Proposal of a mathematical formalism for the EMC, based on tensor analysis of networks

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Proposal of a mathematical formalism for the EMC, based on tensor analysis of networks

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Outline

Motivation - A mathematical formalism in upphases analyses.

A theory for understanding the system behavior and for leading numerical applications.

We use Einstein and tensor algebra notations

$$a_q = \sum_n h_{qnj^n} = h_{qnj^n} \quad [T_{ab}]^{-1} = L^{ba}$$

We observe systems at given scales. Two scales are defined: one by the observation fractal dimension d and one d_i by the wavelength:

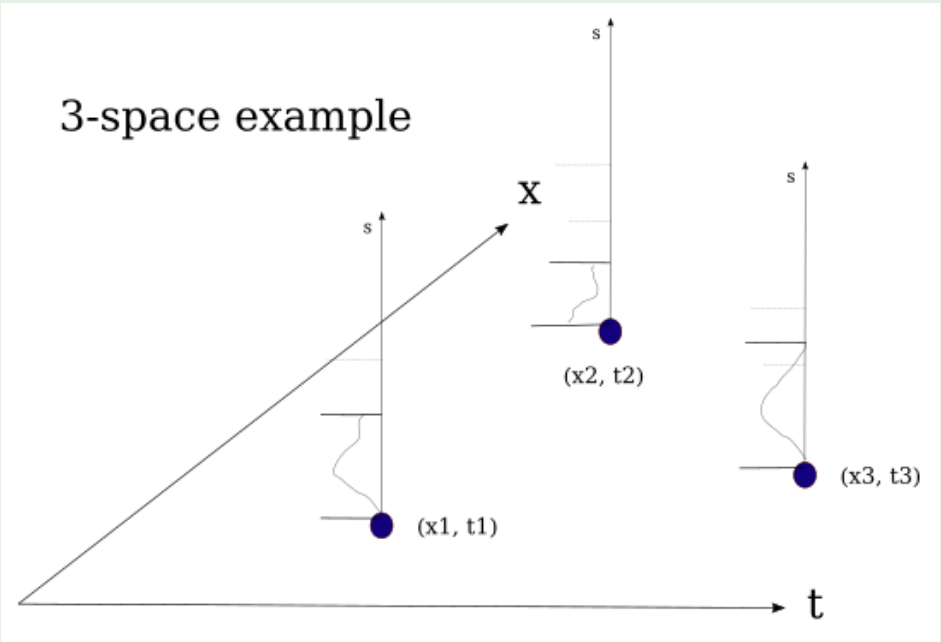
$$d = \frac{log\,(system\,characteristic\,length)}{log\,(associate\,1 - polyhedra\,element)}, \quad d_i = 2\frac{c}{\omega_0}$$

Our Space - Configuration space.

We live in a space with five dimensions: the classical space-time and the Laplace's one. Laplace's transform is exponential order function.

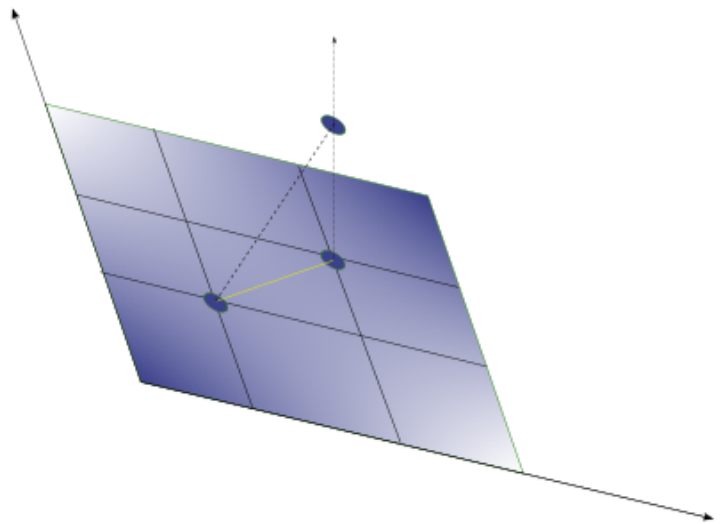
5-space

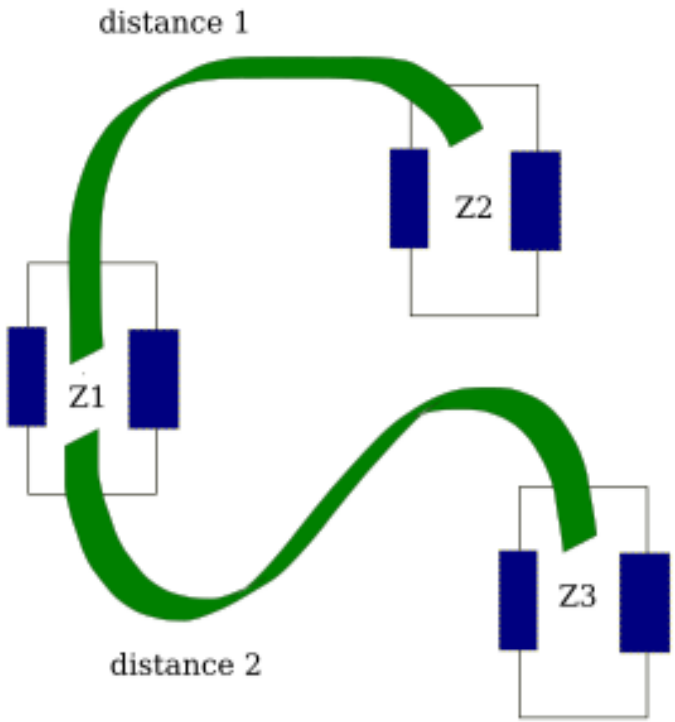
$$\left(s,\, u^k\right) \leftarrow u^0 \leftrightarrow t,\, u^n \quad s = j\frac{2\pi c}{\lambda} + c\alpha$$



Our Space - Topological Space

What means a *Topological Space*? It's a space where we can measure distances.





$$d = \int_t dt \sqrt{g_{ab} v^a v^b}$$

$$d = \sqrt{\frac{1}{2} \sum_0^{\omega_m} \zeta_{ab} k^a e_b^*}$$

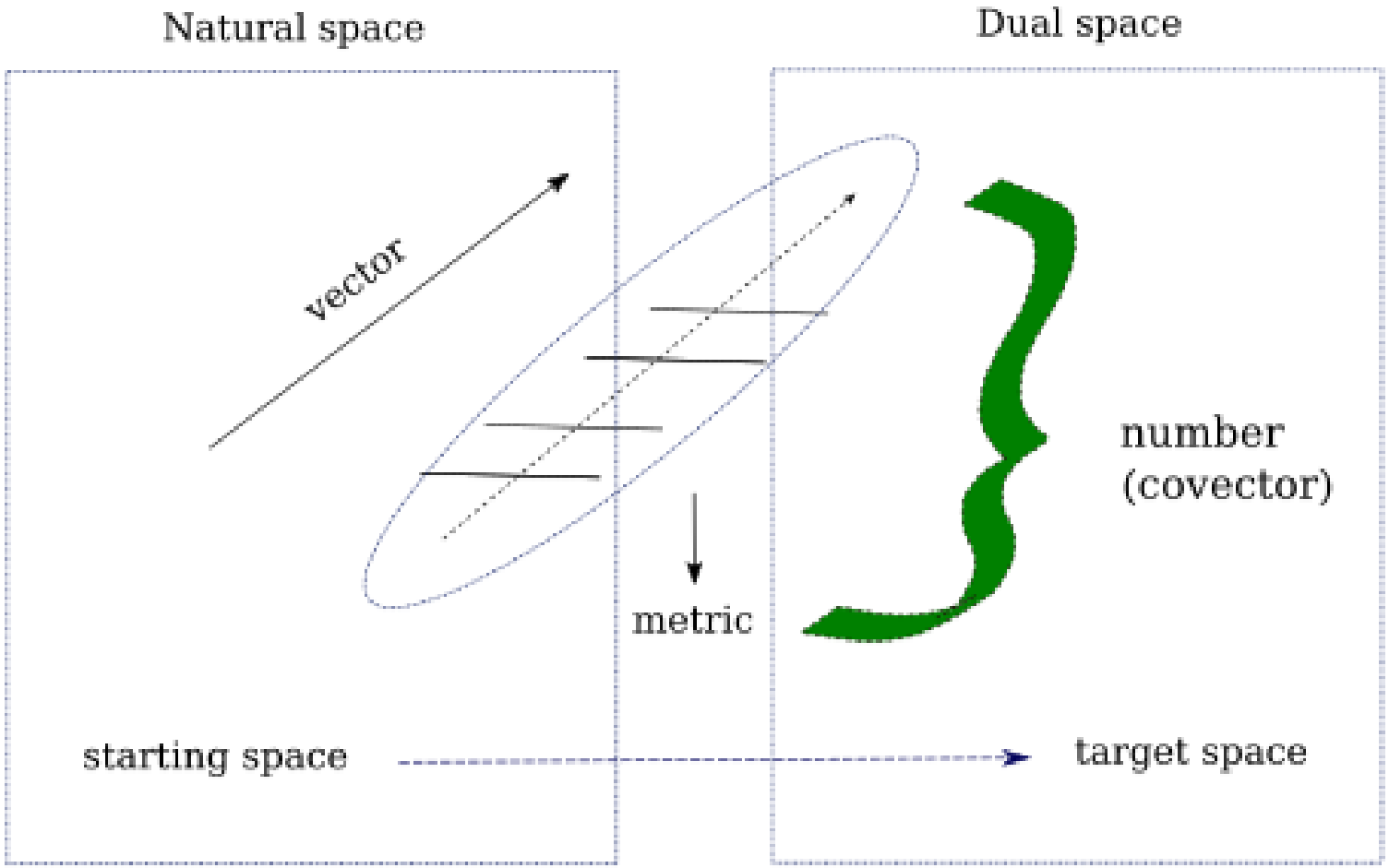
Proposals of metrics for fluids and electronics: the metric is arbitrary: the system properties are intrinsic.

$$g_{\alpha\beta} \leftarrow \rho\frac{x}{A}s,\ \frac{k'}{A^2},\ \frac{1}{s\chi V},\ \dots \qquad \zeta_{\alpha\beta} \leftarrow s\mathcal{L},\ R,\ \frac{1}{sC},\ \dots$$

Power flux density is the invariant which serves us as a repere, whatever the metric chosen.

$$\textit{fluids} \rightarrow \frac{\mathbf{f}}{A}\cdot\mathbf{v} = \mathbf{p}\cdot\mathbf{v} \Leftrightarrow e\frac{k}{A} = eJ \leftarrow \textit{electromagnetism}$$

Our Space - Dual space, vector and covector



Invariant: $\propto V^a C_a$

Golden rule

Index up A^k $\Rightarrow$natural space & vectors $\Leftrightarrow$ currents,
index down $U_n \Rightarrow$ dual space & scalar $\Leftrightarrow$ emf and potential differences.
index up and down: change of referential matrix Λ^a_b

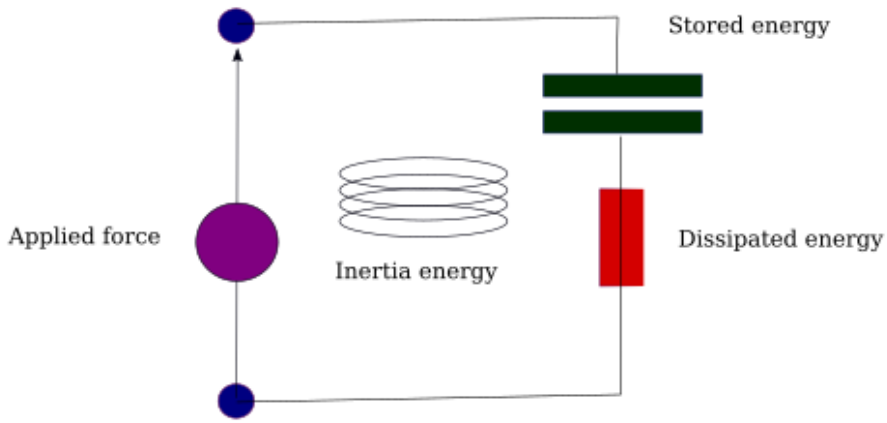
Golden law

The metric is the bridge between both natural and dual space.

$$U_\alpha = \zeta_{\alpha\sigma} A^\sigma, \quad A^\sigma = y^{\sigma\alpha} U_\alpha$$

Currents and emf are equivalents, simply seen in two different spaces. Impedance tensor is the metric for electronics.

$$W = T + U + R$$



Fundamentaly, all comes from energies in a closed cycle:

- force - how energy is transmitted;
- Inertia - a fundamental behavior of Nature;
- stiffness - how energy is stored;
- dissipation - the intrinsic entropic property of Nature.

