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# Construction of a fuel demand function portraying interfuel substitution, a system dynamics approach.

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## Abstract

Most of the recent numerical market equilibrium models of natural gas markets use imperfect competition assumptions. These models are typically embedded with an oversimplified representation of the demand side, usually a single-variable affine function, that does not capture any dynamic adjustment to past prices. To remedy this, we report an effort to construct an enhanced functional specification using the system dynamics-based model of Moxnes (1987, 1990). Thanks to a vintage representation of capital stock, this putty-clay model captures the effect of both past and current energy prices on fuel consumption. Using a re-calibrated version of this model, we first confirm the pertinence of this modeling framework to represent interfuel substitutions at different fuel prices in the industrial sector. Building on these findings, a dynamic functional specification of the demand function for natural gas is then proposed and calibrated.

## 1 Introduction

A casual look at the newspaper headlines suggests that concerns connected with the natural gas sector are now back on top of policy makers' agenda. The reasons for this renewed interest are numerous and include: a rapid globalization of the natural gas trade (Victor et al., 2006), the rising share of gas technologies in the power generation sector inducing, in both Europe and Asia, an increased dependence on foreign sources (Honoré, 2010), and the recent emergence of a Gas Exporting Countries Forum that is often depicted as an embryonic cartel (Massol and Tchong-Ming, 2010).

Unsurprisingly, this context has triggered a renewed interest for energy economics models aimed at analyzing this industry. In particular, several partial equilibrium models in the vein of those pioneered in Mathiesen et al. (1987) have recently been proposed to represent the imperfect competition among gas producers (Boots et al., 2004; Egging and Gabriel, 2006; Holz et al., 2008). Besides the policy-oriented analyses provided in these articles, these detailed numerical models can be very useful for corporate planning purposes. For example, a firm that considers an investment in a large and lumpy gas transmission infrastructure may take advantage of these powerful tools to assess the relative appeal of various alternative routes by comparing the long-run impacts of the proposed infrastructure on the

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markets outcome. Despite their great merits<sup>1</sup>, these models are not exempt from flaws. One of their most striking problems is connected with the relatively rudimentary treatment of the demand side, which is usually oversimplified to an affine inverse demand function. Indeed, a recent meticulous assessment of these models underlines that these contributions rely either on somewhat obsolete information on gas demand function or on more or less arbitrary calibration (Smeers, 2008, p.12). As the outcomes of a market equilibrium model based on the Cournot oligopoly theory are indubitably impacted by the price elasticity of the demand (Tirole, 1988), some further investigations are clearly required to obtain a more satisfactory functional specification of the demand for natural gas. Such a functional form will have to take into account both the substitutability of natural gas by other fuels and the adjustment dynamics of consumption in reaction to fuel prices. This statement provides the motivation for this paper.

Energy demand modeling has become a very productive activity since the 1970s. Indeed, a very large literature has approached the question using econometric analyses. As far as interfuel substitution is concerned, we can distinguish between early empirical specifications based on, for example, discrete choice models as in Joskow and Mishkin (1977), and models predicted on theory where a flexible functional form is aimed at being estimated in coherence with standard microeconomic assumptions (profit or utility maximizing behavior, 0-degree homogeneity of the demand function, symmetry, law of demand, etc.) as in, for example, Fuss (1977), Pindyck (1979), Considine and Mount (1984), or Urga and Walters (2003). Notwithstanding the immense value of these statistical approaches, it must be acknowledged that putting theory to work to model energy demand can turn out to be far from a sinecure because of numerous practical considerations (cf. the informed list reported in Watkins, 1992). For example, practitioners may be compelled to adopt a more simplified dynamic specification to model short- and long-run effects than those recommended by theoretical arguments based on an assumed dynamic optimization behavior (Watkins, 1992). This type of consideration may turn out to be problematical if the obtained demand model is aimed at being embedded within a decision-support tool designed to serve the needs of users (corporate planners and executives) who may have forgotten some of their statistical education and could feel uncomfortable with a modeled dynamics that hardly mimics their *a priori* mental representation of a putty-clay type of dynamics. To increase their confidence with the model's validity, modelers can look for an approach that endeavors to build on their detailed understanding of the gas industry. Given the strong record of applications of system dynamics for strategic modeling purposes (Sterman, 2000; Morecroft, 2007), this technique constitutes an appealing methodology.

Numerous system dynamics-based models have been developed for energy planning purposes. A non-exhaustive list includes: (i) the models originating from research initiated at Dartmouth College in the late 1970s and then refined during nearly two decades to support energy policy analyses conducted by the US federal administration (Naill, 1977; Naill, 1992; Naill et al., 1992; Wood and Geizner, 1997), (ii) the broad approach of Sterman (1981) that analyzed the US energy transition with an integrated energy-economy model and the extended climate-economy model of Fiddaman (1997), (iii) the numerous models surveyed in Ford (1997) that are aimed at informing electric utility policies and (iv) the models dedicated to the oil and/or gas industries such as Davidsen et al. (1990), Olaya and Dyner (2005), Chi et al. (2009) and Ponzo et al. (2011).

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<sup>1</sup>For example, they capture a very detailed representation of the supply side of the natural gas industry including: transmission network, production constraints, etc.

Since the 1980s, an impressive stream of research that encompasses all the facets of natural resources (economic, management, policy) has been conducted in Norway. As far as natural gas is concerned, the affluence and diversity of this "Norwegian school" is well exemplified in Golombek et al. (1987). Amusingly, this book that contains the Cournot equilibrium model of Mathiesen et al. (1987) mentioned above also includes Moxnes (1987), a putty-clay model of OECD-European industrial energy demand that presented a very good explanation of historical fuel substitution during the period 1960-1983. In a subsequent study (Moxnes, 1990), it has been shown that this framework can also provide a very good fit to an historical time-series of fuel choices in OECD-European electricity production. In this contribution, we propose to take advantage of this system dynamics methodology to select and estimate a more satisfactory demand function aimed at being implemented in an imperfect competition model (Abada et al., 2010). To do so, an updated version of this system dynamics-based model is first presented and recalibrated to check the capability of this approach to explain the substitutions between the three main fuels: oil, coal and natural gas. The model is then simulated to generate data that depict the dependence of fuel consumption over fuel prices. Based on these "pseudo data", an interesting functional form is proposed to model the demand function for natural gas, that can be generalized to the three fuels.

The article is organized as follows: section 2 provides a brief review of the methodology presented in Moxnes (1987, 1990). The results obtained with a calibrated model are given in Section 3. In section 4, the system dynamics model is then put to work to construct an adapted demand function. The last section summarizes the conclusions.

## 2 The model

In this section, we briefly review the model detailed in Moxnes (1987). This model aims at predicting the consumption of coal, oil and natural gas observed at time  $t$  using both the historical and current values of fuel prices, and the history and current value of the overall demand for hydrocarbon fuels. In this model, the dynamics of interfuel substitution involves a distinction between the flow of freshly installed equipment, and the stocks of existing equipment that is represented by two vintages of capital. The model is based on a putty-clay framework and assumes that the choice of fuels can be freely adjusted *ex ante*, whereas no substitution is possible *ex post*. Thanks to this decomposition, the model captures the irreversibility associated with the decision to install and operate a durable burning equipment.

To begin with, table 1 clarifies the model boundaries and divides the variables and parameters into those endogenous and those exogenous to the model:

To simplify, the fuel options are indexed by an integer  $i$  and the fuel option coal (respectively oil, and natural gas) is labeled 1 (respectively 2, and 3). The fuel shares in the new burning equipment installed at time  $t$  are assumed to be determined by the relative cost of the three fuel options. The total cost  $C_i$  of fuel option  $i$  is given by the following formula:

$$C_i = \frac{CC_i}{PBT_i} + OO_i + \frac{P_i + Q_{CO_2i} \cdot P_{CO_2}}{E_i} - PR_i, \quad (1)$$

Table 1: An overview of the model boundaries

| Endogenous                                          | Exogenous                |
|-----------------------------------------------------|--------------------------|
| Investment in new burning equipment                 | Total energy consumption |
| Fuel shares in newly installed equipment            | Fuel market prices       |
| Installed burning capacity per fuel option          |                          |
| Capacity utilization factors of installed equipment |                          |
| Consumption of the various fuels                    |                          |

where  $CC_i$  is the capital cost,  $PBT_i$  is the associated payback time,  $OO_i$  denotes the operating cost (fuel and carbon cost excluded),  $P_i$  is the fuel price,  $PCO_2$  is the price of  $CO_2$  if any,  $QCO_{2i}$  is the  $CO_2$  emission factor of fuel  $i$ ,  $E_i$  is the burner efficiency, and  $PR_i$  is a premium, that is, a parameter that reflects the miscellaneous unmodeled features of fuel  $i$  such as flexibility, availability, consumption inertia, etc. In Moxnes (1987,1990), the price of  $CO_2$  has not been taken into account.

