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# Distributed model predictive control using a trigger time search for optimal heating of multizone buildings

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## Abstract

Optimising operation and control of buildings' electric loads is essential to mitigate peak demand and adapt to intermittent production. Predictive control has been applied to building energy management for more than twenty years. Solving an optimal control problem (OCP) allows energy saving and flexibility while maintaining thermal comfort. Resolution methods for OCP require either expert knowledge or intensive computational efforts, which complicates commercial implementation. A simple OCP resolution approach implemented into a distributed model predictive control is presented in this communication. The original contribution of this work stems from the combination of Bellman's and Pontryagin's principles. Results showed a reduction of computation time compared to a reference method with similar energy saving (28 %) when applied to a two-zone building using Time of Use tariff assumptions.

## Key innovations

- Application of Bellman's and Pontryagin's principles
- Trigger time search for optimal storage strategy
- Distributed MPC with short computational times
- Easily implemented for commercial use

## Practical implications

The approach is easily applicable to any type of buildings with sufficient thermal mass; insulation increases the efficiency of heat storage which leads to higher saving potential. Peak hours shifting can also be achieved in some old buildings. The method requires weather and occupancy data for prediction.

## Introduction

The building sector is the highest contributor in final energy consumption in France today with a 45 % share. It is at the centre of the energy and ecological transition due to its environmental impacts, which accounts for a fourth of total greenhouse gases (GHG) emissions [1]. In parallel, there is a growing need to better manage the production and consumption of electricity. The power grid is under pressure due to the temporal variation of consumption, mostly during winter when demand peaks around 7pm. This raises issues of energy supply, and production, export and import capacities. At the grid level, the solutions implemented today to provide balance and flexibility include modular plants (mostly thermal with high GHG emissions), energy storage (hydraulic, hydrogen...) and import and export of electricity [2].

At the consumer level, demand side response can be implemented in order to better manage energy consumption and reduce the tension related to the peak demand. This can be achieved through smart control of flexible loads within a building. It can be applied to space heating [3]–[4], cooling [5], ventilation [6], and solar protection [7]. The goals of the control can range from energy saving [3], improvement of thermal comfort and indoor air quality [8] to energy bill reduction [9].

Model predictive control (MPC) has been rising in building applications throughout the years. It has been applied to peak-load shifting and optimisation of energy consumption through a wide range of applications including different energy systems within the building and different levels of complexity and scale [10]–[14].

In this study, a distributed MPC with a simple resolution method for the optimal control problem (OCP) is presented. The approach is applied to a multizone building model, where the optimal strategy is determined for each zone. Heat transfer between zones are taken into account through the coupling of the dynamical model. Model predictive control is appropriate to take into account criteria that can sometimes be in conflict (example: reduction of energy bill and thermal comfort). The interest is its compliance to constraints, and the integration of errors and uncertainties related to weather forecast as well as occupancy predictions.

The original contribution of this work stems from the combination of the two main paradigms of optimal control theory [15]–[16] which provide a basis to develop a direct method of resolution for a constrained optimal control problem. The focus lies on the management of electrical heating in order to mitigate its contribution to the peak electrical load and minimise its costs in buildings with sufficient thermal mass.

The method offers a flexibility when it comes to adaptation to different types of buildings and especially a robustness against the variability of the cost function (e.g. dynamic tariffs of electricity).

## Models

### Thermal simulation model

The building's dynamics is modelled using the white box model COMFIE to evaluate operative temperature, heating and cooling loads as well as comfort levels. Energy balance is applied to each node of a finite volume meshing of each thermal zone (walls are divided into multiple meshes with uniform temperature, an additional mesh includes air and furniture):

$$C \frac{dT}{dt} = Gains - Losses \text{ \#(Erreur ! Signet non défini.)}$$

$C$  is the thermal capacity of the node,  $T$  is its temperature. The gains and losses account for heat transfer between walls and zones, solar and internal gains as well as air flows.

The dynamic system of equations of all the nodes is defined [17]:

$$\begin{cases} C \cdot \dot{T}(t) = A \cdot T(t) + E \cdot U(t) \\ Y(t) = J \cdot T(t) + G \cdot U(t) \end{cases} \text{ \#(Erreur ! Signet non défini.)}$$

- $T$  represents the node's temperature vector;
- $U$  is the driving forces vector;
- $Y$  is the output vector (contains each zone's temperature)
- $A$  is the state matrix,
- $E$  is the input matrix,
- $J$  is the output matrix,
- $G$  is the feedforward matrix.

The system is reduced by modal analysis in order to run fast simulations using Pleiades+COMFIE tool [17].

### Balanced reduction

The state representation allows another assessment of the dynamic system which simplifies the implementation of an optimal control strategy. System (2) is rearranged into a linear time-invariant system:

$$\begin{cases} \dot{x} = A \cdot x(t) + B \cdot u(t) \\ y(t) = C \cdot x(t) \end{cases} \text{ \#(Erreur ! Signet non défini.)}$$

- $x$  is the state vector,
- $B$  is the input matrix,
- $C$  is the output matrix.