J being a generalized matter movement. *The mesh space is the fundamental space in cellular topology*.

W: external source, T: inertia, U: storage, R: dissipation

$$\begin{cases} s\frac{\partial T}{\partial J^k} + \frac{\partial U}{\partial x^k} + s\frac{\partial R}{\partial J^k} = \frac{\partial W}{\partial x^k} \\ \\ \Leftrightarrow \\ \\ s\mathcal{L}_{\alpha\beta}J^\beta + \frac{1}{sC^{\beta\alpha}}J^\beta + R_{\alpha\beta}J^\beta = e_\alpha \end{cases}$$

The behavior in impedance is leaded by shared factors. Let's consider a simple electrical resistor.

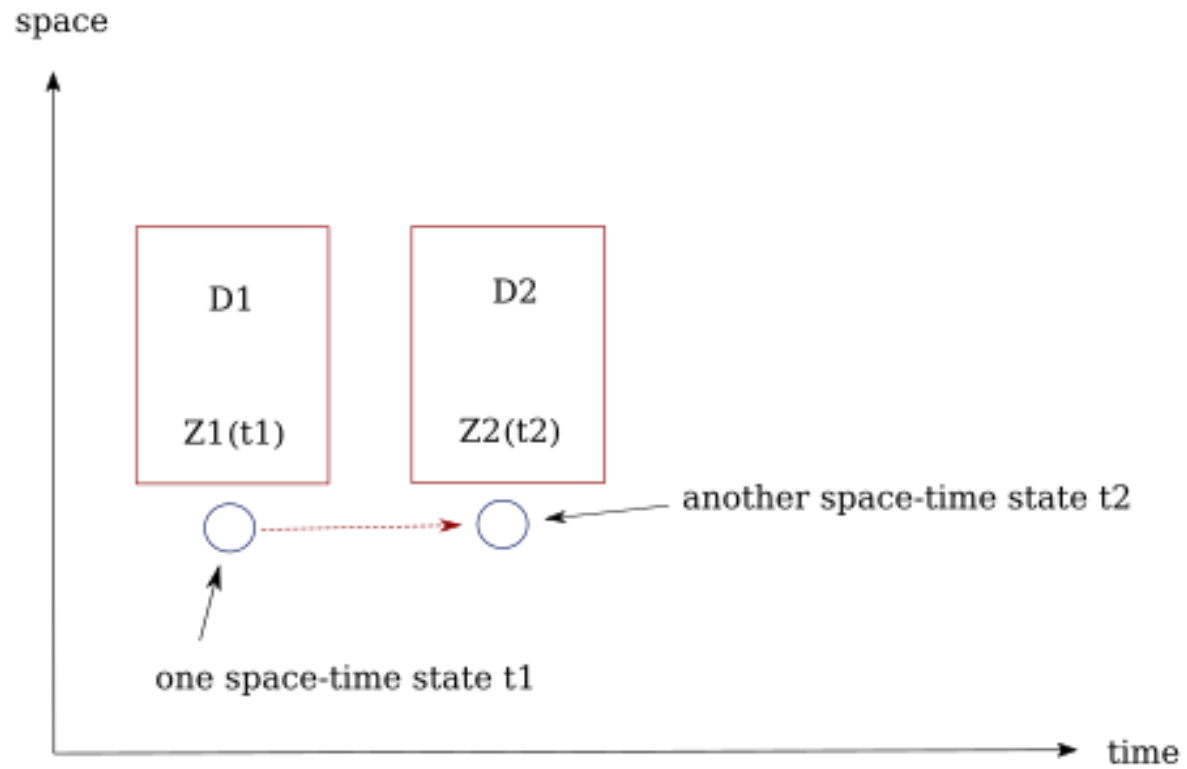
The resistor value depends on temperature θ

$$\zeta\left(R\left[\theta, J\right]\right) = \overset{\theta}{\mathcal{D}}_{i0} R + \overset{\theta}{\mathcal{D}}_{i1} R\left(1 + \alpha\theta\right) + \ldots + \overset{\theta}{\mathcal{D}}_{iq}\left[sC\right]^{-1}$$

$$i0,\ i1,\ \ldots \rightarrow in = \left[\theta_n,\ \ldots,\ \theta_m\right]$$

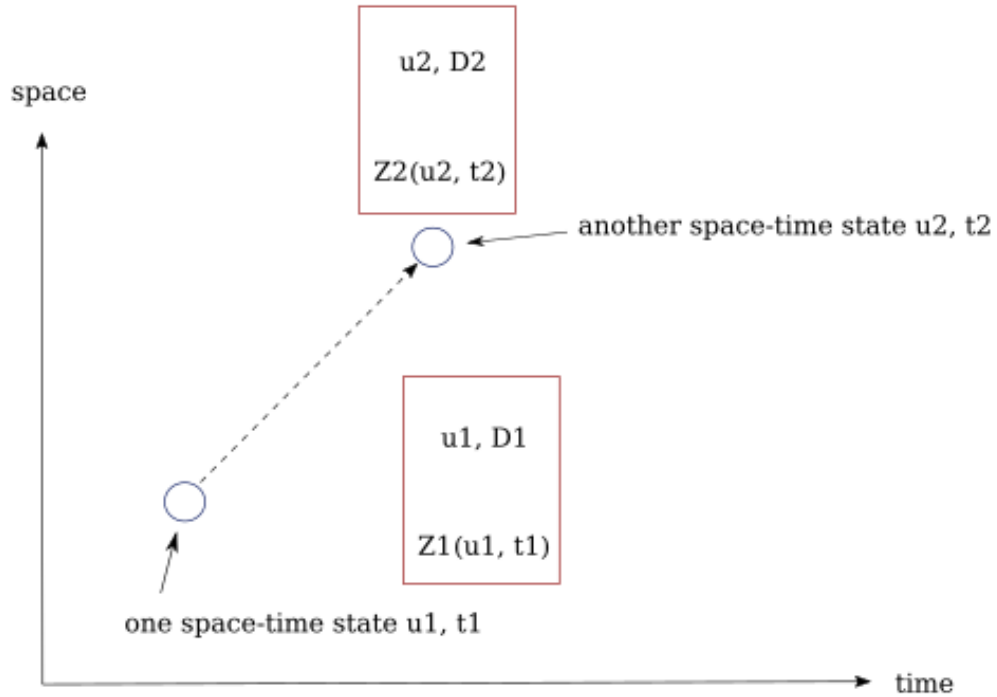
$$\overset{\theta}{\mathcal{D}}_{in} \equiv \left[1 + exp\left(-\alpha(\theta - \theta_n)\right]^{-1}\left\{1 - \left[1 + exp\left(-\alpha(\theta - \theta_m)\right]^{-1}\right\}\right.$$

$$multifactorial\ \ behavior:\ \ \overset{\theta}{\mathcal{D}}_{in} \rightarrow \overset{\theta}{\mathcal{D}}_{in}\overset{p_2}{\mathcal{D}}_{jn}$$



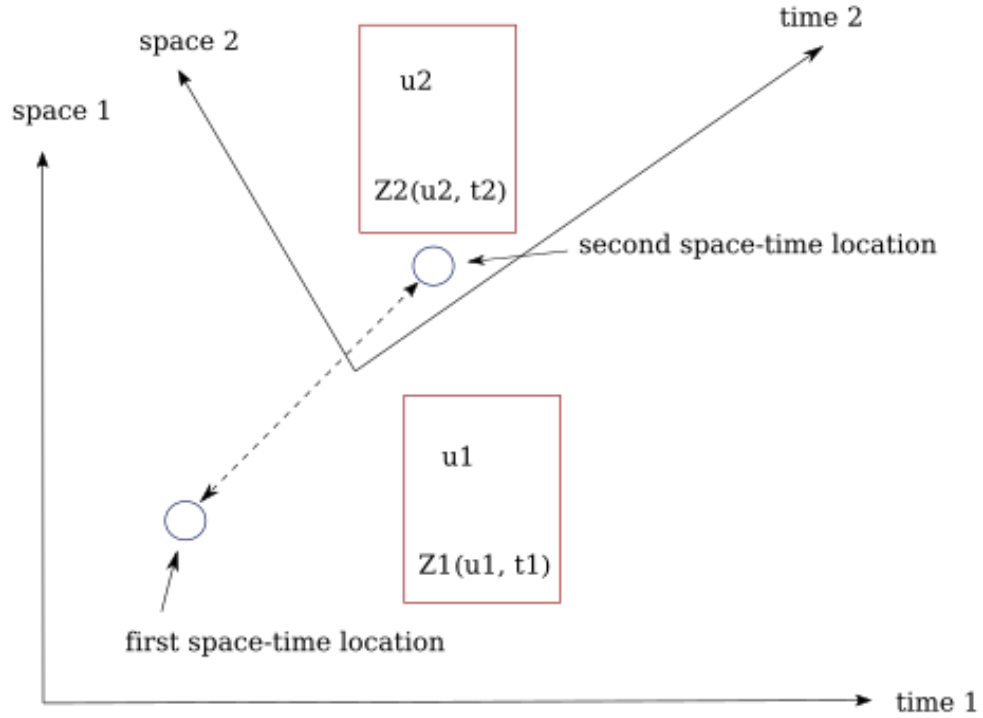
$$\frac{dZ}{dt} = dt^{-1}\left[\overset{p}{\mathcal{D}}_{i2(t2)}Z_2 - \overset{p}{\mathcal{D}}_{i1(t1)}Z_1\right]$$

Lagrange's fundamentals - Changing metrics in space-time: generalization



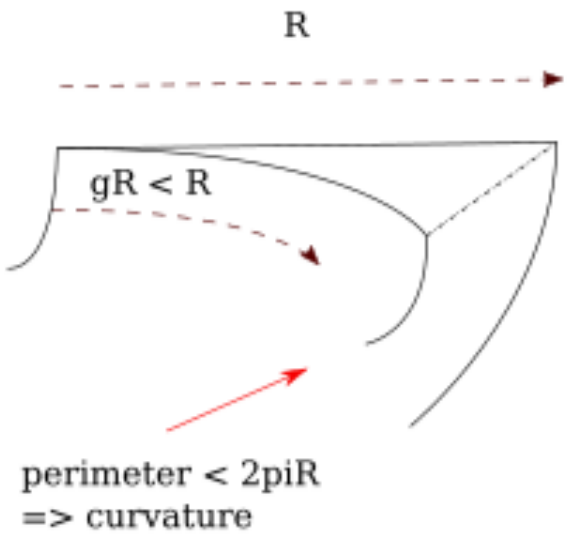
$$\frac{dZ}{dt} \supset \frac{d\mathcal{L}_{\mu\nu}}{dt} = \frac{\partial \mathcal{L}_{\mu\nu}}{\partial J^k}\frac{dJ^k}{dt} = \Gamma_{\mu\nu,\,k}\frac{dJ^k}{dt}$$

