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***Abstract*— This paper addresses the Unknown Input Observer (UIO) design problem for sampled-data Linear Time Invariant systems with time-varying sampling intervals. A discrete-time modeling approach is used. Based on convex optimization techniques, tractable design conditions are proposed and illustrated by numerical examples.**

I. INTRODUCTION

In engineering problems, a large number of physical systems are subject to disturbances that can considered as unknown inputs. A possible solution to take them into account is based on the use of the so-called Unknown Input Observer (UIO) method [1], which was proved to be relevant in a wide variety of application [4]. In the literature there is a large number of contributions addressing the UIO design problem for LTI systems. For pioneering works we point to [1], [14], [23]. The results addressing the case of sampled-data systems are sparse. For LTI sampled-data systems with constant sampling intervals, by using the exact system integration over the sampling interval, the discrete-time UIO approaches [10], [24] can be used (see also some works in the nonlinear case [3], [25]). Here we consider the UIO synthesis problem for the case of sampled-data systems with time-varying sampling intervals. As soon as digital communication network are used, sampling interval variations are ubiquitous ([11], [13], [12]). However, to the best of our knowledge, in the case where the sampling interval is time-varying the UIO design problem has not been addressed yet, not even for the case of LTI systems. This is precisely the case that we consider in this paper.

Although the aforementioned problem statement has not been addressed in the literature, it should be noted that some results exists for problem formulations related to ours. Results for state observers for sampled-data systems with time-varying sampling interval can be found in the literature. A hybrid system modeling approach has been used in [8], [7], [2], [9](see also the results based on continuous-discrete observers in [5], [21]). A time delay method has been proposed in [22]. An approach based on dissipativity and operator models of the sampling interval has been considered in [6]. In this paper a discrete-time modeling approach is used for the UIO design problem for LTI systems with time-varying sampling intervals. We recast the UIO design problem to one for Linear Parameter Varying (LPV) systems where the sampling interval appears as a scheduling parameter. First generic stability conditions are established for observers that depend on the sampling sequence. Next, numerically tractable design conditions are given based on convex optimization algorithm. From a technical point of view the methods used in this paper are related to the results for the UIO design problem in the case of LPV systems [16], [19], [20], time-delay systems [17], [18] and switched systems [7].

The rest of the paper is organized as follow: Section II is devoted to the formulation of the problem. In Section III, generic stability conditions are proposed based as a compact set of LMIs and numerically tractable conditions for robustness UIO design are specified in Section IV. The simulation results are presented in Section V.

Notation:

- v' denote the transpose of v for either a matrix or a vector.
- $\mathbb{R}_{\geq 0}$ corresponds to the positive real numbers.
- The euclidean norm of $x \in \mathbb{R}^n$ is denoted by $||x||$.
- For a symmetric matrix the symbol $\star$ denotes the elements induced by symmetry: $\begin{pmatrix} A & B \\ B' & C \end{pmatrix}$ will be denoted $\begin{pmatrix} A & B \\ \star & C \end{pmatrix}$.
- In the sequel, $\lambda_{\min}(Q)$ (resp $\lambda_{\max}(Q)$) denotes the smallest (resp largest) eigenvalue of a symmetric matrix Q.
- For a square symmetric matrix $P \succ 0$ (resp. $P \prec 0$) means that P is positive definite (resp. negative definite).

II. SYSTEM DEFINITION AND PROBLEM STATEMENT

A. System under consideration

Let us consider a linear time invariant system with unknown input

$$\begin{cases} \dot{x}(t) = A_c x(t) + D_c v_k, \ \forall t \in [t_k, t_{k+1}), \ k \in \mathbb{N} \\ y(t) = C x(t_k), \end{cases} \tag{1}$$

with $x(0) = x_0 \in \mathbb{R}^n$. Here $x(t)$ denotes the system state at time t. $(v_k)_{k\in\mathbb{N}}$ is a sequence describing the unknown system inputs. $y(t)$ represent the sampled output of the system. $A_c \in \mathbb{R}^{n\times n}, D_c \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{q\times n}$. $(t_k)_{k\in\mathbb{N}}$ denotes the sequence of monotonously increasing sampling instants with $t_0 = 0$. In what follows we will consider a class of sampling sequences $(t_k)_{k\in\mathbb{N}}$ where $t_{k+1} - t_k = h_k$ where h_k is assumed to be known at time t_{k+1}.