For the sake of reducing the computation time of the optimisation while maintaining the precision of the building model, a balanced reduction is applied. The full model of order  $n$  is truncated through criteria that requires its states to be completely observable and controllable. The method has been presented in [18] and proved efficient in terms of precision and reliability in building control applications. Robillart et al. [19] used the balanced truncation method in the implementation of model predictive control for load shifting. The order of the reduced model is chosen through frequency analysis and comparison to the reference full building model.

### Optimal control theory

The optimal control problem (OCP) determines a trajectory of a command which minimises a criterion (or objective function) while respecting the state and control constraints. Applied to this article, this means the minimisation of energy or environmental cost of a building through the control of its electrical heating. The optimisation criterion of the OCP can be expressed:

$$\min_{u \in U} \int_0^{t_f} Price(t)u(t)dt \text{ \#(Erreur ! Signet non défini.)}$$

$Price$  represents electricity tariffs (in euros or emission of CO<sub>2</sub> equivalent), and  $u$  is the heating power.

There are two main principles that drive optimal control theory. The first one is Pontryagin's minimum principle introduces a necessary condition of optimality [20]. If a control  $u^*$  associated to the state  $x^*$  which is the solution of the OCP then there exists an adjoint state  $p$  for which:

$$\dot{x}(t) = \frac{\partial}{\partial p} H(x^*(t), u^*(t), p(t)) \#(\text{Erreur ! Signet non défini.})$$

$$\dot{p}(t) = -\frac{\partial}{\partial x} H(x^*(t), u^*(t), p(t)) \#(\text{Erreur ! Signet non défini.})$$

$$H(x^*(t), u^*(t), p(t)) = \min_u H(x^*(t), u(t), p(t)) \#(\text{Erreur ! Signet non défini.})$$

The condition leads to the optimum:

$$u(t) \in \arg \min_u H(x(t), u, p(t)) \#(\text{Erreur ! Signet non défini.})$$

This rewrites the OCP in a two boundary value problem (BVP) and requires continuous indirect methods of resolution. The penalty method will be detailed in the next section of the communication. The continuous time resolution through penalty method guarantees the respect of state constraints. However, convergence problems can occur during the resolution of the two BVP, which makes the method very sensitive to algorithm initialisation. The second principle is the dynamic programming introduced by Bellman [15]. It is a direct method of resolution of the OCP. The basis of the principle stems from the discretisation of the problem into sub-problems where the optimum solution is composed of a succession of sub-optimums such that:

$$J = [V_0^N] \#(\text{Erreur ! Signet non défini.})$$

$$J = \min [v_0(x_0, x_1) + \min[V_1^N]] \#(\text{Erreur ! Signet non défini.})$$

Dynamic programming was applied to building energy management by Favre [21]. This direct approach takes up an important amount of memory as it explores all possible solutions at each time step which results in very high computation time. Any effort to minimise time would lead to a discretisation level that is coarse which can compromise the precision of results.

### Penalty method and centralised model

Pontryagin's principle offers a necessary condition of optimality through a two-boundary value problem by introducing an adjoint state and a condition of minimum of the system's Hamiltonian. Malisani et al. [22] developed an algorithm that solves the OCP for a single zone building using a penalty method. This method consists in penalising the cost function, by introducing a barrier term which increases the cost highly if the state approaches the constraint. The penalised cost function becomes:

$$\min_{u \in U} \left[ J(u) + \varepsilon \int_0^{t_f} \gamma(x(t)) dt \right] \#(\text{Erreur ! Signet non défini.})$$

- $\gamma(s) = \frac{1}{s^{n+1}}$  is the penalty function;
- $\varepsilon$  is the weight of the penalty.

This makes the expression of the problem free from state constraints while forcing the optimum solution to respect the constrained OCP as the solution converges to the optimum with constraints as  $\varepsilon$  tends towards 0.

So far, the penalty method has been introduced for a monozone building model. Frapin et al. [23] applied the method to a two-zone building by developing a single dynamic model which coupled the matrices of contiguous thermal zones resulting in a single OCP with two inputs and two outputs (one for each zone). Four state constraints are therefore taken into consideration which means the cost function includes two penalty functions:

$$\min_{u \in U} \left[ J(u) + \varepsilon \sum_{i=1}^2 \int_{t_0}^{t_f} [\gamma(x_i(t)) dt] dt \right] \#(\text{Erreur ! Signet non défini.})$$

In the same way as for the monozone control, the two penalties (one for each zones) are applied through barrier functions which increase the cost of the strategy when the solution approaches the constraints. The centralised algorithm proceeds in decrements of  $\varepsilon$  by solving a single OCP using Matlab solver bvp5c until the optimum is reached. This yields two control trajectories, one for each zone.