The share  $s_i$  of fuel option  $i$  in the new burning equipment is determined by the relative cost of the three fuel options. The following multinomial logit model is used:

$$s_i = \frac{e^{-\alpha C_i}}{\sum_i e^{-\alpha C_i}}, \quad (2)$$

where  $\alpha$  is a (non-negative) parameter, and  $C_i$  are the total costs defined in (1). By construction, the obtained shares satisfy  $\forall i, s_i \in [0, 1]$  and  $\sum_i s_i = 1$ . In addition, the share  $s_i$  is, *ceteris paribus*, a decreasing function of the fuel price  $P_i$ . Besides, one may notice that shares are determined on the basis of differences in total costs and thus differences in the values of the premiums. As these are adjustable parameters, it may be easier to determine a reference point: hereafter, the premium for coal  $PR_1$  has thus been set equal to 0. It is also interesting to underline that the presence of exponents in the logit formula tends to accentuate the differences in total costs as they are converted into fuel choices. A small value of  $\alpha$  translates into equal shares for all fuels, whereas a large value of  $\alpha$  indicates that minor differences in total cost lead to major differences in the resulting fuel shares.<sup>2</sup> Actually, the validity of this logit model conceptually presupposes a "macroscopic" perspective, meaning that the energy system under scrutiny must contain a large enough number of individual decision-makers.

In this model, capital is measured in units of capacity to burn fuels (that is, in energy unit per unit of time). Thus, the total investment  $I$  represents the overall capacity of new burning equipment. The total investment in new equipment associated with the fuel option  $i$  is denoted  $I_i$  and satisfies:

$$I_i = s_i I. \quad (3)$$

We can now detail the dynamics of fuel substitution. As mentioned above, a vintaging structure is used to portray the aging process of installed equipment. Here, two vintages of capital are kept track of. A more precise description of the aging process should consider

<sup>2</sup>In Moxnes (1990), an informed interpretation is given for  $\alpha$ : if the total costs follow a Weibull distribution,  $\alpha$  is inversely proportional to the standard deviation of this distribution.

more vintages, or continuous aging. However, Moxnes (1987) justifies this choice of a 2-vintages representation by the lack of precise data, and the fact that the model's behavior seems insensitive to the number of modeled vintages. Accordingly, two stock variables are defined for each fuel option  $i$ : the capacity of recently installed equipment, the "new" ones  $KN_i$ , and those of the older ones  $KO_i$ . Investment in new burners  $I_i$  increases the capacity of the new equipment. New equipment becomes old after a use of half the lifetime  $T_i$ . If, as in Moxnes (1987, p. 99), a "fairly wide distribution of lifetimes" can be assumed, the flow variable associated with the transformation of new equipment into old ones can be assumed to be equal to  $\frac{1}{T_i/2}$ <sup>th</sup> of the overall capacity of new-burners  $KN_i$ . Similarly, an old equipment is scrapped after a use of  $\frac{T_i}{2}$  and the flow of scrapped old equipment  $DO_i$  is assumed to be equal to  $\frac{KO_i}{T_i/2}$ . With these assumptions, the dynamics can be formulated as follows:

$$\frac{dKN_i}{dt} = I_i - \frac{KN_i}{\frac{T_i}{2}}, \quad (4)$$

$$\frac{dKO_i}{dt} = \frac{KN_i}{\frac{T_i}{2}} - \frac{KO_i}{\frac{T_i}{2}}. \quad (5)$$

A simple interpretation of these equations can be provided. For each fuel  $i$  at time  $t$ , the change in the overall stock of new equipment with respect to time is given by the inflow of new equipment associated with investment  $I_i$ , and the outflow caused by aging (that is, the equipment that is no longer new and has to be reallocated into the old category). Similarly, the temporal variation of the stock of old burners results from: the inflow of these previously new equipment, and the outflow corresponding to the scrapping of old equipment.

The next step is to model the dependence between the flow of total investment  $I$  and the overall stock of existing equipment. We can first define  $K_i = (KN_i + KO_i)$  the total capacity of installed burning equipment with fuel option  $i$ , and  $K$  the total capacity of installed burning equipment:  $K = \sum_i K_i$ .

At time  $t$ , the overall capacity of scrapped equipment is:

$$DO = \sum_i DO_i = \sum_i \frac{KO_i}{\frac{T_i}{2}}. \quad (6)$$

Let's call  $ED$  the overall demand for the three fuels at time  $t$ , which is an exogenous parameter in this model. Common sense suggests that investment in new equipment should be related in some way to the observed discrepancy between demand and the installed capacity of existing equipment. As this adjustment is usually not instantaneous, Moxnes (1987) introduces the parameter  $TI$ , the time to adjust investments that "determines how fast investments adjust simulated capacity toward exogenous demand." Accordingly, the total investment has to be modeled as an increasing function of  $\frac{ED-K}{TI}$ . In addition, investment has to be connected to the total scrapping of old equipment  $DO$  to allow a regeneration of the stock of equipment. To model these interactions, Moxnes (1987) postulates the following formula that defines the total investment as a function of these parameters:

$$I = DO \cdot f\left(\frac{ED - K}{TI \cdot DO}\right), \quad (7)$$

where  $f$  is a piecewise continuous function that has the following expression:

$$\begin{aligned} f(x) &= x + 1 & \text{if } x \geq 0, \\ f(x) &= e^{a \cdot x} & \text{if } x < 0, \end{aligned} \tag{8}$$

where  $a$  is a non-negative parameter. We can observe that, if the total demand  $ED$  is equal to the installed capacity  $K$  (that is,  $ED = K$ ), the investment will be large enough to compensate for the scrapped equipment  $DO$  ( $f(0) = 1$ ). If  $ED > K$ , investments cause a net rise in the stock of installed equipment ( $f(x) > 1$  if  $x > 0$ ). If  $ED < K$ , some positive investment values can be obtained. However, since  $I < DO$ , they will cause a net drop in the installed capacity ( $f(x) < 1$  if  $x < 0$ ). In the case  $ED < K$ , the chosen functional specification differs slightly from the affine one used in the original model (Moxnes, 1987, fig. 2). This change is guided by the desire to implement a robust formulation to extreme condition testing (Oliva, 2003). With an affine specification, a very large drop in demand  $ED$  could result in a negative investment value, that is, the premature scrapping of "new" equipment (especially those with the most desirable fuel option). To remedy this, an exponential specification is implemented to insure a non-negative investment value.

One then has to determine the capacity utilization to allow the model to track exogenous energy demand in case of large downward variations (compared to total scrapping  $DO$ ). Capacity utilization  $U$  is simply defined as:

$$U = \frac{ED}{K} . \tag{9}$$

Here, capacity utilization is assumed not to be fuel specific as the same capacity utilization figure is posited for the three fuels:

$$\forall i, \quad U_i = U . \tag{10}$$

As a result, the simulated demand for fuel  $i$ , denoted  $\hat{D}_i$ , is:

$$\hat{D}_i = U_i K_i = ED \frac{K_i}{K} . \tag{11}$$

Contrary to Moxnes (1987, 1990), we do not model installed plants with multi-firing capability that, in the short-run, are able to switch from one fuel to another and back again in response to price signals. In this model, all the installed equipment is thus supposed to be inflexible with respect to fuel choice. The decision to abandon this part of the original model was guided by market observations that suggest a phase-out of fuel switching capability in industrial plants after the 1980s. Stern (2007, 2009) provides an informed discussion on the causes of this phase-out based on the extra-cost and inconvenience associated with the maintenance of a multi-firing capability, and the progressive tightening of emission limits on the burning of fossil fuels.

To summarize, the model (equations (1)-(11)) corresponds to a system of non-linear differential equations. The associated initial conditions will be detailed in the next section. Because of its complexity, this system has to be simulated with numerical techniques (Euler's method).

### 3 Calibration and results

In this section, we present the data used in our simulations and detail the calibration of the model before discussing the obtained results.

#### 3.1 Context

The national energy contexts (domestic resource endowment, composition of the industrial sector, energy policies and energy taxation regimes, etc.) vary greatly from one industrial country to another and national specificities play a non-negligible role in the fuel consumption patterns observed in the industrial sector. Accordingly, a country-level perspective has been adopted to analyze the cases of eight industrial countries that are members of the International Energy Agency (IEA): Canada, France, Germany, Italy, Japan, South Korea, the UK and the USA. The hydrocarbon fuel consumption of their industrial sectors are the largest among IEA members and, in 2008, collectively represented 79.6% of the overall industrial fuel consumption of all the IEA member countries (IEA, 2010a).

In this study, we aim at analyzing the adjustment to the relative fuel prices that occurred during the period 1978-2008.<sup>3</sup> This sample period covers the second oil shock, the oversupply-based counter shock associated with the collapse of oil prices that started in 1986 and, more recently, the high-oil price regime that began in late 2003 and unfolded until 2008. During these last 30 years, there has been a net decline in the energy intensity in these eight economies. With exception of South Korea where a net rise in the fuel demand of the industrial sector has been observed, the overall amount of fuel consumption in the energy sector has either diminished (Europe and USA) or has been maintained (Canada, Japan). In terms of fuel substitution, the share of coal remained steady whereas gas consumption increased sharply at the expense of oil (IEA, 2010a).

#### 3.2 Data and model calibration

The data employed in this study consists of time series gathered from the IEA. The fossil fuel consumption data - measured in toe - are those listed in the "Total Industry" category in the IEA World energy balances under the headings "Coal and coal products", "Oil products" and "Gas" (IEA, 2010a). Similarly, the price data refer to the national end-use prices in US dollar reported in IEA (2010b) under the headings "Steam coal", "High sulfur fuel oil" and "Natural gas".<sup>4</sup> All prices are given in 2008 US\$/toe. In South Korea, natural gas consumption began just after the commencement of gas imports in 1987. In that country, an infinite price of natural gas was hence assumed for the period 1978-1986.