*Lucien Etienne and Kokou Anani Agbessi Langueh are with the CERI Systèmes Numerique, IMT-Nord-Europe, 59500 Douai, France, (e-mail: {lucien.etienne,kokou.langueh}@imt-nord.europe.fr)

**Laurentiu Hetel is with Centre de Recherche en Informatique, Signal et Automatique de Lille (CRIStAL, CNRS UMR 9189), Ecole Centrale de Lille, 59650 Villeneuve d'Ascq, France, (e-mail: laurentiu.hetel@ec-lille.fr)

Note that by assumption we consider that the unknown input v_k is constant on $[t_k, t_{k+1})$. This correspond the fact that the unknown input variation can be neglected with respect to the sampling interval. In what follows we consider, a finite set of index $\mathbb{I} \subset \mathbb{N}$. To each index $i \in \mathbb{I}$ we associate a duration τ_i and we assume that the nominal sampling interval h_k takes values in the union of such sets that is,

$$h_k \in \mathcal{T} := \bigcup_{i \in \mathbb{I}} [\underline{\tau_i}, \overline{\tau_i}], \forall k \in \mathbb{N}.$$

The set of sampling sequence under consideration is defined as follows :

$$S(\mathcal{T}) := \{(h_k)_{k \in \mathbb{N}} | h_k \in \mathcal{T}, \forall k \in \mathbb{N}\}.$$

Remark 1: Note that in the case under consideration there is no known input, this assumption is simply for ease of presentation and known input can very easily be integrated in the present work.

B. System remodeling

In this section, we provide a discrete time model of system (1) at sampling times.

Consider the following notations: $x_k = x(t_k), y_k = y(t_k), \forall k \in \mathbb{N}$. For any scalar h, we define the following matrices:

$$\begin{aligned} A(h) &:= e^{A_c h} \\ \Lambda(h) &:= \int_0^h e^{A_c s} ds \\ D(h) &:= \Lambda(h) D_c \end{aligned} \tag{2}$$

In what follows we will recast the original system in continuous time as discrete time LPV system representing the behavior of the system as sampling time.

Lemma 1: Consider notations (2). Then for any scalar h the following relations is satisfied :

$$A(h) = A_c \Lambda(h) + I. \tag{3}$$

Proof: Let us remark that

$$\begin{aligned} A_c \int_0^h e^{A_c s} ds &= \int_0^h A_c e^{A_c s} ds, \\ &= e^{A_c h} - e^{A_c 0}, \\ &= e^{A_c h} - I. \end{aligned}$$

Then using notation (2), $A(h) = A_c \Lambda(h) + I$, that is relation (3) holds. ∎

Lemma 2: Consider system (1) and notations (2). At sampling times $(t_k)_{k \in \mathbb{N}} \in S(\mathcal{T})$, the following relation is satisfied

$$\begin{aligned} x_{k+1} &= A(h_k) x_k + D(h_k) v_k \\ y_k &= C x_k, \, x_0 \in \mathbb{R}^n, \forall k \in \mathbb{N}. \end{aligned} \tag{4}$$

Proof: Integrating (1) between t_k and t_{k+1} and using the notations $t_{k+1} - t_k = h_k$ leads to

$$x(t_{k+1}) = e^{A_c h_k} x(t_k) + \int_{t_k}^{t_{k+1}} e^{A_c (t_{k+1} - \tau)} D_c v_k d\tau.$$

By setting $s = t_{k+1} - \tau$ one has

$$x(t_{k+1}) = e^{A_c h_k} x(t_k) + \int_0^{h_k} e^{A_c s} ds D_c v_k. \tag{5}$$

Using (3) along with notations (2) in (5) one has

$$x(t_{k+1}) = A(h_k) x(t_k) + \Lambda(h_k) D_c v_k,$$

∎

Model (5) will be used in order to derive a UIO which is robust with respect time-varying sampling intervals.

