

# 1 Equilibrium conditions

The following system of equations represents the equilibrium conditions of our economy.

**Households :**

$$C_t = \left( \mu_c^{1/\varepsilon} (C_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_c)^{1/\varepsilon} (C_{g,t})^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}. \quad (1)$$

$$H_t = \left( \mu_h^{1/\varepsilon} (H_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_h)^{1/\varepsilon} (H_{g,t})^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}, \quad (2)$$

$$C_{b,t} + (1 - \tau_{c,t}) P_{g,t} C_{g,t} + D_{h,t} = (1 - \varpi) \sum_{j=b,g} W_{j,t} H_{j,t} + R_{t-1}^d D_{h,t-1} + T_t + (1 - \sigma_B) N_t \quad (3)$$

$$\lambda_t = \mu_c^{\frac{1}{\varepsilon}} C_t^{-\gamma + \frac{1}{\varepsilon}} C_{b,t}^{-\frac{1}{\varepsilon}}, \quad (4)$$

$$(1 - \tau_{c,t}) P_{g,t} \lambda_t = (1 - \mu_c)^{\frac{1}{\varepsilon}} C_t^{-\gamma + \frac{1}{\varepsilon}} C_{g,t}^{-\frac{1}{\varepsilon}}, \quad (5)$$

$$(1 - \varpi) W_{b,t} \lambda_t = \mu_h^{\frac{1}{\varepsilon}} H_t^{\sigma + \frac{1}{\varepsilon}} H_{b,t}^{-\frac{1}{\varepsilon}}, \quad (6)$$

$$(1 - \varpi) W_{g,t} \lambda_t = (1 - \mu_h)^{\frac{1}{\varepsilon}} H_t^{\sigma + \frac{1}{\varepsilon}} H_{g,t}^{-\frac{1}{\varepsilon}}, \quad (7)$$

$$\lambda_t = \beta E_t \{ \lambda_{t+1} R_t^d \}, \quad (8)$$

**Brown sector :**

$$Y_{b,t} = A_{b,t} K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b}, \quad (9)$$

$$A_{b,t} = (1 - d(x_{t-1})) a_{b,t}, \quad (10)$$

$$\log(a_{b,t}) = (1 - \rho_{a_b}) \log(a_b) + \rho_{a_b} \log(a_{b,t-1}) + \varepsilon_{a_b,t}, \quad (11)$$

$$d(x_t) = d_0 + d_1 x_t + d_2 x_t^2, \quad (12)$$

$$x_t = (1 - \delta_x) x_{t-1} + e_t + e_t^{row}, \quad (13)$$

$$e_t = \varphi(1 - \eta_t) Y_{b,t}, \quad (14)$$

$$Z_t = \psi_1 \eta_t^{\psi_2} Y_{b,t}, \quad (15)$$

$$R_{b,t}^l - 1 = \alpha_b \frac{[1 - \psi_1 \eta_t^{\psi_2} - \tau_{e,t} \varphi(1 - \eta_t)] Y_{b,t}}{K_{b,t-1}}, \quad (16)$$

$$W_{b,t} = (1 - \alpha_b) \frac{[1 - \psi_1 \eta_t^{\psi_2} - \tau_{e,t} \varphi(1 - \eta_t)] Y_{b,t}}{H_{b,t}}, \quad (17)$$

$$\eta_t = \left( \frac{\tau_{e,t} \varphi}{\psi_1 \psi_2} \right)^{\frac{1}{\psi_2 - 1}}, \quad (18)$$

$$e_t^{row} = 75 e_t, \quad (19)$$

Green sector :

$$Y_{g,t} = A_{g,t} K_{g,t-1}^{\alpha_g} H_{g,t}^{1-\alpha_g}, \quad (20)$$

$$\log(A_{g,t}) = (1 - \rho_{A_g}) \log(A_g) + \rho_{A_g} \log(A_{g,t-1}) + \varepsilon_{A_g,t}, \quad (21)$$

$$R_{g,t}^l - 1 = \alpha_g \frac{P_{g,t} Y_{g,t}}{K_{g,t-1}}, \quad (22)$$

$$W_{g,t} = (1 - \alpha_g) \frac{P_{g,t} Y_{g,t}}{H_{g,t}}. \quad (23)$$

**Private Bank :**

$$K_{b,t} = \mu E_t \left\{ \left( \frac{(1 - \tau_{b,t+1})(R_{b,t+1}^l - 1)}{R_t^a - 1} \right)^\varepsilon \right\} Asset_t \quad (24)$$

$$K_{g,t} = \eta_B E_t \left\{ \left( \frac{R_{g,t+1}^l - 1}{R_t^a - 1} \right)^\varepsilon \right\} Asset_t + GQE_t, \quad (25)$$

$$B_t = (1 - \mu - \eta_B) E_t \left\{ \left( \frac{R_t^B - 1}{R_t^a - 1} \right)^\varepsilon \right\} Asset_t, \quad (26)$$

$$R_t^a = 1 + [\mu E_t (1 - \tau_{b,t+1})^{\varepsilon-1} (R_{b,t+1}^l - 1)^{\varepsilon-1} + \eta_B E_t (R_{g,t+1}^l - 1)^{\varepsilon-1} + (1 - \mu - \eta_B) E_t (R_t^B - 1)^{\varepsilon-1}]^{\frac{1}{1-\varepsilon}} \quad (27)$$

$$Asset_t = \phi_t N_t, \quad (28)$$

$$D_{h,t} = (\phi_t - 1) N_t, \quad (29)$$

$$N_t = \sigma_B [(R_t^a - R_{t-1}^d) \phi_{t-1} + R_{t-1}^d] N_{t-1}, \quad (30)$$

$$\phi_t = \gamma_t / (\alpha - \gamma_t^a), \quad (31)$$

$$\gamma_t^a = E_t \{ \Lambda_{t,t+1} \sigma_B (R_{t+1}^a - R_t^d) \}, \quad (32)$$

$$\gamma_t = E_t \{ \Lambda_{t,t+1} \sigma_B R_t^d \}, \quad (33)$$

$$\Lambda_{t,t+1} = \beta_{t,t+1} [1 - \sigma_B + \sigma_B (\gamma_{t+1}^a \phi_{t+1} + \gamma_{t+1})], \quad (34)$$

