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Mass-charge metric in curved spacetime

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Abstract

In the context of black holes with spherical symmetry, we derive a solution to Einstein's field equations using a stress-energy tensor that takes into account both the electric field energy of the charge and the gravitational field energy of the mass. The metric, which has a form similar to that of Reisser-Nordström, may be used for predictions in strong fields where it shows better agreement with observation in high z quasars. Our metric could also be relevant in light of the increased interest regarding potential mass loss caused by gravitational waves.

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Key Words: Mass-charge, general relativity, strong gravitational fields, black holes, Reisser-Nordström metric.

Spacetime curvature arises as an effect due to mass, or equivalently energy. Because of the equivalence mass-energy, the energy produced by fields of the same form but different nature, is expected to provide the same effect on the corresponding curvature regardless of the fields different origins.

In pre-relativistic classical mechanics and Maxwell's electromagnetism we have the relations,

$$\frac{F_{New}}{m} = G \frac{M}{r^2} = \frac{M}{r^2} \quad \frac{F_{Coul}}{q} = k \frac{Q}{r^2} = \frac{Q}{r^2}, \quad (1)$$

where the first equation represents Newton's law of gravity with G the constant of gravity, m and M the interacting masses and F_{New} the interacting force, while the second equation represents Coulomb's law with k the constant, q and Q the interacting electric charges and F_{Coul} the interacting force. The last terms in (1) correspond to the choice of units where $G = k = 1$.

If a point charge q is brought from $\mathbf{x} = \infty$ to point \mathbf{x} in a region where there is an electric scalar potential $\Phi(\mathbf{x})$, the work done on the charge (representing its potential energy) is $W = q\Phi(\mathbf{x})$. For a system composed of point charges of total charge Q localized on the mass M , the electric potential energy and corresponding energy density are,

$$W_Q = \frac{1}{8\pi} \int |\nabla\Phi|^2 d^3x = \frac{1}{8\pi} \int |\mathbf{E}|^2 d^3x \quad w_Q = \frac{1}{8\pi} |\mathbf{E}|^2, \quad (2)$$

where for spherical symmetry and at the distance r from the localized mass M , the electric field is $E_r = Q/r^2$. Considering that analogous results are obtained for the gravitational potential energy,

the total energy density of the system $M + Q$ can be conveniently expressed as,

$$w_{Q_M} = \frac{1}{8\pi} |\mathbf{E}_{Q_M}|^2 = \frac{1}{8\pi} \frac{Q_M^2}{r^4} = \frac{1}{8\pi} \frac{M^2}{r^4} + \frac{1}{8\pi} \frac{Q^2}{r^4}, \quad (3)$$

where

$$Q_M^2 = M^2 + Q^2 \quad Q_M = \sqrt{M^2 + Q^2}. \quad (4)$$

The contribution to the total energy densities in (3) that can never disappear is the gravitational, because present even for a perfectly neutral body. Since electromagnetic is much stronger than gravitational interaction, the term with M^2 in (3) and (4) is generally neglected in most physical situations where electric fields are present. However, in the context of black holes, astrophysical objects with a large mass M are generally almost neutral and in proximity of the mass M where r is small the energy density due to M may exceed the one due to Q .

The stress-energy tensor $T^{\mu\nu}$

With the total energy density given by (3) we can form a stress-energy tensor $T^{\mu\nu}$ that takes into account the field energy density due to M and Q . Stress-energy tensors that take into account different forms of energy are common in physics in several contexts outside or within general relativity. As an example, we may consider the tensor $T^{\mu\nu} = \theta^{\mu\nu} + S^{\mu\nu} + \delta_0 U^\mu U^\nu$, used in the literature [1–4] in the context of electrodynamics of charged bodies with internal stresses. This tensor is complemented by the continuity equation, $\partial_\mu T^{\mu\nu} = 0$ while the term $\theta^{\mu\nu}$ represents the usual electromagnetic tensor, $S^{\mu\nu}$ the stress tensor, and δ_0 the proper density of the proper mass.

In our case, we do not consider the stresses inside the mass M as we are interested in the solution of Einstein's field equations outside M . For our purpose we can simply exploit the fact that, for spherical symmetry, the form of the gravitational and electric fields is the same, as shown in (3). Hence, we can formally adopt the stress-energy tensor $T^{\mu\nu} = \theta^{\mu\nu}(Q_M)$ where Q_M is given by (4). Thus, our approach reflects the one by Reisser-Nordström [5, 6] and we use here the formalism adopted by Zee [7] using geometric units $c = G = k = 1$.

The formal properties of $T_{\mu\nu}$ and the related field tensor $F_{\mu\gamma}$ are unchanged, being,

$$T_{\mu\nu} = \frac{1}{\mu_0} (F_{\mu\gamma} F_{\nu\alpha} g^{\alpha\gamma} - \frac{1}{4} g_{\mu\gamma} F_{\gamma\delta} F^{\gamma\delta}).$$

The corresponding Einstein's [8] field equations are ,

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + 8\pi T^{\mu\nu} = 0.$$

In the context of black holes and for spherically symmetric spacetime the metric has the form,

$$ds^2 = -A(t, r) dt^2 + B(t, r) dr^2 + r^2 (d\theta^2 + \sin^2(\theta) d\varphi^2) \quad (5)$$

