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► **To cite this version:**

Séverine Rosat, Nicolas Gillet. Intradecadal variations in length of day: Coherence with models of the Earth's core dynamics. *Physics of the Earth and Planetary Interiors*, 2023, 341, pp.107053. 10.1016/j.pepi.2023.107053 . hal-04126324

HAL Id: hal-04126324

<https://hal.science/hal-04126324v1>

Submitted on 13 Jun 2023

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Highlights

Intradecadal variations in length of day: coherence with models of the Earth's core dynamics

S. Rosat, N. Gillet

- After zonal tides removal, the atmosphere and the core are the main contributors to intradecadal changes in length-of-day.
- Smaller contributions of continental and ocean water mass redistributions are mostly compensated by sea level variations.
- Three periodic oscillations have been highlighted in LOD observations at about 6, 7 and 8.5 years.
- The atmospheric contribution is subdominant at periods longer than 5 yr.
- Coherence with core flow models suggest a core origin for the 6 and 8.5-yr oscillations.
- Our synthetic tests suggest that the previously documented 7-yr line could result from an artefact.
- Two coherent oscillations around 3.5 and 5-yrs are conspicuous in the observed LOD and core-flow predicted LOD.
- Torsional Alfvén waves or QG-MC waves are natural explanations to the presence of various interannual signals in the LOD.

Intradecadal variations in length of day: coherence with models of the Earth's core dynamics

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Abstract

We confirm the presence of interannual oscillations of period about 5.9 and 8.5 years in the Earth's length of day (LOD), better visible after subtracting the atmospheric contribution. Continental water mass redistribution and oceanic angular momentum contribute to a lesser extent, and are furthermore mostly compensated by sea level variations. We show, using a continuous wavelet transform analysis of synthetic oscillators embedded into a random correlated noise, the limits of isolating damped signals presenting nearby periods in time series of limited duration. In particular, we emphasize the possibility that a previously documented 7.3-year oscillation could be an artefact associated with the restricted available time span covered by LOD data. Finally, we perform a wavelet coherence analysis between geodetically observed LOD changes and their prediction from geomagnetically inferred core flow models. It confirms an origin from the fluid outer core for the two signals around 5.9 and 8.5 years. A convincing coherence is also found around 3.5 and 5 years, which may reveal signatures of higher harmonics of torsional modes or of quasi-geostrophic magneto-Coriolis modes.

Keywords: length of day, core flows, damped oscillators, wavelet coherence

1. Introduction

2 In the recent years, several studies have highlighted the presence of interannual peri-
3 odic signals in the variations of length of day (LOD). An oscillation of period close to 6
4 years, hereafter referred as SYO, first detected by Abarca del Rio et al. (2000), has been
5 admitted to originate from the core, in particular given its correlation with the occur-
6 rence of geomagnetic jerks, or abrupt changes in the rate of change of the magnetic field
7 (Holme and De Viron, 2013), and its convincing prediction by core flows inverted from
8 independent magnetic observations (Gillet et al., 2010). The analysis of such motions
9 have led Gillet et al. to propose torsional Alfvén waves propagating in the fluid outer core
10 as a possible physical mechanism behind the SYO in the LOD signal. These constitute a
11 natural oscillatory response within the rapidly rotating spherical core permeated by the

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12 geomagnetic field (Braginskiy, 1970). Organized in cylinders whose axis is aligned with
13 the Earth’s rotation vector, such axisymmetric motions can carry angular momentum
14 in the presence of a non-zero core-mantle coupling (Jault and Finlay, 2015), and thus
15 possibly generate a signal in the LOD series. Several mechanisms have been proposed for
16 the coupling, amongst which a gravitational torque involving a non-spherical inner core
17 and mass anomalies in the mantle (Mound and Buffett, 2006; Chao, 2017), or an electro-
18 magnetic coupling due to a finite conductance of the lowermost mantle (e.g., Jault, 2003;
19 Schaeffer and Jault, 2016).

20 The SYO is best seen after removing the contribution from the atmospheric angular
21 momentum (Abarca del Rio et al., 2000; Duan et al., 2015). Its stability over time has
22 been debated (Holme and De Viron, 2013; Chao et al., 2014; Gillet et al., 2015), in link
23 with the finite duration of accurate LOD series (preventing from precisely characterizing
24 modulations, or dissociating several possible spectral lines around the 6-yr period). In
25 more recent years, thank to the increasing length of the LOD series, several studies have
26 highlighted the presence of distinct oscillations besides the SYO, at periods around 8.6-
27 year (Duan and Huang, 2020) and 7.3-year (Hsu et al., 2021). The 8.6-year signal is
28 hereafter referred to as the EYO (Eight-Year Oscillation).

29 The SYO and EYO are convincingly predicted by recent core flow models inferred
30 from magnetic data (Istas et al., 2023). Torsional modes may be natural candidates, in
31 particular as their periods are not necessarily integer fraction of the fundamental mode
32 (Gillet et al., 2017). With the recent discovery of quasi-geostrophic Magneto-Coriolis
33 modes at interannual periods (Gillet et al., 2022), an alternative possibility arises. These
34 motions are proposed to be at the source of interannual magnetic field changes high-
35 lighted from satellite records over the past two decades (Chulliat and Maus, 2014; Finlay
36 et al., 2016). Predominantly non-axisymmetric, they may nevertheless contain a zonal
37 contribution and thus carry angular momentum (Istas et al., 2023). This would provide
38 an explanation for the concomitance found between jerks and LOD variations (Holme
39 and De Viron, 2013; Duan and Huang, 2020), also observed in geodynamo simulations
40 approaching Earth-like conditions (Aubert and Finlay, 2019; Aubert et al., 2022).

41 For the last signal at 7.3-year period, no mechanism has been discussed so far. The
42 above three signals have been described as damped oscillators, with associated decaying
43 or increasing times longer than the period by about one order of magnitude (about 100
44 and 60 years for respectively the SYO and EYO, see Duan and Huang, 2020). Meanwhile,
45 the finite length of the available series raises the question of the frequency resolution. The
46 time span covered by accurate VLBI data (started in 1962) is today $\Delta T \simeq 60$ years. Fol-
47 lowing the Rayleigh criterion for undamped harmonic oscillations (Godin, 1972), around
48 a given period T_0 the resolution in period is $\delta T \approx 2T_0^2/\Delta T$. For $T_0 = 5.9$ years, it gives
49 $\delta T \approx 1.2$ year: if we can today convincingly distinguish the SYO from the EYO, the
50 7.3-year signal marginally separates from the latter two, and the question of its resolution
51 can be raised.