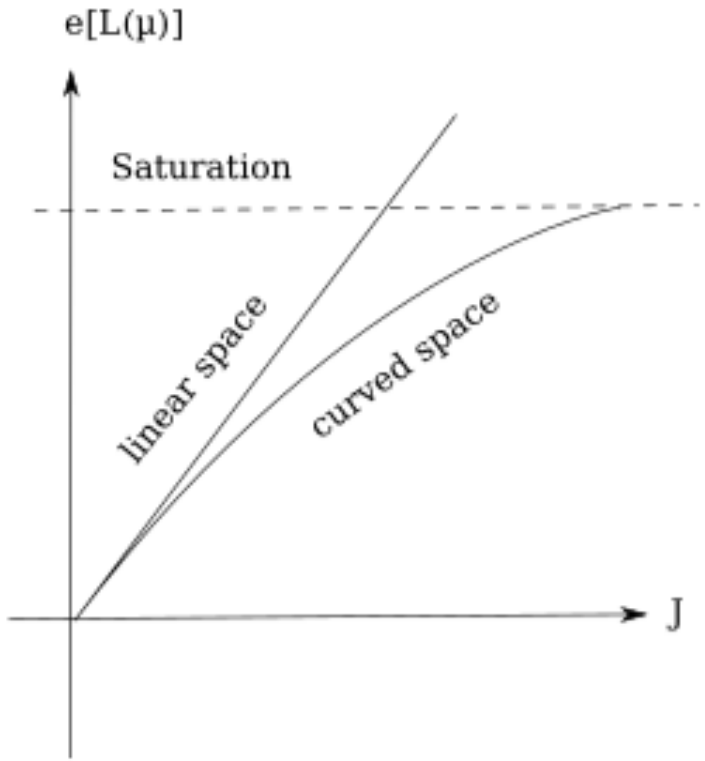
Christoffel's symbols of first kind.



$$\zeta_{\mu\nu} = \frac{d}{dt}\Lambda^{\sigma}_{\mu} z_{\sigma\alpha}\Lambda^{\alpha}_{\nu} \supset \frac{d}{dt}\Lambda^{\sigma}_{\mu} = \frac{d}{du^k}\Lambda^{\sigma}_{\mu}\Omega^k = \Gamma^{\sigma}_{k\mu}\Omega^k$$

Christoffel's symbols of second kind.

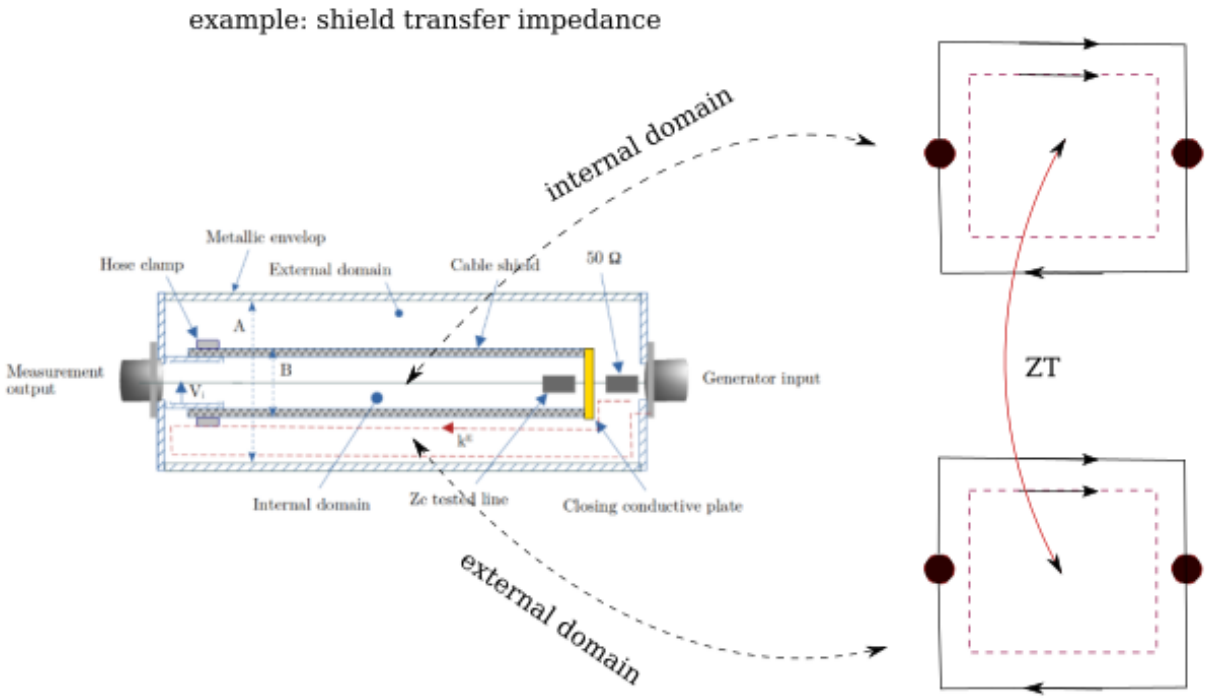




$$x_\alpha = g_{\alpha\beta} u^\beta \leftrightarrow e_\alpha = \zeta_{\alpha\beta} k^\beta$$

$$e_\mu = s L_{\mu\nu} k^\nu \rightarrow s \left(L_{\mu\nu} + \Gamma_{\mu\nu,\sigma} k^\sigma \right) k^\nu \leftarrow \Gamma \text{ works like losses.}$$

All system and geometry can be represented by a graph which is a polyhedra. It's a cellular topology made of nodes, edges, meshes, faces and volumes.
The polyhedral representation gives all the material for defining the scales, the metrics, etc.



Lagrange's fundamentals - Various spaces and transformation matrices

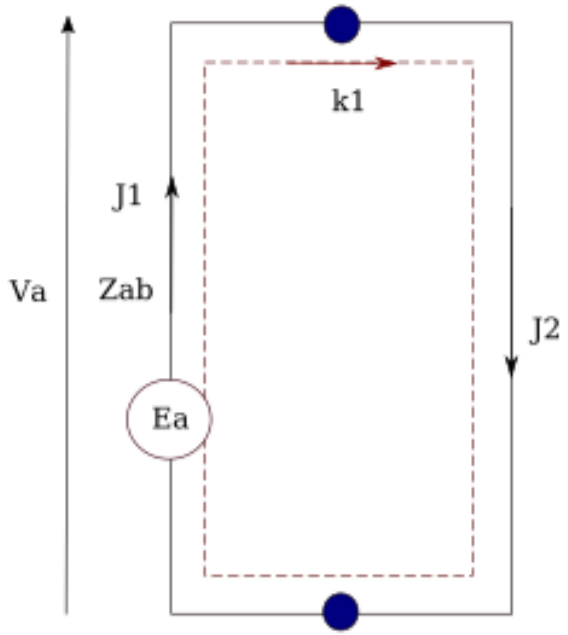
Transformation matrices can express quantities coming from one space in a linear combination of quantities belonging to another space.

Major cellular topology spaces:

- node space $\mathbb{N}$ and nodes pair space $\mathbb{P}$;
- edge space $\mathbb{B}$;
- mesh space $\mathbb{M}$;
- network space $\mathbb{R}$.

Euler - Poincaré's relation: $\mathbb{M} = \mathbb{B} - \mathbb{N} + \mathbb{R}$

$$\Lambda : J^{\alpha} \in \mathbb{B} \rightarrow k^{\sigma} \in \mathbb{M} \,/\, J^{\alpha} = \Lambda^{\alpha}_{\sigma} k^{\sigma}$$



$$E_a = z_{ab} J^b + \psi_a$$
$$E_a = z_{ab} \Lambda^b_{\sigma} k^{\sigma} + \psi_a$$
$$\Lambda^a_{\mu} E_a = \left(\Lambda^a_{\mu} z_{ab} \Lambda^b_{\sigma} \right) k^{\sigma} + \Lambda^a_{\mu} \psi_a$$
$$e_{\mu} = \zeta_{\mu\sigma} k^{\sigma} + 0$$

$\Lambda^a_{\mu} \psi_a = 0$: mesh **law**

$e_{\mu} = \zeta_{\mu\sigma} k^{\sigma}$:

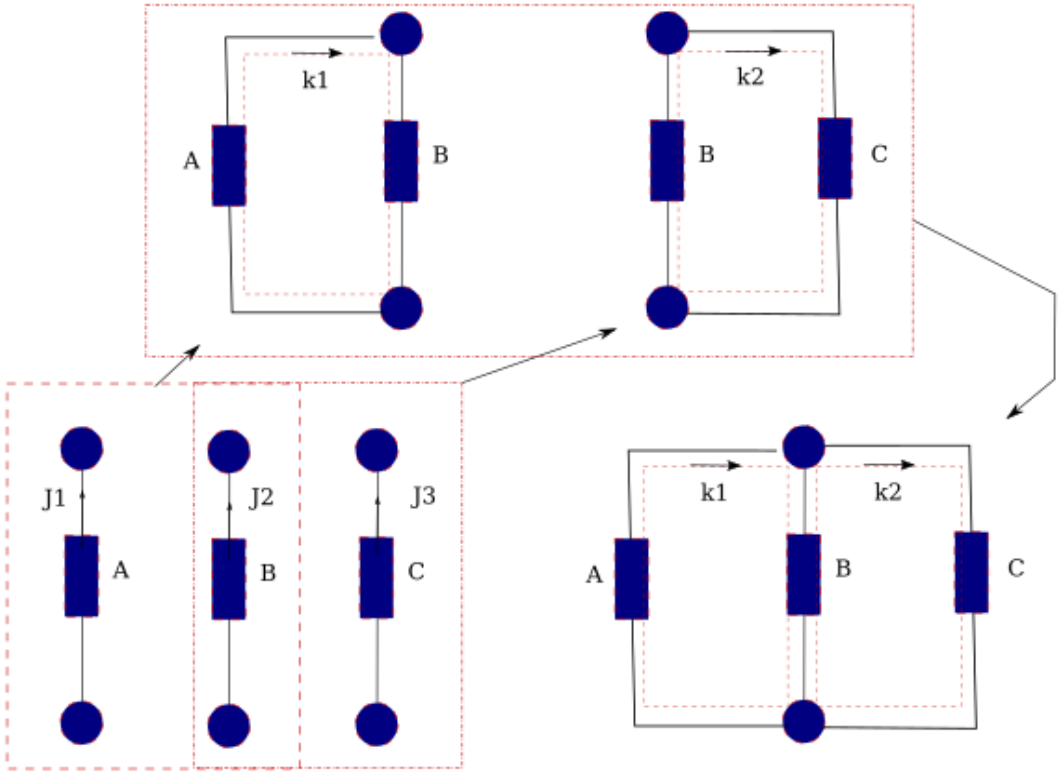
emf, metric and current in the mesh **space**

We define a manifold as a graph G associated with a set of equations $e_\mu = \zeta_{\mu\nu} k^\nu$ in a given configuration space $\mathcal{CS}$. The fundamental tensor ζ is sufficient for defining the set of equations. So, the manifold $\mathcal{M}$ can be written:

$$\mathcal{M}\,(G, \zeta)_{\mathcal{CS}}$$

Note that the configuration space is often implicit (which is often source of errors!).

