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Do markets learn to rationally expect US interest rates? evidence from survey data

Georges Prat[a] and Remzi Uctum[b]

Abstract. Using Consensus Economics survey data on the US 3-month bill rate and the 10 years Treasury bonds expectations for the 3- and 12-month horizons over the period November 1989 – May 2015, this article aims at testing whether a group of rational forecasters coexists with or emerges over time beside a group of forecasters employing the traditional limited information-based rules that are the extrapolative, the adaptive, the regressive and the forward-market premium rules. We estimate the time-varying weights associated with the two groups using the Kalman filter methodology and find that the aggregate expectations fail to exhibit a learning process towards rationality both for short term and long term interest rates. While long term interest rate expectations appear to be explained only by limited information rules at any time, in the case of the short term interest rate a group of rational agents seems to have operated in the market over the whole period with a small but almost constant weight simultaneously with limited information-based forecasters. Overall, for both short and long term interest rates, our results strongly suggest that experts' forecasts are essentially based on a combination of the four traditional processes. This is consistent with the economically rational expectations theory which suggests that information costs and agents' aversion to misestimating future interest rates determine the optimal amounts of information on which they base their expectations.

Keywords : expectation formation, interest rates, dynamic heterogeneity, survey data.
JEL Classification : D84, F31, G14

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1 – Introduction

According to the theory of the term structure of interest rates, the spread between the long term rate and the short term rate is equal to the expected changes in the short rate plus a risk premium. Any empirical examination of this relation involves testing a joint hypothesis based on the latter relation and on hypotheses representing expected changes in the short rate and the risk premium, which are unobservable variables. In the literature, interest rate expectations are either assumed to be rational or determined by the historical values of observed rates, while the specification of the risk premium is either derived from an intertemporal equilibrium condition of the investor (portfolio choice model) or from an ad-hoc representation (constant or time-varying premium represented by an ARCH-in-mean model).

In fact, when the joint hypothesis mentioned above is rejected, it is not possible to conclude whether the rejection comes from the term structure relationship that is not well-specified or from the hypotheses on expectations and risk premium that are not relevant. This is why, in order to solve these indeterminacies, some authors have used interest rate expectations provided by surveys realized among financial experts. Such survey data allow to avoid making hypotheses on expectation formation on the one hand and to measure implicit values of the ex-ante risk premium required by experts on the other hand. In this way, the expected change in the short term rate and the risk premium being directly observable, the term structure relationship can be tested. Froot (1989) and MacDonald & Macmillan (1994) find a significantly time-varying risk premium and conclude that the term structure model based on the pure expectations theory should be rejected[1] while Prat and Uctum (2010) validate the term structure relationship in the 3-month maturity Eurofranc market by using a risk premium representation based on the portfolio choice model. However, the term structure model includes market expectations and not those of the experts involved in the survey, therefore the requirement that survey expectations be a valuable approximation of the market expectations must be satisfied. This issue will be discussed later. In any case, assuming even that the risk premium is well specified, the question of how interest rate expectations are formed remains unsolved and thus introduces indeterminacy in the analysis of the term structure of interest rates. This led some authors to investigate on the formation of interest rate expectations using survey data.

[1] The countries analyzed in the former study are the U.S., Germany, Japan and Australia, while the latter study exploits data from U.K. and uses individual survey data. MacDonald (2000) proposed an overview of the related literature.

A first strand of studies concerned by the formation of interest rate expectations focused on examining the relevance of the rational expectations hypothesis (REH). Using data from various surveys and from various countries and periods, authors found evidence against the unbiasedness of expectations and thus rejected the REH (Friedman, 1980; Froot, 1989; Simon, 1989; Kim, 1997; MacDonald, 2000; Greer, 2003; Jongen and Veschoor, 2008; Prat and Uctum, 2010). These results highlight the relevance of the question of how interest rate expectations are formed. Some studies have reported that each of the three traditional standard expectation rules – namely the extrapolative, the adaptive and the regressive rules - can partially explain interest rate expectations. Using survey data, Kane and Malkiel (1967) found support for extrapolative (bandwagon) and regressive expectations while Malkiel and Kane (1969) and Colletaz (1986) found evidence of adaptive expectations. More recently, using Consensus Economics survey data, Prat and Uctum (2010) have shown that an extrapolative, regressive and adaptive mixed model augmented by expectations in inflation and real GDP growth rates provided by the same surveys is appropriate to explaining expectations in the 3-month maturity Eurofranc rate. In an experimental study framework, Marey (2006) showed, on the basis of the responses provided by final stage M.A. students in economics and business, that a forward-looking component such as the yield curve (i.e. a term spread) and expected GDP growth rate also play a significant role in the interest rate expectations formation. The influence of the spread is in accordance with the expectation hypothesis of the term structure of interest rates which, in its pure form (i.e. without a term premium), says that a positive (negative) spread would imply that investors expect future short interest rates to increase (decrease). This theoretical prediction is confirmed by empirical results since the yield spread is found to have predictive power both at the short and long ends of the maturity spectrum (Campbell, 1995). These results are in line with those of some studies (Joyce et al., 2008; Report of the Sveriges Riksbank, 2013) showing that, for short horizons, the forward rate provides the more reliable forecasts of the repo rate, given that the term premium can be neglected up to one year horizon. The authors conclude that the forward interest rate captures market expectations of the monetary policy conducted by the central bank. This relation is even highlighted by the fact that, over the last two decades, central banks have gradually placed more emphasis on the transparency and predictability of their actions. Among them, the Federal Reserve, the ECB, and the Bank of Japan, have conveyed information through official statements, regular reports or public speeches about the future path of the policy targeting rate. This leads financial market participants to better monitor and thus anticipate central banks' monetary policy (Blattner et al., 2008).

Other studies suggest that the forward premium defined as the difference between the forward and spot interest rates may also be a potential factor of interest rate expectations. According to Shiller at al. (1983), one way of testing the expectation hypothesis consists in regressing the ex-post observed change in interest rate on the forward premium. The hypothesis is accepted if the estimated slope significantly equals unity. Although the two variables are significantly correlated, empirical studies have shown that this hypothesis is rejected. Some authors have responded to this rejection by replacing the ex-post observed change in interest rate by the expected change provided by various surveys for different countries (US, Germany, Japan, UK). As a result, studies by Friedman (1979), Froot (1989) and Batchelor (1990) using aggregated expectations data and Batchelor and Dua (1991), MacDonald and Macmillan (1994) and Mitchell and Pearce (2005) using disaggregated data converged to the same conclusion that expected change in interest rates are significantly correlated with the forward premium although the pure expectation hypothesis is rejected.[2]

Overall, regardless of the source of the survey data, the type of the data used (aggregate or micro data), the maturities of bonds and the time-horizon of expectations, studies of the literature on interest rate expectations formation converge to three conclusions : (i) unbiasedness tests conclude that expectations are not rational; (ii) the traditional backward-looking processes based on the history of interest rates such as the extrapolative, adaptive and the regressive rules contribute to explain interest rate expectations, (iii) forward-looking components such as the term spread, the forward premium or macroeconomic expectations in inflation or real GDP growth rate, may also play a role in the formation of interest rate expectations. However, the standard unbiasedness test based on the regression of the expected change on the observed change has so far been conducted assuming two main assumptions. First, the parameters of this regression are supposed to be time-invariant, so that the REH can be rejected on average throughout the period but not at each point in time of the period. Second, the residuals of this test appear to be far from a white noise and can then be viewed as representing implicitly non rational unspecified factors of expectations. Consequently, a valid detection of a rational behaviour in the market would require: (i) to check for this hypothesis over each point in time rather than on average over the whole period and (ii) to allow that a group of forecasters - instead of the whole market – might be rational, the other group of forecasters relying on rules based on limited information. By examining the evolution of such heterogeneous expectations over time, we especially aim at testing the existence or emergence at some point in time of a learning

[2] Indeed, as in the case of ex-post observed change in interest rate, the estimated slopes using interest rate survey expectations are found to be below unity but significant. This significance suggests that the term premium intervenes in the determination of interest rate expectations.

process towards rationality versus the persistent relevance of explicit rules based on limited information. To this end we propose a state-space expectation model with time-varying stochastic weights that we estimate using the Kalman filter methodology. We use two representative U.S. (short and long term) interest rate expectations provided by Consensus Economics surveys for both 3 and 12 month horizons: the 3-month bills rate and the 10-year bond yield. We find that rational learning is strongly rejected for both the short and long term interest rate expectations whatever the horizon. However, our results suggest that in the case of the short term rate only, the heterogeneity of market expectations is partially explained by the existence of a small but persistent group of rational agents. We explain the evidence of some rational behaviour by the predictability of the short rate over different phases of the period.

2. Theoretical issues

The economically rational expectations theory

According to the Grossman's (1976) paradox, an efficient market leads to prices that are empty of all information on fundamentals. Grossman and Stiglitz (1980) have shown that when information is costly or noisy, this paradox vanishes but the market price does not convey all available information on fundamentals, so that market is inefficient and expectations are not rational.[3] This conclusion is consistent with the implications of the economically rational expectation theory (ERET) introduced by Feige and Pearce (1976), which states that it may be rational for forecasters to use a limited set of information and thus to make biased expectations. Let I_{it}^{j} be the amount of information of type i (i=1,2,…,n) available to forecaster *j* at time *t* and c_{it}^{j} the price of collecting and processing a unit of this information supported by this agent. Assuming constant returns to scale, c_{it}^{j} is a marginal cost. Let $f(.)$ be a twice continuously differentiable function linking the information inputs I_{it}^{j} to the agent's expected quadratic

[3] Following this theoretical line and in the light of Lucas' consumption-based asset pricing model (Lucas, 1978), Cechetti et al. (2000) propose an equilibrium model where the consumer-investor representative is perfectly rational, except in forecasting future consumption, because the appropriate forecasting method is assumed to be too costly to be implemented. Within this framework, using the US stock market data, the authors explain two well-known puzzles in financial markets, the "volatility puzzle" (Shiller, 1981) and the "premium puzzle" (Mehra and Prescott, 1985).