52 In the present study we show, using stochastic damped harmonic oscillators embedded
53 into a random correlated noise, the limits of detecting and isolating quasi-periodic signals
54 with nearby frequencies in series of limited duration. We employ a continuous wavelet
55 transform (CWT) method, which is best suited for the analysis of transient signals.
56 The CWT is then applied to geodetically observed LOD data, and to geomagnetically
57 predicted LOD variations from core flow models. A wavelet coherence analysis is finally
58 applied to estimate the coherency and phase lag between the predicted and observed

59 oscillations. We show conclusive coherences for the SYO and EYO. Higher frequency
60 coherent signals, of weaker amplitude, are also conspicuous around 3.5 and 5 years. We
61 finally conclude that the core flow variations can explain the SYO and EYO found in
62 LOD observations.

63 2. Description of geophysical data and methods

64 2.1. Geodetic LOD data and surface geophysical fluid products

65 LOD data considered here result from the concatenation of the Lunar97 series (Gross,
66 2001) prior to 1962, together with the C04 series (Bizouard and Gambis, 2009) from 1962
67 onwards. This latter is provided by the International Earth Rotation and Reference
68 System Service (IERS). It has been first cleaned for solid zonal tides (following IERS
69 Conventions, 2010). For their combination, Lunar97 and C04 have been shifted in such a
70 way that they share the same average over the common era 1962–1997. The LOD series
71 have been furthermore corrected for contributions from atmospheric angular momentum
72 using NCEP/NCAR model (Barnes et al., 1983) from the IERS. An observed 1.4 ms/cy
73 centennial trend (Stephenson et al., 1984) was fitted and removed for the time-span
74 considered. Surface fluid layers angular momentum functions are provided since 1976
75 by the Earth System Modelling Data at GFZ (Dobslaw et al., 2010), and from the
76 IERS since 1962. For time span between 1948 and 1976, only atmospheric correction
77 can be performed using NCEP/NCAR model (Barnes et al., 1983). The ECMWF and
78 NCEP/NCAR models turn out to be in good agreement at interannual periods on their
79 common time-span with a standard deviation of their difference of 0.01 ms. The LOD
80 and the several contributions from external envelopes are illustrated in Fig. 1(a), starting
81 from 1976.

82 The effective angular momentum functions due to mass transport processes in at-
83 mosphere (AAM – atmospheric angular momentum), oceans (OAM – oceanic angular
84 momentum), and the continental hydrosphere (HAM – hydrological angular momentum)
85 are provided with a temporal resolution of down to 3 hours and include the matter and
86 motion terms. OAM is extracted from the ECCO reanalysis (Estimating the Circulation
87 and Climate of the Ocean) by JPL between 1962 and 1976 (Wunsch et al., 2009), and
88 from the Max-Planck-Institute for Meteorology Ocean Model (MPIOM, Jungclaus et al.,
89 2013). HAM is provided from the Land Surface Discharge Model (LSDM, Dill, 2008).

90 The consideration of the global mass balance effects is important for seasonal varia-
91 tions in LOD (Yan and Chao, 2012). The barystatic sea level angular momentum (SIAM)
92 is computed by globally integrating masses in ECMWF and LSDM so that the total mass
93 in atmosphere, oceans, and all land storages should be constant at any given time. Even-
94 tually missing mass is further assumed to be stored in the oceans only so any net-inflow of
95 water masses into the oceans will cause a barystatic increase in ocean mass (Dobslaw and
96 Dill, 2018). We observe that barystatic sea level changes nearly compensate with HAM
97 and OAM at interannual time scales (see Fig. 1(b)). Besides, they are available only after
98 1976. Consequently, we removed only NCEP AAM from LOD changes (Fig. 1(c)) in the
99 following analyses, keeping in mind that HAM may contain some remaining unbalanced
100 contributions at interannual periods.

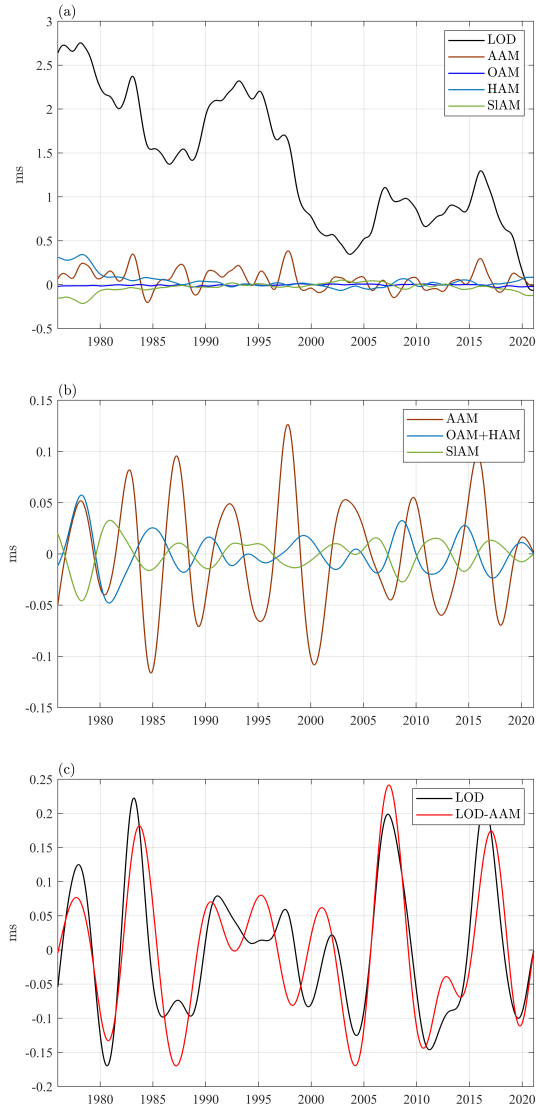


Figure 1: Time series of C04 LOD data, ECMWF atmospheric (AAM), ECCO+MPIOM oceanic (OAM), LSDM hydrologic (HAM) and LSDM-ECMWF sea-level (SIAM) angular momentum functions (1976-2020). (a) Low-pass filtered at 2 year period; (b) AAM, OAM+HAM and SIAM band-pass filtered between 4 and 10 year periods; (c) LOD and LOD-AAM, band-pass filtered between 4 and 10 year periods.

101 *2.2. Synthetic time-series from auto-regressive processes*

102 *2.2.1. Synthetic damped oscillators and background series*

103 For the generation of synthetic series, we consider a set of damped oscillators $y_i(t)$
 104 embedded into a background $y_b(t)$ that mimics the long period changes, plus a correlated
 105 noise $y_n(t)$. All are obtained by the integration of stochastic differential equations, as
 106 described below. We use linear auto-regressive (AR) processes of order 1 (for the noise)
 107 or 2 (for the oscillators and the background). The former (resp. latter) have a spectral
 108 density which evolve as $S(f) \propto f^{-2}$ (resp. f^{-4}) towards high frequencies. For a general
 109 description of such processes, we refer to Yaglom (1962).