C. Observer definition

In this paper we consider a discrete-time UIO of the following form:

$$\left\{ \begin{array}{ll} z_{k+1} & = L(h_k, h_{k-1}) z_k + P(h_k, h_{k-1}) y_k \\ \hat{x}_k & = z_k + K(h_{k-1}) y_k \end{array} \right. \tag{6}$$

$k \in \mathbb{N}, k > 0$. For technical reason we will assume that $z_0 = \hat{x}_0 = 0$. Here z_k is the classical auxiliary variable and $\hat{x}_k$ is the estimation of the system state at sampling instants. The matrix functions $L : \mathcal{T} \times \mathcal{T} \to \mathbb{R}^n \times \mathbb{R}^n$, $P : \mathcal{T} \times \mathcal{T} \to \mathbb{R}^n \times \mathbb{R}^q$ and $K : \mathcal{T} \to \mathbb{R}^n \times \mathbb{R}^q$ are the observer gains which are scheduled according to the known values of the nominal sampling times. In a first step we consider that the scheduling functions are given; next we will provide conditions to design these matrix functions.

D. Problem statement

Since both the system under consideration and the observation scheme have been defined, we are ready to state the observation problem to be solved.

Definition. *(Exponential convergence)* A UIO observer (6) for system (5) is said to be *exponentially convergent* uniformly with respect to the set of sampling sequences $\mathcal{S}(\mathcal{T})$ with decay rate δ and overshoot M if

$$\forall k \in \mathbb{N}, ||\hat{x}_{k+1} - x_{k+1}|| \leq M \delta^k ||\hat{x}_1 - x_1||.$$

The goal of this article is to provide exponential convergence conditions for a given observer of the form (6) as well as to provide numerically tractable design methods.

III. Generic conditions

In this section we present generic conditions for assessing the exponential convergence of the observer (6). The results are formalized in the theorem bellow:

Theorem 1: Consider system (1), the sampled-data representation (5), UIO (6) and the set of sampling sequences $S(\mathcal{T})$. Assume that the following set of matrix equality constraints are satisfied

$$\begin{aligned} L(\theta_1, \theta_2) = \big(I_n - K(\theta_1) C\big) A(\theta_1) \\ + L(\theta_1, \theta_2) K(\theta_2) C - P(\theta_1, \theta_2) C, \end{aligned} \tag{7}$$

$$\big(I_n - K(\theta_1) C\big) D(\theta_1) = 0, \tag{8}$$

for all $\theta_1, \theta_2 \in \mathcal{T}$. Given $\alpha \in (0, 1)$, if there exist a continuous matrix function $\mathcal{P}(\theta)$ such that $\mathcal{P}(\theta) \succ 0, \forall \theta \in \mathcal{T}$ and

$$\begin{pmatrix} \mathcal{P}(\theta_1) & \mathcal{P}(\theta_1) L(\theta_1, \theta_2) \\ \star & (1 - \alpha) \mathcal{P}(\theta_2) \end{pmatrix} \succ 0, \forall \theta_1, \theta_2 \in \mathcal{T}, \tag{9}$$

then observer (6) for system (5) is *exponentially convergent* uniformly with respect to the set of sampling sequences $\mathcal{S}(\mathcal{T})$. In addition an upper bound of the decay rate is given by $\sqrt{1-\alpha}$ while the overshoot is upper bounded by $\sqrt{\dfrac{\max_{\theta\in\mathcal{T}}\lambda_{\max}(\mathcal{P}(\theta))}{\min_{\theta\in\mathcal{T}}\lambda_{\min}(\mathcal{P}(\theta))}}$.