**Public sector :**

$$\begin{aligned} & T_t + R_{t-1} B_{t-1} + \tau_{c,t} P_{g,t} C_{g,t} + \tau_{g,t} (R_{g,t}^l - 1) K_{g,t-1} \\ & = \varpi W_{g,t} H_{g,t} + \varpi W_{b,t} H_{b,t} + \tau_{e,t} e_t + \tau_{b,t} (R_{b,t}^l - 1) K_{b,t-1} + B_t, \end{aligned} \quad (35)$$

**Market clearing :**

$$Y_t = P_{g,t} Y_{g,t} + Y_{b,t} = P_{g,t} C_{g,t} + C_{b,t} + [K_{g,t} - (1 - \delta) K_{g,t} + K_{b,t} - (1 - \delta) K_{b,t}] + Z_t \quad (36)$$

## 2 Derivatives

### 2.1 Households

The Lagrangian of the maximization problem of households is given by:

$$L = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ \left[ \frac{C_{t+\tau}^{1-\gamma}}{1-\gamma} - \frac{H_{t+\tau}^{1+\sigma}}{1+\sigma} \right] - \lambda_{t+\tau} [C_{b,t+\tau} + (1 - \tau_{c,t+\tau})P_{g,t+\tau}C_{g,t+\tau} + D_{h,t+\tau} \right. \\ \left. - (1 - \varpi)W_{b,t+\tau}H_{b,t+\tau} - (1 - \varpi)W_{g,t+\tau}H_{g,t+\tau} - R_{t+\tau-1}^d D_{h,t+\tau-1} - T_{t+\tau} - (1 - \sigma_B)N_{t+\tau}] \right\}$$

which we differentiate with respect to  $\{C_{b,t}, C_{g,t}, H_{b,t}, H_{g,t}, D_{h,t}\}$ . The first order conditions of this maximization problem are:

- differentiation with respect to  $C_{b,t}$  yields:

$$C_t^{-\gamma} \left[ \frac{\varepsilon}{\varepsilon - 1} (\mu_c^{1/\varepsilon} (C_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_c)^{1/\varepsilon} (C_{g,t})^{(\varepsilon-1)/\varepsilon})^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon - 1}{\varepsilon} \mu_c^{1/\varepsilon} C_{b,t}^{\frac{\varepsilon-1}{\varepsilon}-1} \right] - \lambda_t = 0,$$

$$C_t^{-\gamma} \left[ (\mu_c^{1/\varepsilon} (C_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_c)^{1/\varepsilon} (C_{g,t})^{(\varepsilon-1)/\varepsilon})^{\frac{1}{\varepsilon-1}} \mu_c^{1/\varepsilon} C_{b,t}^{\frac{\varepsilon-1}{\varepsilon}-1} \right] - \lambda_t = 0,$$

$$C_t^{-\gamma} C_t^{\frac{1}{\varepsilon}} \mu_c^{1/\varepsilon} C_{b,t}^{-\frac{1}{\varepsilon}} - \lambda_t = 0,$$

We get eq (4):

$$\lambda_t = \mu_c^{1/\varepsilon} C_t^{-\gamma + \frac{1}{\varepsilon}} C_{b,t}^{-\frac{1}{\varepsilon}},$$

- differentiation with respect to  $C_{g,t}$  yields:

$$C_t^{-\gamma} \left[ \frac{\varepsilon}{\varepsilon - 1} (\mu_c^{1/\varepsilon} (C_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_c)^{1/\varepsilon} (C_{g,t})^{(\varepsilon-1)/\varepsilon})^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon - 1}{\varepsilon} (1 - \mu_c)^{1/\varepsilon} C_{g,t}^{\frac{\varepsilon-1}{\varepsilon}-1} \right] - (1 - \tau_{c,t})P_{g,t}\lambda_t = 0,$$

$$C_t^{-\gamma} \left[ (\mu_c^{1/\varepsilon} (C_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_c)^{1/\varepsilon} (C_{g,t})^{(\varepsilon-1)/\varepsilon})^{\frac{1}{\varepsilon-1}} (1 - \mu_c)^{1/\varepsilon} C_{g,t}^{\frac{\varepsilon-1}{\varepsilon}-1} \right] - (1 - \tau_{c,t})P_{g,t}\lambda_t = 0,$$

$$C_t^{-\gamma} C_t^{\frac{1}{\varepsilon}} (1 - \mu_c)^{1/\varepsilon} C_{g,t}^{-\frac{1}{\varepsilon}} - (1 - \tau_{c,t}) P_{g,t} \lambda_t = 0,$$

We get eq (5):

$$(1 - \tau_{c,t}) P_{g,t} \lambda_t = (1 - \mu_c)^{1/\varepsilon} C_t^{-\gamma + \frac{1}{\varepsilon}} C_{g,t}^{-\frac{1}{\varepsilon}},$$