110 A linear AR-2 process $y(t)$ results from an equation of the form

$$d \frac{dy}{dt} + 2\chi dy + \omega^2 y dt = d\zeta, \quad (1)$$

111 with ζ a Wiener process, and (ω, χ) two frequencies. For $\chi^2 < \omega^2$, $y(t)$ takes the form of
 112 a damped oscillator, with a covariance function of the form

$$\rho(\tau) = \sigma^2 \exp(-\chi|\tau|) \left[\cos(\beta\tau) + \frac{\chi}{\beta} \sin(\beta|\tau|) \right], \quad (2)$$

113 where τ is the time lag, σ^2 the variance of the process and $\beta = \sqrt{\omega^2 - \chi^2}$ the pulsation
 114 of the oscillator. When this latter is much higher than the decay rate χ , or $\omega \gg \chi$, one
 115 has $\beta \approx \omega$ and

$$\rho(\tau) \approx \sigma^2 \exp(-\chi|\tau|) \cos(\omega\tau). \quad (3)$$

116 The quality factor of the process,

$$Q = \omega/(2\chi) = \frac{\pi\tau}{T}, \quad (4)$$

117 is a measure of the number of periods $T = 2\pi/\omega$ per decay time $\tau = 1/\chi$. The several
 118 oscillators $y_i(t)$ are each characterized by three parameters: their variance σ_i^2 , frequency
 119 ω_i and decay rate χ_i . The associated power spectral densities (PSD) are

$$S_i(f) = \frac{4\chi_i\omega_i^2\sigma_i^2}{(\omega_i^2 - (2\pi f)^2)^2 + (4\pi\chi_i f)^2}, \quad (5)$$

120 with f the frequency.

121 The background $y_b(t)$ is also modelled with a process defined by Eq. (1), but in the
 122 case where $\omega = \chi$. Then it is only defined by two parameters: its variance σ_b^2 and a
 123 cut-off frequency ω_b . Its correlation function is

$$\rho_b(\tau) = \sigma_b^2 (1 + \omega_b|\tau|) \exp(-\omega_b|\tau|), \quad (6)$$

124 and its PSD is

$$S_b(f) = \frac{4\omega_b^3\sigma_b^2}{(\omega_b^2 + (2\pi f)^2)^2}. \quad (7)$$

125 This choice is made to mimic the behavior of the long period LOD signal (see Fig. 1(a)).

126 Finally the correlated noise is modeled through an AR-1 process, of the form

$$dy + \omega_n y dt = d\zeta. \quad (8)$$

127 It only depends on two parameters, a variance σ_n^2 and a restoring frequency ω_n , which
128 define its correlation function

$$\rho_n(\tau) = \sigma_n^2 \exp(-\omega_n |\tau|). \quad (9)$$

129 Sampled at a frequency $f \gg \omega_n$, the process behaves like a random walk, whereas for
130 $f \ll \omega_n$ it is like a white noise. Such processes are characterized by a PSD of the form

$$S_n(f) = \frac{2\omega_n \sigma_n^2}{\omega_n^2 + (2\pi f)^2}. \quad (10)$$

131 This choice is made to mimic the high frequency behavior of the LOD once cleaned for
132 the contributions from the atmosphere and the solid tides.

133 We shall consider below synthetic series made of the sum of 3 damped oscillators, a
134 background and a noise. This configuration (together with the parameters for each of
135 the processes), is chosen in order to approximately replicate the spectral content of the
136 observed LOD as measured with geodetic data once cleaned for the atmospheric contribu-
137 tion (see Fig 2). The favored parameters are the following: $(T_1, T_2, T_3) = (6, 8.5, 17)$ yr,
138 $(Q_1, Q_2, Q_3) = (20\pi, 20\pi, \pi)$, and $(\sigma_1, \sigma_2, \sigma_3) = (0.1, 0.08, 0.4)$ ms for the three damped
139 oscillators, $\omega_b = 1/15 \text{ yr}^{-1}$ and $\sigma_b = 1$ ms for the background, $\tau_n = 1$ yr and $\sigma_n = 0.04$ ms
140 for the noise. We acknowledge that there is some flexibility in the above parameters (in
141 particular concerning τ_n or ω_b). We consider a single oscillator at decadal periods for the
142 sake of simplicity, even though there may exist more than one (e.g., Chao et al., 2014).
143 However, we attempt here at generating a set-up as simple as possible, still being repre-
144 sentative of the PSD witnessed with observed series. The superposition of the PSD for
145 synthetic and geodetic data from 60 yr long series does not call for a larger complexity.

146 The theoretical PSD for the several synthetic contributions is shown in Fig. 3 (top)
147 for the above choice of parameters. It clearly illustrates the domination of the noise
148 towards short periods, of the background towards long periods, while the sharpness
149 of the three oscillators is much more clear than on the PSD obtained with 60 yr long
150 series. One example of synthetic series, with the parameters chosen above, is provided in
151 Fig. 3. In practice, we generate the discrete synthetic series over a time interval ΔT and
152 with a sampling period δt . These are stored in vectors $\mathbf{y}_i, \mathbf{y}_b, \mathbf{y}_n$ for the oscillators, the
153 background and the noise, of size $N = \Delta T/\delta t + 1$. To this purpose we calculate from the
154 function (2) the covariance matrix $\mathbf{C}_i = \mathbb{E}(\mathbf{y}_i \mathbf{y}_i^T)$, where $\mathbb{E}(\dots)$ means the statistical
155 expectation. The processes are assumed to have a priori a zero mean. Random discrete
156 series are then obtained by multiplying the Choleski decomposition \mathbf{L}_i of $\mathbf{C}_i = \mathbf{L}_i \mathbf{L}_i^T$ with
157 \mathbf{w} a random Gaussian vector of size N , unit variance and zero mean. The same process
158 is used to obtain \mathbf{y}_b and \mathbf{y}_n from Eq. (6) and (9), respectively.