Proof: Let $\varepsilon_k = x_k - \hat{x}_k$ denote the estimation error. Then using (6) :

$$\begin{aligned}\varepsilon_k &= x_k - z_k - K(h_{k-1})Cx_k\\ &= (I_n - K(h_{k-1})C)x_k - z_k,\end{aligned}$$

Using (5) and (6), the dynamics of the observation error is given by

$$\begin{aligned}\varepsilon_{k+1} &= \big(I_n - K(h_k)C\big)x_{k+1} - z_{k+1}\\ &= \big(I_n - K(h_k)C\big)A(h_k)x_k +\\ &\quad \big(I_n - K(h_k)C\big)D(h_k)v_k - L(h_k,h_{k-1})\hat{x}_k\\ &\quad + L(h_k,h_{k-1})K(h_{k-1})Cx_k - P(h_k,h_{k-1})Cx_k.\end{aligned}$$

This can be rearranged in the following way

$$\begin{aligned}\varepsilon_{k+1} = \Bigg(&\big(I_n - K(h_k)C\big)A(h_k)\\ &+ L(h_k,h_{k-1})K(h_{k-1})C - P(h_k,h_{k-1})C\Bigg)x_k\\ &+ \big(I_n - K(h_k)C\big)D(h_k)v_k - L(h_k,h_{k-1})\hat{x}_k.\end{aligned}$$

Using equality constraint (7) one has

$$\varepsilon_{k+1} = L(h_k,h_{k-1})\varepsilon_k + \big(I_n - K(h_k)C\big)D(h_k)v_k.$$

By using (8), the observation error becomes

$$\varepsilon_{k+1} = L(h_k,h_{k-1})\varepsilon_k. \tag{10}$$

Considering a (candidate) Lyapunov function $V : \mathbb{R}^n \times \mathbb{N} \to \mathbb{R}$, $V(\varepsilon_k,k) = \varepsilon_k\mathcal{P}(h_{k-1})\varepsilon_k$, $\forall k > 0$. At time step $k+1$ one has

$$V(\varepsilon_{k+1},k+1) = \varepsilon_{k+1}'\mathcal{P}(h_k)\varepsilon_{k+1}.$$

Using (10) the previous relation becomes

$$V(\varepsilon_{k+1},k+1) = \varepsilon_k' L(h_k,h_{k-1})'\mathcal{P}(h_k)L(h_k,h_{k-1})\varepsilon_k.$$

Therefore,

$$\begin{aligned}V(\varepsilon_{k+1},k+1) - (1-\alpha)V(\varepsilon_k,k)\\ = \varepsilon_k'\Psi(h_k,h_{k-1})\varepsilon_k\end{aligned} \tag{11}$$

with

$$\begin{aligned}\Psi(h_k,h_{k-1}) = L(h_k,h_{k-1})'\mathcal{P}(h_k)L(h_k,h_{k-1})\\ - (1-\alpha)\mathcal{P}(h_{k-1}).\end{aligned}$$

LMI's (9) (by a Shur complement) imply that

$$\Psi(\theta_1,\theta_2) \prec 0,\ \forall\theta_1,\theta_2 \in \mathcal{T},$$

and therefore from (11) one has

$$V(\varepsilon_{k+1},k+1) \leq (1-\alpha)V(\varepsilon_k,k), \forall\varepsilon_k, k > 0.$$

Hence $V(\varepsilon_{k+1},k+1) \leq (1-\alpha)^k V(\varepsilon_1,1), \forall k > 0$.

Since

$$\min_{\theta\in\mathcal{T}}\lambda_{\min}(\mathcal{P}(\theta))||\varepsilon_k||^2 \leq V(\varepsilon_k,k) \leq \max_{\theta\in\mathcal{T}}\lambda_{\max}(\mathcal{P}(\theta))||\varepsilon_k||^2,$$

$\forall\, k > 0, \varepsilon_k \in \mathbb{R}^n$, then

$$||\varepsilon_{k+1}|| \leq \sqrt{\frac{\max_{\theta\in\mathcal{T}}\lambda_{\max}(\mathcal{P}(\theta))}{\min_{\theta\in\mathcal{T}}\lambda_{\min}(\mathcal{P}(\theta))}}\sqrt{(1-\alpha)}^k||\varepsilon_1||, \forall k \in \mathbb{N}.$$

■

Remark 2: Theorem 1 provide generic conditions for checking the exponential convergence of a given UIO described in (6). However the conditions have to be verified for an infinite set of parameter $\theta \in \mathcal{T}$. In what follows we will show how such a generic conditions can be turned into numerically tractable conditions.