- differentiation with respect to  $H_{b,t}$  yields:

$$H_t^\sigma \frac{\varepsilon}{\varepsilon - 1} \left( \mu_h^{1/\varepsilon} (H_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_h)^{1/\varepsilon} (H_{g,t})^{(\varepsilon-1)/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1} - 1} \mu_h^{1/\varepsilon} \frac{\varepsilon - 1}{\varepsilon} H_{b,t}^{\frac{\varepsilon-1}{\varepsilon} - 1} + \lambda_t (1 - \varpi) W_{b,t} = 0,$$

$$-H_t^\sigma \left( \mu_h^{1/\varepsilon} (H_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_h)^{1/\varepsilon} (H_{g,t})^{(\varepsilon-1)/\varepsilon} \right)^{\frac{1}{\varepsilon-1}} \mu_h^{1/\varepsilon} H_{b,t}^{-\frac{1}{\varepsilon}} + \lambda_t (1 - \varpi) W_{b,t} = 0,$$

$$-H_t^\sigma H_t^{\frac{1}{\varepsilon}} \mu_h^{\frac{1}{\varepsilon}} H_{b,t}^{-\frac{1}{\varepsilon}} + \lambda_t (1 - \varpi) W_{b,t} = 0,$$

We get eq (6):

$$\lambda_t (1 - \varpi) W_{b,t} = \mu_h^{\frac{1}{\varepsilon}} H_t^{\sigma + \frac{1}{\varepsilon}} H_{b,t}^{-\frac{1}{\varepsilon}},$$

- differentiation with respect to  $H_{g,t}$  yields:

$$H_t^\sigma \left( \mu_h^{1/\varepsilon} (H_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_h)^{1/\varepsilon} (H_{g,t})^{(\varepsilon-1)/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1} - 1} (1 - \mu_h)^{1/\varepsilon} H_{g,t}^{\frac{\varepsilon-1}{\varepsilon} - 1} + \lambda_t (1 - \varpi) W_{g,t} = 0,$$

$$-H_t^\sigma \left( \mu_h^{1/\varepsilon} (H_{b,t})^{(\varepsilon-1)/\varepsilon} + (1 - \mu_h)^{1/\varepsilon} (H_{g,t})^{(\varepsilon-1)/\varepsilon} \right)^{\frac{1}{\varepsilon-1}} (1 - \mu_h)^{1/\varepsilon} H_{g,t}^{-\frac{1}{\varepsilon}} + \lambda_t (1 - \varpi) W_{g,t} = 0,$$

$$-H_t^\sigma H_t^{\frac{1}{\varepsilon}} (1 - \mu_h)^{1/\varepsilon} H_{g,t}^{-\frac{1}{\varepsilon}} + \lambda_t (1 - \varpi) W_{g,t} = 0,$$

We get (7):

$$\lambda_t (1 - \varpi) W_{g,t} = (1 - \mu_h)^{1/\varepsilon} H_t^{\sigma + \frac{1}{\varepsilon}} H_{g,t}^{-\frac{1}{\varepsilon}},$$

- differentiation with respect to  $D_{h,t}$  yields:

$$-\lambda_t + \beta E_t \{ \lambda_{t+1} R_t^d \} = 0,$$

We get eq (8):

$$\lambda_t = \beta E_t \{ \lambda_{t+1} R_t^d \},$$

## 2.2 Green sector

The green firm maximizes its profits and solves the following problem :

$$\max_{K_{tp-1}, H_{g,t}} \{ P_{g,t} Y_{g,t} - (R_{g,t}^l - 1) K_{g,t-1} - W_{g,t} H_{g,t} \},$$

Subject to

$$Y_{g,t} = A_{g,t} K_{g,t-1}^{\alpha_g} H_{g,t}^{1-\alpha_g},$$

- differentiation with respect to  $K_{g,t-1}$  yields:

$$0 = A_{g,t} \alpha_g K_{g,t-1}^{\alpha_g - 1} H_{g,t}^{1-\alpha_g} P_{g,t} - (R_{g,t}^l - 1)$$

$$R_{g,t}^l - 1 = P_{g,t} A_{g,t} \alpha_g K_{g,t-1}^{\alpha_g - 1} H_{g,t}^{1-\alpha_g}$$

$$R_{g,t}^l - 1 = P_{g,t} A_{g,t} \alpha_g K_{g,t-1}^{\alpha_g} K_{g,t-1}^{-1} H_{g,t}^{1-\alpha_g}$$

We get eq (22)

$$R_{g,t}^l - 1 = \alpha_g P_{g,t} \frac{Y_{g,t}}{K_{g,t-1}}$$

- differentiation with respect to  $H_{g,t}$  yields:

$$\begin{aligned}
0 &= A_{g,t}(1 - \alpha_g) K_{g,t-1}^{\alpha_g} H_{g,t}^{1-\alpha_g-1} P_{g,t} - W_{g,t} \\
W_{g,t} &= P_{g,t} A_{g,t} (1 - \alpha_g) K_{g,t-1}^{\alpha_g} H_{g,t}^{1-\alpha_g} H_{g,t}^{-1} \\
W_{g,t} &= P_{g,t} A_{g,t} (1 - \alpha_g) K_{g,t-1}^{\alpha_g} H_{g,t}^{1-\alpha_g} H_{g,t}^{-1}
\end{aligned}$$