159 2.2.2. Prior insights from continuous wavelet transform

160 Duan et al. (2015) have extracted the SYO from the LOD series between 1962 and
161 2012, using a normal Morlet wavelet method (Morlet et al., 1982; Liu et al., 2007). Then,
162 using the same wavelet method, Duan et al. (2017) estimated the damping rate of the

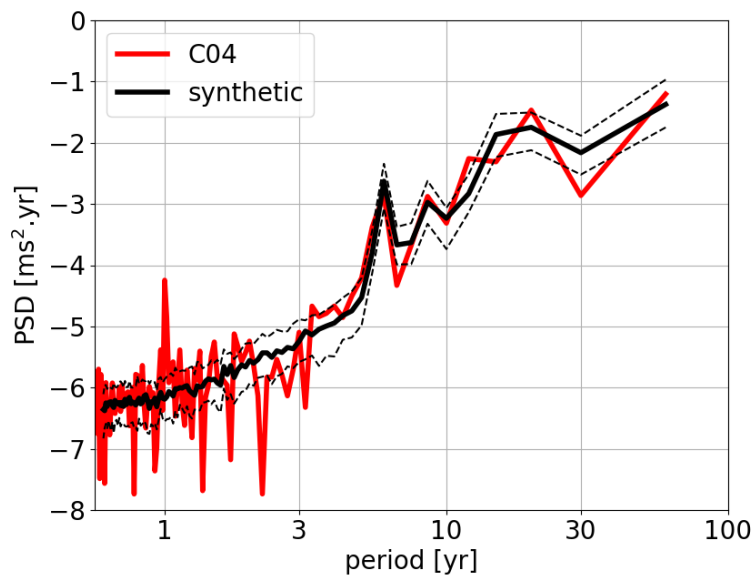


Figure 2: In red the PSD of the C04 series (1962–2022) cleaned for NCEP AAM and solid tides (IERS Conventions, 2010). It is superimposed with the median PSD (in black) within an ensemble of 100 synthetic stochastic series with the same 60 yr duration. The dotted black lines indicate the 25% and 75% quantiles within the ensemble of synthetic series. Parameters of the stochastic processes have been chosen so as to match the LOD data PSD with that of the median synthetic series (see text for details). The PSD is computed after applying a Kaiser window on each of the series, with shape parameter $\beta = 2.5$.

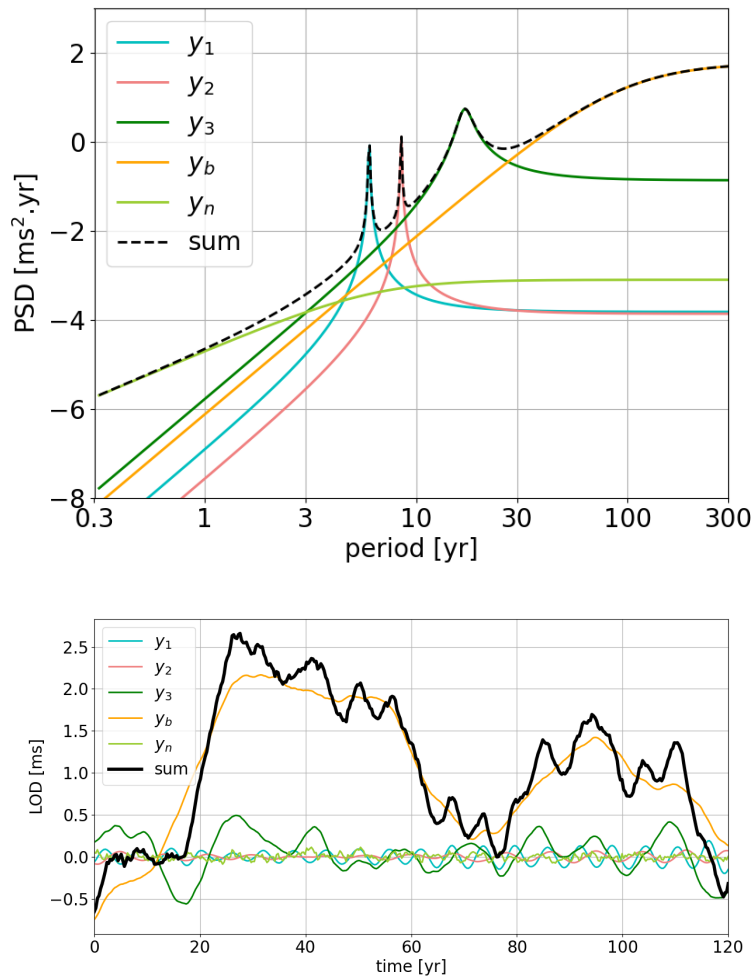


Figure 3: Analytical PSD (top) and one example of synthetic stochastic series (bottom) corresponding to the set up favored from Fig. 2. The several contributions from the 3 damped oscillator ($y_1 \dots 3$, of periods 6, 8.5 and 17 yr), the noise and background have been isolated (see text for details).

163 SYO with a quality factor of $Q \sim 50$ (a decay time of about 100 yr) by least-squares
 164 fitting a decaying sinusoidal function in the time domain. Duan and Huang (2020)
 165 highlighted, on top of a SYO, the presence of a EYO presenting a good concordance
 166 with the occurrence of geomagnetic jerks. They however obtained an increasing signal by
 167 fitting an exponential function in time-domain with a rate of about 1/60 yr (or $Q \approx 20$).
 168 They provided values for the average amplitudes of the SYO and EYO, at respectively
 169 ~ 0.12 and ~ 0.08 ms over 1962–2018. Hsu et al. (2021) have proposed a frequency
 170 domain stepwise regression method to identify and characterize periodic signals. They
 171 have tested their method on synthetic data composed of three oscillators at 5.9, 7.3
 172 and 8.5-year periods with respective amplitudes 0.12, 0.05 and 0.08 ms. They have also
 173 tested the case of varying amplitudes with a quality-factor of $Q \sim 50$ for the SYO and
 174 an increasing amplitude for the EYO, while the 7.3-year oscillation was kept steady. For
 175 some tests, these oscillations were buried in a white noise of standard deviation 0.025 ms.
 176 They have employed their method on actual LOD data, and proposed the existence of
 177 three components at 5.9, 7.3 and 8.5-year periods. They further used the normal Morlet
 178 wavelet transformation to check the consistency with their protocole.

179 With short series, estimating Q from a PSD is subject to uncertainties (see Fig. 2).
 180 In this context, values of Q proposed so far from the analysis of LOD records have been
 181 obtained by fitting analytical functions to the observed series, after filtering. We illus-
 182 trate below that approaching Q this way might lead to biased estimates. We generated
 183 1000 noise-free series $y(t)$ for a single damped oscillator of frequency f^* , amplitude A^* ,
 184 and quality factor Q^* , over a duration $\Delta T = 7/f^*$. This time-span is similar to the
 185 configuration of a 8.5 yr signal recorded over 60 yr of geodetic LOD data, and has been
 186 previously considered short enough for the oscillator to present a behavior characteristic
 187 of its theoretical growth or decay (e.g. Duan et al., 2017; Duan and Huang, 2020). We
 188 consider below two values $Q^* = 20\pi$ and $Q^* = 40\pi$. We fitted the synthetic series with
 189 functions of the form

$$f(t) = A \exp(\alpha t) \cos(2\pi f(t - t_0)) . \quad (11)$$

190 Adjustable parameters are the amplitude A and frequency f of the oscillator, its phase
 191 (through t_0) and its decay (or growth) time α . The recovered quality factor is then
 192 $Q = \pi f / |\alpha|$. In this relatively simple configuration, the fit allows to retrieve accurately
 193 the frequency (with $\pm 1\%$), while the amplitude is over-estimated by $\approx 20\%$ (from the
 194 median value, see Table 1). Meanwhile, the inferred Q significantly under-estimates the
 195 reference Q^* , and this mis-estimation is stronger for larger values of Q^* ; in the case
 196 where $Q^* = 40\pi$ the targeted value is only marginally included within the first three
 197 quantiles. In many occasions, the sampled oscillator is not in a period characteristic
 198 of the expected growth/decay. For the synthetic tests presented in our study, we thus
 199 considered instead as geophysically representative empirical series obtained with values
 200 of Q producing PSD statistically comparable to the PSD of the observed LOD series (see
 201 Fig. 2). In comparison with previous studies, we consider here relatively larger values of
 202 Q (which is coherent with Table 1), in particular for the EYO.