IV. TRACTABLE CONDITIONS FOR A SPECIFIC CLASS OF SAMPLING

In the following section we will consider a specific class of sampling sequences. More precisely we consider sampling sequences described by a finite number of sampling intervals (i.e. $\forall k \in \mathbb{N}, h_k$ belongs to a finite set $\mathcal{T}_f$). First tractable conditions for stability of the UIO are provided, next conditions for designing a UIO of the form (6) are given.

Consider $S(\mathcal{T}_f)$ with

$$\mathcal{T}_f = \bigcup_{i\in\mathbb{I}_f}\{\tau_i\}$$

where $\mathbb{I}_f \subset \mathbb{N}$ a finite set of index. For $i,j \in \mathbb{I}_f$ let us define $A_i = A(\tau_i)$, $D_i = D(\tau_i)$, $K_i = K(\tau_i)$, $L_{i,j} = L(\tau_i,\tau_j)$, $P_{i,j} = P(\tau_i,\tau_j)$. In this specific case Theorem 1 becomes :

Corollary 1: Consider system (1), the sampled-data representation (5), UIO (6) and the set of sampling sequences $S(\mathcal{T}_f)$. Assume that the following set of matrix equality constraints are satisfied

$$\forall i,j \in \mathbb{I}_f,\ L_{i,j} = \big(I_n - K_iC\big)A_i + L_{i,j}K_jC - P_{i,j}C \tag{12}$$

$$\big(I_n - K_iC\big)D_i = 0. \tag{13}$$

Given $\alpha \in (0,1)$, if there exist a set of matrices $\mathcal{P}_i$ such that $\mathcal{P}_i \succ 0, \forall i \in \mathbb{I}_f$ and

$$\begin{pmatrix}\mathcal{P}_i & \mathcal{P}_iL_{i,j}\\ \star & (1-\alpha)\mathcal{P}_j\end{pmatrix} \succ 0,\ \forall i,j \in \mathbb{I}_f. \tag{14}$$

Then observer (6) for system (5) is *exponentially convergent* uniformly with respect to the set of sampling sequences $\mathcal{S}(\mathcal{T}_f)$. In addition an upper bound of the decay rate is given by $\sqrt{1-\alpha}$ while the overshoot is upper bounded by $\sqrt{\dfrac{\max_{i\in\mathbb{I}_f}\lambda_{\max}(\mathcal{P}_i)}{\min_{i\in\mathbb{I}_f}\lambda_{\min}(\mathcal{P}_i)}}$.

Proof: Corollary 1 is a special case of Theorem 1 where $L(\theta_1,\theta_2) = L_{i,j}$ (resp $P(\theta_1,\theta_2) = P_{i,j}$) when $\theta_1 = \tau_i, \theta_2 = \tau_j$ and $\mathcal{P}(\theta_1) = \mathcal{P}_i$ (resp $K(\theta_1) = K_i$) when $\theta_1 = \tau_i$

■

Remark 3: Note that conditions (12)-(14) represent a tractable set of LMIs for stability analysis when the gain matrices $K_i, L_{i,j}$ and $P_{i,j}$ are given. Here only the matrices $\mathcal{P}_i$ are LMIs variables.

In the following corollary we provide design conditions for a UIO of the form (6) in the specific case where the sampling interval take value in a finite set.