In the solution of the Reisser-Nordström form, the relevant component of $T^{\mu\nu}$ is $T_{22} = (1/2\mu_0)r^2 F_{01} F^{01}$. As we have only a radial component of the electric field, we may write: $E_r = F_{01} = F_{10} = Q_M/r^2$. Then, $T_{22} = (1/2\mu_0)r^2(Q_M^2/r^4)$ and we find that $A(t, r)$ and $B(t, r)$ obey the equations (see [7]),

$$\begin{aligned} A(r, t) &= f(t) + \frac{C(t)}{r} + \frac{Q_M^2}{r^2} \\ AB &= f(t). \end{aligned}$$

So far, the solution for (5) is expressed in terms of mere algebraic quantities and does not represent physical reality. In order to be used to reflect physical reality and provide a falsifiable theory that

can be tested experimentally, we need to relate the algebraic quantities to observable physical quantities. The quantities f and C can be determined by taking the limit to the Schwarzschild metric $Q_M \rightarrow 0$ and,

$$g_{00} = 1 - \frac{2M}{r} \quad f(t) = 1 \quad C(t) = -2M \equiv r_S$$

Finally, by means of (4) we find the *horizon function* $\Delta = A(r)$,

$$\Delta = A(r) = 1 - \frac{2M}{r} + \frac{M^2}{r^2} + \frac{Q^2}{r^2} = 1 - \frac{2M}{r} \left(1 - \frac{M}{2r}\right) + \frac{Q^2}{r^2}$$

and the resulting metric is,

$$ds^2 = -\Delta dt^2 + \Delta^{-1} dr^2 + r^2 d\Omega^2, \quad (6)$$

which, for almost neutral bodies where the charge $Q \simeq 0$ can be neglected, can be expressed as,

$$ds^2 = -\left[1 - \frac{2M}{r} \left(1 - \frac{M}{2r}\right)\right] dt^2 + \left[1 - \frac{2M}{r} \left(1 - \frac{M}{2r}\right)\right]^{-1} dr^2 + r^2 d\Omega^2 \quad (7)$$

Expressing the energy of gravitational fields as mass contributions to M .

A possible interpretation of the additional mass term in the new metric (7) is obtained by comparing the Schwarzschild mass M_S with the mass $M(1 - \frac{M}{2r}) = M - \frac{M^2}{2r}$ of our metric (6) or (7). Integrating the gravitational energy density given by (3) starting from the observer position r up to $r = \infty$ we find,

$$W_M = 4\pi \int_r^{r=\infty} \left(\frac{1}{8\pi} \frac{M^2}{r'^4}\right) r'^2 dr = \frac{M^2}{2r}, \quad (8)$$

which is the mass, or equivalent energy, stored in the gravitational field M/r^2 outside r . The stored field energy in the space from r to $r = \infty$ must come from the original mass corresponding to absence of field energy. Hence, the effect of the field energy from r to $r = \infty$ is to reduce the Schwarzschild mass M_S (and corresponding space curvature) by $\frac{M^2}{2r}$.

Consequently, the resulting metric (6), or (7), can be obtained directly from the Schwarzschild (or Reisser-Nordström) metric by inserting into it the contribution to M_S given by (8). Our metric indicates that the energy of gravitational fields plays a role analogous to that of electromagnetic fields. Nevertheless, the additional mass term in our metric has to be considered in a wider context than that of simple analogy between Newton's gravity law and Coulomb's law. Of course, we have to go beyond Newton's gravity theory but we know today that the analogy between features of gravitational fields and those of electromagnetic fields is holding and is supported by developments in gravitational waves. A relevant confirmation is provided by the observed orbital decay of the Hulse-Taylor binary pulsar that matched the decay foreseen by general relativity as gravitation is lost to gravitational radiation, see [14]. Bondi et al. [15, 16] in 1960 were likely the first to suggest that gravitational objects could lose mass due to the emission of gravitational waves, see also [17, 18]. Since the detection of gravitational waves from the merger of a binary black hole system by LIGO in 2016, the relationship between mass loss and gravitational waves has garnered increased interest, see for example [19–21] for more information. In conclusion, the added mass term in the exact solution to Einstein's field equation given by our metric (7) is not surprising because based essentially on the equivalence between mass and field energy, regardless of whether the field source is the charge Q or the mass M . Therefore, our metric could potentially provide new insights in this area and, as shown below, in the interpretation of observational data in the context of strong fields.

Horizons of the metric

The metric gives the following horizons:

$$r_{\pm} = M \pm \sqrt{M^2 - M^2 - Q^2} = M \pm Q \quad (9)$$

In the case of a neutral charge ($Q = 0$), the metric only yields one horizon, which is equal to half the Schwarzschild radius, i.e., $r = M$, instead of $r = r_s = 2M$. For comparison, the horizons in the Reissner-Nordström metric are given by $r_{\pm} = M \pm \sqrt{M^2 - Q^2}$, as shown in [9–11] for example. The well-known extremal solution of the Reissner-Nordström metric (see [7]) occurs when $Q = M$. In this case, it predicts the same outcome as our metric when the charge is neutral.

Predictions of the metric in strong fields

The metric (Eq. 7) predicts that the event horizon for black holes without charge is only half of what is predicted by the Schwarzschild metric. This would result in a stronger predicted gravitational redshift for light emitted just outside the horizon (from the accretion disk). The accretion disk will be able to approach closer to the center of the black hole compared to the Schwarzschild metric. This finding could potentially help explain the absence of observed velocity time dilation in high- z quasars (see [12, 13]), as more of the red-shift could be simply due to gravitational redshift. Although a detailed study would be necessary before drawing any definite conclusions, it seems that the new metric can be of help for explaining these observed phenomena.

Declarations

Ethical Approval

Not applicable as a pure theoretical study.

Competing interests

The authors have no competing interests to report.

Authors' contributions

Both authors contributed to the research writing and review of the manuscript.

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