203 2.2.3. Analysis tools

204 In the following, the influence of the long-period background is reduced using a high-
 205 pass filtering with a cut-off period at 10-year. The time-frequency analysis is based on
 206 the CWT using an elongated Morlet wavelet. The wavelet equation is written in the

Q^*	A/A^*			f/f^*			Q/Q^*		
20π	0.70	1.19	1.73	0.99	1.00	1.01	0.37	0.70	1.51
40π	0.79	1.21	1.68	0.99	1.00	1.01	0.24	0.48	1.05

Table 1: Evaluation of the exponential fit for the recovery of the parameters of a single damped oscillator of amplitude A^* , frequency f^* , and quality factor Q^* , for two values of Q^* . We considered a set of 1000 series, covering a time-span of $7/f^*$. The three values in each case represent the 1st quantile, the median and the third quantile within the ensemble of series.

207 frequency domain as

$$h(af > 0) = 2 \exp\left(-4\sigma^2\pi^2\frac{(af - s_0)^2}{2}\right), \quad (12)$$

208 where s_0 is the frequency of the mother wavelet (with $s_0 = 1$ Hz). σ is an elongation
209 parameter that enables to improve the spectral resolution of the Morlet wavelet, resulting
210 in a better separation of close-by periodic oscillations (we chose $\sigma = 4$). In the meantime,
211 the temporal resolution is degraded when the spectral resolution is improved because
212 of Heisenberg’s uncertainty principle (or inequality of Gabor). The CWT transform is
213 performed in the Fourier domain following the method described for instance in Torrence
214 and Compo (1998). Contour lines shown below in the scalograms (Fig. 4, 5, 6, etc.)
215 correspond to the 95% confidence levels and the shaded areas represent the cones of
216 influence (masking the regions of the wavelet spectrum for which edge effects become
217 important).

218 2.3. Core flow models

219 The link between subdecadal changes in LOD data and core flows inverted from
220 geomagnetic field observations has been demonstrated using various algorithms for the
221 reconstruction of core motions (Gillet et al., 2015, 2019; Istas et al., 2023). Focusing
222 on band-pass filtered flows, Gillet et al. (2015) had not dissociated the SYO from the
223 EYO, presenting modulated interannual LOD series. Only recently Istas et al. (2023)
224 has shown that both signals, considered separately, are convincingly fitted by core flow
225 reconstructions. We consider below two core flow models, obtained using two versions of
226 the `pygeodyn` software (Huder et al., 2019) and two different kinds of geomagnetic data, as
227 detailed below. `pygeodyn` is a data assimilation tool for the reanalysis of the core surface
228 dynamics. It is based on an ensemble Kalman filter (Evensen, 2003), in which the forward
229 integration of the core flow model is operated using stochastic equations. The parameters
230 entering these empirical equations are extracted from the spatio-temporal statistics of a
231 geodynamo simulation (which solved for the primitive momentum, induction and heat
232 equations). For both considered models, the prior statistics have been extracted from
233 the 71%-path dynamo (Aubert and Gillet, 2021). Subgrid induction and diffusion in the
234 model are accounted for via an augmented state approach.

235 A first model has been obtained from observations that consist of geomagnetic field
236 Gauss coefficients from the CHAOS-7 geomagnetic field model (Finlay et al., 2020) over
237 1999–2020, warmed-up prior to 1999 using the COV-OBS.x2 model (Huder et al., 2020),
238 starting in 1880. It consists in an ensemble of 50 realisations obtained using `pygeodyn`
239 with the algorithm described in Gillet et al. (2019). It correspond to a model published
240 by Gillet et al. (2022), hereafter referred to as ‘GGJx22’.

241 The second model (hereafter referred to as ‘IGFx23’) consists in an ensemble of 400
242 realizations covering the period 1880–2022. It uses an improved version of `pygeodyn`
243 where the “graphical Lasso” has been implemented for the estimation of the forecast
244 covariance matrix that enters the Kalman filter (Istas et al., 2023). For this reanalysis,
245 observations consist of ground records (annual means, since 1997), virtual observatory
246 data since 2000 (see Hammer et al., 2021b,a), and prior to 1997 of Gauss coefficient series
247 from the COV-OBS.x2 model (Huder et al., 2020). Virtual observatory data considered
248 here are annual means from the CHAMP, Oersted and Cryosat-2 missions, and quarter-
249 monthly values from Swarm (after 2014).

250 Both reanalyzed core flow series have been interpolated using a 0.2 year sampling.

251 3. Results

252 3.1. Geophysical data analysis

253 Among surface envelopes, the atmosphere possesses the largest contribution to LOD
254 changes. Then come the hydrology and oceans signals (see Fig. 1 and Fig. 4). While
255 the AAM shows enhanced energy at periods shorter than 6 years, the hydrology exhibits
256 a broader frequency content in the period range between 2 and 10 years (Fig. 4). We
257 also observe that HAM excitation function has a small but non-negligible contribution to
258 the interannual LOD fluctuations (Fig. 4(c)). Continental hydrology presents an isolated
259 contribution, with an amplitude of 0.01 ms, at a period close to the 7.3-yr documented
260 by Hsu et al. (2021) for LOD data (though of much weaker magnitude).

261 As expected, subtracting the atmospheric angular momentum contribution consider-
262 ably reduces the spectral content of geodetic LOD series at periods shorter than 5 years,
263 while it magnifies the EYO and SYO (see Fig. 1(c) and Fig. 5). This suggests that LOD
264 and AAM are predominantly anti-correlated at these periods (as noted already by Requier
265 et al., 2022), i.e. the atmosphere and the mantle overall rotate in the same direction at
266 these time-scales. As stated in Section 2.1, OAM and HAM are mostly compensated
267 by SIAM. Consequently, the signal seen in HAM around 7.3 yr is not retrieved when
268 considering the LOD series corrected for the AAM only. We recall that the AAM is the
269 only surface correction that is applied in the following.