We get eq (23)

$$W_{g,t} = (1 - \alpha_g) P_{g,t} \frac{Y_{g,t}}{H_{g,t}},$$

### 2.3 Brown sector

The brown firm maximizes its profits and solves the following problem :

$$\max_{K_{b,t-1}, H_{b,t}, \eta_t} Y_{b,t} - Z_t - \tau_{e,t} e_t - W_{b,t} H_{b,t} - (R_{b,t}^l - 1) K_{b,t-1}$$

subject to

$$Y_{b,t} = A_{b,t} K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b},$$

- differentiation with respect to  $K_{b,t-1}$  yields:

$$\begin{aligned}
&A_{b,t} \alpha_b K_{b,t-1}^{\alpha_b-1} H_{b,t}^{\beta_b} - \psi_1 \eta_t^{\psi_2} A_{b,t} \alpha_b K_{b,t-1}^{\alpha_b-1} H_{b,t}^{1-\alpha_b} \\
&- \tau_{e,t} \varphi(1 - \eta_t) A_{b,t} \alpha_b K_{b,t-1}^{\alpha_b-1} H_{b,t}^{1-\alpha_b} - (R_{b,t}^l - 1) = 0
\end{aligned}$$

$$\begin{aligned}
&A_{b,t} \alpha_b K_{b,t-1}^{-1} K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b} - \psi_1 \eta_t^{\psi_2} A_{b,t} \alpha_b K_{b,t-1}^{-1} K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b} \\
&- \tau_{e,t} \varphi(1 - \eta_t) A_{b,t} \alpha_b K_{b,t-1}^{-1} K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b} - (R_{b,t}^l - 1) = 0
\end{aligned}$$

$$\alpha_b \frac{Y_{b,t}}{K_{b,t-1}} - \psi_1 \eta_t^{\psi_2} \alpha_b \frac{Y_{b,t}}{K_{b,t-1}} - \tau_{e,t} \varphi(1 - \eta_t) \alpha_b \frac{Y_{b,t}}{K_{b,t-1}} - (R_{b,t}^l - 1) = 0$$

We get (16)

$$R_{b,t}^l - 1 = \alpha_b \frac{[1 - \psi_1 \eta_t^{\psi_2} - \tau_{e,t} \varphi(1 - \eta_t)] Y_{b,t}}{K_{b,t-1}},$$

- differentiation with respect to  $H_{b,t}$  yields:

$$\begin{aligned} 0 &= A_{b,t} (1 - \alpha_b) K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b-1} - \psi_1 \eta_t^{\psi_2} A_{b,t} (1 - \alpha_b) K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b-1} \\ &\quad - \tau_{e,t} \varphi(1 - \eta_t) A_{b,t} (1 - \alpha_b) K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b-1} - W_{b,t} \end{aligned}$$

$$\begin{aligned} 0 &= A_{b,t} (1 - \alpha_b) K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b} H_{b,t}^{-1} - \psi_1 \eta_t^{\psi_2} A_{b,t} (1 - \alpha_b) K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b} H_{b,t}^{-1} \\ &\quad - \tau_{e,t} \varphi(1 - \eta_t) A_{b,t} (1 - \alpha_b) K_{b,t-1}^{\alpha_b} H_{b,t}^{1-\alpha_b} H_{b,t}^{-1} - W_{b,t} \end{aligned}$$

$$(1 - \alpha_b) \frac{Y_{b,t}}{H_{b,t}} - \psi_1 \eta_t^{\psi_2} (1 - \alpha_b) \frac{Y_{b,t}}{H_{b,t}} - \tau_{e,t} \varphi(1 - \eta_t) (1 - \alpha_b) \frac{Y_{b,t}}{H_{b,t}} - W_{b,t} = 0$$

We get (17)

$$W_{b,t} = (1 - \alpha_b) \frac{[1 - \psi_1 \eta_t^{\psi_2} - \tau_{e,t} \varphi(1 - \eta_t)] Y_{b,t}}{H_{b,t}},$$

differentiation with respect to  $\eta_t$  yields:

$$\begin{aligned} -\psi_1 \psi_2 \eta_t^{\psi_2-1} Y_{b,t} + \tau_e \varphi Y_{b,t} &= 0 \\ -\psi_1 \psi_2 \eta_t^{\psi_2-1} + \tau_e \varphi &= 0 \\ \psi_1 \psi_2 \eta_t^{\psi_2-1} &= \tau_e \varphi \\ \eta_t^{\psi_2-1} &= \frac{\tau_e \varphi}{\psi_1 \psi_2} \end{aligned}$$

We get eq (18)

$$\eta_t = \left( \frac{\tau_e \varphi}{\psi_1 \psi_2} \right)^{\frac{1}{\psi_2-1}},$$



## 2.4 Banks

The initial guess of the solution of the optimization problem is :

$$V_t = \gamma_t^a Asset_t + \gamma_t N_t \quad (37)$$

and the incentive condition therefore simplified as :

$$\phi_t N_t \geq Asset_t \quad (38)$$

where  $\phi_t = \gamma_t / (\alpha - \gamma_t^a)$  is defined as the bank's leverage ratio. We substitute the binding constraint in the guessed solution and it yields  $V_t = (\gamma_t^a \phi_t + \gamma_t) N_t$ . Plugging the expression into the value function of accumulation of capital  $N_{t+1}$ , we have

$$V_t = E_t \{ \Lambda_{t,t+1} N_{t+1} \} \quad (39)$$

$$= E_t \{ \Lambda_{t,t+1} \sigma_B [(R_{t+1}^a - R_t^d) Asset_t + R_t^d N_t] \} \quad (40)$$

where  $\Lambda_{t,t+1} = \beta_{t,t+1} [1 - \sigma_B + \sigma_B (\gamma_{t+1}^a \phi_{t+1} + \gamma_{t+1})]$ . This allows to identify the arguments of the value function:

$$\gamma_t^a = E_t \{ \Lambda_{t,t+1} \sigma_B (R_{t+1}^a - R_t^d) \} \text{ and } \gamma_t = E_t \{ \Lambda_{t,t+1} \sigma_B R_t^d \}$$