We can now detail the model calibration. In Moxnes (1987, 1990), a Bayesian approach is used where most of the parameters' values are derived from direct observations (costs, efficiency of the burners, etc.) and educated guesses (the coefficient for the logit specification  $\alpha$ ). Some parameters (especially those pertaining to preferences and the initial values of the stocks), are then revised thanks to an iterative procedure aimed at improving the fit between simulated and historical behavior. Arguably, such an iterative procedure may

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<sup>3</sup>The non-inclusion of the earlier period has been imposed by practical considerations on data availability. Indeed, the IEA no longer provides time series on end-user prices for the period 1960-1977.

<sup>4</sup>For periods with missing price data, end-use prices have been reconstructed using the indices of energy prices by sector reported by the IEA.

somehow involve some subjectivity (Oliva, 2003). To remedy this, an automatic calibration (AC) procedure is applied to minimize the deviation between a simulated outcome and historical data. According to Oliva (2003), a parsimonious approach should guide the practical implementation of AC. Accordingly, the use of AC has been restricted to "the smallest possible calibration problems." In line with Moxnes' Bayesian approach, *a priori* information has thus been used for the observable parameters (costs, efficiency of the burners, etc.) and the AC procedure has been applied to solely adjust the value of the most uncertain parameters (initial values of the stocks,  $\alpha$ , etc.).

Our assumptions are based on Moxnes (1987, Table 2) and are summarized in Table 2 (Note: the CO<sub>2</sub> emission factors are drawn from IEA (2010a)):

Table 2: Cost assumptions for the industrial sector

|                                                                     | Year      | Coal  | Oil   | Gas   |
|---------------------------------------------------------------------|-----------|-------|-------|-------|
| Capital costs $CC_i$ (\$/utoe <sup>a</sup> per year)                | all       | 410   | 200   | 200   |
| Payback time $PBT_i$ (years)                                        | all       | 5     | 5     | 5     |
| Other operating costs $OO_i$ (\$/utoe <sup>a</sup> per year)        | all       | 70    | 40    | 40    |
| Burner efficiencies $E_i$<br>(% useful)                             | 1978-1982 | 70    | 75    | 75    |
|                                                                     | 1983-2008 | 71    | 76    | 76    |
| Lifetime of burners $T_i$ (years)                                   | all       | 25    | 25    | 25    |
| CO <sub>2</sub> emission factor $Q_{CO_2i}$ (tCO <sub>2</sub> /toe) | all       | 3.881 | 3.207 | 2.337 |

<sup>a</sup>Hereafter, utoe is used to denote useful toe.

In addition, the parameters associated with the investment function have to be defined. The coefficient  $a$  in the function  $f$  defined in equation (8) has been set equal to 0.231, a value interpolated from the extreme left point in Moxnes (1987, Fig. 2). Besides, the time to adjust total investments  $TI$  is assumed to be equal to 1 year because, contrary to the 1960s (Moxnes used 0.25 year), we can reasonably posit that industrial investment during the period 1978-2008 was not primarily guided by "building ahead of demand" motives.

We can now detail the main features of the AC procedure. Here, we rely on the model reference optimization (MRO) method described in Oliva (2003). We first specify an error function capable of measuring the distance between the observed and simulated behavior as a function of the model's parameter values. For each fuel  $i$ , a fuel-specific distance can be evaluated with the absolute error, that is, the sum of the absolute discrepancies between historical  $D_i^t$  and estimated  $\hat{D}_i^t$  fuel consumption. The model's error function is thus defined as the sum of these three fuel-specific distances:

$$e = \sum_i \sum_t |D_i^t - \hat{D}_i^t| . \quad (12)$$

We note that this function gives an equal weight to each fuel and each observation no matter when it was recorded. Using the model's equations above, it is possible to specify the error  $e$  as a multivariate function of the parameters to be estimated, namely the non-negative values of the initial stocks  $(KO_i^0)_{\forall i}$  and  $(KN_i^0)_{\forall i}$ , the non-negative coefficient for the logit specification  $\alpha$ , and the premiums for both oil  $PR_2$  and natural gas  $PR_3$  ( $PR_1$

is set equal to 0\$/toe).

Following Oliva (2003), the AC procedure is then specified as an optimization problem: finding the parameter values that minimize this distance subject to feasibility constraints (the non-linear equations presented in the preceding section). The optimization problem at hand is a nonconvex, nonlinear mathematical program that can be successfully attacked by modern global solvers.<sup>5</sup> Table 3 reports the parameters' values obtained thanks to the AC procedure, for the countries studied.

Table 3: Calibrated values of the parameters

|         | Initial capacities<br>(Mtoe/year) |               |              |              |              |              | alpha<br>(utoe/\$) | Premiums<br>(\$/utoe) |            |
|---------|-----------------------------------|---------------|--------------|--------------|--------------|--------------|--------------------|-----------------------|------------|
|         | $KN_{coal}^0$                     | $KO_{coal}^0$ | $KN_{oil}^0$ | $KO_{oil}^0$ | $KN_{gas}^0$ | $KO_{gas}^0$ | $\alpha$           | $PR_{oil}$            | $PR_{gas}$ |
| Canada  | 3.63                              | 0.58          | -            | 13.56        | 13.95        | -            | 0.0073             | 55.1                  | 140.2      |
| France  | 8.35                              | -             | -            | 20.33        | 6.17         | -            | 0.0220             | 0.0                   | 119.0      |
| Germany | 13.35                             | 34.90         | -            | 30.83        | -            | 23.16        | 0.0112             | -                     | 317.7      |
| Italy   | 3.52                              | -             | -            | 19.56        | 8.14         | -            | 0.0107             | 105.5                 | 229.0      |
| Japan   | 14.95                             | 22.01         | -            | 98.02        | 5.57         | -            | 0.0047             | -5.8                  | 124.1      |
| Korea   | 2.74                              | -             | 10.31        | 0.00         | -            | -            | 0.0128             | -80.7                 | 48.0       |
| UK      | 9.13                              | -             | -            | 16.68        | 13.92        | -            | 0.0087             | 352.2                 | 400.3      |
| USA     | 59.35                             | -             | 11.56        | 116.39       | -            | 176.90       | 0.0304             | -25.2                 | 99.0       |

From these calibration results, several facts stand out. First, the initial stocks of new burners in 1978 suggest that, with the exception of Korea, the installed oil burning capacities mainly consist of old burners, revealing a limited investment in oil burning appliances in the previous years. This finding looks coherent with the oil diversification policies initiated after the first oil shock. Then, Moxnes (1987, 1990) underlines that the multinomial logit model used for the investment shares involves an implicit assumption: that the total costs follow a Weibull distribution. Thus,  $\alpha$  the coefficient for the logit specification is inversely proportional to  $\sigma$  the standard deviation of the cost distribution: ( $\sigma = \frac{\pi}{\alpha\sqrt{6}}$ ). According to the obtained values, the standard deviations of total costs range from \$42.2 per useful toe in the USA to \$270.0 per useful toe in Japan. Finally, the relatively large values of the natural gas premiums (compared to the oil ones) reveal a strong preference regarding that fuel in investments. Several features of natural gas can justify this preference, such as the wish to diversify energy sources in oil-importing economies after the two oil shocks, and the cleanliness of natural gas at a time of raising environmental concerns.

### 3.3 Results and validation

The validation of a system dynamics model usually involves two dimensions: (i) structural validity, and (ii) behavioral validity. The purpose of the former is to check whether the implemented structure constitutes, or not, an adequate representation of the phenomenon to be modeled, whereas the aim of the latter is to compare the model generated behavior

<sup>5</sup>Here, the LINDOGlobal optimization procedure is applied.

to the observed behavior (Barlas, 1989; Qudrat-Ullah and Seong, 2010).

In this study, the modeling framework is derived from a classical approach and is thus firmly grounded in previous knowledge. Nevertheless, a meticulous check of its structural validity is carried out. The model at hand has a moderate complexity which considerably eases these verifications (logical coherence of the set of modeled equations, the dimensional consistency of each equation, the robustness against extreme parameter values, etc.). Following a recommendation in Qudrat-Ullah and Seong (2010), this model was also submitted to the judgment of a group of practitioners (corporate planners, executives) and academics whose research is focused on energy issues. All these assessments confirmed the logical soundness of this model built to capture the main drivers of the fuel substitutions dynamics. Accordingly, we can feel confident in the model's ability to "generate the right behavior for the right reasons."

Concerning behavioral validity, Figure 1 and Figure 2 show both the historical and simulated demand behavior for the eight countries. A visual inspection of these plots suggests that the calibrated models satisfactorily capture the history of fuel consumption in these countries.

In addition, some quantitative tools for the analysis of fit are reported in Table 4. The root mean square errors (RMSE) measure the magnitude of the errors. To ease comparisons across series/countries, a normalized measure of these errors is also presented: the mean absolute percent error (MAPE). According to these findings, the fit to historical behavior is quite good, particularly for Canada, Italy and Japan. The large MAPE figure obtained for South Korea's industrial gas demand can be explained by the formulation chosen for the AC procedure. Indeed, our objective function pays attention to absolute differences between historical and simulated values, whereas the MAPE is a relative average normalized measure. For Korea, the MAPE is heavily twisted by the presence of large relative errors during the decade 1987-1997. During that period, gas consumption was ramping up in South Korea and the resulting consumed volumes remained small relative to the generated error. For the UK, most of the discrepancies are observed on the gas and coal series between 1996 and 2003, a period of very low gas prices underpinned by increased competition and upstream developments in the North Sea. During that period, many market observers documented a "dash for gas" causing the premature scrapping of coal burning equipment replaced by gas-fired ones, a behavior that has not been modeled here. Arguably, the observed discrepancies between the simulated and historical series for both gas and coal provide an order of magnitude of the amplitude of this unmodeled phenomenon. For Germany, the model poorly explains the oil and coal consumption monitored in the 1980s. For Germany, the model hardly explains oil and coal consumption in the 1980s but performs significantly better in the subsequent period. Several explanations can be proposed for this poor performance including *(i)* the possibility of under-optimal fuel choices in GDR industries prior to German reunification, *(ii)* the unmodeled subsequent modernization of these industries, *(iii)* the possibly debatable quality of the "reconstructed" energy statistics for the aggregate country in the 1980s (especially those on energy prices), and *(iv)* the unmodeled coal-friendly policy conducted in West Germany that resulted in a net rise in coal consumption between 1979 and 1983 (Renou-Maissant, 1999).