270 3.2. Uncertainties of geodetic LOD series and geophysical products

271 Assessing uncertainties on LOD series and geophysical products is a difficult task,
272 furthermore when searching for errorbars at specific frequencies. Here we consider the
273 dispersion between time-series and products to provide an estimate of the uncertainty
274 level. Several LOD series and two atmospheric models are available from the IERS web-
275 site. The uncertainty σ related to the AAM reconstruction is computed as the standard
276 deviation obtained by integrating, between 4 and 9.5 years, the Power Spectral Density
277 of the time series residuals. We obtain a value of $\sigma = 0.01$ ms between ECMWF and
278 NCEP/NCAR AAM products. The paired differences between various LOD series are
279 summarized in Table 2. As for the two AAM models, we have computed the variance
280 by integrating, between 4 and 9.5 years, the PSD of the difference on the common time-
281 span. It thus encompasses the periods of both the SYO and the EYO. We also provide
282 for comparison the variance computed in the same way for the individual time-series.

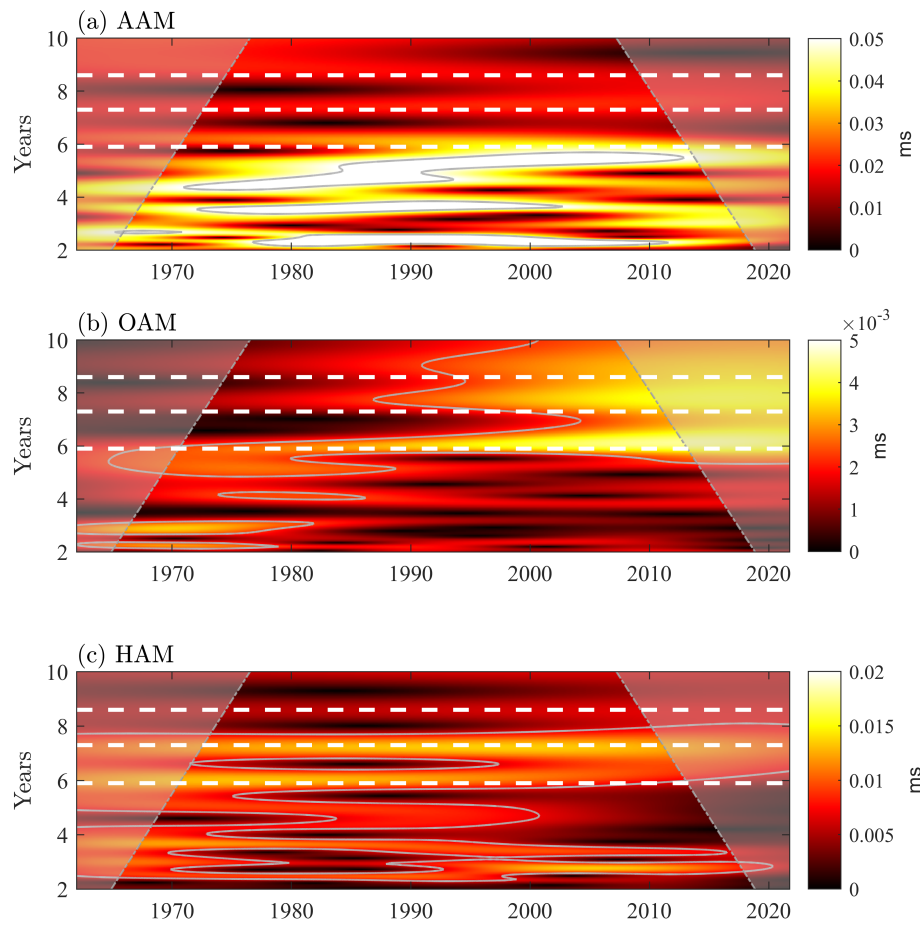


Figure 4: Wavelet transform coefficients of (a) NCEP atmospheric, (b) ECCO+MPIOM oceanic and (c) LSDM hydrologic angular momentum functions (1962-2021). The white dashed lines correspond to the three periods of 5.9, 7.3 and 8.5 yr. Thin white lines represent the 95% confidence level.

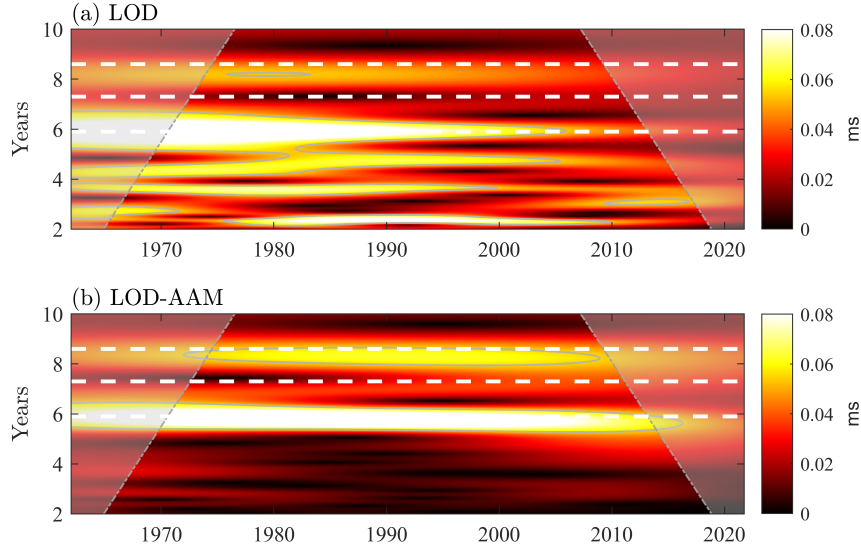


Figure 5: Wavelet transform coefficients of C04 LOD variations (a) only zonal tides were corrected, (b) NCEP atmospheric angular momentum contribution was further subtracted (1962-2021).