For the private banks, their FOCs are determined as following :

Equation (24) - (25) are derived from the cost minimization of asset, given the CES integration of different assets. We assume that different assets provide identical contingent dividend  $D_t = 1$  to shareholders. This is a strong assumption. However, it is not surprising to assume that the dividend is contingent and exogenous as in [Dib \(2010\)](#). By having this assumption, the price of each asset is directly linked to the asset's interest rate. From the

bond valuation theory, we have

$$q_t^a = \frac{D_t}{R_t^a - 1} \quad (41)$$

$$q_t^b = \frac{D_t}{R_{b,t}^L - 1} \quad (42)$$

$$q_t^g = \frac{D_t}{R_{g,t}^L - 1} \quad (43)$$

Equation (28) determines the asset, which comes from the binding condition equation (38).

Equation (29) comes from the definition that asset equals to the sum of deposit and capital, replace the asset by the binding condition of equation (38), we get the first order condition for  $D_{h,t}$  equation (29).

Equation (30) is the dynamic of bank's capital, which comes from the evolution of capital on function of asset and the leverage constraint (38).

Equation (31) is the leverage ratio which is determined by the incentive condition together with equation (37).

Equation (32) and (33) are to determine  $\gamma_t^a$  and  $\gamma_t$  from equation (37) and the bank's value function.

Equation (34) is derived from equation (40) and (37).

### 3 Steady-State

To find the steady state solution, we proceed in two steps: - First, we calibrate the parameters and assign an empirical value to the capital stock  $x$  and we set  $e^{row} = 75e$  based on the ratio of emissions in France to emissions in the rest of the world (see calibration section); second, we solve analytically the system of equations of our equilibrium conditions (appendix ). We get the value of  $R_b^L$ ,  $R_g^L$ , and  $R^B$  from historical data. Note also that, the steady state value

of  $a_b$  and  $A_g$  are equal to 1.

Then, from (12):

$$dx = d_0 + d_1x + d_2x^2, \quad (44)$$

From (13), we can get :

$$\begin{aligned} x &= (1 - \delta_x)x + e + e^{row}, \\ x &= (1 - \delta_x)x + e + 75e, \\ 76e &= x - (1 - \delta_x)x, \\ 76e &= \delta_x x, \end{aligned}$$

$$e = \frac{\delta_x}{76}x \quad (45)$$

From (14) and (18) we can find the value of  $Y_b$  :

$$\begin{aligned} e &= (1 - \eta)\varphi Y_b, \\ Y_b &= e/(1 - \eta)\varphi, \end{aligned} \quad (46)$$

with :

$$\eta = \left( \frac{\tau_e \varphi}{\psi_1 \psi_2} \right)^{\frac{1}{\psi_2 - 1}}, \quad (47)$$

From (24) :

$$\begin{aligned} R_b^L - 1 &= \alpha_b \frac{[1 - \psi_1 \eta^{\psi_2} - \tau_e \varphi (1 - \eta)] Y_b}{K_b}, \\ K_b &= \alpha_b \frac{[1 - \psi_1 \eta^{\psi_2} - \tau_e \varphi (1 - \eta)] Y_b}{R_b^L - 1} \end{aligned} \quad (48)$$

From these results, we can get the values of  $A_b$ ,  $H_b$ ,  $Z$  and  $W_b$  respectively from (10), (9) and

(15).

Recall that  $a_b = 1$ . Thus :

$$A_b = (1 - dx)a_b, \quad (49)$$

and then :

$$\begin{aligned} Y_b &= A_b K_b^{\alpha_b} H_b^{1-\alpha_b}, \\ H_b^{1-\alpha_b} &= \frac{Y_b}{A_b K_b^{\alpha_b}}, \\ H_b &= \left( \frac{Y_b}{A_b K_b^{\alpha_b}} \right)^{\frac{1}{1-\alpha_b}}, \end{aligned} \quad (50)$$

and

$$Z = \psi_1 \eta^{\psi_2} Y_b, \quad (51)$$

$$W_b = (1 - \alpha_b) \frac{[1 - \psi_1 \eta^{\psi_2} - \tau_e \varphi(1 - \eta)] Y_b}{H_b}, \quad (52)$$

From (8) we get :

$$R^d = \frac{1}{\beta}, \quad (53)$$

From (27):

$$R^a = 1 + [\mu(1 - \tau_b)^{\varepsilon-1} (R_b^l - 1)^{\varepsilon-1} + \eta_B (R_g^l - 1)^{\varepsilon-1} + (1 - \mu - \eta_B) (R^B - 1)^{\varepsilon-1}]^{\frac{1}{\varepsilon-1}}, \quad (54)$$

Then, we can rewrite the net worth equation from (30) and (31) :

$$\begin{aligned}
N &= \sigma_B[(R^a - R^d) \phi N + R^d N], \\
N &= \sigma_B((R^a - R^d) \phi + R^d) N, \\
1 &= \sigma_B((R^a - R^d) \phi + R^d), \\
1 &= \sigma_B(R^a - R^d) \phi + \sigma_B R^d \\
1 - \sigma_B R^d &= \sigma_B(R^a - R^d) \phi, \\
\phi &= \frac{1 - \sigma_B R^d}{\sigma_B(R^a - R^d)}, \tag{55}
\end{aligned}$$

