right) + \beta * \ln \left( \frac{p^{neg}}{1 - p^{pos}} \right)$$

Using the exponential function, we finally get the Bayesian revision of belief after  $\alpha$  positive messages and  $\beta$  negative messages :

$$X_t = Rev_{\alpha, \beta}(X_0) = \frac{\left( \frac{1 - p^{neg}}{p^{pos}} \right)^\alpha * \left( \frac{p^{neg}}{1 - p^{pos}} \right)^\beta * X_0}{1 - X_0 * \left( 1 - \left( \frac{1 - p^{neg}}{p^{pos}} \right)^\alpha * \left( \frac{p^{neg}}{1 - p^{pos}} \right)^\beta \right)} \quad (22)$$

#### A.4 Firm's optimal pricing strategy when information on product quality is perfect



*Discounted sum of expected profit (vertical axis) as a function of Invention's Price (horizontal axis)*

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