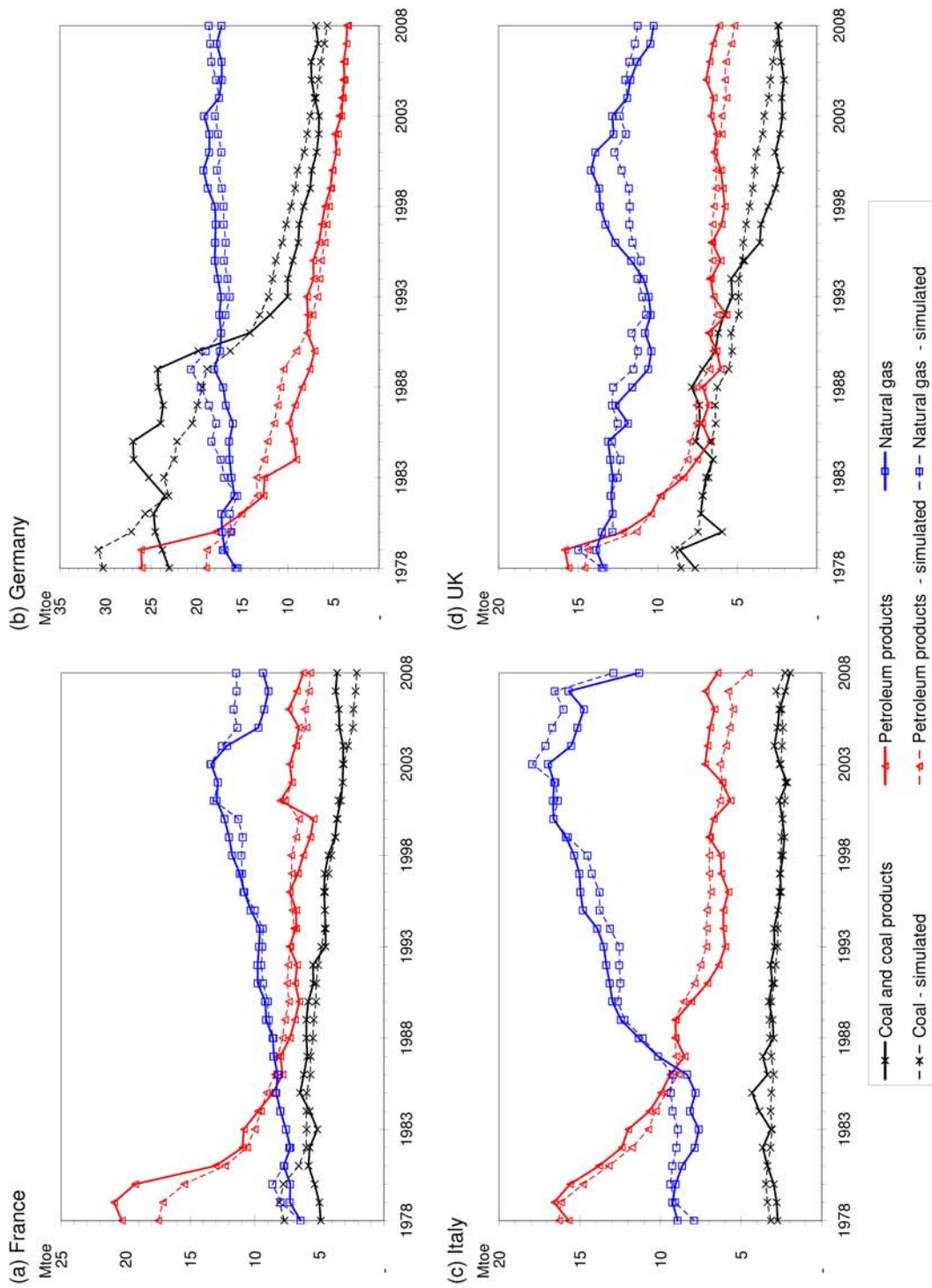


Figure 1:  
*Historical and simulated consumption behavior of industrial annual fuel demand in France, Germany, Italy and UK.*

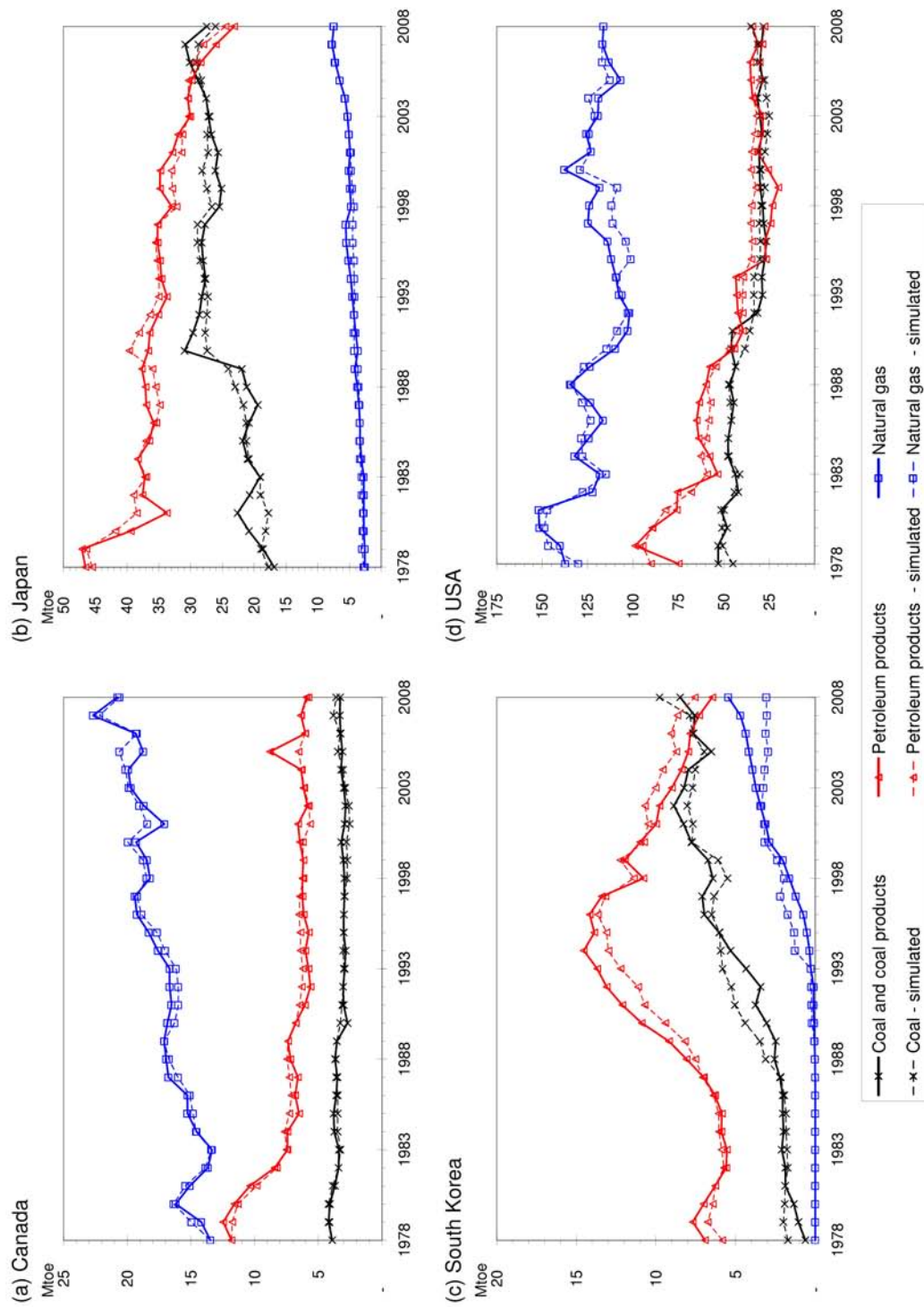


Figure 2:  
*Historical and simulated consumption behavior of industrial annual fuel demand in Canada, Japan, South Korea and USA.*