### New approach

The direct method to solve the OCP proposed here applies both principles of optimality of Pontryagin and Bellman.

Primarily, the two-boundary value problem using Pontryagin principle is no longer used. The result of the principle for this type of problem is applied: the solution of equation (7) is situated in the limits of the admissible commands. In that sense, two solutions are explored; heat storage and load shifting. The storage describes the increase of heating up to the maximum state constraint  $T_{max}$ . If the temperature of the zone during storage exceeds  $T_{max}$ , then the maximum command constraint kicks in and will limit the power supply to  $P_{max}$ . In the same manner, load shifting takes into account the minimum state constraint  $T_{min}$ , and operates a setpoint control. If the zone temperature goes below the minimum  $T_{min}$  heating is provided in order to maintain it. Otherwise, the heating load is considered completely shifted ( $P_{min}$ ).

Secondly, Bellman's principle is applied through the discretisation and the exploration of the two possible solutions at each time step in contrast with dynamic programming which explores all admissible command.

The criterion remains the minimisation of the energy bill for the chosen period during which the optimisation will be implemented.

## Methodology

This section describes the methodology followed in order to operate an optimal control strategy for energy management in a building.

The first step involves the modelling of the building, the extraction of system matrices and dynamic coupling, and the balanced reduction.

The second step focuses on the optimal control method developed. The following subsection details the approach and describes the optimisation algorithm.

Finally, the strategy using the new method will be validated by comparison with the centralised method using penalty method [23].

## Modelling

The building is modelled using Pleiades+COMFIE. matrices A, B and C are extracted for each thermal zone. The order of the full model will depend on the meshing which takes into account the wall compositions and orientations of the zones. Internal gains reflect the type of occupancy and use of the building, occupancy is considered in the form of a time-based scenario describing dissipated power. External driving forces correspond to weather conditions (data can be typical or real) and include the external temperature and solar radiation which are the same for all zones. The solar heat fluxes on walls and windows are calculated. The detailed dynamic system is:

$$\begin{cases} \dot{x}(t) = Ax(t) + BP_{ch}(t) + Bd_{int}(t) + B_{ext}d_{ext}(t) \\ T(t) = Cx(t) \end{cases} \#(\text{Erreur ! Signet non défini.})$$

- $P_{ch}$  is the heating power;
- $d_{int}$  is the internal gains (occupation);
- $d_{ext}$  are the external driving forces;
- $T$  is the operative temperature of the zone.

In the case of a multizone model, the heat transfer between zones needs to be taken into consideration. Two methods are applied and compared in this article. Firstly, the centralised method which considers the multizone building as a whole and calculates the strategy as one OCP. The thermal model is the result of the coupling of zone models into one multizone model. This is done by connecting the meshes of intermediate walls between zones and introducing an equivalent temperature in the system as an input for adjacent zones. The centralised model predictive control is solved using the penalty method based on Pontryagin principle.

For the new approach, the OCP is distributed and the optimal strategy is calculated at the level of each zone while taking into account the heat transfers from adjacent zone. This introduces a new term in the equation:

$$\begin{cases} \dot{x}(t) = Ax(t) + BP_{ch}(t) + Bd_{int}(t) \\ \quad + B_{ext}d_{ext}(t) + B_{adj}T_{adj} \\ Y(t) = Cx(t) \end{cases} \#(\text{Erreur ! Signet non défini.})$$

- $d_{ext}$  now includes the solar flux that comes in from the adjacent zone through the shared wall,
- $T_{adj}$  is the adjacent zone temperature.

The linear ordinary differential equation is solved using the variation of constants general method. This enables the calculation of the state as well as the heating power needed to reach a setpoint at any given time step.

## The multizone optimal control

The novel approach stems from the result of Pontryagin's principle and explores at each time step two possible solutions; minimum and maximum command. The optimum is determined by evaluating a strategy's energy cost reduction when applying heat storage as opposed to temperature setpoint control. In order to better assess the energy bill, two time horizons are introduced when solving the OCP. The optimisation horizon is considered to be the period during which the OCP is solved and an optimum trajectory for heating is calculated. The effect horizon is the amount of time in the future during which the optimal strategy will have an impact. Indeed, when heat is stored and destored, the heating load of the building is affected during a period of time linked to its time constants.

Figure 1 illustrates two strategies; one where optimal control is applied during 5 days and one where a 19 °C setpoint control is applied, constituting a reference. Starting from the 5<sup>th</sup> day the heating load are evaluated. When heat storage occurs, the energy level of the building is modified. In Bellman's perspective for two strategies to be compared, the initial and the final states need to be the same for both strategies. The effect horizon takes that into consideration. A sensitivity analysis showed that the effect of storage in prevision of peak load shifting impacts significantly the

building's state over a period equal to three times the building's highest time constant. Once the elapsed horizon of effect, the heating load difference goes under 1 %.