283 We consider the series C01 (IERS), C04 (IERS) and Space (JPL) (defined in the
284 ITRF2020; Altamimi et al. (2023)) that combine various space techniques: VLBI (Very
285 Long Baseline Interferometry), SLR (Satellite Laser Ranging) and GNSS (Global Navigation
286 Satellite Systems). On top of this, the C04 series incorporate Doppler orbitography
287 by radiopositioning integrated on satellite (DORIS) since 2015 (Bizouard et al., 2019),
288 while the Space series consider LLR (Lunar Laser ranging) data (Ratcliff and Gross,
289 2022). However, the C04 series results mostly from a combined smoothing of LOD GNSS
290 data and UT1 VLBI data. The input of SLR and DORIS LOD values in that combined
291 solution is negligible in reason of their larger uncertainties. We also compare with al-
292 ternative series, each obtained from a specific technique: the IVS series from VLBI and
293 the JPL series from GNSS. LOD from GNSS provides the smallest differences with other
294 series because of their highest sampling of the Earth’s rotation. As expected, combined
295 series C01 and C04 are overall consistent to each other, with a difference coming from
296 a distinct sampling interval (C01 being designed for long-term studies). Larger differ-
297 ences are obtained between IERS C01/C04 and the JPL Space series with $\sigma = 0.12$ ms,
298 only slightly weaker than the r.m.s. of the interannual LOD signal. This estimate is in
299 agreement with previously observed differences (Ratcliff and Gross, 2022), which can be
300 attributed to the following: the C01/C04 series adopt a weighted-average approach to
301 combine geodetic data (Bizouard et al., 2019) while the Space series is based on a Kalman
302 filter, accounting for the growth in the uncertainty of the Earth orientation parameters
303 between measurements (Gross, 2015). Combining various measurement methods is nec-
304 essary to reduce as much as possible possible technical bias in the estimation of the
305 Earth’s rotation parameters. Nevertheless, the difference between C01/C04 and Space is

	C01 (IERS) 1846-2023	C04 (IERS) 1962-2023	Space (JPL) 1993-2023	GNSS (JPL) 1996-2023	VLBI (IVS) 1984-2020
C01 (IERS)	8×10^{-2}	1×10^{-3}	0.12	5×10^{-3}	1×10^{-2}
C04 (IERS)		0.17	0.12	6×10^{-3}	1×10^{-2}
Space (JPL)			0.25	7×10^{-2}	0.11
GNSS (JPL)				0.21	7×10^{-3}
VLBI (IVS)					0.20

Table 2: Standard deviations σ (in ms) of the paired differences between astro-geodetic LOD series. σ is computed by integration between 4 and 9.5 years of the PSD of the difference between two time-series on the common time-span. The diagonal values (in bold) are the σ for the corresponding LOD series in the band 4-9.5 years computed by integrating the PSD of those series.

Case No.	Background ($y_3 + y_b$)	high-pass filtered at $T \leq 10$ yr	Oscillators	Figure No.
(1)	Yes	No	y_1	Fig. (6) and (A.3)
(1)	Yes	Yes	y_1	Fig. (A.1) (top)
(1)	No	No	y_1	Fig. (A.2) (top)
(2)	Yes	No	$y_1 + y_2$	Fig. (7)
(2)	Yes	Yes	$y_1 + y_2$	Fig. (A.1) (bottom)
(2)	No	No	$y_1 + y_2$	Fig. (A.2) (bottom)

Table 3: Summary of synthetic signals considered in the different cases, with reference to the corresponding figures. All tests were performed with the same noise properties (y_n), and using same time-span $\Delta T = 60$ yr.

306 not negligible in comparison with the SYO and EYO oscillations described above, of res-
307 pective amplitudes 0.1 and 0.08 ms. These later are however larger than the differences
308 between C01/C04 and GNSS or VLBI series between 4 and 9.5 years.

309 3.3. Synthetic data analysis

310 We have performed several runs on the synthetic series described in section (2.2.1),
311 using various durations ΔT of the time windows. We highlight below two main configu-
312 rations with $\Delta T = 60$ years, in the presence of the long-period background y_b , the noise
313 y_n and of the the 3rd oscillator of period 17 yr:

314 ~~C~~Case (1) one damped oscillator (y_1) of period 6-yr, amplitude 0.1 ms and quality factor
315 $Q = 20\pi$.

316 ~~C~~Case (2) two damped oscillators (y_1 and y_2) of periods 6 and 8.5-yr, with respective ampli-
317 tudes 0.1 and 0.08 ms, and presenting the same quality factor $Q = 20\pi$.

318 Further comparisons are presented in Appendix A, either after filtering the long period,
319 or in the absence of a background. The various configurations shown here are summarized
320 in the Table (3).

321 In the configuration of Case (1), we are able to detect the existing subdecadal os-
322 cillator above the confidence level. However, we also detect a parasitic oscillator (false
323 positive) over the whole considered time-span, at a period around 7.5 yr (see Fig. 6).

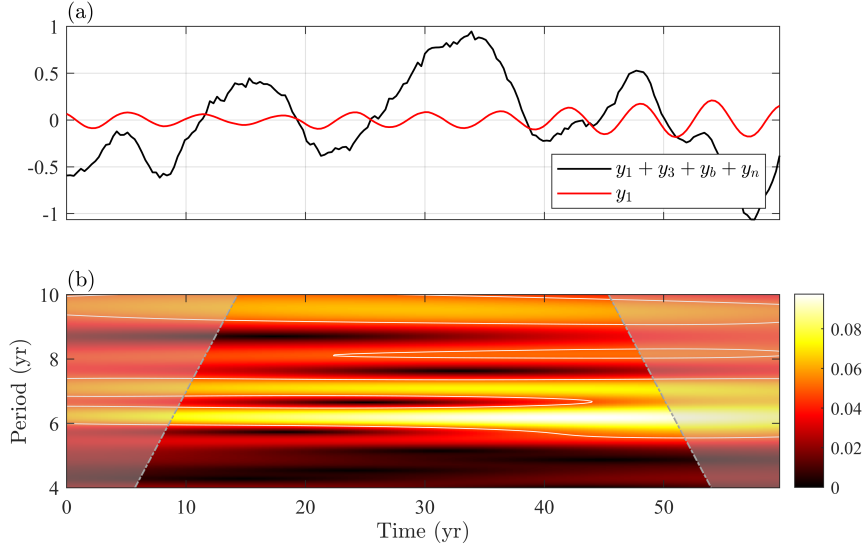


Figure 6: Synthetic signal of Case (1), composed of an auto-regressive stochastic background (y_b) plus two damped oscillators at 6 and 17-yr periods, with $Q = 20\pi$ and $Q = \pi$ (y_1 and y_3) embedded in colored noise (y_n). (a) Time series, (b) Continuous Wavelet Transform.