forecast error, that is $E_t(E_t^j s_{t+\tau} - s_{t+\tau})^2 = f(I_{1t}^j,...,I_{it}^j,...I_{nt}^j)$. The forecaster is supposed to minimize at any time the total cost:

$$C_t^j = \pi_t^j f(I_{1t}^j, I_{2t}^j,...I_{nt}^j) + \sum_{i=1}^{n} c_{it}^j I_{it}^j, \qquad f_i^{'} < 0, \quad f_i^{''} > 0 \quad i = 1,...n \tag{1}$$

where $\pi_t^j > 0$ is the agent's aversion to misestimating future interest rates, so that $\pi_t^j f(.)$ represents the agent's loss function.[4] The signs of the first and second derivatives of $f(.)$ mean that the more an agent collects information the more s/he expects to reduce the squared forecast error. Hence, the marginal efficiency of information decreases as the amount of information increases. At the equilibrium, equation (1) leads to:

$$c_{it}^j = -\pi_t^j df / dI_{it}^{*j}, \qquad i = 1, 2,\ldots, n \tag{2}$$

According to (2), the optimal amount of information I_{it}^{*j} used by agent *j* is such that the unit cost equals the marginal gain (i.e. the marginal decrease in the loss function). When at time *t* the cost/aversion ratio c_{it}^j / π_t^j is negligible for any type of information *i* ($c_{it}^j \to 0$ or $\pi_t^j \to \infty \ \forall i$), the forecaster *j* uses all relevant information since s/he reaches the condition $df / dI_{it}^{*j} \to 0$ meaning that information is used until the marginal gain vanishes, leading the forecast error variance to be minimal. This case corresponds to Muthian rationality (Muth, 1961). At the opposite, if the value of the cost/aversion ratio is prohibitively high (typically when information is too costly or agents have little aversion to forecast errors), the forecaster ignores any information other than the (costless) observed market price and thus behaves as a noise-trader by forming naive expectations. However, in real markets there exists a continuum of non-zero cost/aversion ratios so that the economically rational forecaster accumulates information until the condition $-df / dI_{it}^j = c_{it}^j / \pi_t^j$ is met, which is true under none of the REH and the naïve hypothesis. The existence of such boundary costs of information suggests that it may be rational for agents not to anticipate rationally, because purchasing all available information whatever its price may be more costly than the utility loss implied by an increase of the forecast error. At the equilibrium, the lower the cost/aversion ratio associated with a given type of information, the larger the proportion of agents using the corresponding information and thus the higher the

[4] Note that π_t^j is similar to the risk aversion coefficient, which is sometimes assumed to be time-varying, as in equation (1). For example, Barberis et al. (2001) explain the equity premium puzzle by the fact that the risk aversion coefficient depends on the states of the nature.

weight associated with this information. Accordingly, expectations may be both heterogeneous and biased, although these expectations are economically optimal.[5]

However, the cost/aversion ratio c_{it}^{j} / π_{t}^{j} is not an observable magnitude and therefore its implication in terms of information included in the forecasting process is not directly measurable. For any type of information *i*, the ratio c_{it}^{j} / π_{t}^{j} in equation (2) is determined by individual characteristics of agent *j*, which can themselves change over time according to the state of nature. At the aggregate level, the heterogeneous expectations is described by a mixture of expectation rules stemming from two kinds of situations: (i) the market comprises different groups of agents each of them using an expectation rule based on a given type of information (group-heterogeneity effect); (ii) all forecasters combine different types of information (individual weighting effect). Because groups in case (i) may also be made of forecasters using processes based on mixed information, the effects (i) and (ii) may operate simultaneously.

The hypothesis of a learning process towards rationality

The REH posits that agents endowed with all the relevant information have a complete and accurate knowledge about the true model of the economy and the value of the parameters contained therein. These strong assumptions have been widely subject to criticism in the literature. In particular, it is not clear how agents in the economy can know the model and the value of the parameters underlying rational expectations when economists who postulate this hypothesis do not themselves have this knowledge and must seek the model and estimate its parameters econometrically (Shiller, 1978; Sargent, 1993). It seems thus more appropriate to assume that agents estimate their models and generate their forecasts using econometric tools, updating their forecast model as new data becomes available over time. Hence, agents in the economy are no more assumed to be fully informed and rational at each point in time but are considered as econometricians who process limited information at any time and learn about rationality asymptotically (bounded rationality).[6] Such a forecast generating process based on a

[5] Note that any forecast bias could be interpreted as a "peso effect" (Kraster 1980; Kaminsky, 1993). However, this effect implies that expectations are homogeneous since the states of nature and the associated "true" probabilities are known to all agents who all compute the same mathematical expectation. This makes the peso effect inappropriate to explaining the features of interest rate expectations which are heterogeneous (see Table 1).

[6] As suggested by Simon (1978, 1986), "substantive" rationality (that is, making rational expectations) may not prevail for an economic agent whereas the latter may be involved in a "procedural" rationality (i.e. a rationality learning process). For a derivation of the conditions for the convergence of the learning process towards rational expectations, see for example Fourgeaud et al (1986).

parameter vector that may be updated in response to new data is the key feature of the adaptive learning approach towards rational expectations (Evans and Honkapohja, 2001).

In this dynamic framework, the rule for updating the parameters is generally of the recursive form, implying that agents act as econometricians who implement recursive least squares learning. Suppose that the model of the economy in its reduced form (actual law of motion, or ALM) is a dynamic stochastic linear equation describing the endogenous variable (say, the interest rate) as a function of non-rational expectations of the future value of this variable and a vector of all relevant exogenous variables assumed to follow AR(1) processes. Agents do not have knowledge about the ALM but they believe that the price dynamics is of the form of the rational expectation equilibrium (REE), of which they seek to learn the unknown parameters recursively using available observations. This gives their initial perceived law of motion (PLM) at, say, time t-1, which agents use to form their new forecasts for t. Given these forecasts, and provided new observations on exogenous variables and new draw for the reduced form stochastic term, a time-t value of the price can then be determined by the ALM (which is in fact referred to as an *implied* ALM due to its self-referential feature depending on the PLM). Using the complete set of new information at time t, agents revise the estimates of their PLM by recursive least squares, which in turn will allow them to form new forecasts for t+1, and so forth. Bray and Savin (1986), Marcet and Sargent (1989), Evans and Honkapohja (2001) show that if the slope of the expected price in the reduced form model is less than one, then the parameters of the PLM converge asymptotically to those of the REE. One can alternatively think the agents in the economy as being Bayesian econometricians instead of classical econometricians. In this case, agents are supposed to have prior beliefs about the distribution of the parameters of the PLM and these beliefs are updated with each realization of the economy via Bayes' rule, so that the updating process also takes a recursive form. In any event, the stochastic dynamic system describes the revision of expectations (measurement equations) along with the adjustment of coefficients (state equations) over time. Following Sargent (1993, 1999), McCulloch (2005) provides theoretical foundations for the adaptive or recursive least squares learning as a special case of the Kalman filter solution of a time-varying parameter model (Harvey, 1992 and Hamilton, 1994). Also, under the conditions that measurement and state equations are linear and noises Gaussian, some authors show that the Kalman filter corresponds to a Bayesian process of revision of parameters as new information is available (Doan et al., 1984; Racette and Raynauld, 1994; Bullard, 1992).[7] Nevertheless, adaptive or Bayesian, all learning models involve two questionable assumptions. First, it is not clear how agents, who are endowed with bounded

[7] An example of study using the Kalman filter methodology to estimate Bayesian updating is McGough (2003) who examines the dynamics of the US inflation.

rationality while they are learning the parameters, can know the economic structure given by the rational expectation solution. Second, these models are concerned with the convergence of the parameters only, letting totally aside the possibility that the PLM may be based on an incomplete set of exogenous variables that itself may evolve over time.[8] This can happen indeed as a result of temporal adjustments in information costs and in agents' forecast error aversions, which are the ingredients of our ERET framework discussed above: decreasing cost/aversion ratios of all types of information allow more and more agents to switch over time from bounded rationality to rationality by accumulating the relevant types and quantities of information and thus enhancing their forecast rules. However, from the investigator's point of view, it clearly is not feasible to specify a PLM whose initial exogenous variables are not necessarily the same as the one of the REE and whose coefficients and variables are modified as new information is processed so as to converge asymptotically to the REE model. We thus voluntarily leave implicit the PLM underlying the formation of expectations and we rather endeavor to describe how the rational expectations component, if any, of the market expectations evolves over time. Inference of this time-varying rational component of aggregate expectations can be made using the Kalman filter methodology, as in the case of adaptive or Bayesian learning processes, although the state variable is different: whereas in standard learning models the state equation describes the convergence of the parameters towards REE, in our model it will represent the dynamics of the proportion of rational agents. A continuous increase in this proportion would then mean that the market learns more and more rationality. To our knowledge, no empirical study using survey data has so far sought to highlight the possible existence of a rational component in aggregate interest rate expectations and the evolution of its weight over time.

Specifying a learning process towards market rationality

Although the ERET and the hypothesis of a learning process toward rationality both describe the forecaster's behavior at the individual level, such a framework is also useful to analyze the aggregate (market) expectation. The market expectation in interest rates is even more relevant to be analyzed as it largely contributes to determine the prices of bonds, as particularly shown by the expectations theory of the term structure of interest rates. With this aim, we will consider the market expectation as represented by the average ("consensus") of experts' opinions about the future value of interest rate provided by surveys. We distinguish at any time two main

[8] Evans and Honkapohja (2001) examine the case of a PLM model whose misspecification results not only from the non-optimal parameters but also from an omitted subset of exogenous variables. They show that the parameters of this model converge to some "restrictive perceptions equilibrium", but not to the REE.

groups of agents in the market expectations, each of them representing a typical behavior. The first group, denoted RAT, represents agents acting as rational forecasters at time *t* while the second group, referred to as LIM, represents agents basing their forecasts on a limited information rule. According to the ERET, the cost/aversion ratios for all type of information are supposed to be negligible for the RAT group, leading agents to use all the relevant information; at the opposite, agents in the LIM group use information essentially related to interest rates, which can be supposed to be costless. In this framework, the market learning process is characterized by a growing number of agents making rational expectations. The development and the upgrowing availability of database services on the one hand and the improvement in forecasting methods and technology on the other hand could explain a downward tendency in the cost/aversion ratio, fostering agents to undertake learning. For a given time-horizon τ and a bond with a maturity θ, the market learning hypothesis can be examined by expressing the aggregate expected change in interest rate $E_{t\,\theta} r_{t+\tau} - {}_{\theta} r_t$ as a time-varying weighted average of expectations made by the groups of rational agents (RAT) and limited information-user agents (LIM):[9]

$$E_{t\,\theta} r_{t+\tau} - {}_{\theta} r_t = (1 - {}_{\theta}\lambda_{t,\tau\,\theta})\, RAT_{t,\tau} + {}_{\theta}\lambda_{t,\tau\ \theta} LIM_{t,\tau} \qquad 0 \le {}_{\theta}\lambda_{t,\tau} \le 1 \ \ \forall t, \tau \qquad (3)$$

where $RAT_{t,\tau}$ and $LIM_{t,\tau}$ stand for the rational agents' and the limited information users' expected changes respectively[10], while ${}_{\theta}\lambda_{t,\tau}$ is the time-varying weight associated to the latter group. If ${}_{\theta}\lambda_{t,\tau} = 1$, the market expectation is only determined on rules based on information limited to the bond market, while if ${}_{\theta}\lambda_{t,\tau} = 0$ the market expectation is based on all the relevant information, making the market expectations rational. In the case $0 < {}_{\theta}\lambda_{t,\tau} < 1$ and when the weigh ${}_{\theta}\lambda_{t,\tau}$ is decreasing over time, one can consider that the market is involved in a learning process with a growing proportion of rational forecasters, and this also means that there is a decreasing proportion of forecasters of group LIM.[11]

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[9] Prat and Uctum (2015) used a similar approach to model foreign exchange expectations.