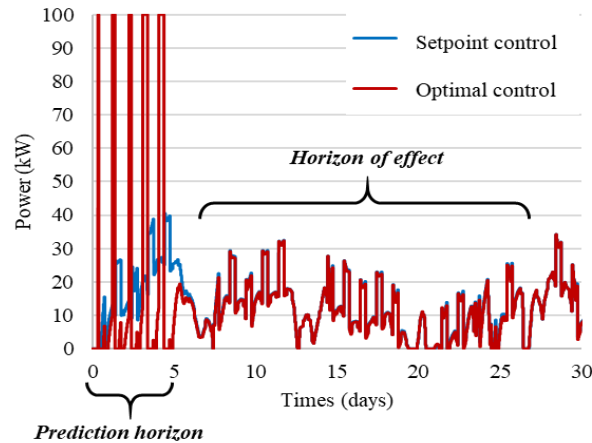


Figure 1: Effect of heat storage on future load

The resolution of the OCP requires the minimisation on the total period including the optimisation and the effect horizons.

### Algorithm

For a two zone case study, two similar OCP models are developed and thermally coupled. They are resolved separately considering an adjacent temperature accounting for the heat exchanges with the contiguous zone, and considering different disturbances (occupancy and weather). The OCP coupling is implemented in an extra step and the optimisation is reiterated with updated estimations of the adjacent temperatures.

The optimisation algorithm operates as follows:

1. The table of tariffs is studied and a first categorisation of the different ranges of energy prices is identified. If a time-of-use tariff is used with high peak, and off peak periods throughout a day, then high peak hours will be considered for peak load shifting. This would make the off peak hours heat storage periods. On the contrary if a dynamic pricing is used with an hourly tariff of energy then all ranges will be considered for storage or peak load shifting.
2. The trigger time for the beginning of the heat storage at maximum capacity ( $P_{max}$ ) is determined for each price range identified as a possible storage period in step 1. The trigger time explores both solutions [ $P_{min}$ ,  $P_{max}$ ] and defines the transition time between the application of the heating load and the storage at  $P_{max}$ . Matlab solver *fmincon* is used in order to calculate all trigger times within the optimisation period, the criterion being the minimisation of the energy bill throughout the period including the optimisation and the effect horizons. Figure 2 illustrates for a day the trigger time, where setpoint control is applied before as well as after the storage period. Here two cases are identified; when the temperature is higher than the setpoint no additional heating is needed and the load is erased ( $P=0$ ). Otherwise, when the temperature is lower than the setpoint, heat is supplied in order to maintain the minimal temperature constraint.

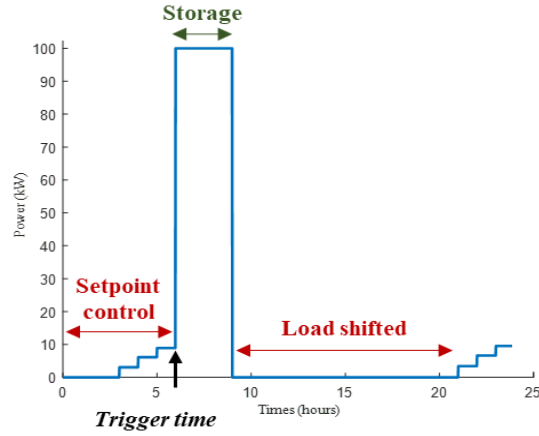


Figure 2: Optimal control strategy

3. The adjacent temperatures are initialised at 19 °C for the first OCP resolution. They are then updated based on the output of the optimisation of each zone and calculations are reiterated until convergence, corresponding to a tolerance of 0.1°C on the adjacent temperature (cf Figure 3).
4. The feedback loop is activated; part of the optimal sequence is applied during a control horizon after which the state, weather and occupancy are updated moving forward by one sampling period. The calculation of the next sequence of the OCP is repeated in the model predictive control logic to restart at step 1. This step simulates the practical application of the optimal strategy after its estimation. Tools such as an asymptotic observer and a controller for trajectory tracking can be added but are not implemented in this study.