Table 4: Error analysis of the model

|         |      | MAPE<br>(%)        | RMSE<br>(ktoe) | $U_M$<br>(%) | $U_S$<br>(%) | $U_C$<br>(%) |
|---------|------|--------------------|----------------|--------------|--------------|--------------|
| Canada  | Coal | 5.4%               | 232            | 0.1%         | 3.5%         | 96.4%        |
|         | Oil  | 4.9%               | 557            | 0.0%         | 9.4%         | 90.6%        |
|         | Gas  | 2.4%               | 574            | 0.1%         | 1.2%         | 98.8%        |
| France  | Coal | 14.5%              | 1045           | 0.1%         | 30.1%        | 69.7%        |
|         | Oil  | 8.4%               | 1244           | 3.8%         | 68.7%        | 27.5%        |
|         | Gas  | 5.7%               | 891            | 5.3%         | 0.1%         | 94.6%        |
| Germany | Coal | 15.5%              | 3002           | 0.0%         | 1.9%         | 98.1%        |
|         | Oil  | 12.0%              | 2230           | 0.1%         | 23.8%        | 76.1%        |
|         | Gas  | 6.2%               | 1250           | 0.3%         | 2.5%         | 97.2%        |
| Italy   | Coal | 9.5%               | 385            | 3.9%         | 16.7%        | 79.4%        |
|         | Oil  | 9.9%               | 865            | 1.1%         | 2.0%         | 97.0%        |
|         | Gas  | 6.9%               | 924            | 3.2%         | 1.9%         | 94.9%        |
| Japan   | Coal | 5.4%               | 1700           | 0.8%         | 0.1%         | 99.0%        |
|         | Oil  | 3.5%               | 1553           | 2.6%         | 0.6%         | 96.8%        |
|         | Gas  | 6.8%               | 400            | 5.7%         | 4.3%         | 90.0%        |
| Korea   | Coal | 22.4%              | 781            | 7.6%         | 6.6%         | 85.8%        |
|         | Oil  | 8.2%               | 927            | 1.9%         | 12.7%        | 85.3%        |
|         | Gas  | 63.2% <sup>a</sup> | 721            | 1.5%         | 31.3%        | 67.2%        |
| UK      | Coal | 21.4%              | 950            | 2.6%         | 24.9%        | 72.5%        |
|         | Oil  | 7.8%               | 693            | 0.5%         | 12.9%        | 86.6%        |
|         | Gas  | 6.0%               | 916            | 1.3%         | 15.7%        | 82.9%        |
| USA     | Coal | 8.3%               | 3783           | 1.6%         | 3.4%         | 95.0%        |
|         | Oil  | 15.3%              | 6519           | 5.5%         | 3.3%         | 91.2%        |
|         | Gas  | 4.0%               | 6066           | 3.0%         | 0.0%         | 97.0%        |

<sup>a</sup>In Korea, gas consumption began in 1987. To avoid a division by 0, this figure corresponds to the period 1987-2008.

In addition, the Theil inequality statistics detailed in Sterman (1984) provide a useful decomposition of the mean square errors in terms of bias ( $U_M$ ), unequal variation ( $U_S$ ), and unequal co-variations ( $U_C$ ). In most cases, the largest share of the MSE can be ascribed to  $U_C$  the imperfect covariation component of the Theil inequality statistics. The low bias and variation components of these statistics indicate that the errors are unsystematic, meaning that the models can replicate the observed behaviors.

These results together with the graphical representations suggest that the model does an excellent job of tracking the observed interfuel substitutions.

## 4 The demand function

The system dynamics model above offers great appeal for the prospective analysis of industrial energy demands. Given the poor representation of the demand side included in most natural gas market models, one could thus wish to embed this system dynamics-based

model within a partial equilibrium model of the natural gas markets. Unfortunately, all these models (c.f. the introduction) require the formal specification of a single-equation function of the demand for natural gas. In this section, this system dynamics-based representation is put to work to construct such a single-equation demand function.

Hereafter, the reference year is assumed to be  $t_0 = 2008$  and we focus on the future annual consumption of a given fuel, in one of the countries listed above, in year  $t > 2008$ . For the sake of clarity, we detail the case of natural gas consumption in Canada but this approach is general and can be used to model the industrial demand of any two other fuels in any country. In addition, we assume the availability of an exogenous scenario that details the evolution of future total final energy consumption and both coal and oil domestic prices in any future year  $t > 2008$ .

#### 4.1 Modeling next year's demand

To begin with, we focus on the first future year (that is,  $t_0 + 1 = 2009$ ) and detail the construction of a single-equation demand function for that year. To do so, a series of simulations of the system dynamics model are conducted with, *ceteris paribus*, various values of the 2009 price of natural gas. Using a large sample (1000 values) of possible 2009 gas prices regularly drawn over a wide range, we can generate a large data set that depicts the instantaneous change in the quantity of natural gas demanded in 2009 as a function of the 2009 price of that fuel.

As an illustration, Figure 3 depicts the results of a series of numerical simulations conducted for the case of Canada with the following exogenous parameters:  $FP_{coal}(2009) = 165\$/toe$  (coal price in 2009),  $FP_{oil}(2009) = 1030\$/toe$  (oil price in 2009), and  $ED(2009) = 30Mtoe$  (total final energy consumption in 2009).

These "pseudo data" can in turn be used to estimate the parameters of a single-variable, single-equation, demand function for the year 2009. This empirical demand function aims at providing an easy-to-handle representation of the quantity of fuel consumed in 2009 (the response variable) as a function of the own fuel price that year (the explanatory variable). Our simulation results (given in Figure 3) suggest that the quantity of fuel demanded should be modeled as a smooth and monotonically decreasing function of that fuel's price. For a very low price level, the fuel under scrutiny nearly captures all the new investments whereas the quantity demanded saturates at large values of this fuel prices, and this saturation level is set by the capacity of previously installed burning equipment. As all our simulations suggested the presence of an "S" shaped pattern, we explored the possibility of modeling these simulation results with an empirically determined sigmoid curve. Among the set of mathematical functions with an S-shaped curve (*e.g.*, logistic function, Gompertz function, etc.), our experiments lead us to consider the hyperbolic tangent. For each year  $t$ , we thus propose to fit the relation between simulated demand and price with the following functional form:

$$\hat{q}(p) = \beta + \delta \cdot (1 - \tanh(\gamma \cdot (p - pc))) \quad (13)$$

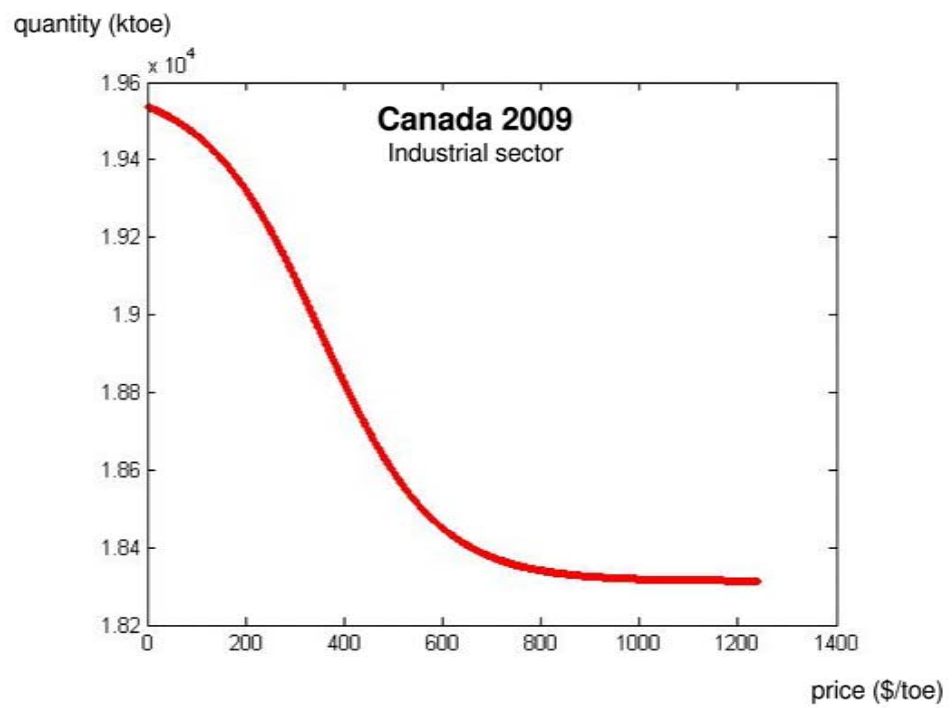


Figure 3:  
*The numerical demand function. Canada, industrial sector, natural gas, year 2009.*

where  $\hat{q}$  is the approximated quantity of fuel demanded in 2009,  $p$  is the 2009 fuel price (the explanatory variable),  $t$  is the time, and  $\beta$ ,  $\delta$ ,  $\gamma$ , and  $pc$  are non-negative parameters. The function  $\tanh$  is the hyperbolic tangent:

$$\forall x \in \mathbb{R}, \quad \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (14)$$

According to Formula (13), the proposed approximated demand function is monotonically decreasing. This specified demand does not rise to  $+\infty$  when the price is very low. This is principally due to the fact that the total final energy demand is exogenous to our model. Hence, the demand for natural gas remains upper-bounded. When the price is very high, we can notice that the quantity demanded converges towards a finite positive value  $\beta$ , that captures the "clay" effect, that is, the remaining demand originating from all the previous investments done in that fuel.

An interesting interpretation can be associated with the proposed approximation. This specification can be decomposed in two components, a constant term  $\beta$  that captures the rigidity associated with past decisions, and a price-variable term that measures the instantaneous reaction of demand to the current price (that is,  $\delta \cdot (1 - \tanh(\gamma \cdot (p - pc)))$ ). Concerning the latter term, the parameter  $pc$ , which is the inflexion point of the curve (cf. Figure 4), can be interpreted as a measure of the price of an alternative composite energy utilizing both coal and oil. Thus, the value of this parameter is influenced by the prices of both coal and oil products. The curvature parameter  $\gamma$  represents how fast the natural gas usage drops within a year, if the gas price rises. It is directly linked to the derivative of the demand function at the competing energy price  $pc$ . The amplitude parameter  $\delta$  is connected with the share of the total annual fuel demand that is subject to interfuel substitutions.