Homotopy group	Homology group
$\mathcal{M}(G,\zeta) \in \{\mathcal{M}_i(G_i,\zeta_i)\}$ *if* $G_i \equiv G$	$\mathcal{M}(G,\zeta) \in \{\mathcal{M}_i(G_i,\zeta_i)\}$ *if* $\zeta_i = \zeta$



$$\{J^1, J^2, J^3\}, \Lambda_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Lambda_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2' \end{bmatrix},$$

$$\Lambda_2' = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \Rightarrow \zeta = \begin{bmatrix} A+B & -B \\ -B & B+C \end{bmatrix}$$

- We start from a collection of basic manifolds (metric diagonal components): $\zeta = \begin{bmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_3 \end{bmatrix}$;
- We make a circuit by sharing (by a way sometimes not desired) edges: $\zeta = \begin{bmatrix} z_1 & -a & -\beta \\ -a & z_2 & 0 \\ -\beta & 0 & z_3 \end{bmatrix}$.

Each time a force f_b is induced in a mesh b by a current of a mesh a, the coupling impedance

$$\zeta_{ba} = \frac{f_b}{k^a}$$

is symbolized by a string linking both meshes.
The coupling impedance appears as an extra-diagonal component in the metric ζ.

$$\rightarrow \zeta = \begin{bmatrix} \zeta_a & 0 \\ \zeta_{ba} & \zeta_b \end{bmatrix}$$

This can be generalized to a relation defined by:
$\varphi : \nu_2 \in M_2(\mathbb{M}) \leftarrow \nu_1 \in M_1(\mathbb{M}),\ /\nu_2 = \varphi(\nu_1)$.

Electronics - Basic components in edge space and intrinsic inertia

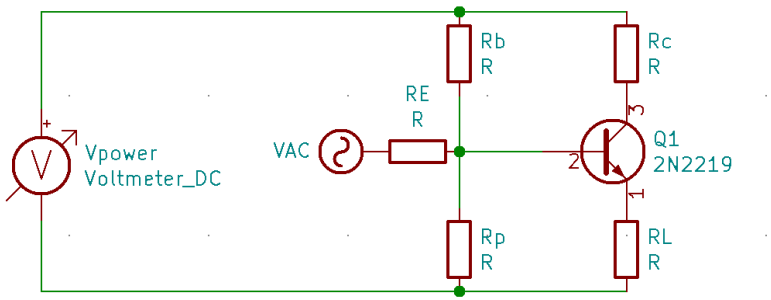
$\mathbb{B} : \{R,\ C,\ \textit{local } L,\ \textit{diodes},\ \ldots\}$

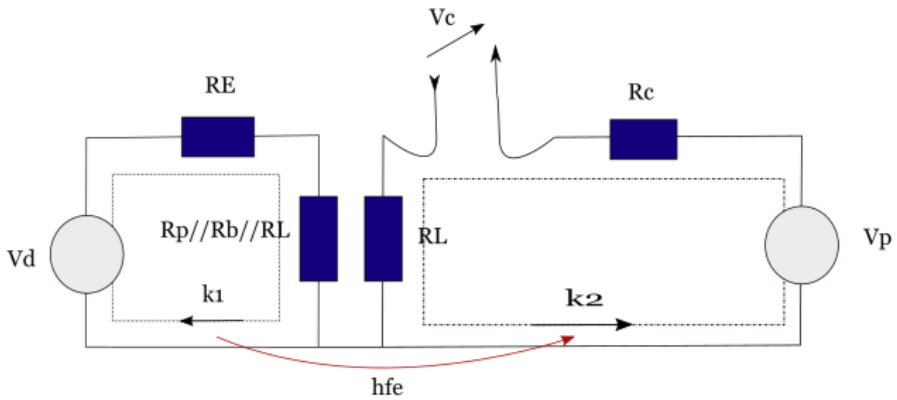
Each time we connect two edges for making a mesh, we create an inductance that translates the electromagnetic inertia, associated with the mesh loop.

Inertia

$$(a,\ b) \in \mathbb{B},\ z = \begin{bmatrix} a & 0 \\ & \\ 0 & b \end{bmatrix}\ \Lambda = \begin{bmatrix} 1 \\ \\ 1 \end{bmatrix} / \zeta = \Lambda^T z \Lambda + \mathcal{L} = a + b + sL$$

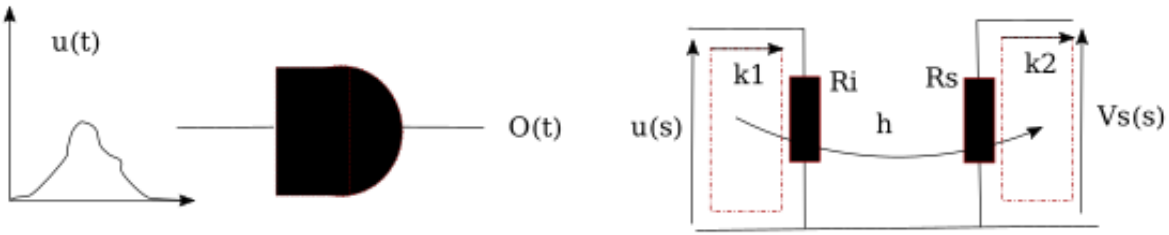
The metric is set by the DC polarization (s reduced to α). Under a linear (class A) polarization, the graph becomes:





$$\begin{cases} V_\alpha = \overset{DC}{\mathcal{D}}_{>0,6}\left[\zeta_{\alpha\sigma}\right] k^\sigma, \ \forall t \in [t_1,\ t_2] \\ V_d = \left[(R_E + R_{pbL}) + sL_1\right] k^1 \\ V_p - V_c = \left[(R_c + R_L) + sL_2\right] k^2 = h_{fe}\left[(R_c + R_L) + sL_2\right] k^1 \end{cases}$$

Electronics - Example of a binary operator

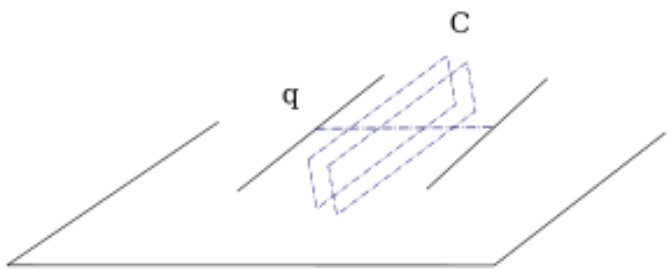


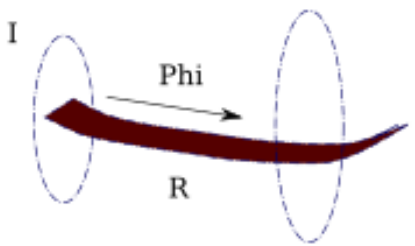
$$\begin{cases} u(s) = \sum_t \frac{\nu(t)}{s} e^{-s\tau_t},\ \nu(t) \in [-1,0,1],\ \lambda > \lambda_c,\ t = nT\ \ T : \textit{clock} \\ O(s) = \overset{u}{\mathcal{D}}_{>u_h}\left(\frac{\alpha}{s}\right) + \overset{u}{\mathcal{D}}_{<u_l}(0) \Leftrightarrow O(s) = \overset{k^1}{\mathcal{D}}_{>k_h}\left(\frac{\beta}{s}\right) + \overset{k^1}{\mathcal{D}}_{<k_l}(0) \\ \overset{k^1}{\mathcal{D}}_{>k_h} = \left[1 + exp\left(-\sigma(k^1 - k_h)\right)\right]^{-1},\ \zeta = \begin{bmatrix} R_i & 0 \\ h & R_s \end{bmatrix} \end{cases}$$

We consider for near fields, longitudinal and reactive fields.

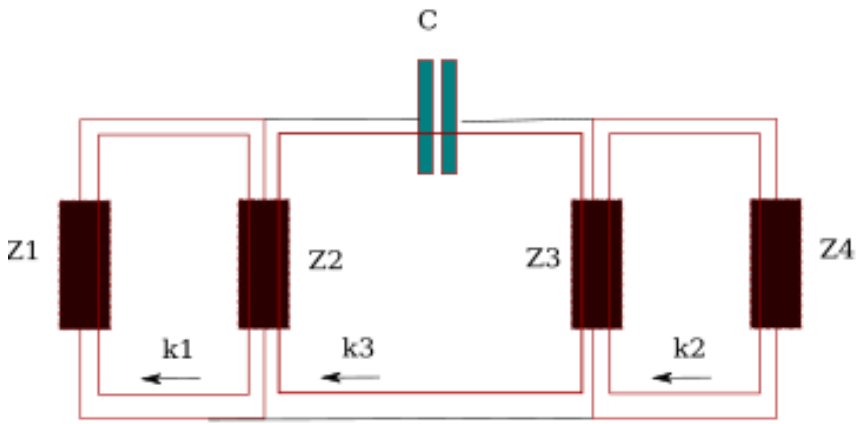
- Electric near field E: coulombian fields and capacitive interactions C.
- Magnetic near field: magnetic circuits and reluctances $\mathcal{R}$.

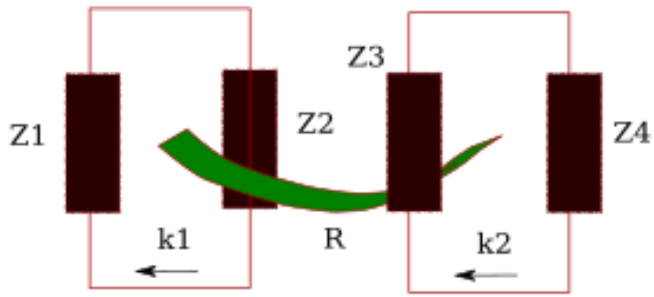
$$-\int_z dzE = \frac{q}{C} \qquad I \cdot N = \left(\mathcal{R} \oiint_S dS^z \cdot B\right), \ \mathcal{R} = \frac{1}{\mu}\frac{z}{S^z}, \ S^z \perp z$$





Electrical coupling:	Magnetic coupling:
$\zeta = \begin{bmatrix} z_1 + z_2 & 0 & -z_1 \\ 0 & z_3 + z_4 & -z_3 \\ -z_1 & -z_3 & z_2 + \frac{1}{sC} + z_3 \end{bmatrix}$	$\begin{cases} e_2 = -s\Phi = -s\frac{k^1}{\mathcal{R}} \Leftarrow k^1 = \mathcal{R}\Phi \\ M \equiv (\mathcal{R})^{-1} \end{cases}$





Only stored energy: the active power is zero.

$$\frac{1}{4}\left\{\frac{1}{sC}k^3k^{*3} + \left(\frac{-1}{sC}k^{*3}\right)k^3\right\} = 0,\ \frac{1}{4}\left\{\mathcal{R}\Phi(-j\omega)\Phi^* + \mathcal{R}\Phi^*(j\omega)\Phi\right\} = 0$$

Transverse (Coulomb's) gauge under the vector potential field. It's the field which wears energy (photon).