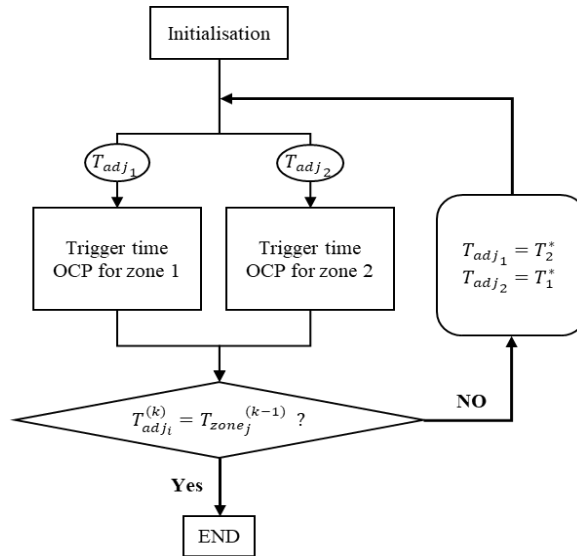


Figure 3: Resolution of the two zone OCP and coupling

### Validation

The distributed MPC developed in this article is compared to the centralised MPC using a perfect feedback (assuming perfect knowledge of state and future internal gains and external driving forces i.e. no error in state estimation nor in occupancy or weather predictions). This way, their performances can be more easily assessed without adding unnecessary complexity.

### Case study

#### Building type

The studied building is a six-storey passive building situated in Paris climate with an annual heating load of 13 kWh/m<sup>2</sup>. The total heated area is 4800 m<sup>2</sup>. The structure is concrete with external insulation (Table 1). It benefits from 45 % glazing in the south façade, 15 % in the north and 20 % for east and west façades. Argon double glazing windows is used with a solar factor of 0.54 and a  $U_{gw} = 1.6W/m^2.K$ .

The building is divided into two zones: the first three storeys are offices and the last three storeys are residential.

Table 1: Envelope composition

	External walls	Ground	Intermediate floor	Ceiling
Composition	20 cm glass wool 20 cm concrete	20 cm glass wool 20 cm concrete	16 cm concrete	26 cm glass wool 1 cm plaster
U(W/m <sup>2</sup> .K)	0.2	0.2	11	0.16

### Occupancy data

A standard occupancy scenario is applied to both areas of the building. The office zone is occupied by 169 workers; metabolic heat is fixed at 110 W/occupant. The residential area is occupied by 73 people; metabolic gain is considered at 80 W/occupant. ThBCE (French regulation calculation tool) scenarios are used for internal gains. Figure 4 represents the internal gains throughout a weekday.

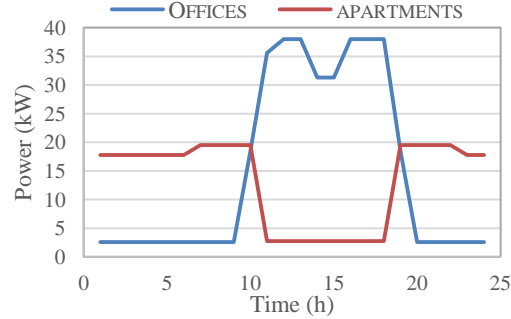


Figure 4: Daily internal gains

### Weather data

Meteorological data for the Paris climatic zone was chosen, providing hourly values on a typical year for: external air temperature, global and diffuse horizontal solar radiation, and direct normal radiation. Information on wind and humidity were not used in this calculation, air flows being fixed by mechanical ventilation

### Optimisation parameters

The prediction horizon choice for the resolution of the OCP determines a crucial role in the overall performance of the MPC. A sensitivity analysis was therefore conducted in order to choose the prediction horizon which offers the best compromise between precision of results and computation time. Different horizons were applied ranging from 3 to 10 days to determine the optimal heating trajectory. The focus was on the results of the 1<sup>st</sup> day of the optimal control as the sampling period in the MPC was fixed at 24 h. While a short horizon provides smaller computation time, the strategy and so its cost will depend on the view into the future disturbances (weather and occupancy). The longer the horizon the better the chances to anticipate any changes that occur and will affect the building dynamic and hence the control. That being said the results stabilise and the control converges towards a cost that does not fluctuate much further. In this case, a 5-day horizon is enough to provide a good result with a reasonable computation time.

The horizon of effect is 15 days; it was calculated as three times the highest time constant of the building (99 h).

The horizon of control is 24 h; it represents the sampling period after which the predictive control is updated to reiterate the calculation advancing forward in a receding time horizon.

State constraints describe thermal comfort:

$$19\text{ }^{\circ}\text{C} \leq T_{zone} \leq 24\text{ }^{\circ}\text{C} \quad \forall t\#(\text{Erreur ! Signet non défini.})$$

Command constraints account for the heating system's capacity:

$$0 \leq U \leq 100\text{ kW} \quad \forall t\#(\text{Erreur ! Signet non défini.})$$

### Electricity tariff

The time-of-use pricing of electricity as well as the time slots are reported in Table 2. The cost structure is virtual and does not represent real pricing applied in France. Tariffs vary depending on the type of subscription at building and company levels. The lowest cost is the off peak period, the peak hour tariff is intermediate and is doubled during high peak hours.