If we denote by  $q(p)$  the simulated demand provided by the system dynamics model and  $\hat{q}(p)$  the one given in equation (13), the *error* (distance between  $q$  and  $\hat{q}$ ) can be defined as follows:

$$error = \frac{\langle |q(p) - \hat{q}(p)| \rangle}{\langle q(p) \rangle} \quad (15)$$

The  $\langle . \rangle$  is the mean value. The mean value of a one-variable function  $f$  is defined as follows:

$$\langle f \rangle = \lim_{a \rightarrow +\infty} \frac{\int_{-a}^a f(x) dx}{2a} \quad (16)$$

The values of the parameters  $\beta$ ,  $\delta$ ,  $\gamma$  and  $pc$  are derived from a minimization of the *error* function.

Table 5: Optimal parameters, Canada, industrial sector, natural gas, year 2009

|          |                         |                   |
|----------|-------------------------|-------------------|
| $\beta$  | (ktoe)                  | $1.84 \cdot 10^4$ |
| $\delta$ | (ktoe)                  | $0.65 \cdot 10^3$ |
| $\gamma$ | (\$/toe <sup>-1</sup> ) | 0.0043            |
| $pc$     | (\$/toe)                | 352               |

As an illustration, Table 5 details the optimal values of the parameters  $\beta$ ,  $\delta$ ,  $\gamma$  and  $pc$  found for the case of natural gas industrial consumption in Canada for the year 2009. Figure 4 illustrates the quality of the numerical fit for that case. Apparently, this empirical model does an excellent job of tracking the simulated gas consumption as it is almost impossible to distinguish between the simulated pseudo-data and the proposed S-shaped approximation. This finding is also confirmed by the numerical value of the associated *error* which is extremely low:  $10^{-8}$ .

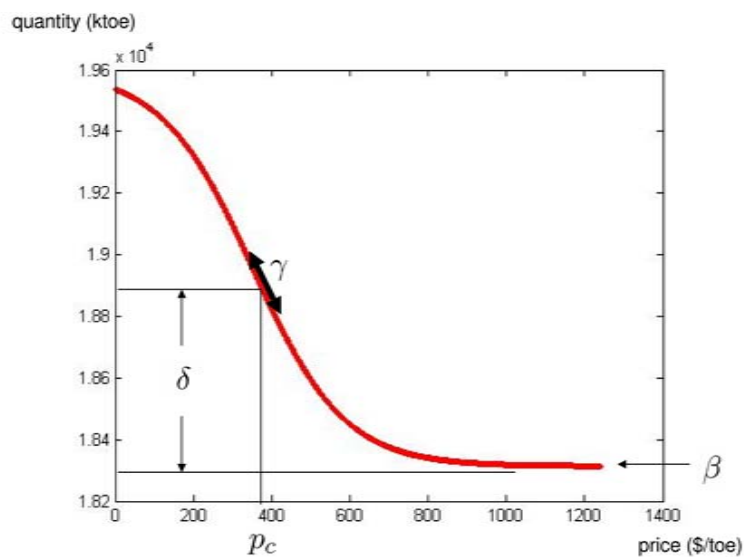


Figure 4:  
*The numerical fit. Canada, industrial sector, natural gas, 2009.*

Given the good fit offered by this specification, we can discuss the implied short-run price elasticity of natural gas demand. This elasticity is given by the following function (issued

from equation (13):

$$\epsilon(p) = -\frac{p\gamma\delta \cdot (1 - \tanh^2(\gamma(p - pc)))}{\beta + \delta \cdot (1 - \tanh(\gamma(p - pc)))} \quad (17)$$

*Ceteris paribus*, this short-run elasticity is a decreasing function of the addiction parameter  $\beta$ .

With usual numerical values, the graph of this function has the shape depicted in Figure 5. From the example of Canada in 2009, the magnitude of the short-run price elasticity of natural gas demand remains extremely low. Our experiments conducted with the other seven countries systematically confirmed the fact that, in the short-run, industrial consumers appear to be very little responsive to natural gas price increases. Of course, such a low price-response can have far-reaching consequences when analyzing security of supply issues (Abada and Massol, 2011) or the possibility to exert market power in the short-run with the help of an imperfect competition model.

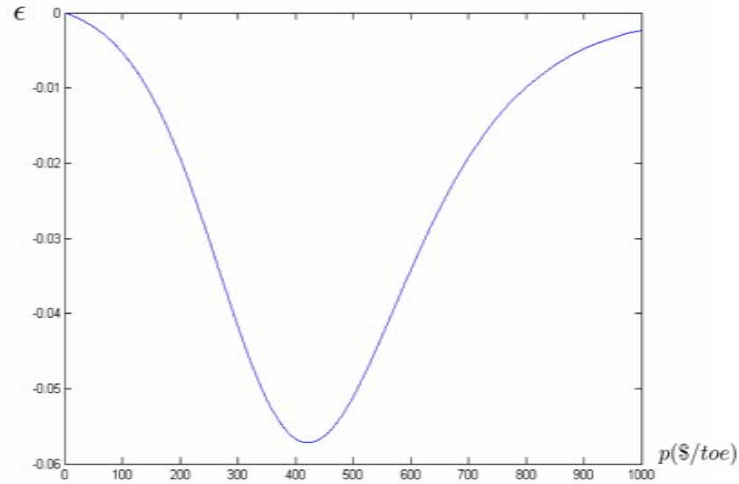


Figure 5:  
*The short-run price elasticity of industrial demand for natural gas. Canada, 2009.*

Of course, the values of the parameters  $\beta$ ,  $\delta$ ,  $\gamma$  and  $pc$  are conditioned by the chosen scenario (that is.  $ED(2009)$ ,  $FP_{oil}(2009)$ , and  $FP_{coal}(2009)$ ). Some sensitivity analysis can

thus be conducted to analyze the influence of the assumptions embedded in the scenario. As an example, we can study how the value of  $pc$  varies with the assumed coal and oil prices. Figure 6 gives the evolution of  $pc$  over the oil price  $FP_{oil}(2009)$ , in Canada in 2009. The coal price  $FP_{coal}(2009)$  is fixed at 163\$/toe. Our findings show that the price of the alternative energy is an increasing function of the oil price. The saturation effect observed is due to the coal price that remains constant.

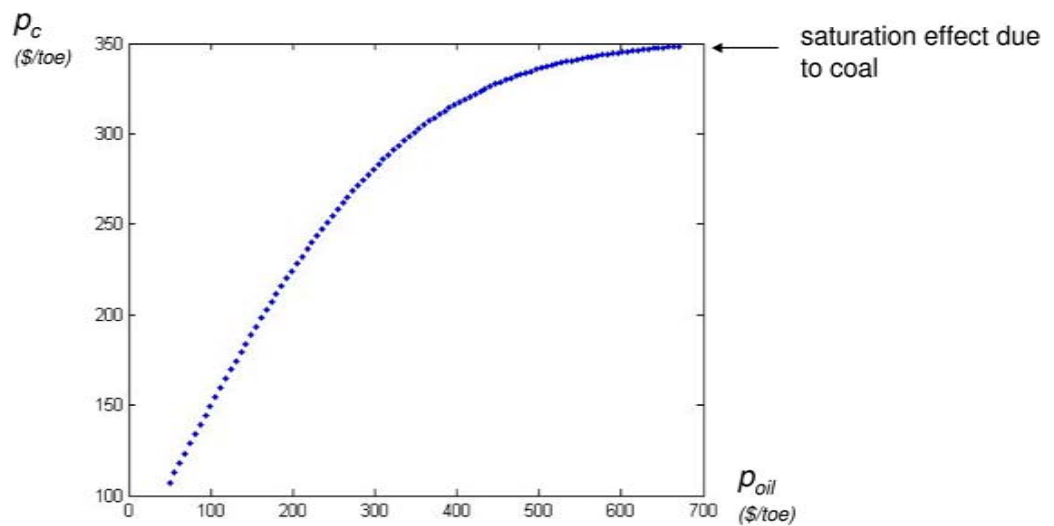


Figure 6:

*The evolution of  $pc$  over  $FP_{oil}(2009)$ . Canada, industrial sector, natural gas, year 2009.*

Similarly, we can analyze the influence of the global energy demand  $ED(2009)$  on the natural gas addiction quantified by the parameter  $\beta$ . Hence, Figure 7 gives the evolution of  $\beta$  for Canada over the assumed global energy demand  $ED(2009)$  for the year 2009. One can notice that there is always a remaining addiction  $\beta \neq 0$  even if there is no global energy demand  $ED = 0$ . This is due to the previous (to 2009) investments in natural gas.