[10] The RAT component may or may not share information imbedded in the LIM component. We will show below that there is a weak empirical correlation between these two components whatever θ and τ (see footnote (14)).

[11] A growing relevant information used by agents at the individual level could also lead ${}_{\theta}\lambda_{t,\tau}$ to increase since this would mean that forecasters at a whole include increasingly the relevant information underlying REH. However, to capture in a rigorous manner this informational effect, it would be necessary to introduce explicitly the set of the relevant variables corresponding to REH and to affect to each of them a time-varying coefficient. This is not a feasible specification especially regarding the difficulty of defining the set of all relevant variables and with respect to the large number of state variables that should be estimated.

3. Specifying the rational and limited information components

We now turn to the questions of how to represent the components ${}_{\theta}RAT_{t,\tau}$ and ${}_{\theta}LIM_{t,\tau}$ in equation (3). The first component can be represented by the ex-post interest rate plus a white noise error ${}_{\theta}\xi_{t+\tau}$, which is $N(0, {}_{\theta}\nu_{\xi,\tau})$ and which represents a forecast error that is uncorrelated with any observable variable:

$$ {}_{\theta}RAT_{t,\tau} = {}_{\theta}r_{t+\tau} - {}_{\theta}r_t + {}_{\theta}\xi_{t+\tau} \tag{4} $$

The ${}_{\theta}LIM_{t,\tau}$ component is expressed as a linear combination of the standard extrapolative, adaptive, regressive and forward-market processes that have been considered in the literature devoted to interest rate expectations (see introduction). For a maturity θ of the debt and a horizon of expectation τ, these four simple expectation rules are denoted as ${}_{\theta}EXT_{t,\tau}$, ${}_{\theta}ADA_{t,\tau}$, ${}_{\theta}REG_{t,\tau}$ and ${}_{\theta}FOR_{t,\tau}$, respectively. The choice of the forecaster to use any of these rules depends on their perceived law of motion concerning the dynamics of interest rates.[12] For example, if the change in the interest rate is perceived as a sequence of stochastic observed shocks, it can be shown that the adaptive process is the relevant forecast rule (Muth, 1960). If, alternatively, change in the interest rate is perceived as an autoregressive process, the relevant expectation rule will be of the extrapolative form (Baillie and MacMahon, 1992). If interest rate is perceived as exhibiting a mean-reversion feature towards a given level considered as "normal", the forecaster will base their forecasts on a regressive expectation rule (Holden et al., 1985). If the forward interest rate premium is perceived to be a leading indicator of future change in interest rates, the forecaster will consider this premium as a guidance to form their expectation. Of course, the law of motion perceived by a forecaster may be a combination of the preceding types of dynamics, so that s/he would be stimulated to employ a mixing of the four components. The fact that any forecaster may use a basic rule or a combination of rules supports the relevance of a mixed expectation process at the aggregate level. However, all the rules comprised in the LIM component are essentially based on observable information that is directly linked to the debt market: actual and lagged observed interest rates and lagged expected rates. This allows us to

[12] In any case, if the perceived dynamics corresponds to the true one, this will allow for forming rational expectations since the expectation process would be optimal in the sense that the forecast error variance is minimal. Of course, if the perceived dynamics does not reflect the true one, expectations generated based on these dynamics are biased.

assume that the cost/aversion ratio corresponding to the use of each rule is stable over time, implying that the coefficients associated with each rule is constant.

We now specify the basic rules in the component ${}_{\theta}LIM_{t,\tau}$. Let ${}_{\theta}r_t$ be the interest rate at time *t* served to a debt with maturity $\theta = S, L$, where *S* stands for the short term maturity (*S* = 3 months) and *L* for the long term maturity (*L* = 10 years). The *extrapolative component* is defined as a linear function of the past changes in the interest rate. For the short and long term interest rates and for the two horizons of expectations ($\tau =$ 3 and 12 months), preliminary results led us to select three lags whose parameters were found to be non-significantly different to each other, so that our extrapolative component is given by the change observed during the last three months:

$$ {}_{\theta}EXT_{t,\tau} = {}_{\theta}a_{\tau}\ ({}_{\theta}r_t - {}_{\theta}r_{t-3}) \tag{5} $$

Although the theoretical sign of the parameter ${}_{\theta}a_{\tau}$ is more likely to be positive ("bandwagon" effect), a negative value is conceivable in the extent that it can reflect a naive regressive process, describing a systematic turning tendency or “contrarian” expectations

The *adaptive component* is proportional to the last observable forecast error, that is the difference between the interest rate forecast generated τ months before and the observed rate. But it is possible, indeed very likely, that experts will not wait until the τ-month horizon is completed to revise their expectations. When, during the survey procedure, the interest rate at the beginning of the month is known, the individuals will most probably compare this rate to the interest rate which they had expected during the last survey, i.e., a month before, and not τ-months before as the standard adaptive model assumes.[13] Our adaptive component based on an “early revision” mechanism of forecast errors is therefore written as:

$$ {}_{\theta}ADA_{t,\tau} = {}_{\theta}b_{\tau}\,(E_{t-1}\ {}_{\theta}r_{t+\tau-1} - {}_{\theta}r_t) \qquad 0 < {}_{\theta}b_{\tau} < 1 \tag{6} $$

The *regressive component* states that the expected change in interest rate depends on the deviation between the long-run target value ${}_{\theta}\bar{r}_t$ and the actual rate, such that

[13] For a given interest rate ${}_{\theta}r_t$, the standard adaptive model $E_t\,{}_{\theta}r_{t+\tau} - r_t = {}_{\theta}b_{\tau}(E_{t-\tau}\,{}_{\theta}r_t - {}_{\theta}r_t)$ defines the expected value $E_t\,{}_{\theta}r_{t+\tau}$ as a weighted average of ${}_{\theta}r_t$, ${}_{\theta}r_{t-\tau}$, ${}_{\theta}r_{t-2\tau}$,…, with exponentially decreasing weights. For monthly survey data and for all $\tau > 1$ month, it is unlikely that agents refer to a weighted average of observations spaced by τ months. Such a hypothesis is all the more unrealistic that our τ values are 3 and 12 months while our data have a monthly frequency. Our early revision model defines $E_t\,{}_{\theta}r_{t+\tau}$ as a weighted average of actual and past monthly values of ${}_{\theta}r_t$, which seems more appropriate with our data.

$$ {}_{\theta}REG_{t,\tau} = {}_{\theta}c_{\tau}({}_{\theta}\bar{r}_t - {}_{\theta}r_t) \tag{7} $$

In the standard form of the regressive component, we have $0 < {}_{\theta}c_{\tau} < 1$. When the interest rate is undervalued (overvalued) with respect to its target level, forecasters who believe in a temporary misalignment expect that the rate will follow a mean–reverting path and therefore will increase (decrease). Nevertheless, the case ${}_{\theta}c_{\tau} < 0$ is also allowed, which indicates that a majority of forecasters expect a deviation from the target value to be amplified in the same direction (see Ellen et al, 2013 for exchange rate expectations). This characterizes an explosive process in expectation formation over the horizon τ, after which beliefs can be reversed. We assume that the target value for the short term interest rate ${}_{S}\bar{r}_t$ depends on the long term interest rate ${}_{L}r_t$; in accordance to the results of the literature (see introduction), this means that the term spread of interest rates intervenes in the expectations formation. After preliminary analysis, we posit that the target value of the long term interest rate is determined by the expected rate of inflation and expected production growth rate (GNP and industrial production).

We finally consider a fourth expectation rule based on the observed forward premium since, as suggested by empirical studies of the literature, the expected change in interest rate is found to be significantly linked with this premium. The *forward market component* can be written as follows:

$$ {}_{\theta}FOR_{t,\tau} = {}_{\theta}d_{\tau}({}_{\theta}f_{t,\tau} - {}_{\theta}r_t) \qquad {}_{\theta}d_{t,\tau} \geq 0 \tag{8} $$

where ${}_{\theta}f_{t,\tau}$ is the τ-month ahead forward interest rate on a θ-maturity debt. It is worth noting that the forward market component may describe a kind of mimetic behaviour: to form their opinion, a significant number of agents may rely on a common knowledge market opinion about the future value of the short rate represented by the forward rate. This component will be considered only for the short term rate (i.e. ${}_{\theta}\delta_{\tau} = 0$ for $\theta = L$) since it depends on monetary policy expectations (see introduction). Indeed, contrarily to short term rates, the market of futures in long term interest rates relates to notional bonds and not to the bonds quoted on the spot market. Moreover, the difference between the maturity ($\theta = L = 10$ years) and the horizons of expectations (τ = 3 or 12 months) is much too large for the presence of a forward component to make sense.