Emission:

$$dA_\alpha = \mu_{\alpha\sigma}\left[\frac{du^\gamma\left(J^\sigma + sC^{\sigma\nu}V_\nu\right)}{4\pi r^\gamma}\right] exp\left(-s\tau\right), \mathbf{A} = A_\alpha \mathbf{u}_\perp^\alpha$$

$$\frac{1}{2}R_{ad}J^\sigma J_\sigma^* = \frac{1}{2}\iint_S dS\left(\frac{A_\alpha}{\eta}A^{*\alpha}\right)$$

Reception:

$$e = -s\oint_u d\mathbf{u}_{R'}\cdot\mathbf{A}_{R\to R'}, \quad de = -sdu^\alpha\Lambda_\alpha^\sigma A_\sigma$$

In EMC we study undesired antennas with small radiation resistances.

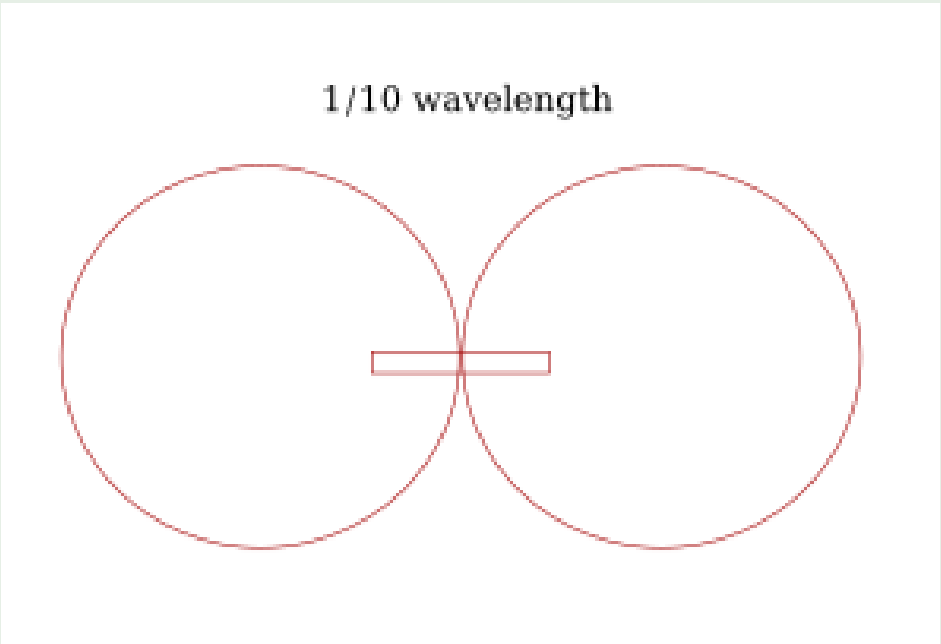
Their efficiency is low because their geometrical dimensions are not matched with the resonances and their impedances conditions are rarely very high or very low.

Nevertheless they are the source of uncompliances with the EMC requirements.

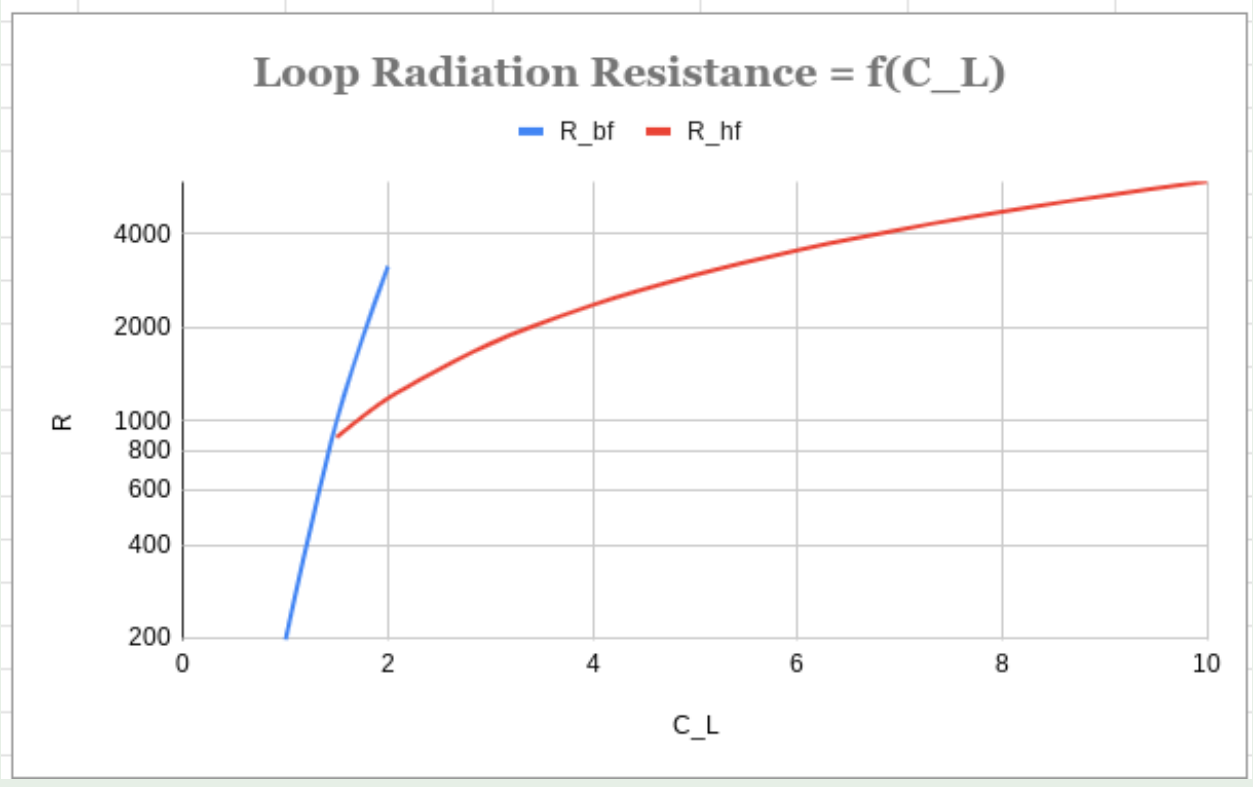
Emission of a mesh of diameter Φ, $\Phi << \lambda$

$$A_\phi = -\frac{120\pi^2 J^0 \Phi sin\theta}{sr\lambda^2} e^{-s\frac{r}{c}}$$

$$R_{ad} = 31,171 \left(\frac{\Phi}{\lambda^2}\right)^2$$

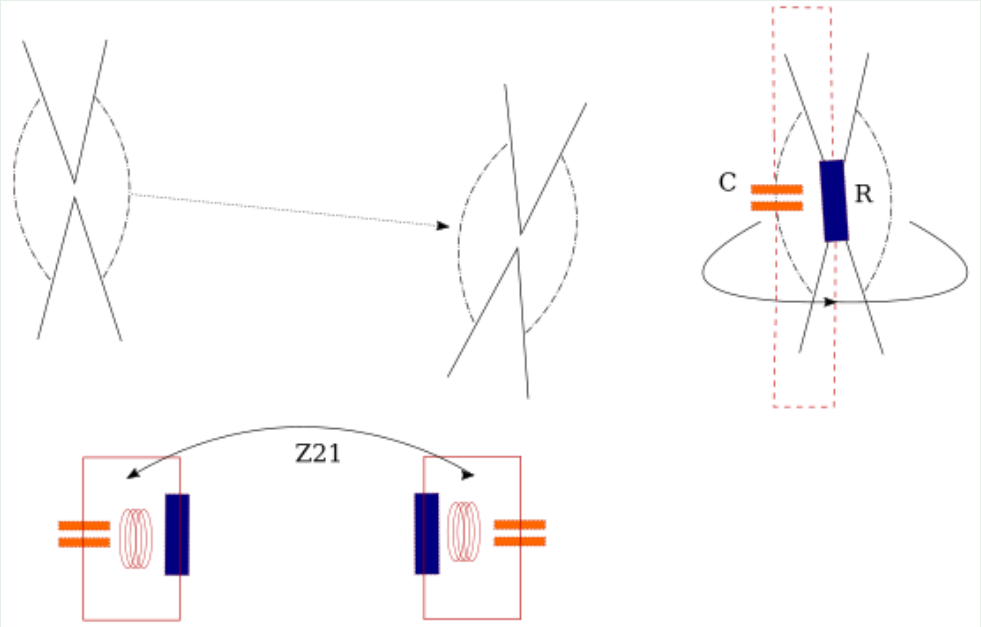


Emission of a mesh of diameter Φ, $\Phi \geq \lambda$ and radius a/ $C_\lambda = \frac{2\pi a}{\lambda}$



$$C_\lambda \leq 1,5 \rightarrow R = 197 C_\lambda^4, \quad C_\lambda \geq 1,5 \rightarrow R = 592 C_\lambda$$