According to these findings, the empirical approach at hand provides an acceptable approximation of the relation between simulated fuel demand and the fuel own price in year

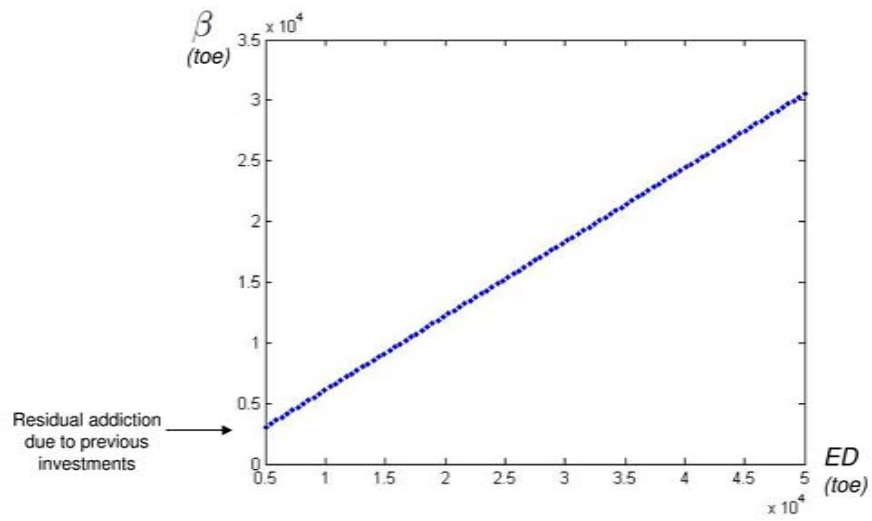


Figure 7:  
*The evolution of  $\beta$  over ED(2009). Canada, industrial sector, natural gas, year 2009.*

$t_0 + 1$  for a wide range of possible scenarios. Such a specification could usefully be put to work to refine the demand-side treatment embedded in most static oligopolistic models of the natural gas industry. For example, the popular models detailed in Golombek et al. (1995), Boots et al. (2004), and Holz et al. (2008) systematically postulate a simple, downward sloping, affine function to represent the connection between gas price and the volume demanded during the base year. Nevertheless, one could rightly feel uncomfortable with a model solely based on a static vision of the natural gas industry. At least two types of arguments can be advanced to consider a dynamic specification. Firstly, on the supply side, natural gas is an exhaustible resource and gas producers typically have to decide an intertemporal policy (investment, extraction path, etc.). Secondly, on the demand side, the magnitude of the long-run price elasticity of fuel demand is notoriously larger than its short-run counterpart. Any sudden rise in the price of a given fuel can, *ceteris paribus*, have far reaching negative consequences on both the volumes of fuel demanded during actual and future time periods. Concerning the supply-side, some progress has been made as a couple of recent imperfect competition models propose a dynamic treatment of the supply-side (Lise and Hobbs, 2008; Egging et al., 2010). On the contrary, the dynamic adjustment of volumes demanded to prices has, to our knowledge, never been taken into consideration within an imperfect competition model of the natural gas industry. In most cases, demand behaviour is simplified to an affine demand function depicting an instantaneous relationship between current prices and volumes demanded without any reference to past prices. Such a statement obviously calls for some investigation.

## 4.2 Modeling future demands

By construction, the system dynamics approach presented above is coherent with the fact that, *ceteris paribus*, the occurrence of a large gas price rise at a given future time  $t'$  will result in a lower demand for that fuel during the subsequent periods. In this subsection, we aim at putting this model to work to specify a single-equation demand function that captures such a dynamic adjustment. To begin with, we report how a meticulous analysis of a large number of simulated demand outcomes has guided us in the construction of such a dynamic specification. Then, a numerical example is detailed to illustrate the performances of the proposed specification.

### 4.2.1 Simulations: paving the way to a multivariate specification

Now, we focus on the demand for natural gas at a given future time period  $t$ . We assume that an exogenously defined scenario gives, for each time period  $t' \leq t$ , the overall energy demand  $ED(t')$  and the prices of the two alternative fuels  $FP_{coal}(t')$  and  $FP_{oil}(t')$ . Our approach can be decomposed into three successive steps.

1. A large number (10,000) of scenarios have been generated for the future prices of natural gas at any future time  $t'$  with  $t' < t$ . Hereafter,  $J$  is used to denote the set of scenarios. If  $j$  is used to index the generated scenarios, a gas price scenario can thus be written as  $(p_{t'}^j)_{t' < t}$  a vector with  $t - 1$  components. From a practical perspective, these future prices have been randomly generated assuming that future gas prices are i.i.d. random variables that follow an uniform distribution on the in-

terval  $[0, 700] \$/\text{toe}$ . These assumptions allow us to explore a large domain of possible future price scenarios<sup>6</sup>.

2. Then, we propose to analyze, for each scenario  $j$ , the instantaneous relationship between the current price of natural gas  $p_t$  and  $q_t^j$  the volume of natural gas demanded at time  $t$ . To do so, the current price  $p_t$  is varied so as to generate by simulation, for each scenario  $j$ , a data set of 1000 observations of the volume demanded. Unsurprisingly, these observations suggested the presence of a downward sloping, "S" shaped relation between the price  $p_t$  and  $q_t^j$ . Such a statement called for further investigations.
3. Each of these data sets has in turn been used to fit the following "S-shaped" specification. As a result, we have estimated, for each scenario  $j$ , the parameters  $\beta_t^j$ ,  $\delta_t^j$ ,  $\gamma_t^j$  and  $pc_t^j$  (according to equation (13)):

$$\hat{q}_t^j(p_t) = \beta_t^j + \delta_t^j \cdot \left( 1 - \tanh \left( \gamma_t^j \cdot (p_t - pc_t^j) \right) \right) . \quad (18)$$

For each parameter, we can gather the values  $(\beta_t^j)_{j \in J}$ ,  $(\delta_t^j)_{j \in J}$ ,  $(\gamma_t^j)_{j \in J}$  and  $(pc_t^j)_{j \in J}$  obtained for the various scenarios  $j$  and analyze their distributional properties. Two interesting findings emerged from this analysis. Firstly, the "dispersion", measured either in absolute terms (with the sample standard deviation) or in relative terms (with the coefficient of variation) was extremely low for the series  $(\delta_t^j)_{j \in J}$ ,  $(\gamma_t^j)_{j \in J}$  and  $(pc_t^j)_{j \in J}$ . Accordingly, the values of these three parameters are not influenced by previous gas prices. Secondly, on the contrary, the values  $(\beta_t^j)_{j \in J}$  are intimately connected with those of previous gas prices. Moreover, we systematically observed that, with two scenarios  $j_1$  and  $j_2$  that are such that  $p_{t'}^{j_1} \leq p_{t'}^{j_2}$  for all  $t' < t$ , a comparison of the values  $\beta_t^{j_1}$  and  $\beta_t^{j_2}$  provided  $\beta_t^{j_1} \geq \beta_t^{j_2}$ ,  $\forall t' < t$ . This latter observation is coherent with the interpretation given for  $\beta_t^j$  in the previous subsection, *i.e.*, a parameter that captures the "clay" effect associated with past investment decisions.

This three-step approach has been replicated for several time horizons  $t$  (in the range  $t_0 + 2$  and  $t_0 + 30$  years), for various countries, various alternative scenarios for both the overall energy demand and the prices of the two alternative fuels (coal and oil). Our empirical findings systematically confirmed the fact that: (i) the parameters  $\delta_t^j$ ,  $\gamma_t^j$  and  $pc_t^j$  do not depend on previous gas prices, whereas (ii)  $\beta_t^j$  exhibits a clear dependence on past values of the natural gas prices. From these investigations, it appears that: the index  $j$  can be dropped on the parameters  $\delta_t$ ,  $\gamma_t$  and  $pc_t$ , and that  $\beta_t^j$  can be viewed as the value taken by  $\beta_t$  a multivariate function of past gas prices evaluated at the particular point  $(p_{t'}^j)_{t' < t}$ , *i.e.*:

$$\hat{q}_t^j(p_t) = \beta_t \left( (p_{t'}^j)_{t' < t} \right) + \delta_t \cdot (1 - \tanh (\gamma_t \cdot (p_t - pc_t))) . \quad (19)$$

In addition, one may wish to elaborate on the path-dependency that is at work for the parameter  $\beta_t$ . As this parameter reflects the rigidity associated with past decisions, it is

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<sup>6</sup>More subtle probabilistic models fitted on historical time series, including alternative distributions and autocorrelation, have also been considered. Given that the obtained results did not differ from those detailed here, we have decided to maintain these rough assumptions.

tempting to relate it to  $q_{t-1}^j$  the volumes demanded at time  $t - 1$  in the scenario  $j$  (modulo some aging/scrapping of installed equipment). These latter volumes can, in turn, be approximated by the "S" shaped function  $\hat{q}_{t-1}^j$  evaluated at the particular price  $p_{t-1}^j$ :

$$\hat{q}_{t-1}^j(p_{t-1}^j) = \beta_{t-1}^j + \delta_{t-1} \cdot \left( 1 - \tanh \left( \gamma_{t-1} \cdot (p_{t-1}^j - pc_{t-1}) \right) \right) . \quad (20)$$

Here, the overall volume  $\hat{q}_{t-1}^j(p_{t-1}^j)$  can also be decomposed into: those precisely decided at date  $t - 1$ , and those encapsulated within the term  $\beta_{t-1}^j$  that reflects earlier decisions. This latter term can in turn be related, modulo some aging/scrapping of installed equipment, to  $q_{t-2}^j$ , a volume that can be approximated by  $\hat{q}_{t-2}^j(p_{t-2}^j)$  and so on...