Weighting rules (5) to (8), we obtain the following mixed LIM equation:

$$ {}_{\theta}LIM_{t,\tau} = {}_{\theta}\kappa_{\tau} + {}_{\theta}\alpha_{\tau}({}_{\theta}r_t - {}_{\theta}r_{t-3}) + {}_{\theta}\beta_{\tau}(E_{t-1}\ {}_{\theta}r_{t+\tau-1} - {}_{\theta}r_t) + {}_{\theta}\gamma_{\tau}({}_{\theta}\bar{r}_t - {}_{\theta}r_t) + {}_{\theta}\delta_{\tau}({}_{\theta}f_{t,\tau} - {}_{\theta}r_t) \quad (9) $$

where ${}_{\theta}\kappa_{\tau}$ is an intercept representing a possible systematic bias and ${}_{\theta}\alpha_{\tau}$, ${}_{\theta}\beta_{\tau}$, ${}_{\theta}\gamma_{\tau}$ and ${}_{\theta}\delta_{\tau}$ are composite coefficients equal to the structural parameters of the basic expectation rules (${}_{\theta}a_{\tau}$, ${}_{\theta}b_{\tau}$, ${}_{\theta}c_{\tau}$ and ${}_{\theta}d_{\tau}$) times the weighting coefficients associated with the rules. Note that, by construction, these composite coefficients do not sum to 1.

Reporting the rational expectation component (4) and the limited information expectations component (9) in equation (3) yields to the aggregate expectation model to be estimated: [14]

$$ \begin{aligned} & E_t\,{}_{\theta}r_{t+\tau} - {}_{\theta}r_t = {}_{\theta}\kappa_{\tau} + (1 - {}_{\theta}\lambda_{t,\tau})\,({}_{\theta}r_{t+\tau} - {}_{\theta}r_t + {}_{\theta}\hat{\xi}_{t+\tau}) \\ & + {}_{\theta}\lambda_{t,\tau}\left\{{}_{\theta}\alpha_{\tau}({}_{\theta}r_t - {}_{\theta}r_{t-3}) + {}_{\theta}\beta_{\tau}(E_{t-1}\ {}_{\theta}r_{t+\tau-1} - {}_{\theta}r_t) + {}_{\theta}\gamma_{\tau}({}_{\theta}\bar{r}_t - {}_{\theta}r) + {}_{\theta}\delta_{\tau}({}_{\theta}f_{t,\tau} - {}_{\theta}r_t)\right\} + {}_{\theta}\varepsilon_{t,\tau} \end{aligned} \quad (10) $$

with $\theta = S, L$ [15], ${}_{\theta}\varepsilon_{t,\tau}$ a *Nid* error term with mean zero and constant variance. ${}_{\theta}\lambda_{t,\tau}$ is a continuous time-varying parameter such that $0 \leq {}_{\theta}\lambda_{t,\tau} \leq 1$; the more it approaches zero, the more the market learns rationality.

4. The data

Our empirical analysis covers the period November 1989 – May 2015. At the beginning of each month, *Consensus Economics* (CE, London) asks about 200 economists, financial market operators and executives in various institutions (commercial and investment banks, forecasting agencies and industrial corporations) in over 30 countries to forecast future values of principal

[14] The $\bar{R}^2$ of the regression of ${}_{\theta}RAT_{t,\tau}$ over the set of the components of ${}_{\theta}LIM_{t,\tau}$ for horizons $\tau = (3, 12$ months) are (0.19, 0.35) and (0.02, 0.13) in the case of the short and long term interest rates, respectively. This shows that rational agents and limited information-based forecasters share little common information. See Sethi and Franke (1995) for a similar approach based on evolutionary competition between adaptive and rational forecasters in the foreign exchange markets. Also, using a model with constant weights, Heinemann and Ullrich (2006) find evidence of complementarity between adaptive and rational expectations in inflation expectation formation.

[15] According to the expectations hypothesis of the term structure, the long term interest rate is a weighted average of the actual and expected values of the short rates over the maturity of the long term bond (Shiller et al, 1983). In our model, because the long term rate appears among the exogenous variables in the equation of expected change in long term interest rates (10), there exists an implicit link between the dependent variable and actual and expected changes in short rates.

macroeconomic variables for the three and the twelve month horizons. The short rate is represented by the US 3-month Treasury bill rate while the long rate is represented by the 10-year constant to maturity Treasury bonds yield. The CE newsletter gives every month the "consensus" corresponding to the arithmetic average of individual expected values of these two variables, denoted $E_{t\,\theta} r_{t+\tau}$, where τ stands for the 3 and 12 month time-horizons and θ the short (3 months) and long (10 years) maturities. About 30 financial institutions are asked to predict these two variables. These institutions are, by their own activity, directly concerned by forecasting US interest rates and include essentially major American banks (Bank of America, Goldman Sachs, Barclays, Wells Fargo, JP Morgan, Northern Trust…), investment advisory firms (First Trust Advisor, Wells Capital Management…), research organizations or academic institutions (The Conference Board, Moody's Analytics, RDQ Economics, Georgia State University, University of Michigan, University of Maryland…), and industrial companies (General Motors, Eaton Corporation…). The experts answer only when they think they have a good knowledge about the variable of interest, and this allows assuming that those who respond are informed agents. Since the individual answers are confidential (only the consensus is disclosed to the public, with a time lag) and since each individual is negligible within the consensus, it is difficult to claim that, for reasons which are inherent to speculative games, individuals might not reveal their « true » opinion. Note that these considerations only suggest that the responses are not distorted but they do not imply that the consensus represents an unbiased proxy of market expectations. However, regarding the existence of forward interest rates, one can argue that there is an incentive for experts to compare their expected rate to the forward rate as a guideline. Doing so, they introduce a clear distinction between their expectations and the forward rates. Finally, to interpret the consensus as a market expectation, we only need to suppose that the latter equals the former plus an intercept and a white noise representing the systematic and random terms of the measurement error, respectively. For all these reasons, one can reasonably assume that the expectations provided by the respondent experts are representative of market expectations.

On the other hand, about half of the respondents remain unchanged over the period. The turnover in the other half can therefore lead to a bias due to a lack of homogeneity in the average responses over time. However, this bias can be considered as being negligible regarding the fairly moderate dispersion of the opinions. Table 1 presents the descriptive statistics of the coefficients of variation of experts' forecasts (defined as the ratios of the standard-deviations of the responses to their means at a given time) for the two maturities and for the two horizons. As

the mean values do not exceed 0.12, one can conclude that the heterogeneity of individual expectations is moderate enough for the aggregation not to raise serious problems.

[Insert Table 1]

The CE requires a very specific day for the answers. As a rule, this day is the same for all respondents.[16] Accordingly, we consider actual interest rates ${}_{\theta}r_t$ ($\theta = S, L$) at the same day as the expected values $E_{t\,\theta}r_{t+\tau}$ (τ=3, 12 months). Actual values of the 3-month bills rate and the 10 year Treasury bond yield are directly published in the CE bulletin while interest rates for other maturities used to calculate forward interest rates are extracted from the Board of Governors of the US Federal reserve System at a daily frequency. We also consider the expected values of the rate of inflation calculated using the CPI (p_t) and the growth rate of real GNP (g_t) or of the industrial production (q_t), that are provided by CE surveys (arithmetic averages). For each variable, CE provides two expected values for different time horizons. The first set of values comprises the expected rates for the current remaining year but that are revised at each month of the same year ($E_t(p_{t+c})$, $E_t(g_{t+c})$ and $E_t(q_{t+c})$). The second set of values provides at each month of the year the expected values for the following year ($E_t(p_{t+f})$, $E_t(g_{t+f})$ and $E_t(q_{t+f})$).

[Insert Figure 1]

Figure 1 displays the time series of the short and long term interest rates ${}_{\theta}r_t$: it can be seen that the evolutions of these two variables differ substantially - especially after the severe and long lasting financial turmoil that began in summer 2007 with the subprime crisis. Figure 2 and Figure 3 compare, for the short and long term interest rates respectively, the expected changes $E_{t\,\theta}r_{t+\tau} - {}_{\theta}r_t$ for horizons τ = 3-month and τ =12-months: it can be seen that expected changes exhibit stark differences according to horizons. These time patterns raise the question whether the formation of expectations may or may not be characterized by similar processes both according to the maturity of the debt and to the horizon of expectations.

[16] This day is the first Monday of the month until March 1994, and the second Monday since April 1994, except the closed days (in this last case, the survey is dated at the following day). The effective horizons however always remain equal to 3 and 12 months. If, for instance, the answers are due on the 3rd of May (which was the case in May 1993), the future values are asked for August 3, 1993 (3 months ahead expectations) and for January 3, 1994 (12 months ahead expectations). The individual responses are then concentrated on the same day.

[Insert Figure 2]

[Insert Figure 3]

5. Empirical results

5.1 – Lessons from the standard unbiasedness test

Our first concern is to check whether CE experts form their expectations rationally in average over the whole period. Testing for the REH requires performing unbiasedness and orthogonality tests. If the null of unbiasedness is rejected, then there is no need to check for orthogonality to conclude that expectations are not rational. The test equation is the following:

$$E_t\,{}_\theta r_{t+\tau} - {}_\theta r_t = {}_\theta\omega_\tau + {}_\theta\varphi_\tau({}_\theta r_{t+\tau} - {}_\theta r_t) + {}_\theta\upsilon_{t+\tau} \qquad (11)$$

where the conditions ${}_\theta\omega_\tau = 0$ and ${}_\theta\varphi_\tau = 1$ must be jointly satisfied and ${}_\theta\upsilon_{\xi+\tau}$ must be white noise under the null. In addition, since our 3- and 12-month ahead expectations are observed at a monthly frequency, an overlapping data bias may affect the OLS variance-covariance matrix of estimates. In order to adjust the OLS standard errors in the presence of overlapping data, Estrella and Hardouvelis (1991) suggest using the Newey-West methodology, which is robust to autocorrelation and heteroskedasticity in residuals. In testing (9) we follow their suggestion and estimate parameters ω_τ and φ_τ accordingly. Table 2 provides the unbiasedness test results.

[Insert Table 2]

For the short term and long term interest rates, the Wald test strongly rejects for the two horizons the hypothesis that $(\omega_\tau, \varphi_\tau)$ is jointly equal to (0,1), i.e. that expectations of interest rates are rational over the period.[17] However, although the null of unbiasedness is rejected, the slopes φ_τ are significantly positive except for the 3-month ahead expected change in long term interest rate. These results suggest that some rationality may occur during sub-periods, as it would be the case if a learning process towards rationality was in action in the bond market. In

[17] This result is consistent with Muth's (1985) findings that firms' production forecasts, provided by a survey of current business prospects (University of Pittsburgh), are not rational. Rather, the author finds evidence that firms' expectations are explained by an adaptive-type rule. These conclusions take on an increased significance as they emanate from the pioneer of the rational expectations hypothesis.

fact, in all cases, if a learning process is ongoing over time, a test with a constant slope is a priori inappropriate. Moreover, the residuals of the unbiasedness tests are far from being a white noise (the DW statistics indicate substantial autocorrelation) and this suggests that these residuals reflect some factors of market expectations that are omitted in the regression, especially due to the group LIM's expectations. For these reasons, we now propose a new test based on equation (10) allowing both to describe a possible evolution of market expectations towards rationality and to capture other factors of expectations through the LIM component.