From (28), (24), (26), (31), and bank's balance sheet  $Asset = D_h + N$ , we can get respectively  $Asset$ ,  $N$ ,  $D_h$ ,  $K_g$  and  $B$  :

$$\begin{aligned}
K_b &= \mu \left( \frac{(1 - \tau_b)(R_b^l - 1)}{R^a - 1} \right)^\varepsilon Asset, \\
Asset &= K_b / \mu \left( \frac{(1 - \tau_b)(R_b^l - 1)}{R^a - 1} \right)^{-\varepsilon}, \tag{56}
\end{aligned}$$

$$K_g = \eta_B \left( \frac{R_g^l - 1}{R^a - 1} \right)^\varepsilon Asset + GQE, \tag{57}$$

with  $GQE = 0$ .

$$B = (1 - \mu - \eta_B) \left( \frac{R^B - 1}{R^a - 1} \right)^\varepsilon Asset, \tag{58}$$

$$N = \frac{Asset}{\phi}, \tag{59}$$

$$\begin{aligned}
Asset &= D_h + N, \\
D_h &= Asset - N, \\
D_h &= \phi N - N, \\
D_h &= (\phi - 1)N,
\end{aligned} \tag{60}$$

From (22),

$$Y_g = (1 - \tau_g) (R_g^l - 1) \frac{K_g}{\alpha_g P_g}, \tag{61}$$

Knowing  $Y_g$ , we can get  $H_g$  from (20),

$$\begin{aligned}
Y_g &= A_g K_g^{\alpha_g} H_g^{1-\alpha_g}, \\
H_g^{1-\alpha} &= \frac{Y_g}{A_g K_g^{\alpha_g}}, \\
H_g &= \left( \frac{Y_g}{A_g K_g^{\alpha_g}} \right)^{\frac{1}{1-\alpha}},
\end{aligned} \tag{62}$$

Then, from (7) and (2) :

$$W_g = (1 - \alpha_g) P_g \frac{Y_g}{H_g}, \tag{63}$$

$$H = \left( \mu_h^{1/\varepsilon} (H_b)^{(\varepsilon-1)/\varepsilon} + (1 - \mu_h)^{1/\varepsilon} (H_g)^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}, \tag{64}$$

From the government budget constraint, we have :

$$T = RB + \tau_c P_g C_g + \tau_g (R_g^l - 1) K_g - \varpi W_g H_g - \varpi W_b H_b - \tau_e e - \tau_b (R_b^l - 1) K_b - B,$$

Combining (4) and (5) we get,

$$\mu_c^{\frac{1}{\epsilon}} C^{-\gamma+\frac{1}{\epsilon}} C_b^{-\frac{1}{\epsilon}} = \frac{(1-\mu_c)^{\frac{1}{\epsilon}}}{P_g} C^{-\gamma+\frac{1}{\epsilon}} C_g^{-\frac{1}{\epsilon}}, \quad (65)$$

$$\mu_c^{\frac{1}{\epsilon}} C_b^{-\frac{1}{\epsilon}} = \frac{(1-\mu_c)^{\frac{1}{\epsilon}}}{P_g} C_g^{-\frac{1}{\epsilon}}, \quad (66)$$

$$C_g = \mu_c^{-1} C_b P_g^{-\epsilon} (1 - \mu_c),$$

We replace the result in the household budget constraint :

$$\begin{aligned} C_b(1 + P_g \mu_c^{-1} P_g^{-\epsilon} (1 - \mu_c)) &= (1 - \varpi) W_b H_b + (1 - \varpi) W_g H_g + (R^d - 1) D_h \\ &+ T + (1 - \sigma_B) N, \end{aligned}$$

$$C_b = \frac{(1 - \varpi) W_b H_b + (1 - \varpi) W_g H_g + (R^d - 1) D_h + T + (1 - \sigma_B) N}{1 + \mu_c^{-1} P_g^{1-\epsilon} (1 - \mu_c)},$$

and then :

$$C = (\mu_c^{1/\epsilon} (C_b)^{(\epsilon-1)/\epsilon} + (1 - \mu_c)^{1/\epsilon} (C_g)^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)}. \quad (67)$$

$$\lambda = \mu_c^{\frac{1}{\epsilon}} C^{-\gamma+\frac{1}{\epsilon}} C_b^{-\frac{1}{\epsilon}} \quad (68)$$

From (34), (33), (32) and (55), we get :

$$\begin{aligned}
\Lambda\sigma_B R^d &= \frac{\alpha(1 - \sigma_B R^d) - \Lambda\sigma_B(R^a - R^d)(1 - \sigma_B R^d)}{\sigma_B(R^a - R^d)}, \\
\Lambda\sigma_B R^d &= \frac{\alpha(1 - \sigma_B R^d) - \Lambda\sigma_B(R^a - R^d)(1 - \sigma_B R^d)}{\sigma_B(R^a - R^d)}, \\
\Lambda\sigma_B R^d &= \alpha \frac{(1 - \sigma_B R^d)}{\sigma_B(R^a - R^d)} - \frac{\Lambda\sigma_B(R^a - R^d)(1 - \sigma_B R^d)}{\sigma_B(R^a - R^d)}, \\
\Lambda\sigma_B R^d &= \alpha \frac{(1 - \sigma_B R^d)}{\sigma_B(R^a - R^d)} - \Lambda(1 - \sigma_B R^d), \\
\Lambda\sigma_B R^d + \Lambda(1 - \sigma_B R^d) &= \alpha \frac{(1 - \sigma_B R^d)}{\sigma_B(R^a - R^d)}, \\
\Lambda\sigma_B R^d + \Lambda - \Lambda\sigma_B R^d &= \alpha \frac{(1 - \sigma_B R^d)}{\sigma_B(R^a - R^d)}, \\
\Lambda &= \alpha \frac{1 - \sigma_B R^d}{\sigma_B(R^a - R^d)} = \alpha\phi = \alpha \frac{Asset}{N}, \tag{69}
\end{aligned}$$