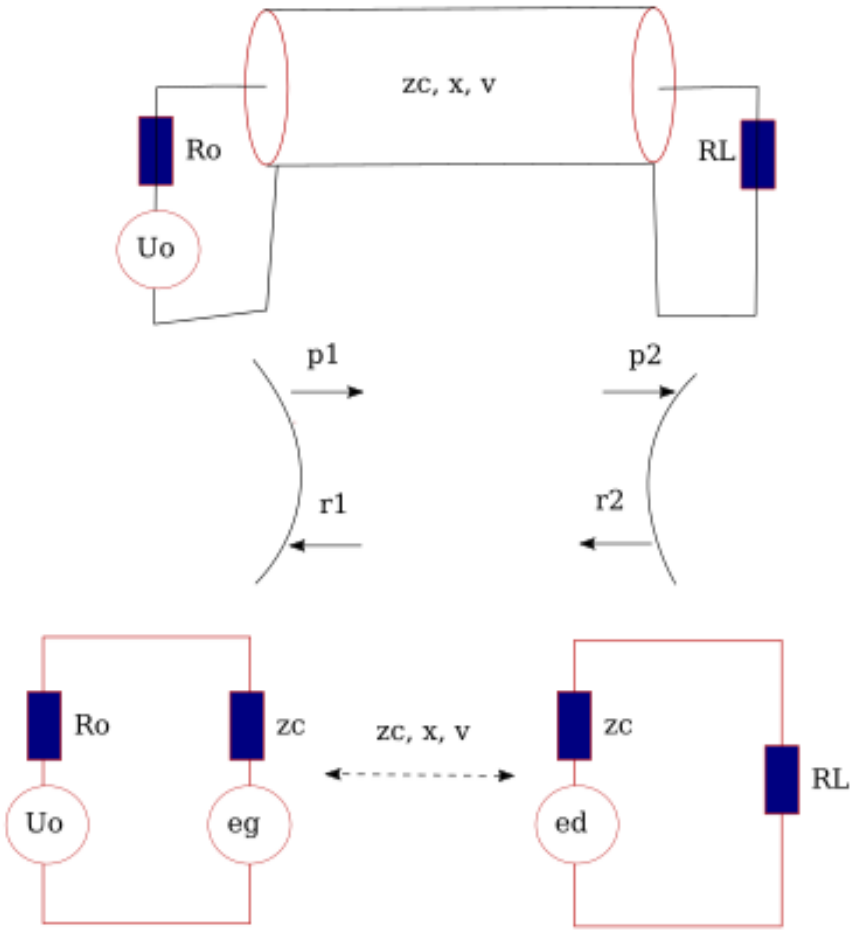
Example with two dipole antennas



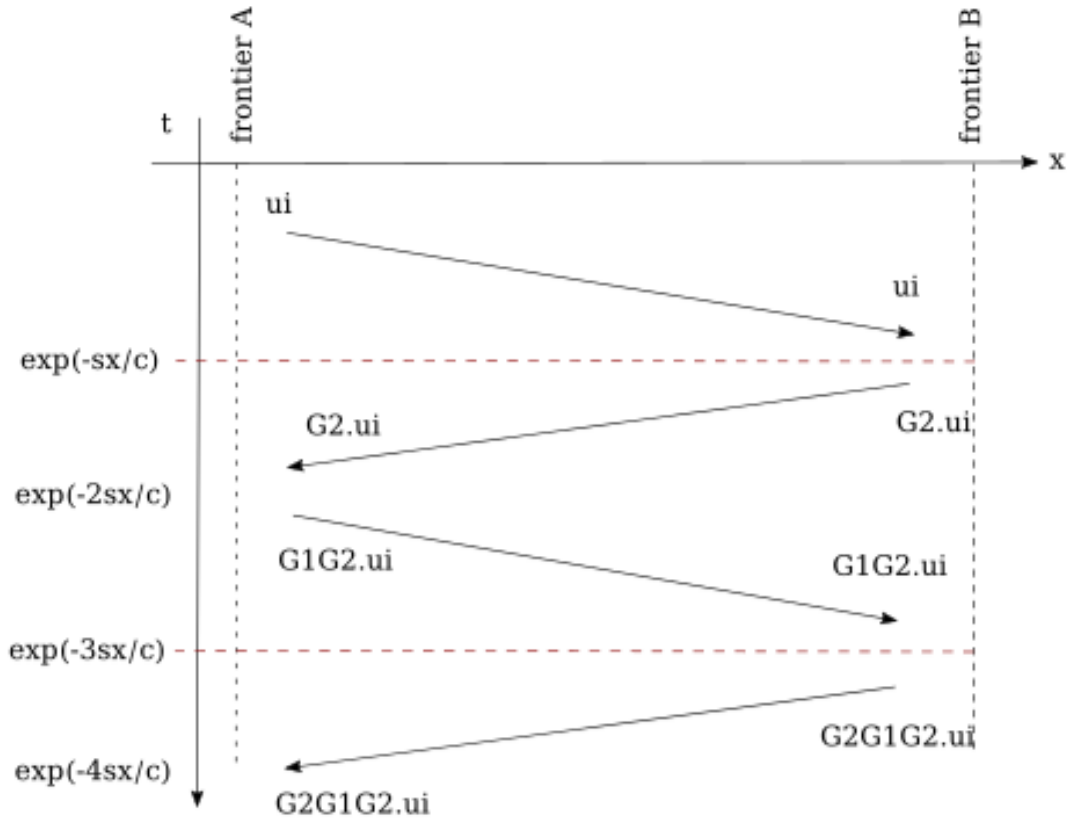
$$\zeta_{21} = -\frac{d}{dt}\Phi_2\Lambda_1^{\alpha}\left[\frac{\sqrt{30G_1R_1}}{cr_{12}}e^{-s\frac{r_{12}}{c}}\right]_{\alpha}$$

$$\zeta = \begin{bmatrix} R_1 + R_0 + \frac{1}{sC} + sL & -\zeta_{21} \\ -\zeta_{21} & R_1 + R_0 + \frac{1}{sC} + sL \end{bmatrix}$$

Guided waves are near field waves travelling along a conductive media. As their are perfectly located, we can define a wave space and its connection to the mesh one.



$$\Gamma_1 = \frac{R_0 - z_c}{R_0 + z_c}, \quad \Gamma_2 = \frac{R_L - z_c}{R_L + z_c}$$



	p_1	r_1	p_2	r_2
p_1	0	Γ_1	0	0
r_1	0	0	0	$exp\left(-s\frac{x}{c}\right)$
p_2	$exp\left(-s\frac{x}{c}\right)$	0	0	0
r_2	0	0	Γ_2	0

$$\Rightarrow \quad \gamma = \begin{bmatrix} 0 & \Gamma_1 & 0 & 0 \\ 0 & 0 & 0 & exp\left(-s\frac{x}{c}\right) \\ exp\left(-s\frac{x}{c}\right) & 0 & 0 & 0 \\ 0 & 0 & \Gamma_2 & 0 \end{bmatrix}$$

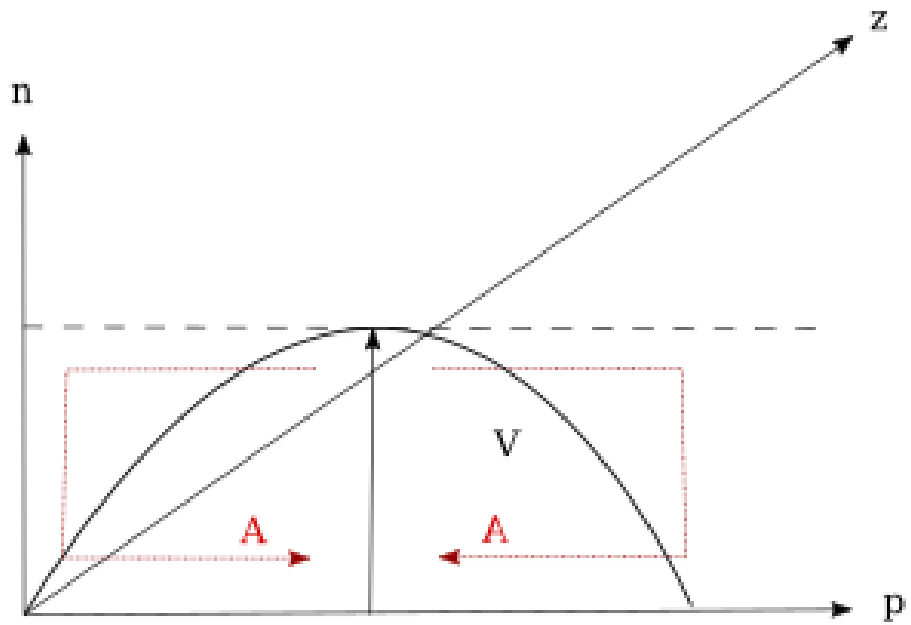
$$\text{wavevector: } O_\alpha = \begin{bmatrix} u_i \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \gamma O_\alpha = \begin{bmatrix} 0 \\ 0 \\ u_i e^{-sx/c} \\ 0 \end{bmatrix}, \quad \gamma\gamma O_\alpha = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \Gamma_2 u_i e^{-sx/c} \end{bmatrix}, \quad \dots$$

$$(1 + \gamma + \gamma\gamma + \gamma\gamma\gamma + \ldots) = \bar{\gamma}, \quad e_g = 2\Lambda_1^\alpha \bar{\gamma} O_\alpha, \quad e_d = 2\Lambda_2^\alpha \bar{\gamma} O_\alpha$$

Λ here is the wave to mesh connectivity. e_g and e_d are the reported sources in the source covector of the mesh space.

A cavity is a closed waveguide where a 2D wave propagates following a given mode $(u_\eta,\, u_\perp) \rightarrow \Gamma_1 = \Gamma_2 = -1$

$$u_i \rightarrow \psi_{\eta\perp}\left(u_\eta,\, u_\perp\right),\ v_{\eta\perp} = \frac{c}{\sqrt{1-\left(\frac{\lambda\eta\perp}{\lambda_c}\right)^2}},\ z_c = \alpha\frac{\psi_\eta}{J^\perp}$$



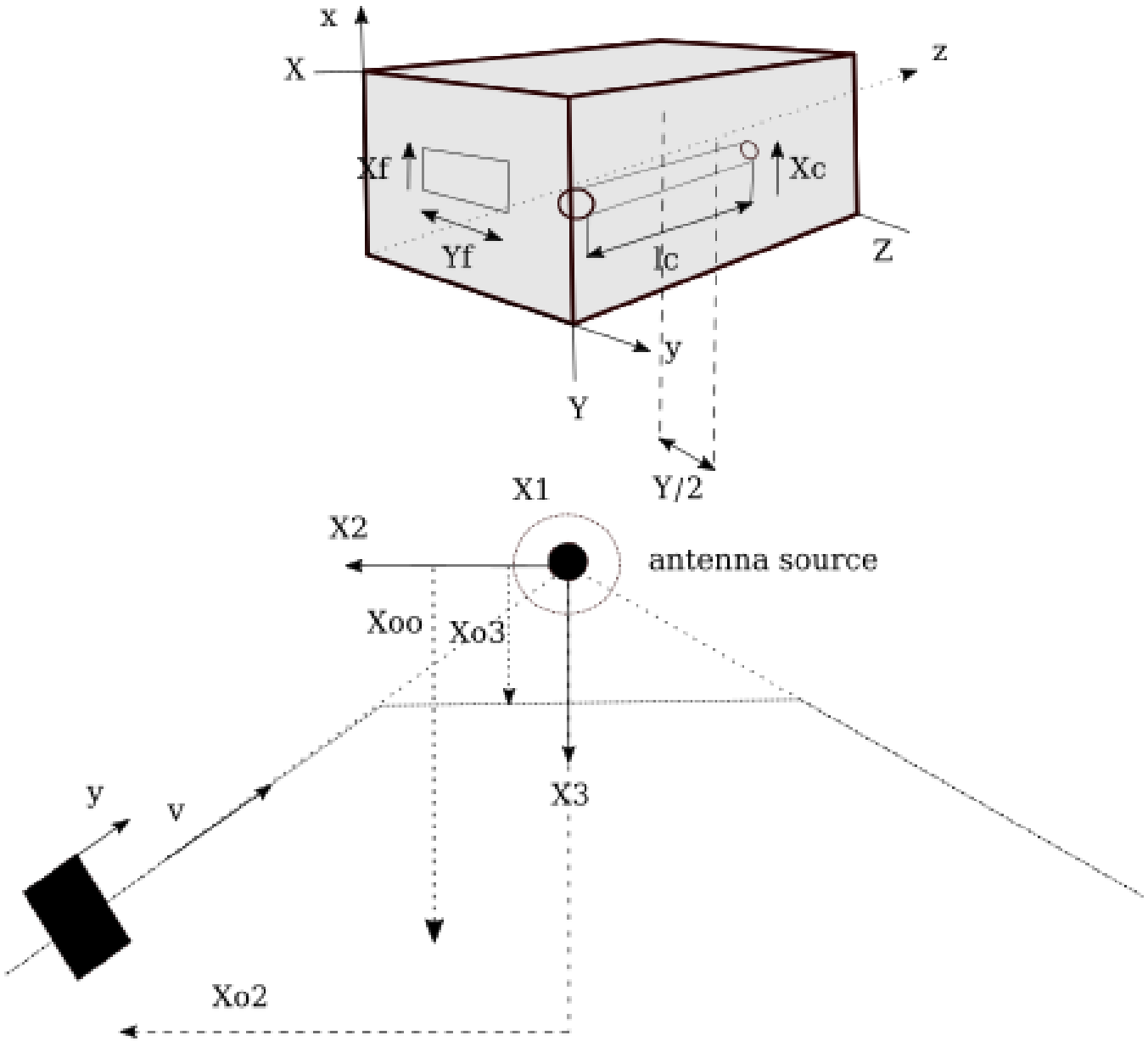
The modes interact with objects through near fields between the waveguide and the object.