5.2 - Testing a rational component in market expectations

The weights associated with the groups of forecasters RAT and LIM are constant

For each interest rate, we estimate a system of two equations (10) reflecting the 3- and 12-month horizons, under the condition ${}_{\theta}\lambda_{t,\tau} = {}_{\theta}\lambda_{o,\tau}\ \forall t, \theta, \tau$. Note that the error terms for each horizon are likely to be contemporaneously correlated since the model includes common variables for the two horizons. Moreover, to account for the possible autocorrelation of residuals resulting from overlapping bias as our 3- and 12-month horizon expectations data are observed at the one-month frequency, we estimate our two-equation model using the system GMM method with heteroskedasticity and autocorrelation consistent (HAC) covariance matrix. The system GMM-HAC method has also the advantage of avoiding simultaneity bias due to the fact that expectations depend on actual interest rates and the latter depends on expected rates. The forward rate for the 3-month rate and the 3-month horizon is given by ${}_{3}f_{t,3} = 2\ {}_{6}r_t - {}_{3}r_t$ corresponding to the 3-month ahead forward interest rate on the 3 months maturity bill. For the 12-month horizon, the maturities of bills do not allow a rigorous measure of the 12-month ahead forward interest rate on the 3 month maturity bill. We thus considered the approximation ${}_{3}f_{t,12} \approx {}_{12}f_{t,12} = 2\ {}_{24}r_t - {}_{12}r_t$. In fact, by regressing ${}_{3}r_t$ on ${}_{12}r_t$ using the Newey-West method, a Wald test showed that the joint hypothesis of a slope equal to 1 and an intercept equal to 0 is acceptable at the 5% level with $R^2 = 0.983$; if we suppose that this result for the spot rates prevails also for the forward rates, we can admit that the bias due to the measurement error in ${}_{3}f_{t,12}$ is likely to be negligible. Concerning the representations of the target ${}_{\theta}\bar{r}_t$ in the regressive processes, we supposed ${}_{S}\bar{r}_t = {}_{L}r_t + h_S$ and ${}_{L}\bar{r}_t = {}_{L}\hat{r}_t + h_L$, where h_S and h_L stand for target calibration constants to be estimated whereas ${}_{L}\hat{r}_t$ was obtained as the fitted values of the

bond yield ${}_{L}r_t$ from its regression on the expected rate of inflation and expected rates of growth in GNP and in industrial production.[18]

Table 3 presents the GMM estimation results of equation (10) for the short and long term interest rates and the two horizons, where the intercept ${}_{\theta}\kappa_{\tau}$, which was systematically found to be insignificant, has been omitted.

[Insert Table 3]

The estimated values of ${}_{\theta}\hat{\lambda}_{0,\tau}$ are found to be significantly positive whatever the maturity of the debt and the horizon. This result strongly confirms that aggregate expectations in interest rates are not rational. However, the results are contrasted in that ${}_{\theta}\hat{\lambda}_{0,\tau}$ significantly lies between 0 and 1 for the short term interest rate whatever the horizon whereas for the long term rate it is not significantly different from 1 whatever the horizon. These conflicting outcomes can be explained as follows. Contrary to the long term interest rate, the short term interest rate is directly dependent on open market operations based on the Fed funds rate, so that monetary policy targeting announcements released by the Fed may help agents to forecast the future values of the short term interest rate. Related episodes, where the dynamics of the interest rate were rather regular, can be observed especially during the trending sub-periods 1990.07-1992.10, 2001, 2004.04-2006.07, 2007.07-2008.04, during the sub-period of stability 1996.02-1998.08 and after the late 2008 where the interest rate was close to zero (Figure 1). Such periods makes the short rate more likely to be rationally forecasted. Things are less clear-cut for the long rate: even though the quantitative easing policies conducted by the Fed since 2008 may have led agents to expect a decrease in long term rates, our results suggest that they could not forecast the latter rationally.

Concerning the ${}_{\theta}LIM_{t,\tau}$ component of the short and long term interest rates, all parameters are significant for the rules at the 5% level. Estimates have the positive expected sign for the adaptive rules and the same result holds for the forward component in the short term

[18] Using the Newey-West method, which provides standard error of estimate that are robust to residual autocorrelation and hereroskedasticity, the estimated target equation for the long term interest rate is ${}_{L}\hat{r}_t = \underset{(4.9)}{0.66}\left[E_t(g_{t+c}) - E_t(q_{t+c})\right] + \underset{(4.5)}{0.36}E_t(q_{t+c}) + \underset{(17.3)}{2.26}E_t(p_{t+f}) - \underset{(-4.1)}{1.84}$ ($R^2 = 0.804$), where g_t, q_t and p_t stand for the rate of change in the GNP, the industrial production IP and the CPI, respectively. The expected values of these three rates of change are issued from Consensus Economics' surveys. The expected growth rate of the ratio GNP/IP can be viewed as representing separately the influence of the non-industrial production sector. It can be seen that the three exogenous variables have the positive traditional expected sign. The ADF test (not reported) shows that residuals are stationary at the 1% level, implying that ${}_{L}\hat{r}_t$ can statistically be considered as a target.

interest rate equation. It can be noted that the parameters of the extrapolative rules, which we argued that they can take any sign, indicate that the forecasters have used these processes consistently with the conventional bandwagon behavior, whatever the horizon and the maturity of the debt. Concerning the mean-reverting strategy, the positive values of estimates found for the long term interest rate indicate for the two horizons that the forecasters have used this rule consistently with the conventional regressive behavior; for the short rate, the estimate is positive for $\tau = 12$ but negative for $\tau = 3$, hence suggesting that a majority of forecasters expect a deviation from the target value to be amplified in the short run before starting a mean reversion. Overall, whatever the maturity of the debt, the results suggest that the rules based on limited information play a central role in the formation of expectations over our sample period.

The findings presented in Table 3 do not prove that there is no learning, since they do not prove that the weight between the group of rational forecasters and those using a limited information is stable over time. For example, the significance of the rational component obtained on average for the short term rate could be due to the polar case that no rationality existed at the beginning of the period but the market became rational at the end of the period. For this reason, the relaxation of the time independence assumption for this weight is necessary to explore the market learning hypothesis, and this is consistent with the theoretical analysis developed in section 2, since the cost/aversion ratios can themselves change over time. More generally, if the weight between the RAT and the LIM components were time-varying, the hypothesis of constant values for the parameter ${}_{\theta}\lambda_{0,\tau}$ (Table 3) would lead to biased estimates in the aggregate expectation model (10). It would especially explain the lack of significance of the long term interest rate rational component.

In order to check for the existence of such biases, we next examined the statistical properties of the residuals of the model. The system residual Portmanteau test suggests that residuals are autocorrelated at any level of significance for the two maturities of the debt. Moreover, the K statistic indicates an excess kurtosis with respect to normality which may be due to system heteroskedasticity. To gauge the evidence of a time-varying weight, we performed Wald tests which unambiguously rejected the null of stability. Overall, these results show that the residuals are not well-behaved for both interest rates. These undesirable properties of residuals may be due to the overlapping feature of our data or/and to the instability of ${}_{\theta}\lambda_{0,\tau}$, and this strongly points towards the relaxation of the hypothesis of a constant weight between the RAT and the LIM components.

The weights associated with the groups of forecasters RAT and LIM are time-varying

In this section, the weight ${}_{\theta}\lambda_{t,\tau}$ in Eq. (10) is assumed to evolve according to a stochastic dynamics and the model is given a state-space representation that can be estimated using the Kalman filter methodology. More specifically, we assume that the weight for each horizon can be represented as an unobservable stochastic variable drawn from an AR process, whose innovations reflect the influence of the change in the cost/aversion ratio upon the revision of the coefficient. These innovations are assumed to be drawn from Gaussian white noise, the variance of which belongs to the set of hyperparameters to be estimated. For each interest rate, we then estimate our two horizons aggregate expectation model in the form of a four equation state-space representation consisting in two equations (10) representing the measurement equations and two AR(1) equations for ${}_{\theta}\lambda_{t,\tau}$ representing the state equations:

$$ {}_{\theta}\lambda_{t,\tau} = {}_{\theta}\omega_{0,\tau} + {}_{\theta}\omega_{1,\tau}\,{}_{\theta}\lambda_{t-1,\tau} + {}_{\theta}\eta_{t,\tau} \qquad \theta = S, L \qquad \tau = 3, 12 \qquad (12) $$

where ${}_{\theta}\eta_{t,\tau} = N(0, {}_{\theta}\sigma_{t,\tau})$ and $E({}_{\theta}\eta_{t,3}, {}_{\theta}\eta_{12}) = {}_{\theta}\rho$. We determined the value of the standard error ${}_{\theta}\nu_{\xi,\tau}$ of the rational forecast error (see Eq. (4)) by fitting an ARMA(p, q) on the ex-post change in interest rate ${}_{\theta}r_{t+\tau} - {}_{\theta}r_t$. We gave to ${}_{\theta}\nu_{\xi,\tau}$ the value of the residual standard error of the optimal ARMA obtained for p=q=1. We checked for the robustness of this value using a grid search over ${}_{\theta}\nu_{\xi,\tau}$ to which we assigned different values from zero (i.e. the case of perfect foresight) to a reasonably high limit value provided by the standard deviation of the ex-post change in interest rate ${}_{\theta}r_{t+\tau} - {}_{\theta}r_t$. We found that the estimates of the signal equations for each of these values were not significantly different from those obtained when ${}_{\theta}\nu_{\xi,\tau}$ is measured by the standard error of the ARMA(1,1). Furthermore, we accounted for the overlapping bias resulting from the difference between our expectation horizons and frequency of observations by introducing a moving average specification of order $\tau - 1$ for the residuals (Hansen and Hodrick, 1980), that is, 2 lagged residuals for the 3-month horizon and 11 lagged residuals for the 12-month horizon in equation (10). The moving average terms were generally found to be significant, indicating that our estimates would have been biased if the overlapping problem was not accounted for. Table 4 provides for the short and the long term interest rates the estimates of the state-space representation for the 3-month and 12-months horizons. In this paper we are interested in a structural interpretation of the model, the values of the measurement and state variables are therefore calculated at each time using the whole sample of observations (smoothed