From the last result, we obtain :

$$\gamma^a = \Lambda\sigma_B(R^a - R^d), \tag{70}$$

$$\gamma = \Lambda\sigma_B R^d, \tag{71}$$

The steady state value of the weight of work in the brown sector is given by the combination of eq (7) and eq (6)

$$\begin{aligned}
(1 - \varpi)\lambda &= \mu_h^{\frac{1}{\epsilon}} H^{\sigma + \frac{1}{\epsilon}} H_b^{-\frac{1}{\epsilon}} / W_b, \\
(1 - \varpi)\lambda &= (1 - \mu_h)^{\frac{1}{\epsilon}} H^{\sigma + \frac{1}{\epsilon}} H_g^{-\frac{1}{\epsilon}} / W_g,
\end{aligned}$$



$$\begin{aligned}
\mu_h^{\frac{1}{\epsilon}} \frac{H^{\sigma+\frac{1}{\epsilon}} H_b^{-\frac{1}{\epsilon}}}{W_b} &= (1-\mu_h)^{\frac{1}{\epsilon}} \frac{H^{\sigma+\frac{1}{\epsilon}} H_g^{-\frac{1}{\epsilon}}}{W_g}, \\
\mu_h^{\frac{1}{\epsilon}} \frac{H_b^{-\frac{1}{\epsilon}}}{W_b} &= (1-\mu_h)^{\frac{1}{\epsilon}} \frac{H_g^{-\frac{1}{\epsilon}}}{W_g}, \\
\frac{H_b^{-\frac{1}{\epsilon}} W_g}{H_g^{-\frac{1}{\epsilon}} W_b} &= \frac{(1-\mu_h)^{\frac{1}{\epsilon}}}{\mu_h^{\frac{1}{\epsilon}}}, \\
\left(\frac{H_b}{H_g}\right)^{-\frac{1}{\epsilon}} \frac{W_g}{W_b} &= \left(\frac{1-\mu_h}{\mu_h}\right)^{\frac{1}{\epsilon}}, \\
\frac{H_g}{H_b} \left(\frac{W_g}{W_b}\right)^{\epsilon} &= \frac{1-\mu_h}{\mu_h},
\end{aligned}$$

$$\mu_h = \left(\frac{H_g}{H_b} \left(\frac{W_g}{W_b}\right)^{\epsilon} + 1\right)^{-1}, \quad (72)$$

From (4) and (6) we find :

$$\begin{aligned}
\lambda &= \mu_c^{\frac{1}{\epsilon}} C^{-\gamma+\frac{1}{\epsilon}} C_b^{-\frac{1}{\epsilon}}, \\
(1-\varpi)W_b\lambda &= \mu_h^{\frac{1}{\epsilon}} H^{\sigma+\frac{1}{\epsilon}} H_b^{-\frac{1}{\epsilon}},
\end{aligned}$$

$$\mu_c^{\frac{1}{\epsilon}} C^{-\gamma+\frac{1}{\epsilon}} C_b^{-\frac{1}{\epsilon}} = \frac{\mu_h^{\frac{1}{\epsilon}}}{(1-\varpi)W_b} H^{\sigma+\frac{1}{\epsilon}} H_b^{-\frac{1}{\epsilon}},$$

$$\frac{1}{\epsilon} \log \mu_c + \left(-\gamma + \frac{1}{\epsilon}\right) \log C - \frac{1}{\epsilon} \log C_b = \frac{1}{\epsilon} \log \mu_h - \log((1-\varpi)W_b) + \left(\sigma + \frac{1}{\epsilon}\right) \log H - \frac{1}{\epsilon} \log H_b,$$

$$\gamma \log C = \frac{1}{\epsilon} \log \mu_c - \frac{1}{\epsilon} \log \mu_h + \log((1-\varpi)W_b) + \frac{1}{\epsilon} \log C - \frac{1}{\epsilon} \log C_b - \sigma \log H - \frac{1}{\epsilon} \log H + \frac{1}{\epsilon} \log H_b,$$

$$\begin{aligned}\gamma \log C &= \frac{1}{\epsilon} \log \left( \frac{\mu_c}{\mu_h} \right) + \log((1 - \varpi)W_b) + \frac{1}{\epsilon} \log \left( \frac{C}{H} \right) - \frac{1}{\epsilon} \log \left( \frac{C_b}{H_b} \right) - \sigma \log H, \\ \gamma &= \frac{\frac{1}{\epsilon} \log \left( \frac{\mu_c}{\mu_h} \right) + \log((1 - \varpi)W_b) + \frac{1}{\epsilon} \log \left( \frac{C}{H} \right) - \frac{1}{\epsilon} \log \left( \frac{C_b}{H_b} \right) - \sigma \log H}{\log C},\end{aligned}\quad (73)$$