Table 2: Time-of use tariff

	Off peak hours	Peak hours	High peak hours
Time slot	0 h to 9 h	9 h to 17 h 22 h to 0 h	17 h to 22 h
Tariff per kWh	0,0863 €	0,1275 €	0,255 €

## Results and discussion

The algorithm previously detailed is applied to the case study during the coldest period of the year: 2<sup>nd</sup> week of January. A perfect feedback is used; i.e. after the sampling period the initialisation of the calculation does not take into consideration any prediction error in the boundary conditions. This means that weather and occupancy data are known and fixed in advance. The MPC simply advances by one sampling period.

The centralised MPC using the penalty method developed by Frapin [24] is considered as the reference based on which the newly developed distributed MPC will be validated. The centralised method is based on Pontryagin's principle and requires the expression of a Jacobian in order to solve the two boundary value problem by MATLAB's `bvp5c` solver. In some cases, a singular Jacobian was encountered. In an MPC loop, this put an abrupt end to the calculation of the strategy and the results did not converge. But this did not occur in the case study presented here.

Figure 5 and Figure 6 report the results of the heating strategy for the office and residential zones using the centralised and distributed MPC for three weekdays (Wednesday through Friday). The dashed grey line represent the tariffs of electricity applied daily. The optimal strategy is to store heat during off peak hours where the tariff is lowest, the high peak hours are completely erased as well as a majority of peak hours. Outside of the storage hours the minimal temperature constraint is controlled through setpoint strategy. If the zone temperature falls below the minimal state constraint, the heating load is calculated and applied in order to respect the constraint. This transitional power supply is referred to as setpoint control. The first notable difference stems from the comparison of a continuous versus a discretised method; the centralised MPC leads to smoother curves. In terms of storage, both methods consider somewhat equivalent amounts. The centralised method progresses slowly towards the maximum power whereas the distributed method will apply setpoint control before switching to heat storage instantly. The trigger time is calculated as the optimum time at which storage should resume while maintaining the minimum comfort constraint at the beginning of the off peak hours.

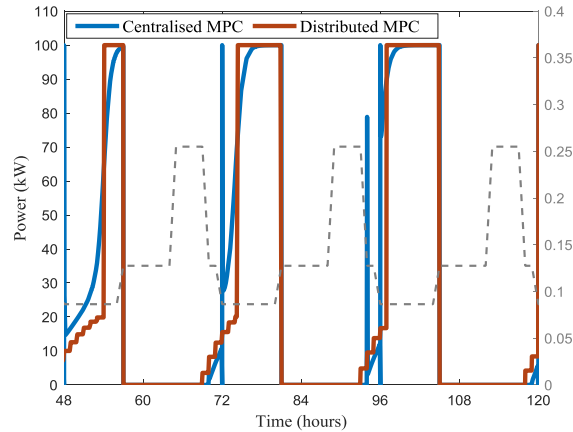


Figure 5: Optimal heating trajectory for office zone

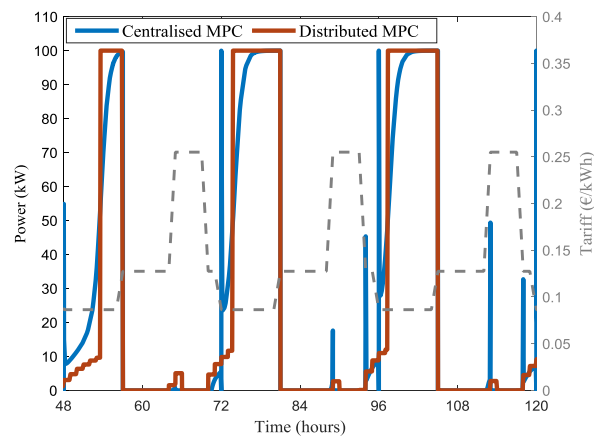


Figure 6 : Optimal heating for residential zone

The resulting temperatures are represented in Figure 7 **Erreur ! Source du renvoi introuvable.** and Figure 8. The effect of the heat storage can be seen at night (off peak hours) where temperature rises without reaching the maximum temperature constraint. The storage periods are followed by a decrease throughout the day. For the office zone, the temperature increases slightly during working hours (between 10 am and 5 pm) due to the internal gains produced by this activity. During the weekend, this is not observed due to the lack of activity and so the temperature follows a decreasing dynamic until it reaches 19 °C at the end of the day. The occupation peak in the residential area occurs at around 7 pm, the impact of internal gains is not as important due to the lower number of occupants and their activities which differ from the working offices. The effect of the instant peak power applied in the centralised MPC can be seen at the beginning of the off peak period where the temperature is raised because it was too close to the minimum constraint.

The week long strategy cost is 653 € for the centralised MPC and 657 € for the distributed MPC (0.65 % difference). The slight difference is mainly due to the difference between the centralised method which finds a global optimum whereas the distributed method runs one optimisation for each zone and then takes into account the error related to the consideration of heat transfer. The optimum depends on the estimation of the fluxes through the adjacent walls. The effect horizon also has an impact as the distributed MPC minimises the long term cost.