inference) rather than only past observations (filtered inference). The results show that the estimates for the limited information rules remain significant with the same signs as those obtained with the GMM method. Concerning the state equations and whatever θ and τ, the drifts ${}_{\theta}\omega_{0,\tau}$ were systematically found to be insignificantly different from zero (they were thus excluded at the final estimations), while the slopes ${}_{\theta}\omega_{1,\tau}$ appear to be insignificantly different from one indicating that ${}_{\theta}\lambda_{t,\tau}$'s follow random walk processes. Note that, for each of the two interest rates, the covariance ${}_{\theta}\rho$ of the measurement equation residuals between the two horizons is very significant, justifying ex-post the relevance of jointly estimating our model for the two horizons. We further checked the relevance of the fits using Harvey's (1992) modified coefficient of determination R_D^2 that assesses the goodness of the fit with respect to a simple random walk plus drift process.[19] In the case of short term interest rate, the R_D^2 values indicate that the residual variance of the signal equation is 0.21 and 0.30 times the one of the random walk model for the 3-month and 12-month horizons, respectively. The corresponding proportions for the long term interest rate are 0.26 and 0.24, respectively. These results imply that our aggregate expectation model with time varying weights strongly outperforms the random walk. To check now that our time-varying weights model also allows for better fits than the constant-weights model, we perform the likelihood ratio test $LR = 2[L(\hat{\Theta}) - L(\tilde{\Theta})]$ that is distributed as a χ_m^2, where $\hat{\Theta}$, $\tilde{\Theta}$ and *m* are the vector of parameters of the unrestricted model (time-varying weights model), the vector of parameters of the restricted model (constant weights model) and the number of restrictions[20], respectively. The test statistic *LR* equals 57.94 in the case of short term interest rate and 33.30 in the case of the long term interest rate. These values, when compared to the statistic χ_{18}^2 that is equal to 34.8 at the 1% level, indicate that the time-varying parameters model (10) significantly outperforms the constant-parameters model for the short term rate but not for the long term rate.

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[19] Harvey's (1992) goodness of fit measure is given by $R_D^2 = 1 - SSR \,/ \sum_{t=2}^{T} (\Delta y_t - \overline{\Delta y})^2$ where y_t and *SSR* are the dependent variable and the sum of the squared residuals of the measurement equation, respectively. A negative R_D^2 would imply that the estimated model is beaten by a simple random walk plus drift.

[20] The restrictions allow for reducing the time-varying parameters model into constant parameters model. Setting to zero the residual variances of the state variables (i.e. setting ${}_{\theta}k'_{\tau} = +\infty$, $\theta = S, L$, $\tau = 3, 12$), the slope parameters in the state equations and the coefficients of MA residual terms fulfills this condition and leads to *m*=18.

Figures 4 and 5 display the time-patterns of the parameters ${}_S\hat{\lambda}_{t,\tau}$ (short term interest rate) and ${}_L\hat{\lambda}_{t,\tau}$ (long term interest rate) for $\tau = 3, 12$ months. Since these parameters do not exhibit a decreasing trend, we can conclude that aggregate expectations fail to describe a learning process towards rationality for the two interest rates. For the long term rate, it can be seen that the state variables ${}_L\hat{\lambda}_{t,3}$ and ${}_L\hat{\lambda}_{t,12}$ include uninterruptedly the line 1 into their 95% confidence bounds, which implies that the hypothesis that the rational component never contributes in generating forecasts at any points in time is acceptable (Figure 5). This result confirms the one of the insignificance of the constant weight over the period. On the contrary, for the short term rate and the 3-month horizon, ${}_S\hat{\lambda}_{t,3}$ is found to be significantly smaller than 1 (Figure 4a), suggesting the existence of a significant group of rational agents in the market at any time (in other words, the absence of a rational group of agents is statistically rejected at the 5% level). Nevertheless, it can be seen that the estimated values of ${}_S\hat{\lambda}_{t,3}$ are rather stable over time, which contradicts the existence of a learning effect. Concerning the 12-month horizon, the values of ${}_S\hat{\lambda}_{t,12}$ includes uninterruptedly the line 1 into the 95% confidence bounds after June 1993, indicating that the hypothesis of the absence of a rational group of agents is acceptable for most of the period (Figure 4b). By contrast, before this date, the values are significantly smaller than 1, suggesting that the hypothesis of absence of rational agents is statistically not acceptable. Moreover, if we omit the period of the global financial crisis (2007-2008), values less than 1 are also plausible, suggesting the existence of a rational group of agents consistently with the results of the constant weights model (GMM). Overall, these results are in favor of rationality over most of the period. However, a striking feature of the time-pattern of ${}_S\hat{\lambda}_{t,12}$ is that its increasing trend contradicts the learning hypothesis.

[Insert Table 4]

[Insert Figure 4]

[Insert Figure 5]

Figure 6 compares, for the two horizons, the observed values of the expected change in the short term interest rate with the fitted values from the state-space representation estimated using the Kalman filter (Table 4). For both horizons, major fluctuations are well reproduced and it can especially be seen that there is no systematic lag between observed and fitted values.

Figure 7 compares the observed and calculated expected changes in the long term interest rate for both horizons:[21] again, we can conclude that our model allowed for rather good fits.

[Insert Figure 6]

[Insert Figure 7]

6. Conclusion

Using Consensus Economics survey data on US 3-month bill rate and 10 years Treasury bonds yield expectations for the 3- and 12-month horizons over the period November 1989 – May 2015, we estimate a market expectation process that combines expectations made by a group of rational forecasters and a group of forecasters relying on a combination of limited information rules, that are the traditional extrapolative, adaptive, regressive and forward-market rules. We allow the weights between the two groups to be time-varying, so that our model can account for a possible learning process towards rationality. Regarding the standard unbiasedness test, we find that market interest rate expectations for both horizons and both maturities are not rational on average over the period. Kalman filter inference of a heterogenous expectations model with time-varying weights between a group of rational agents and a group of limited information-based forecasters do not reveal either the evidence of learning process towards rationality. However, we find evidence that a group of agents forecast rationally the short term interest rate over the whole period at the 3-month horizon, and over a part of the period when the forecast horizon is of 12 months. By contrast, no group of agents rationally forecasts the long term rate, whatever the horizon. We interpret these contrasting results between long and short term interest rates as (i) the effect of Federal Reserve's monetary policy announcements scrutinized by financial investors prior to forming their short term interest rate expectations, and (ii) the effect of periods of trended swings in the short rate, making this rate easier to forecast compared to the long rate. Overall, our results strongly suggest that experts form their forecasts in both the short and the long term interest rates by basically using a linear combination of the four traditional rules. We show that this is consistent with the economically rational expectations theory which states that information costs and agents' aversion to misestimating future interest rates determine the optimal amounts of information on which they base their expectations.

[21] To avoid noisy variations in the fits, the fitted values were calculated after estimating model (10) under the restriction ${}_L\hat{\lambda}_{t,\tau}$ =1. The resulting constant parameter estimates in the LIM component were not significantly different from those estimated in the model with no restriction on ${}_L\lambda_{t,\tau}$ (Table 4), which is in accordance with the result that the latter is not significantly different from 1 at any time.

However, relaxing our assumption of time-invariance of the weights between these limited information forecast rules – which, in our rational learning hypothesis framework, do not question our main results - seems to be a natural extension of this study to understand the formation of interest rate expectations.

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Table 1. Descriptive statistics of expected changes in US short and long term interest rates provided by Consensus Economics surveys

	$E_{t\ S} r_{t+\tau} - r_t$		$E_{t\ L} r_{t+\tau} - r_t$	
τ	3 months	12 months	3 months	12 months
Minimum	0.013	0.032	0.012	0.027
Maximum	0.300	0.333	0.222	0.250
Mean	0.067	0.113	0.056	0.086
Number of observations	224	224	303	303

Notes. The statistics have been calculated over the period 1989.10 – 2008.09 so as to avoid distortions due to the very low values of short term interest rates in the recent period. τ is the time horizon of expectations. *S* and *L* denote the short term interest rate (US 3-month Treasury bill rate) and the long term interest rate (10 years Treasury bond yield), respectively.

Table 2. Unbiasedness tests

	$E_{t\ S} r_{t+\tau} - r_t$		$E_{t\ L} r_{t+\tau} - r_t$	
τ	3 months	12 months	3 months	12 months
ω	0.11 (3.58)	0.45 (4.40)	0.11 (4.32)	0.40 (4.75)
φ	0.16 (2.18)	0.14 (2.39)	-0.004 (-0.12)	0.09 (1.87)
$\overline{R}^2$	0.11	0.16	-0.003	0.04
DW	0.70	0.22	0.99	0.32
Wald test : $Ho : (\omega = 0, \varphi = 1)$	F(2,304)=80 (p=0.00)	F(2,295)=187 (p=0.00)	F(2,304)=494 (p=0.00)	F(2,295)=194 (p=0.00)
Sample size	306	297	306	297

Notes. Numbers in brackets represent *t*-values. For each maturity, estimations cover the period starting at 1989.10 and ending at 2014.06 or at 2015.03 depending on whether the horizon of expectations is 12 or 3 months, respectively. To account for possible overlapping bias, the Newey-West heteroscedasticity and autocorrelation consistent estimator is used (quadratic-spectral kernel, Andrews' automatic bandwith). *S* and *L* denote the short term interest rate (US 3-month Treasury bill rate) and the long term interest rate (10 years Treasury bond yield), respectively.