Corollary 2: Given a scalar $\alpha \in (0,1)$, if there exists positive definite matrices $\mathcal{P}_i$ as well as matrices $\mathcal{S}_i, \mathcal{Q}_{ij}, i,j \in \mathbb{I}_f$ of appropriate dimension such that

$$\begin{pmatrix} \mathcal{P}_i & \mathcal{P}_i A_i - \mathcal{S}_i C A_i - \mathcal{Q}_{ij} C \\ \star & (1-\alpha)\mathcal{P}_j \end{pmatrix} \succ 0 \qquad (15)$$

$$\mathcal{P}_i D_i - \mathcal{S}_i C D_i = 0, \forall\, i,j \in \mathbb{I}_f, \qquad (16)$$

Then observer (6) with

$$K(\tau_i) = \mathcal{P}_i^{-1}\mathcal{S}_i \qquad (17)$$

$$L(\tau_i,\tau_j) = (I_n - K(\tau_i)C)A(\tau_i) - \mathcal{P}_i^{-1}\mathcal{Q}_{ij}C \qquad (18)$$

$$P(\tau_i,\tau_j) = \mathcal{P}_i^{-1}\mathcal{Q}_{ij} - L(\tau_i,\tau_j)K(\tau_j), \forall i,j \in \mathbb{I}_f \qquad (19)$$

is *exponentially convergent* uniformly with respect to the set of sampling sequences $\mathcal{S}(\mathcal{T}_f)$. In addition an upper bound of the decay rate is given by $\sqrt{1-\alpha}$ while the overshoot is upper bounded by $\sqrt{\dfrac{\max_{i\in\mathbb{I}_f}\lambda_{\max}(\mathcal{P}_i)}{\min_{i\in\mathbb{I}_f}\lambda_{\min}(\mathcal{P}_i)}}$.

Proof: Recall the notations $A_i = A(\tau_i)$, $D_i = D(\tau_i)$, $K_i = K(\tau_i)$, $L_{i,j} = L(\tau_i,\tau_j)$, $P_{i,j} = P(\tau_i,\tau_j)$. Note that using (18), one has

$$\mathcal{P}_i L_{i,j} = \mathcal{P}_i A_i - \mathcal{P}_i K_i C A_i - \mathcal{Q}_{ij} C.$$

Hence (15) imply that (9) holds. Similarly, using (17)

$$\mathcal{P}_i D_i - \mathcal{S}_i C D_i = 0 \leftrightarrow \mathcal{P}_i(D_i - K_i C D_i) = 0$$

Since $\mathcal{P}_i$ is invertible $D_i - K_i C D_i = 0$ so (13) is satisfied. Using (18) and (19) one has

$$L_{i,j} = \big(I_n - K_i C\big)A_i + L_{i,j}K_j C - P_{i,j} C.$$

Hence (12) holds.

■

Remark 4: The previous corollary provides simple design conditions for a UIO of the form (6). By formulating the UIO synthesis problem as a specific convex optimization problem (in this case both linear matrices equalities and inequalities) it is possible to directly derive both $L(.,.)$ and $K(.)$ as variables to be computed. Note that no particular assumption is needed concerning the structure of $K(.)$.

V. Simulation

In this section we present two illustrative numerical examples.

A. Example 1

We consider the linear model of a two room building [26]. The equivalent RC model is being used with

$$A_c = \begin{pmatrix} -\frac{1}{C_1 R_{12}} & \frac{1}{C_1 R_{12}} \\ \frac{1}{C_2 R_{12}} & -\frac{1}{C_2 R_{23}} - \frac{1}{C_2 R_{12}} \end{pmatrix}, \quad D_c = \begin{pmatrix} \frac{w_1}{C_1} \\ \frac{w_2}{C_2} \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