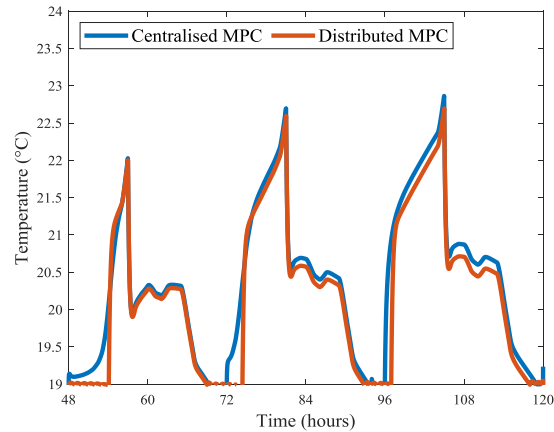


Figure 7 : Optimal temperature profile for office zone

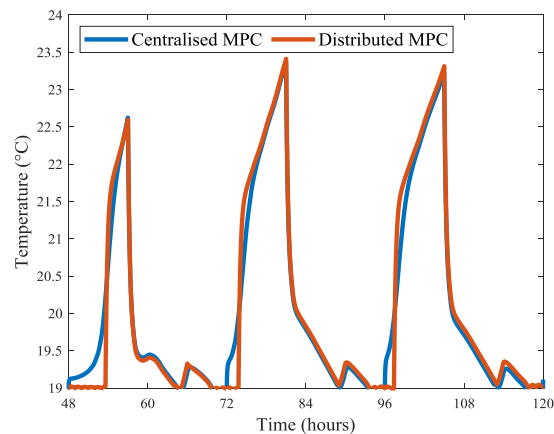


Figure 8: Optimal temperature for housing zone

In terms of computation time the distributed MPC is faster by a factor of 40; a one-week calculation takes up to 45 seconds against 28 minutes for the centralised MPC.

The optimal strategy in the coldest week of the heating season amounts to a 27 % energy bill saving. This is calculated in comparison to a thermostat type control where the heating load to satisfy a 19 °C setpoint at all times.

Over the whole heating season, a 29 % decrease in the energy cost is observed when applying the distributed MPC using the trigger time method to solve the OCP.

## Conclusion

A new approach to solve the OCP using a trigger time search was proposed in this article. A distributed MPC was adapted and applied to study heating management strategies on a two-zone building model.

The approach consisted in running an optimisation at zone level with a coupled multizone model that integrated heat transfer between zones in the form of additional gain (adjacent temperature and solar flux exchanges through the adjoining walls). This method was validated using the work of [24] as a reference to evaluate the approach and its relevance.

The optimal heating strategies can represent up to 28 % saving on the energy bill. This solution allows effective management through passive storage in building thermal mass, without compromising comfort. Results show that the distributed MPC reduces computation time by a factor of 40 compared to the reference centralised MPC with equivalent performances in terms of heating and temperature profiles. Thanks to its simplicity, this model predictive control approach did not demonstrate any convergence problems contrary to the reference.

The method is suitable for all types of building, but saving will be depend on building envelope, weather conditions and occupancy as well as optimisation objectives (heating or cooling, cost reduction or GHG emissions).

In perspective, it will be useful to include a real time feedback application using observed and updated data. This will allow the consideration of the disturbances that arise from weather and occupancy prediction models, making the strategy more robust to input and simulation errors. From a practical standpoint the assumption of a perfect feedback is not viable. The model predictive control is based on forecasted input data (weather and occupancy). Moreover, the

operative temperature is used to estimate the state of the building during the feedback. On the one hand, a state observer is needed in order to take into account observation error and calculate the real state of the building for the initialization of the OCP. On the other hand, a simple controller should be used (PI for example) in order to operate a trajectory tracking that takes into account forecast error and applies corrections to the control. The consideration of unforeseen events is essential in order to ensure the reliability of the algorithm in terms of disturbances. With these two tools the model predictive control can be implemented in real time in a practical building application. Potential additional cost can occur (when overestimating gains) but are minimised in by the MPC feedback using the tools afore mentioned.

It is believed that this approach could very easily become available for commercial use thanks to its short computation time.