(p_1', r_1', p_2', r_2') are the ports associated with the victim line, L, C the victim line inductance and capacitance per meter, M, γ the differential line mutual inductance and capacitance per meter:

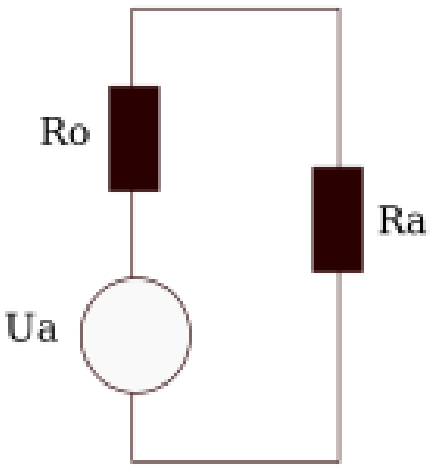
$$\begin{cases} \alpha = \gamma \left(C + \gamma\right)^{-1}, \quad K = M \left(L\alpha\right)^{-1} \\ u(r_1') = \alpha \left(\frac{K+1}{2}\right) U_0(s) \left[1 - exp\left(-2s\frac{x}{c}\right)\right] \\ u(p_2') = -\alpha \left(K - 1\right) \frac{x}{c} s U_0(s) exp\left(-s\frac{x}{c}\right) \end{cases}$$

The wave vector at start is $O = \begin{bmatrix} 0 & u(r_1') & u(p_2') & 0 \end{bmatrix}^T$

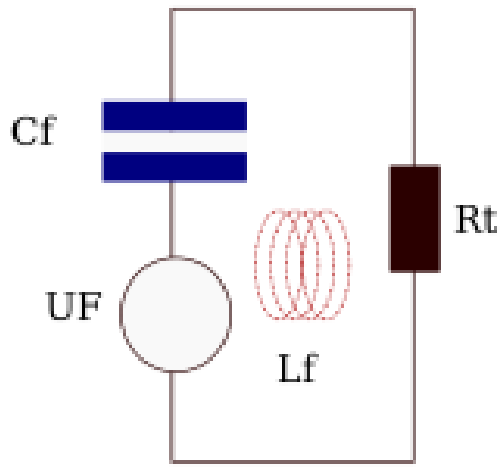
Example - Let's detail an illustration



Three basic models: the source antenna and the box window.

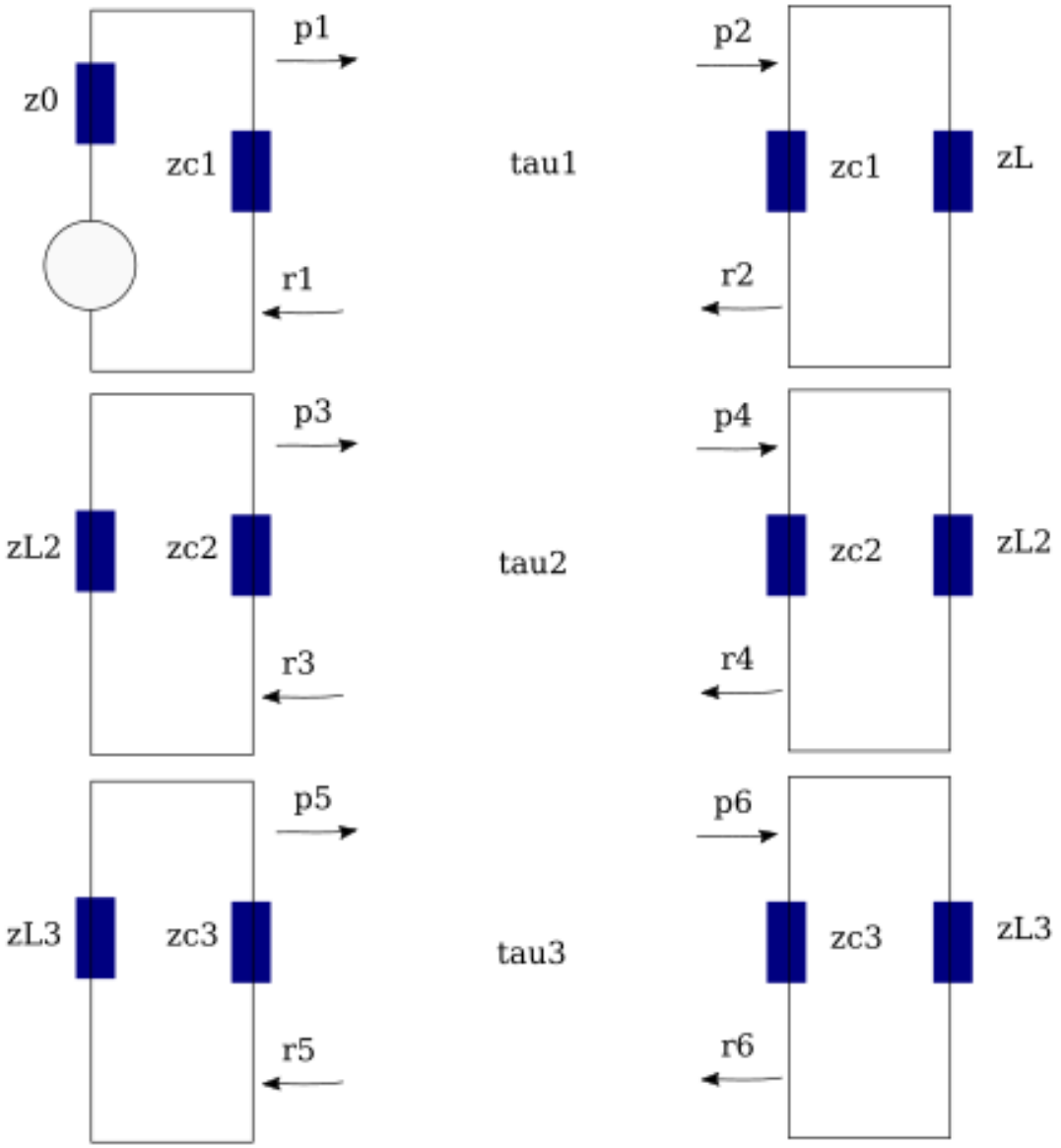


source graph

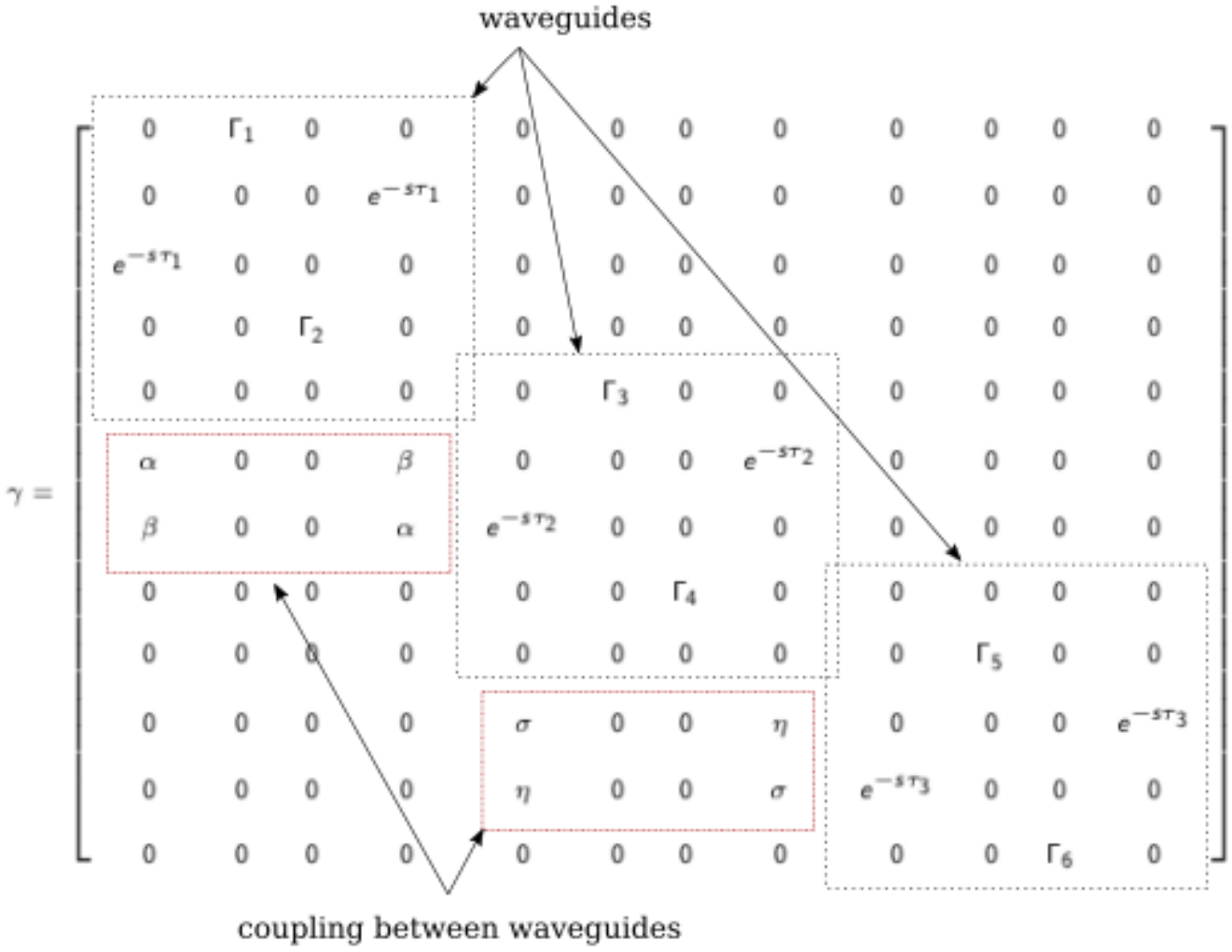


opening graph

Example - Cavity + external line + internal line: wave space



Example - A 3 times propagator



Example - Whole problem construction - I

First we enter basic manifolds $\oplus_i \mathcal{M}_i\left(G_i, \zeta_i\right)$:

$$\zeta = \begin{bmatrix} R_0 + R_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_t + sL_f + \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta + zc_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & zc_1 + z_L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & zL_2 + zc_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zc_2 + zL_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & zL_3 + zc_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & zc_3 + zL_3 \end{bmatrix}$$

$\beta = (sC_f)^{-1}$

Example - Whole problem construction - II

Then we add the couplings through strings.

Coupling between the antenna and the opening:

$$E(x,r) = \frac{\sqrt{30GP}}{r(t)}\Lambda_X^x = \frac{\sqrt{30GR_a}}{r(t)}J^1 e^{-s\frac{r(t)}{c}}$$

$$e_2 = \left(h_f - X_f Y_f \frac{d}{cdt}\right) E \Rightarrow \zeta_{21} = \frac{e_2}{J^1} = \left(h_f + \frac{X_f Y_f}{ct} + sX_f Y_f \frac{v_m}{c^2}\right) \frac{\sqrt{30GR_a}}{v_m t} e^{-s\frac{v_m t}{c}} = \zeta_{12}$$

Between the opening and the cavity (for given modes):

$$e_3 = \frac{1}{sC_f}J^2 \Rightarrow \zeta_{32} = \zeta_{23} = \frac{1}{sC_f}$$

Example - Whole problem construction - III

Coupling between the cavity and the shield line:

$$\alpha = \frac{h_c}{\chi}\int_x dx e^{-s\frac{x}{c}},\ \beta = \frac{h_c}{\chi}\int_x dx e^{-s\left(\frac{X-x}{c}\right)}$$

Coupling through the cable shield:

$$\sigma = \int_x dx \frac{z_T}{zc_2} e^{-s\frac{x}{c}},\ \eta = \int_x dx \frac{z_T}{zc_2} e^{-s\left(\frac{X-x}{c}\right)}$$

Example - Whole problem construction - IV

The strings are added to the metric:

$$\zeta = \begin{bmatrix} R_0 + R_a & \zeta_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ \zeta_{21} & R_t + sL_f + \beta & \zeta_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & \zeta_{32} & \beta + zc_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & zc_1 + z_L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & zL_2 + zc_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zc_2 + zL_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & zL_3 + zc_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & zc_3 + zL_3 \end{bmatrix}$$

The couplings between the waveguides are supported by the source covector.