In the above matrices C_i is the thermal capacitance of room i in kWh/K. $R_{i,j}$ is the thermal resistance K/kW between room i and room j, $i,j \in \{1,2\}$ and R_{23} represente the thermal resistance of room 2 and the outside. Subsequently w_j is a weighting factor (e.g., solar aperture) relating the heat input and the heat entering room i. x_i represents the temperature of room i, with $i \in \{1,2\}$. The unknown input, v, represents the heat induced by the solar irradiance. We consider that the variation of the heat due to solar irradiance is slow with respect to the sampling time (therefore the unknown input is considered constant within a sampling interval). The numerical values used for simulation are based on [15]: $C_1 = 8*10^6, C_2 = 10^7, w_1 = 3, w_2 = 3, R_{12} = 4*10^{-3}, R_{23} = 4*10^{-3}$. The heat due to the solar irradiance, v, has a base a value of 800 Watts and an added oscillation with a magnitude of 100 Watts. The system output is nominally sampled every ten minutes and send to a communication network. However, the communication can be subject to data loss. We consider that we may have up to 11 successive packet dropout. We assume that data packet are time-stamped, therefore h_k can be considered as a known parameter. Communication delays are assumed to be neglectable. Hence h_k is arbitrarily selected in the set $\{6*10^2 l, l \in \{1...12\}\}$.

We apply Corollary 2 with a decay rate$\sqrt{0.1}$ to derive a sampled-data observer (6). By solving the proposed conditions one obtain 144 matrices L_{ij}, 144 matrices P_{ij} and 12 matrices K_i. The observer gains are switched according to the sampling sequence h_k using a lookup table. Figure 1 illustrate the continuous-time evolution of the non measured state x_2 from the initial condition $x_0 = (22\,22)'$ as well as the discrete-time observer state $\hat{x}_2$ from the initial condition $\hat{x}_0 = (22\,17.5)'$. The norm of the observation error in discrete-time is given in Figure 2

VI. Conclusion

In this work we have studied the design of unknown input observers for linear time invariant systems with sampled output measurements. Both theoretical and tractable conditions have been presented for the synthesis of an unknown input observer under aperiodic sampling. First generic conditions have been derived to verify the convergence of the observer. Then, for a specific class of sampling sequences, it was possible to formulate the generic results as a tractable convex optimization problem. In this case the resulting sufficient conditions lead to a finite set of linear matrix inequalities

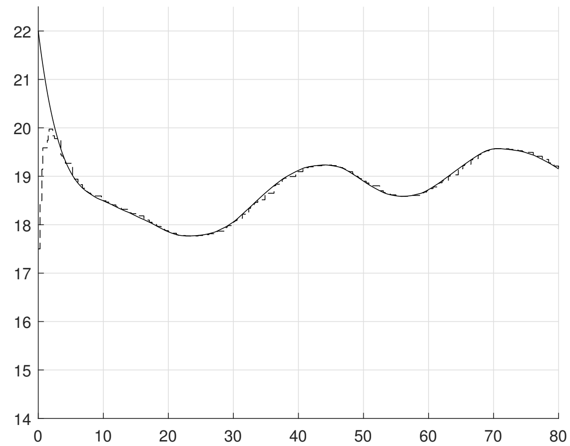

Fig. 1: Non measured state $x_2(t)$ of (plain) and observer state $\hat{x}_{2,k}$ (dashed).

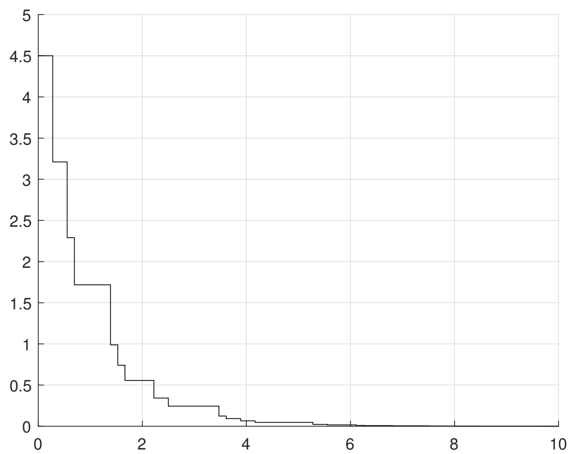

Fig. 2: Norm of the observation error with initial error $(0,\, 4.5)'$.

which allow to design an observer with prescribed convergence rate. The proposed results are illustrated on a numerical example.

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