Also, we find  $\alpha$  as follows :

$$\begin{aligned}\phi &= \gamma / (\alpha - \gamma^a), \\ \phi(\alpha - \gamma^a) &= \gamma, \\ \phi\alpha - \phi\gamma^a &= \gamma, \\ \phi\alpha &= \gamma + \phi\gamma^a, \\ \alpha &= \frac{\gamma}{\phi} + \gamma^a,\end{aligned}\quad (74)$$

To find  $\delta$  we use (36) :

$$P_g Y_g = P_g C_g + 0.3(K_g - (1 - \delta)K_g + K_b - (1 - \delta)K_b + T),$$

$$P_g Y_g = P_g C_g + 0.3(K_g + K_b - (1 - \delta)(K_g + K_b) + T),$$

$$(P_g Y_g - P_g C_g) / 0.3 = K_g + K_b - (1 - \delta)(K_g + K_b) + T,$$

$$\delta = 1 - \frac{0.3(K_g + K_b + T) - (P_g Y_g - P_g C_g)}{0.3(K_g + K_b)},$$

$$\delta = 1 + \frac{-0.3(K_g + K_b) - 0.3T + (P_g Y_g - P_g C_g)}{0.3(K_g + K_b)},$$

$$\delta = \frac{P_g Y_g - (P_g C_g + 0.3T)}{0.3(K_g + K_b)}.\quad (75)$$

## 4 The IRFs with endogenous GQE

In this subsection, we try to address the scenario in which the Green Quantitative Easing (GQE) is endogenous and enters into the government's budget constraint :

$$\begin{aligned} & T_t + R_{t-1}^B B_{t-1} + \tau_{c,t} P_{g,t} C_{g,t} + GQE_t \\ & = \varpi W_{g,t} H_{g,t} + \varpi W_{b,t} H_{b,t} + \tau_{e,t} e_t + \tau_{b,t} (R_{b,t}^l - 1) K_{b,t-1} + B_t \end{aligned} \tag{76}$$

Figure 1 shows the results. The light green curves represent IRFs of endogenous GQE. With endogenous GQE, the inflationary effects become even more evident, with a strong rise of relative price for green goods  $P_{g,t}$ . Consequently, the consumption of green goods  $C_{g,t}$  decreases, and the  $CO_2$  emissions rise.

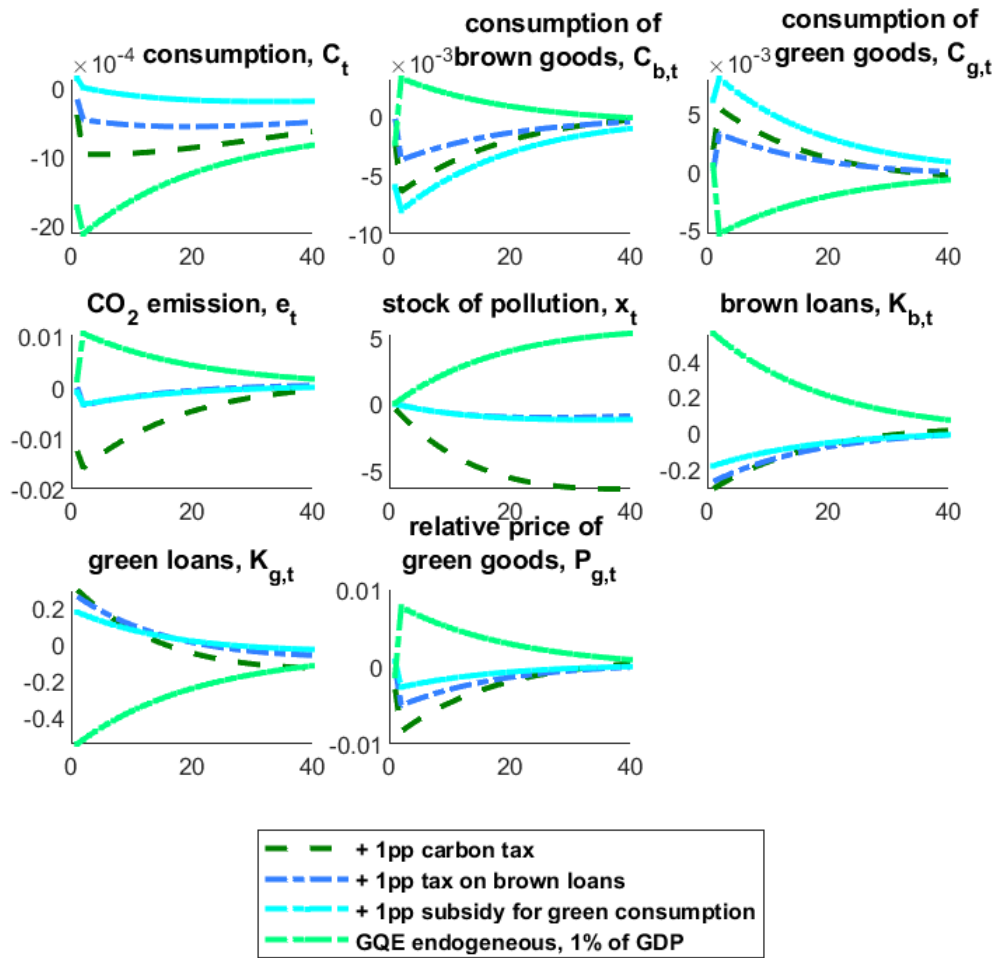


Figure 1: Comparison among GQE (exogeneous and endogeneous), carbon tax, tax on brown loans and subsidy for green consumption. The responses are quarterly log-deviations from the variables' steady state values.

## References

Dib, Ali, "Banks, credit market frictions, and business cycles," *Bank of Canada working paper 2010-24*, 2010.