Table 3. GMM (HAC) system estimation results

	$E_{t\ S} r_{t+\tau} - r_t$		$E_{t\ L} r_{t+\tau} - r_t$	
τ	3 months	12 months	3 months	12 months
λ	0.96*** (138.70)	0.95*** (86.99)	1.02*** (69.67)	1.00*** (156.83)
α	0.14*** (10.27)	0.13*** (5.97)	0.09*** (5.77)	0.16*** (10.58)
β	0.58*** (23.44)	0.86*** (42.86)	0.59*** (22.68)	0.84*** (39.41)
γ	-0.015*** (-4.26)	0.01* (1.84)	0.03*** (4.13)	0.03*** (3.71)
δ	0.28*** (16.00)	0.09*** (10.26)	-	-
h	-3.53*** (-8.83)		1.79*** (4.09)	
$\overline{R}^2$	0.83	0.90	0.73	0.89
Q(4) p-value	0.00		0.00	
K p-value	0.00		0.00	
AIC	-3.14		-3.35	
SC	-3.00		-3.24	
HQC	-3.09		-3.31	
L	485.09		515.27	
Wald p-value	0.00; 0.00		0.00; 0.00	

Notes. Numbers in parentheses are the Student t-values. The estimated model is given by (10), where the intercept, which has been systematically found to be insignificant, has been omitted. The instruments chosen are the contemporaneous and/or lagged values of the components of the system. The data used cover the period February 1990 – Mai 2015. To account for possible overlapping bias, the GMM with heteroskedasticity and autocorrelation consistent (HAC) covariance matrix is performed using a quadratic-spectral kernel and Andrews' automatic bandwidth. *Q(4)* and *K* represent the Portmanteau system test for residual autocorrelation for 4 lags and the Kurtosis system test, respectively. *S* and *L* denote the short term interest rate (US 3-month Treasury bill rate) and the long term interest rate (10 years Treasury bond yield), respectively. *AIC*, *SC*, *HQC* and *L* stand for Akaike, Schwarz and Hannan-Quinn information criteria and the log-likelihood value for each system estimation, respectively. The first (second) entry of the Wald p-value is the probability of the null that the set of coefficients of the system estimated over the first (second) half of the period is significantly equal to that of the system estimated over the whole period. Symbols *, ** and *** represent 10, 5 and 1% levels of significance, respectively.

Table 4. Kalman filter estimation results

	$E_{t\ S} r_{t+\tau} - r_t$		$E_{t\ L} r_{t+\tau} - r_t$	
τ	3 months	12 months	3 months	12 months
Measurement equations				
α	0.12*** (0.01)	0.11*** (0.03)	0.10*** (0.02)	0.07** (0.03)
β	0.55*** (0.03)	0.67*** (0.03)	0.57*** (0.03)	0.64*** (0.04)
γ	-0.02*** (0.007)	0.04*** (0.016)	0.006** (0.003)	0.02** (0.01)
δ	0.28*** (0.02)	0.15*** (0.02)	-	-
k	-4.85*** (0.07)	-4.21*** (0.08)	-4.39*** (0.08)	-4.35*** (0.09)
h	-3.04*** (0.58)		5.54** (2.68)	
ρ	0.005*** (0.001)		0.01*** (0.001)	
State equations				
ω_0	-	-	-	-
ω_1	1.00*** (0.00)	1.00*** (0.00)	1.00*** (0.00)	1.00*** (0.00)
k'	-13.80* (8.50)	-10.06*** (1.00)	-12.94*** (2.30)	-12.54*** (2.47)
R_D^2	0.79	0.70	0.74	0.76
AIC	-3.31		-3.46	
SC	-2.95		-3.12	
HQC	-3.16		-3.32	
L	514.06		531.92	

Notes. Numbers in parentheses are the standard deviations. The data used cover the period February 1990 – Mai 2015. For each interest rate, the estimated state-space model is given by the measurement equations (10) and the state equations (11). *S* and *L* denote the short term interest rate (US 3-month Treasury bill rate) and the long term interest rate (10 years Treasury bond yield), respectively. Dashes in the ω_0 row indicate that the estimates of the system are obtained by setting these intercepts to zero as a result of prior estimations. To ensure positivity, the constant variances of ${}_\theta\varepsilon_{t,\tau}$ and ${}_\theta\eta_{t,\tau}$ $\theta = \{S, L\}$ and $\tau = 3, 12$, are estimated as $\exp({}_\theta k_\tau)$ and $\exp({}_\theta k'_\tau)$. ${}_\rho$ is the contemporaneous covariance between the two signal residuals. R_D^2 is a measure of goodness of fit with respect to a random walk with drift (Harvey, 1992) (see footnote 19). AIC, SC and HQC stand for Akaike, Schwarz and Hannan and Quinn information criteria for the two-horizon system estimation. Symbols *, ** and *** represent 10, 5 and 1% levels of significance, respectively.

Figure 1. Short and long term US interest rates

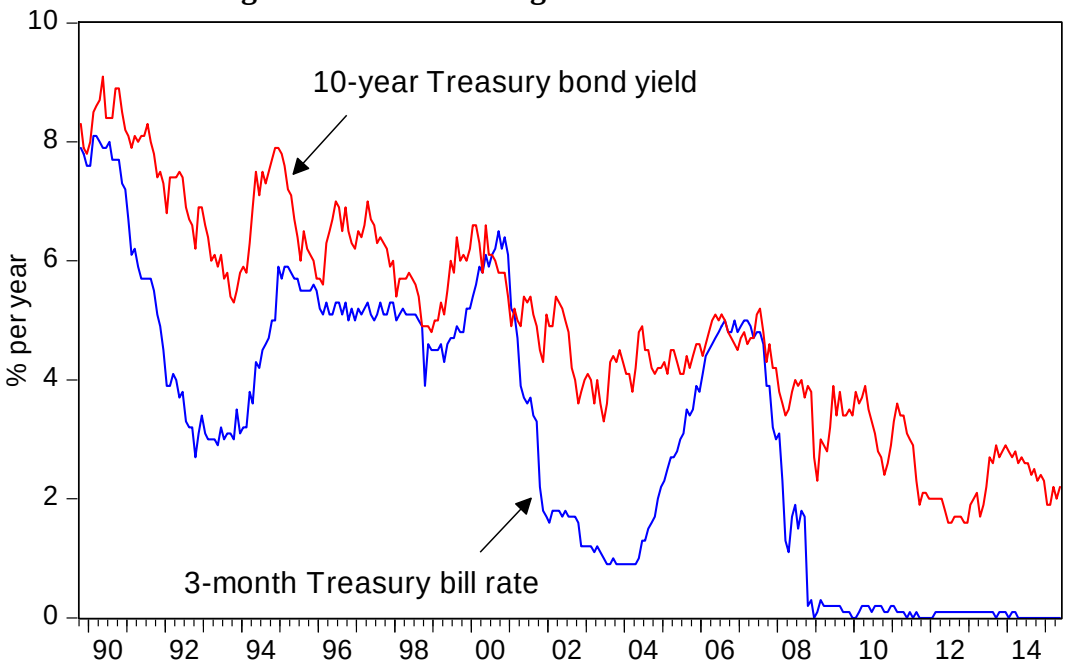


Figure 2. 3-month and 12-month horizon expected changes in the 3-month Treasury bill rate

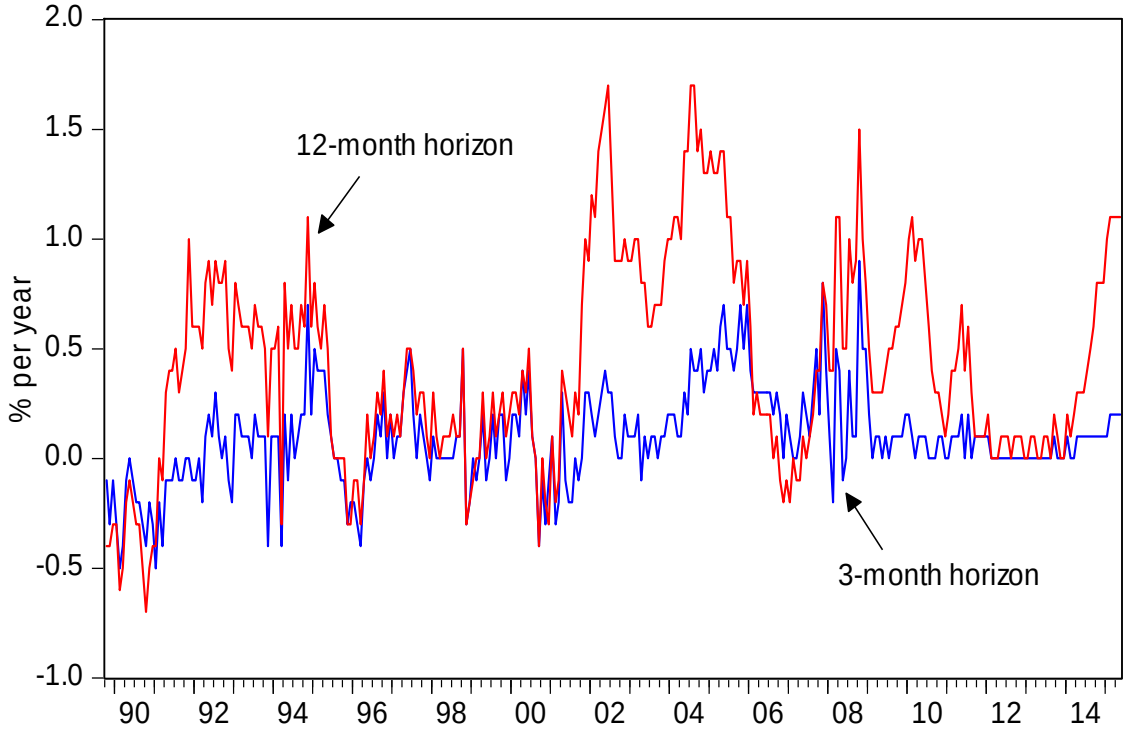


Figure 3. 3-month and 12-month horizon expected changes in the 10-year Treasury bond yield