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## References

- [1] ADEME. (2018). Climat, Air et Energie. (France).
- [2] ADEME, E-CUBE Strategy Consultants, and CEREN. (2017). L'effacement de consommation électrique en France.
- [3] L. Mauri, A. Vallati, and P. Ocloń, 'Low impact energy saving strategies for individual heating systems in a modern residential building: A case study in Rome. *Journal of Cleaner Production* vol. 214, 791–802.
- [4] M. Robillart, P. Schalbart, and B. Peuportier. (2014). Extraction of heating control rules from the dynamic programming method for load shifting in energy-efficient building. *Proceedings of the 9th International Conference on System Simulation in Buildings (SSB2014)*. Liège (Belgium). Dec. 2014.
- [5] R. Tang, S. Wang, K. Shan, and H. Cheung. (2018). Optimal control strategy of central air-conditioning systems of buildings at morning start period for enhanced energy efficiency and peak demand limiting. *Energy*, vol. 151, 771–781.
- [6] E. Rezaei and H. Dagdougui. (2020). Optimal Real-Time Energy Management in Apartment Building Integrating Microgrid With Multizone HVAC Control. *IEEE Transactions on Industrial Informatics*, vol. 16, no. 11, 6848–6856.
- [7] M. del Mar Castilla, J. D. Álvarez, J. E. Normey-Rico, F. Rodríguez, and M. Berenguel. (2013). A multivariable nonlinear MPC control strategy for thermal comfort and indoor-air quality. *IECON 2013 - 39th Annual Conference of the IEEE Industrial Electronics Society*. Vienna (Austria). Nov. 2013.
- [8] E. Shen, J. Hu, and M. Patel. (2014). Energy and visual comfort analysis of lighting and daylight control strategies. *Building and Environment*, vol. 78, 155–170.
- [9] R. Kamal, F. Moloney, C. Wickramaratne, A. Narasimhan, and D. Y. Goswami. (2019). Strategic control and cost optimization of thermal energy storage in buildings using EnergyPlus. *Applied Energy*, vol. 246, 77–90.
- [10] S. Rastegarpour, L. Ferrarini, and S. Gros. (2020). Economic NMPC for Multiple Buildings Connected to a Heat Pump and Thermal and Electrical Storages. *IFAC-PapersOnLine*. Jan. 2020.
- [11] S. Kuboth, F. Heberle, A. König-Haagen, and D. Brüggemann. (2019). Economic model predictive control of combined thermal and electric residential building energy systems. *Applied Energy*, vol. 240, 372–385.
- [12] I. Cupeiro Figueroa, M. Cimmino, and L. Helsen. (2020). A Methodology for Long-Term Model Predictive Control of Hybrid Geothermal Systems: The Shadow-Cost Formulation. *Energies*, vol. 13, no. 23, 6203.
- [13] Y. Löhr, D. Wolf, C. Pollerberg, A. Hörsting, and M. Mönnigmann. (2021). Supervisory model predictive control for combined electrical and thermal supply with multiple sources and storages. *Applied Energy*, vol. 290, 116742.
- [14] S. Ma, Y. Zou, and S. Li. (2022). Distributed model predictive control with priority coordination for limited supply multi-zone HVAC systems. *Journal of Process Control*, vol. 117, 157–168.
- [15] R. Bellman and R. Kalaba. (1957). Dynamic Programming and Statistical Communication Theory. *Proceedings of the National Academy of Sciences*. (USA). Aug. 1957.
- [16] R. Gamkrelidze, L. S. Pontrjagin, and V. G. Boltjanskij. (1964). *The mathematical theory of optimal processes*. Macmillan Company.
- [17] B. Peuportier and I. Blanc-Sommereux. (1990). Simulation tool with its expert interface for the thermal design of multizone buildings', *International Journal of Solar Energy*, vol. 8, no. 2, 109–120.
- [18] E. Palomo Del Barrio, G. Lefebvre, P. Behar, and N. Bailly. (2000). Using model size reduction techniques for thermal control applications in buildings. *Energy and Buildings*, vol. 33, no. 1, 1–14.
- [18] M. Robillart, P. Schalbart, F. Chaplais, and B. Peuportier. (2019). Model reduction and model predictive control of energy-efficient buildings for electrical heating load shifting. *Journal of Process Control*, vol. 74, 23–34.
- [19] P. Malisani. (2012). Pilotage dynamique de l'énergie du bâtiment par commande optimale sous contraintes utilisant la pénalisation intérieure. Thèse de doctorat, Ecole Nationale Supérieure des Mines de Paris, 2012. Paris (France).
- [20] B. Favre. (2013). Étude de stratégies de gestion énergétique des bâtiments par l'application de la programmation dynamique. *Doctoral dissertation, École nationale supérieure des mines de Paris*. Paris (France).

- [21] P. Malisani, F. Chaplais, and N. Petit. (2016). An interior penalty method for optimal control problems with state and input constraints of nonlinear systems. *Optimal Control Applications and Methods*, vol. 37, no. 1, 3–33.
- [22] M. Frapin, F. Chaplais, P. Schalbart, and B. Peuportier. (2018). Optimal control of heating in a two-zone building using price decomposition-coordination method (2018). *Proceedings from the 31st International Conference On Efficiency Cost, Optimization, Simulation And Environmental Impact Of Energy Systems*. Guimarães (Portugal). Jun. 2018.
- [23] M. Frapin. Étude de stratégies de gestion énergétique en temps réel à l'échelle multizone. (2018). *Doctoral dissertation, Mines ParisTech- PSL Research University*. Paris (France).