Because of this nested scheme, one could wish to model the function  $\beta_t \left( (p_{t'}^j)_{t' < t} \right)$ , given in (19), with an additive specification that explicitly tracks the contributions of earlier vintages:

$$\beta_t \left( (p_{t'}^j)_{t' < t} \right) = \beta_{0,t} + \sum_{t' < t} h_{t' \rightarrow t} \left( \delta_{t'} \cdot \left( 1 - \tanh \left( \gamma_{t',t} \cdot (p_{t'}^j - pc_{t',t}) \right) \right) \right) , \quad (21)$$

where  $\beta_{0,t}$  denotes the contribution of burners initially present at time  $t_0$ , and  $h_{t' \rightarrow t}$  is a function that models the aging of burning appliances installed at date  $t'$ . Rather than specifying these aging processes, we consider that the aging function only alters the amplitude parameters  $\delta_{t'}$  so that equation (21) can be rewritten as follows:

$$\beta_t \left( (p_{t'}^j)_{t' < t} \right) = \beta_{0,t} + \sum_{t' < t} \delta_{t',t} \cdot \left( 1 - \tanh \left( \gamma_{t',t} \cdot (p_{t'}^j - pc_{t',t}) \right) \right) . \quad (22)$$

Since this formula, which will be confirmed in the following section, holds for a huge number of possible values of  $(p_{t'}^j)_{t' < t}$ , we can drop the scenario index  $j$ , and write:

$$\forall (p_{t'})_{t' < t}, \beta_t \left( (p_{t'})_{t' < t} \right) = \beta_{0,t} + \sum_{t' < t} \delta_{t',t} \cdot \left( 1 - \tanh \left( \gamma_{t',t} \cdot (p_{t'} - pc_{t',t}) \right) \right) . \quad (23)$$

If we denote  $\delta_{t,t} = \delta_t$ , both equations (22) and (19) suggest to model the volumes demanded at date  $t$  thanks to the following multivariable specification:

$$\hat{q}_t \left( (p_{t'})_{t' \leq t} \right) = \beta_{0,t} + \sum_{t' \leq t} \delta_{t',t} \cdot \left( 1 - \tanh \left( \gamma_{t',t} \cdot (p_{t'} - pc_{t',t}) \right) \right) , \quad (24)$$

where  $\beta_{0,t}$ ,  $(\delta_{t',t})_{t' \leq t}$ ,  $(\gamma_{t',t})_{t' \leq t}$  and  $(pc_{t',t})_{t' \leq t}$  are unknown parameters to be determined numerically.

## 4.2.2 Estimation and performance

We can now clarify the calibration procedure used to fit the approximation specified in equation (24). As in subsection 4.1, we need to define a distance between the simulated demand function  $q(t)$  and the theoretically proposed one  $\hat{q}(t)$ . Let us re-write the functions while showing the main variables:  $q(t, (p_{t'})_{t' \leq t})$  and  $\hat{q}(t, (p_{t'})_{t' \leq t})$ . It is difficult to define a distance because of the multivariable aspect of the functions, the variables being  $(p_{t'})_{t' \leq t}$  and  $t$ . Therefore, we define the time-depending error as follows:

$$error(t) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \frac{|q(t, (p_{t'}^j)_{t' \leq t}) - \hat{q}(t, (p_{t'}^j)_{t' \leq t})|}{q(t, (p_{t'}^j)_{t' \leq t})} \quad (25)$$

where the variables  $p_1^j, p_2^j \dots p_t^j$  are randomly selected between 0 and 700\$/toe (uniform distribution), for all  $j \in \mathbb{N}$ . Thanks to the strong law of large numbers, we know that  $\frac{1}{n} \sum_{j=1}^n \frac{|q(t, (p_{t'}^j)_{t' \leq t}) - \hat{q}(t, (p_{t'}^j)_{t' \leq t})|}{q(t, (p_{t'}^j)_{t' \leq t})}$  converges when  $n \rightarrow \infty$ .

Here again, our method minimizes the (time-depending) errors in order to estimate the parameters  $\beta_{0,t}$ ,  $\delta_{t',t}$ ,  $\gamma_{t',t}$  and  $pc_{t',t}$ . In the following, we report an illustration obtained for Canada, in year 2013. The following scenario has been used: constant fuel prices  $FP_{coal}=165$  \$/toe,  $FP_{oil}=1030$  \$/toe and a constant overall energy demand  $ED=30$  Mtoe. Table 6 gives the values of the parameters  $\beta_{0,2013}$ ,  $\gamma_{t',2013}$ ,  $pc_{t',2013}$  and  $\delta_{t',2013}$  for  $t' \in \{2009 \dots 2013\}$ .

Table 6: Optimal parameters, Canada, industrial sector, natural gas, year 2013

| time ( $t'$ )                              | 2009   | 2010   | 2011   | 2012   | 2013   |
|--------------------------------------------|--------|--------|--------|--------|--------|
| $\delta_{t',2013}$ (ktoe)                  | 595    | 611    | 625    | 636    | 644    |
| $\gamma_{t',2013}$ (\$/toe <sup>-1</sup> ) | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 |
| $pc_{t',2013}$ (\$/toe)                    | 352    | 352    | 352    | 352    | 352    |

|                         |                   |
|-------------------------|-------------------|
| $\beta_{0,2013}$ (ktoe) | $1.55 \cdot 10^4$ |
|-------------------------|-------------------|

The error we find numerically is very small:  $\forall t, error(t) \leq 10^{-7}$ . This is a validation of the use of the functional form provided in equation (24) to mathematically describe the demand function.

The new parameter  $\beta_{0,t}$  decreases with time. Indeed, formula (24) indicates that  $\beta_{0,t}$  is the residual demand in year  $t$ , when all the previous natural gas prices are very high. This residual consumption is expected to decrease with time if no investments are made in natural gas (which is the case when the natural gas prices are high, considering equation (2)).

At a fixed time  $t$ , the parameter  $\delta_{t',t}$  increases with  $t'$ , which is intuitive: the consumption dependence on natural gas price in year  $t'$  is less and less important in the future. If the global demand remains constant over time, the parameters  $\delta_{t',t}$  behave like the following:

$$\delta_{t',t} = \delta_0 \kappa^{t-t'}$$

where  $\delta_0$  and  $\kappa$  are constants. We found out that the new parameter  $\kappa$  is roughly the same for all the countries we studied:  $\kappa = 0.95$ .

There are many advantages to using our model to make a demand forecast. First, we take into account the inertia present in energy consumption, which is due to all the past investments in coal, oil and natural gas. Second, the demand function estimated for gas naturally depends on the other fuel prices. Thus, a competition between fuels, thanks to the substitution aspect, appears in the demand function. Finally, this technique takes into consideration the intertemporal dependence between consumption and prices. Indeed, fuel prices in year  $t$  will influence the demand in future years  $t' \geq t$ . Obviously, if the natural gas price is high in 2010, for instance, compared to the other fuels, few investments will be made in that fuel and the corresponding demand will therefore be low in the future years. In Abada et al. (2010), it is shown that this functional form can be used for building imperfect competition models of natural gas markets.

It has been stated before that the addiction parameter  $\beta_0$  decreases with time. Figure 8 shows the evolution of  $\beta_0$  between 2009 and 2023.

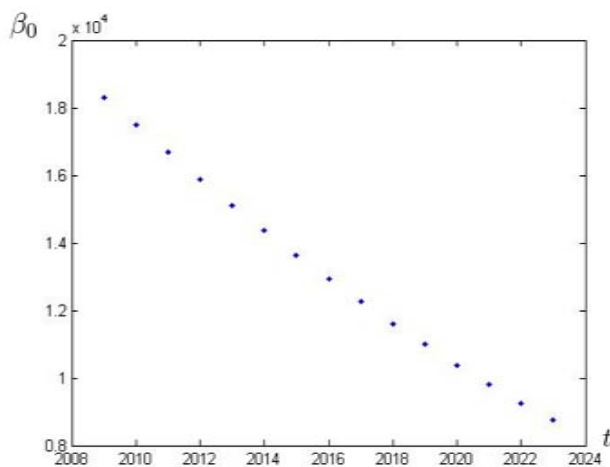


Figure 8:  
*The evolution of  $\beta_0$  over time. Canada, industrial sector, natural gas.*

The decrease of  $\beta_0$  is quasi exponential. We can numerically estimate the following dependence:  $\beta_{0,t} = K e^{-(t-2009)/\tau}$ . The values of the constants  $K$  and  $\tau$  in the case we studied (Canada) are  $K = 1.83 \cdot 10^4$  ktoe and  $\tau = 19$  years, which is roughly the investments depreciation time factor  $T_{natural\ gas}$ .

## 5 Conclusion

The extent to which alternative fuels can substitute for natural gas in the industrial sector is an issue of substantial interest to both energy policy analysts and corporate planners alike. It has recently been underlined that most of the large-scale representations of the natural gas industry embed a rudimentary representation of the demand side.

To remedy this, a revisited version of the system dynamics model proposed by Moxnes (1987) is put to work to analyze fuel choices in the industrial sector. This model emphasizes the role of prices in analyzing interfuel substitutions and captures the dynamic adjustment of demand to relative fuel prices using a vintaging structure. Using data on eight of the OECD countries for the period 1978-2008, we found that this model can satisfactorily replicate past patterns of fuel consumption. These performances make the model an appealing tool to examine fuel substitution possibilities in industrial energy demand. As result, a large number of simulations have been conducted with the aim to propose an adapted single-equation specification for the demand for fuel. From these investigations, it appears that a smooth, S-shaped, function can be used to represent the instantaneous reaction of fuel demand to price. In addition, this approach provides the ingredients necessary to capture the dynamic influence of past fuel prices on current consumption level. An extended multivariable specification has thus been derived and successfully tested.

This paper demonstrates the potential of that system dynamics-based method for deriving demand curves and thus offers a promising approach to further enhance the relevance of existing large-scale models of the natural gas industry.

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