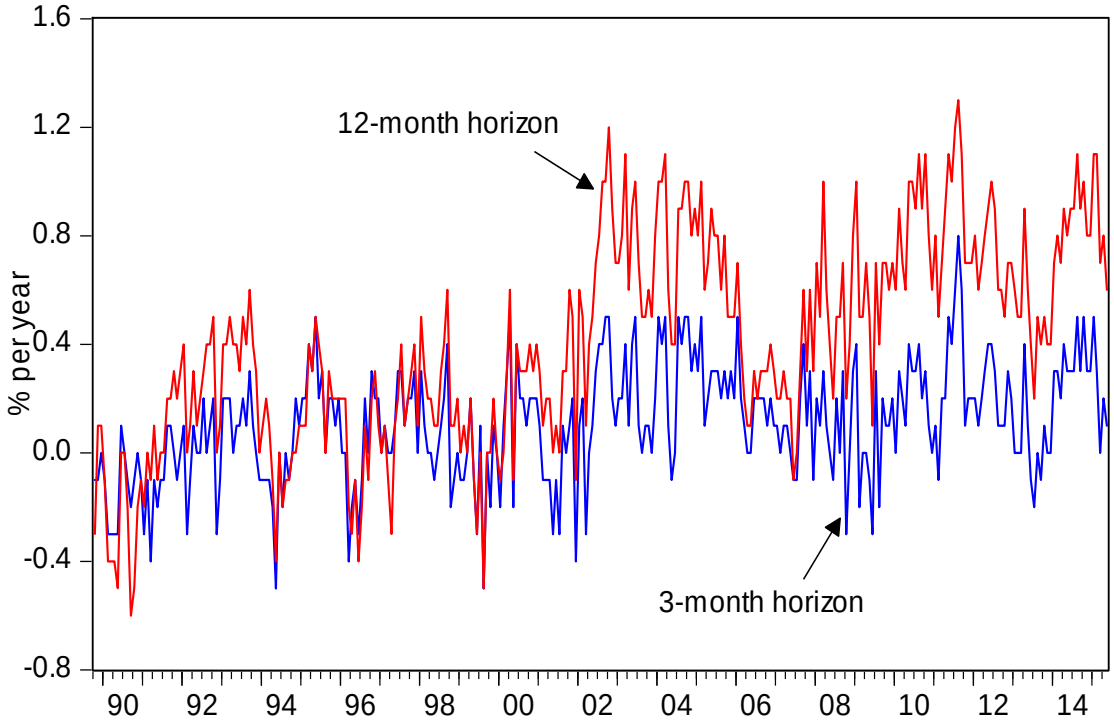
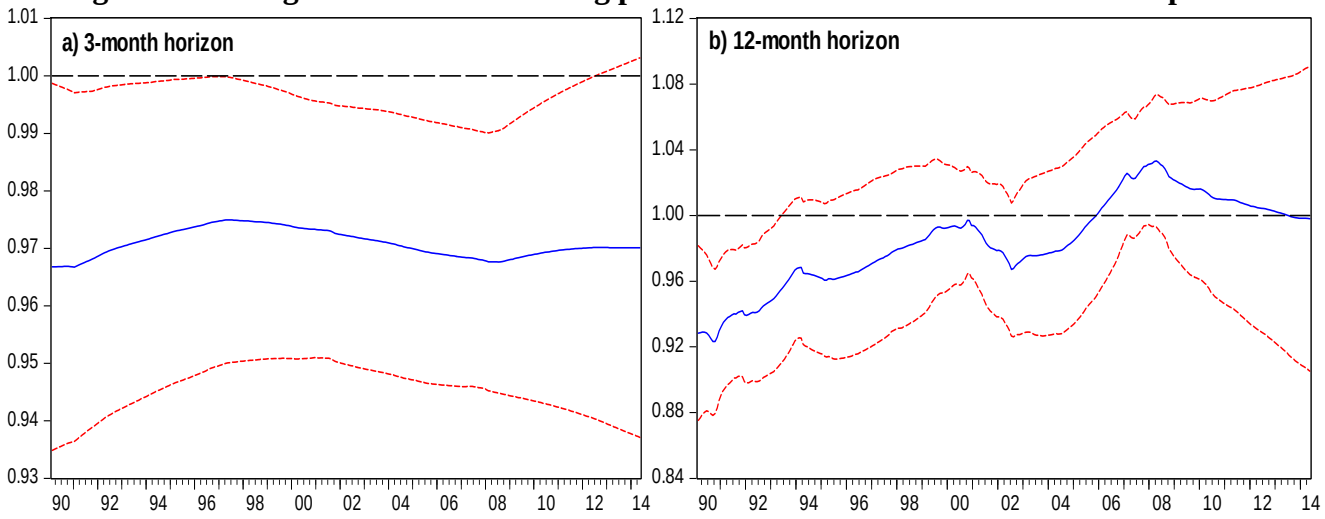
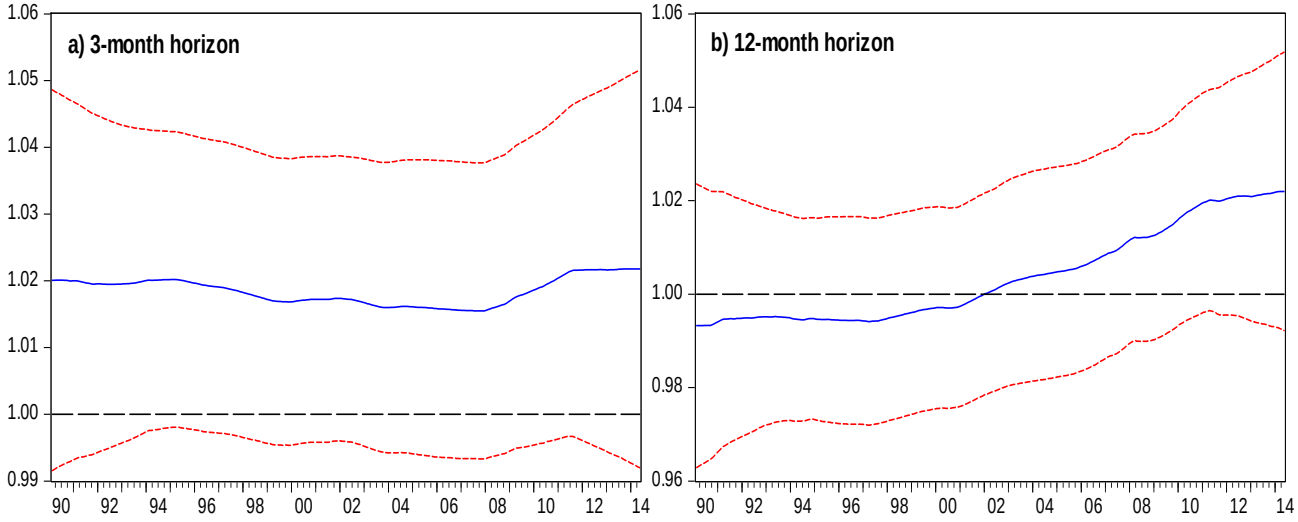


Figure 4: Testing for a rational learning process in US short term interest rate expectations



Notes: Solid lines represent the smoothed estimates of the state variable ${}_{S}\lambda_{t,\tau}$ in Equation 12, estimated using the Kalman filter methodology for τ =3 (left panel) and 12 (right panel) month ahead expectations. Dashed lines stand for these state estimates ± 2 standard errors.

Figure 5: Testing for a rational learning process in US long term interest rate expectations



Notes: Solid lines represent the smoothed estimates of the state variable ${}_{L}\lambda_{t,\tau}$ in Equation 12, estimated using the Kalman filter methodology for τ =3 (left panel) and 12 (right panel) month ahead expectations. Dashed lines stand for these state estimates ± 2 standard errors.

Figure 6: Observed and fitted values of the expected change in the short term interest rate

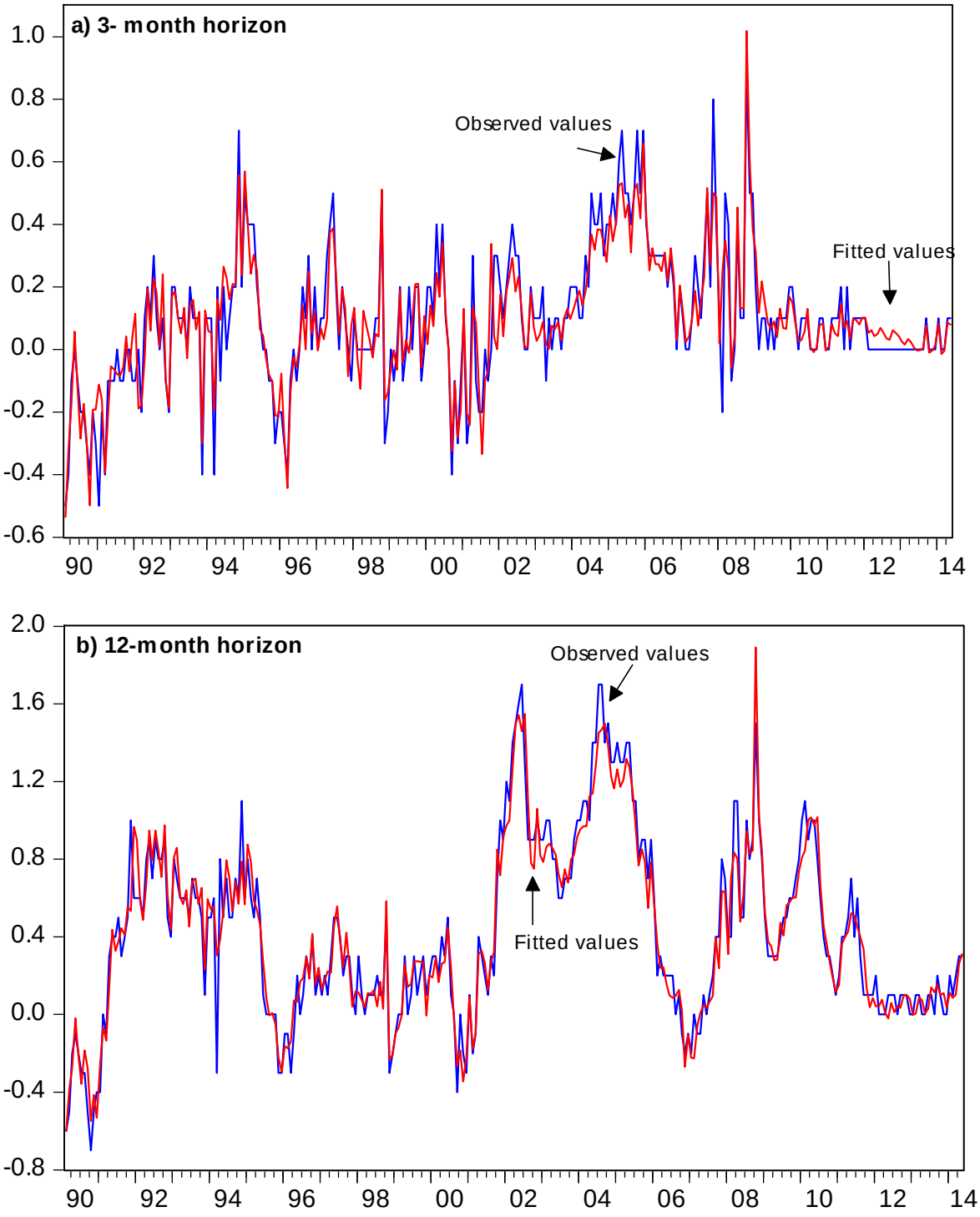


Figure 7: Observed and fitted values of the expected change in the long term interest rate