We define a wave vector by:

$$\mathcal{V}_\sigma = \begin{bmatrix} zc_1 J^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Then:

$$T_\alpha = 2\Lambda^\sigma_\alpha \bar{\gamma} \mathcal{V}_\sigma$$

In final we solve $T_\alpha(t) = \zeta_{\alpha\beta}(t) J^\beta(t)$. The constraint is given by: $C_o(t) = zL_3 J^8(t) = zL_3 y^{8\alpha}(t) T_\alpha(t)$.

Example - You speak of margin?

For various conditions, missions, etc., we find three possible constraints $C_o(1)$, $C_o(2)$, $C_o(3)$: C_{oi}. The target component present three major thresholds S_j (depending on its source, etc.).

	S_1	S_2	S_3
C_{o1}	μ_{11}, P_{11}	μ_{12}, P_{12}	$\dots$
C_{o2}	μ_{21}, P_{21}	$\dots$	
C_{o3}	$\dots$		

$\mu_{ij} = S_j / C_{oi}$: disturbance margin.
$P_{ij} = P\left(C_{oi} |\ \textit{mission}, \dots\right) \times P\left(S_j |\ \textit{source}, \dots\right)$ the combination probability.

$$\text{Effective margin: } \sum_{i,j} \mu_{ij} \times P_{ij}.$$

Basic Risk Law: $R = \frac{WH}{S}$. Protection approach when playing on the transfer function H (using filters) or on the component threshold S. Prevention function when playing on the source W or on the distance (segregation using shields) through H.

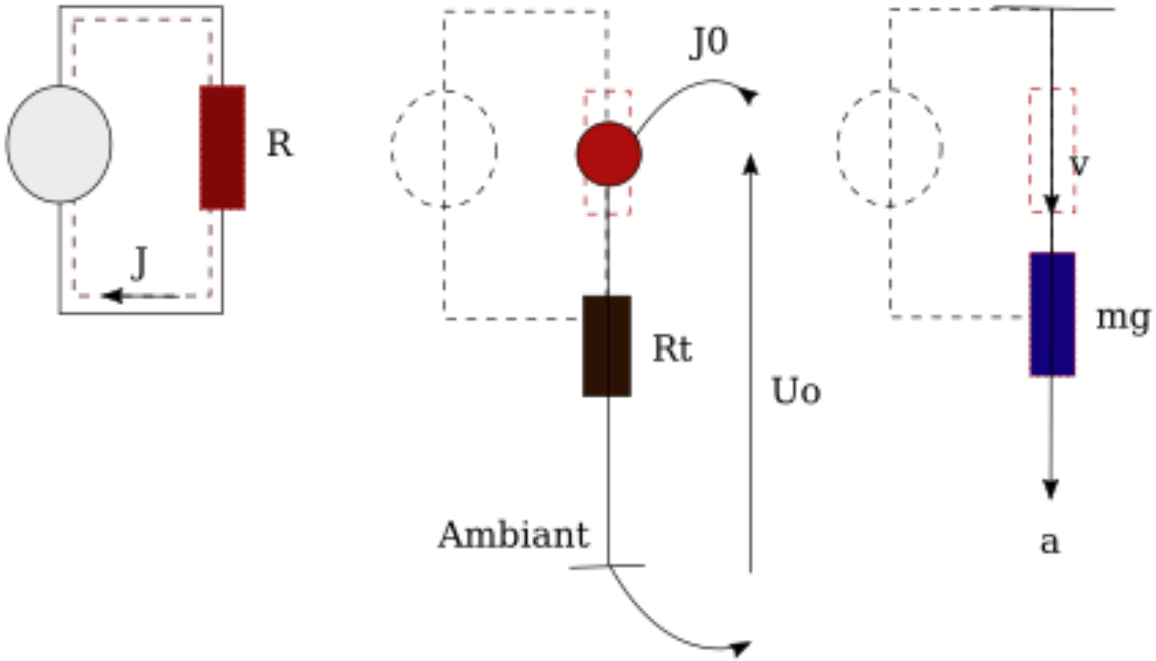
Having the problem equations, we can make theoretical studies:

$$\frac{d}{dt}C_o(t) = \frac{d}{dt}\left(zL_3 y^{8\alpha}(t)\, T_\alpha(t)\right) = zL_3\left[\frac{dy^{8\alpha}}{dt}T_\alpha + y^{8\alpha}\frac{dT_\alpha}{dt}\right]$$

We can manage uncertainties:

$$Co = \left[zL_3 \pm \delta zL_3\right]\left[y^{8\alpha} \pm \delta y^{8\alpha}\right]\left[T_\alpha \pm \delta T_\alpha\right]$$

$$\Rightarrow \delta C_o \approx \pm\left(\delta zL_3 y^{8\alpha} T_\alpha + zL_3 \delta y^{8\alpha} T_\alpha + zL_3 y^{8\alpha}\delta T_\alpha\right)$$

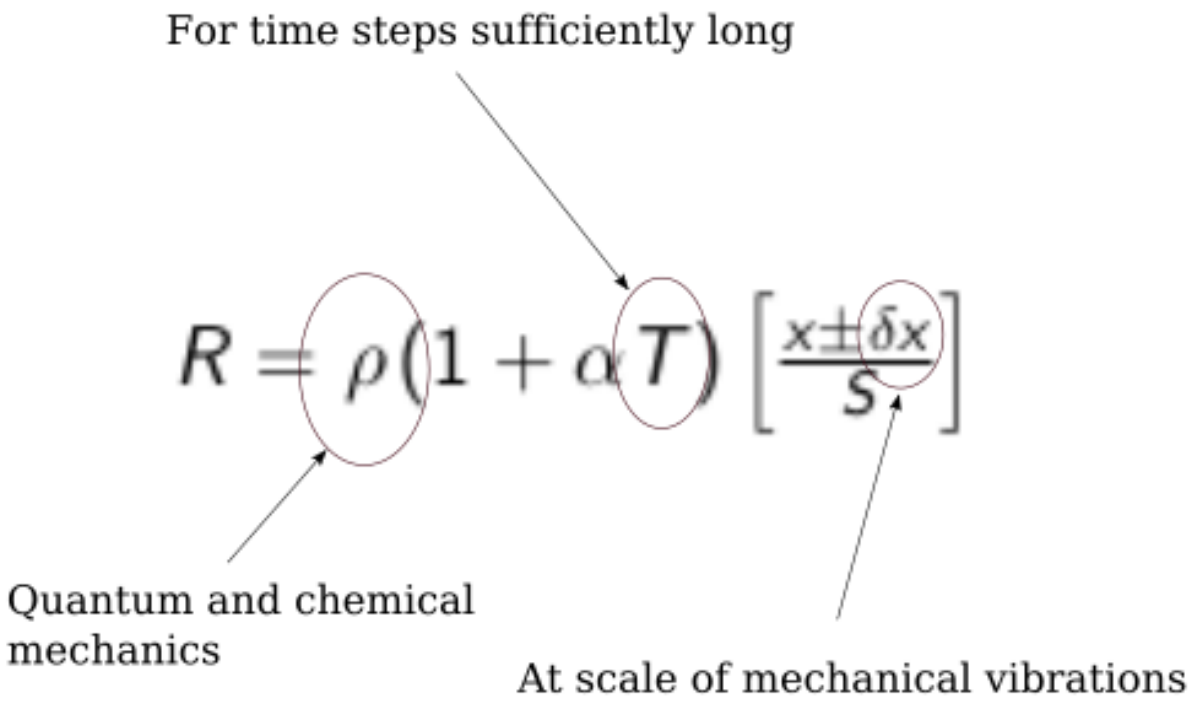


$$\begin{cases} U_1 = \rho_{11}\left(\frac{x^1+\delta x^1}{\phi_1^1}\right)\left(1+\alpha_{11}\delta\theta^1\right)J^1 \\ \delta\theta^1 = \mathcal{R}^{11}\left(\frac{1}{4}\left[RJ^1J^{*1}+RJ^{*1}J^1\right]\right)_1 \\ f_1 = m_{11}\frac{d}{dt}v^1 + k_{11}\int_t dtv^1,\ \delta x^1 = \int_t dtv^1 \end{cases}$$

$$\begin{cases} U_1 = \rho_{11}\left(\frac{x^1+\delta x^1}{\phi_1^1}\right)\left(1+\alpha_{11}\delta\theta^1\right)J^1 \\ \delta\theta^1 = \mathcal{R}^{11}\left(\frac{1}{4}\left[RJ^1J^{*1}+RJ^{*1}J^1\right]\right)_1 \\ f_1 = m_{11}\frac{d}{dt}v^1 + k_{11}\int_t dtv^1,\ \delta x^1 = \int_t dtv^1 \end{cases}$$

$$\Leftrightarrow \mathcal{T}_\alpha = \chi_{\alpha\beta}\mathcal{J}^\beta \leftrightarrow \begin{cases} T = \begin{bmatrix} u_1 & P_1 & f_1 \end{bmatrix} \\ \chi = \begin{bmatrix} R_{11} & \frac{\rho_{11}}{\phi_1^1} & R_{11}\alpha_{11} & \frac{\rho_{11}}{\phi_1^1}\alpha_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{Y}_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{11}\frac{d}{dt} + k_{11}\int_t dt \end{bmatrix} \\ \mathcal{J} = \begin{bmatrix} J^1 & \delta x^1 J^1 & \delta\theta^1 J^1 & \delta x^1\delta\theta^1 J^1 & \delta\theta^1 & v^1 \end{bmatrix}^T \end{cases}$$

Example for scale mechanism:



In all physics, polarization (or *static* state) is obtained taking the inertia contribution in the lagragian $\mathcal{L}_{\alpha\beta}$, $m_{\alpha\beta}$ equal to zero.

The stiffness behaves as if the energy was already stored ($C = 0,\ k = 0$).

If $\Theta[0]$ is the static observable and $\Theta[s]$, $\omega \neq 0$ the dynamic one, at each time step the rms value is defined by:

$$rms = \sqrt{\frac{1}{2}\left(\Theta[0]\Theta^*[0] + \sum_{1}^{N}\Theta[s]\Theta^*[s]\right)}$$

C & S - Maxwell's 3D solver

3D solvers are based on weak order models. Once the problem equations are obtained through the tensorial analysis of network, we can confirm that our numerical tool can solve the problem with an adequat accuracy. If 3D tools have a higher geometrical accuracy, they have poor description of the physical models coming from their low order schematics. The numerical applications directly extracted from the system equations can also give pertinent results. In practice, what leads to robust analyses is:

1. make theoretical analysis using TAN;
2. make numerical application of the system of equations;
3. make 3D tool simulation;
4. if a new physical and unknown component used: make an experiment;
5. all these exercizes should converge.

- The tensor analysis of networks (TAN) give engineers a tool for writing the equations of any EMC (physical) problem.
- The TAN analysis is the first action to do before any 3D simulation or physical experiment.

- Outlook
  - Thermo-electronic analysis should be more and more usefull for optimized systems.