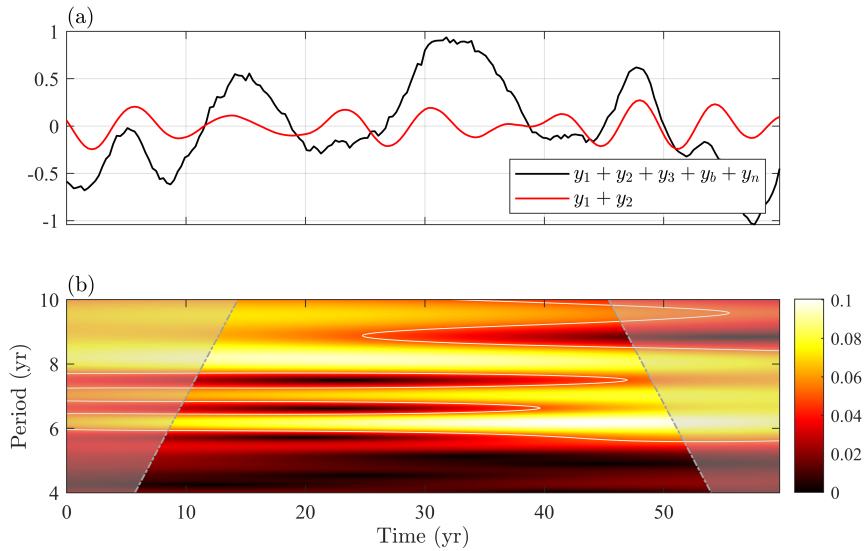


Figure 7: Synthetic signal of Case (2), composed of an auto-regressive stochastic background (y_b) plus three damped oscillators at 6, 8.5 and 17-yr periods embedded in colored noise (y_n). (a) Time series; (b) Continuous Wavelet Transform.

324 After filtering the low frequency content, the amplitude of the 7.5-year oscillation
 325 is reduced but still visible (Fig. A.1). When there is no background, the parasitic 7.5-
 326 year oscillation is not visible any more (Fig. A.2). We have considered one hundred
 327 runs for generating synthetic oscillators with random starts. Depending on the relative
 328 amplitudes of the 6-yr oscillations and the background at the considered epochs, the
 329 detection of a parasitic oscillation is more or less obvious (see Fig. A.3). Reducing the
 330 duration of the time window and/or decreasing the quality factor Q of the damped
 331 oscillator tend to favor the occurrence of false positives.

332 We now turn to the configuration of Case (2), when two damped oscillators are
 333 present at 6 and 8.5 years. We are able to isolate both of them above the confidence
 334 level. However, as in Case (1) a parasitic subdecadal oscillation is detected above the
 335 confidence level, at an intermediate period ≈ 7 yr (see Fig. 7). When the time-series are
 336 high-pass filtered, the parasitic 7-year oscillation is still detected although attenuated
 337 (Fig. A.1, bottom), as in Case (1). Here also, when in the absence of a background, we
 338 do not see any clear signal emerging around 7-yr periods (Fig. A.2, bottom).

339 Theoretically, with undamped harmonic signals, we would need $\Delta T > 2T_0^2/\delta T \simeq$
 340 60 years of data in order to separate a 5.9-yr oscillation from a 7.3-yr oscillation in
 341 spectral domain, and $\Delta T \gtrsim 100$ years to separate 8.6-yr and 7.3-yr oscillations, based
 342 on the Rayleigh criterion. Our synthetic tests with time duration of 60 years mimic the
 343 time duration of LOD records and only marginally respect this criterion for the 5.9 and
 344 and 7.3 yr signals. Considering damped oscillators (here with $Q = 20\pi$) renders more
 345 difficult the separation of the various spectral lines. In the considered synthetic case, the
 346 presence of a background (y_b) and of a decadal oscillator (y_3), both presenting reduced
 347 but non-zero power at the subdecadal periods of interest, favors the detection of false
 348 positive at a period between 6 and 8.5 yr.

349 3.4. Coherence analysis with core flows

350 3.4.1. Oscillations in the period range 4–10 years over 1880–2020

351 The wavelet coherence is a measure of the correlation between two signals in the
 352 time-scale (here time-frequency) domain (t, f) used for the CWT. If we denote $C_x(t, f)$
 353 and $C_y(t, f)$ the wavelet coefficients of the CWT of signals $x(t)$ and $y(t)$, then the wavelet
 354 coherence is

$$W_c(t, f) = \frac{C_x^*(t, f)C_y(t, f)}{|C_x(t, f)||C_y(t, f)|} = w_c(t, f) \exp(i\varphi_c(t, f)), \quad (13)$$

355 with the coherence module $w_c(t, f) \in [0, 1]$ and its phase $\varphi_c \in [-\pi, \pi]$. The superscript
 356 * denotes the complex conjugate. As for the CWT, we follow the method described in
 357 Grinsted et al. (2004) to implement the wavelet coherence analysis. Coherent signals
 358 are characterized by a large module. In the figures below, the phase information is
 359 represented via arrows: for two signals in phase, or $\varphi_c = 0$ (resp. opposition phase,
 360 $\varphi_c = \pi$), the arrow is oriented to the right (resp. left). For two signals in quadrature
 361 ($\varphi_c = \pi/2$) the arrow is oriented upward. In the following, $x(t)$ corresponds to observed
 362 LOD variations and $y(t)$ to its prediction from a core flow model. The phase (arrows)
 363 are shown only when $w_c \geq 0.3$.

364 Previous core flow studies (Gillet et al., 2019; Istas et al., 2023) have emphasized
 365 the cogent fit between observed and predicted LOD time series. Fig. 8 confronts the

366 two predictions from models GGJx22 and IGFx23 to geodetic records over 1880–2022.
 367 Despite the different ways of accounting for magnetic observations (see Section 2.3),
 368 decadal changes are well recovered (except for the early part of the series, when magnetic
 369 models are arguably less accurate). It is also the case for interannual variations since
 370 1940: both models overall convincingly predict both the phase and amplitude of the SYO
 371 and the EYO observed in LOD data. We pursue the confrontation of LOD data and core
 372 flow predictions, and show below the results of the wavelet analysis in the period range
 373 [4, 10] years over the time-span 1880–2020 covered by the flow models. In the scalograms
 374 of the LOD data (Fig. 9, top), the cleaning of the atmospheric signal after 1948 suppresses
 375 a significant part of the observed variability at periods shorter than ~ 5.5 years. The
 376 EYO is present starting from ~ 1900 , with of amplitude ~ 0.08 ms. Contrary to Hsu
 377 et al. (2021) and Duan and Huang (2020), our analysis does not reveal any obvious
 378 increase in the amplitude for the EYO. Signals of shorter period are also clearly there
 379 from the beginning of the XXth century, with periods ranging from 6 to 7 years and
 380 amplitudes similar to that of the EYO. As for the EYO, their signature is less at the
 381 beginning of the XXth century. Over the most recent epochs, the period shifts towards
 382 that of the SYO.

383 Despite confidence intervals that are less tightened in period, LOD predictions from
 384 both investigated flow models also show enhanced power around 8 and 6 years over
 385 the whole time-span, with amplitudes comparable to that of the observed signal – see
 386 Fig. 9 (b,c). For both considered models, the period of the predicted EYO is closer to
 387 8 years than 8.5 years. Its amplitude is comparable to the observed one, ~ 0.08 ms
 388 (resp. 0.06 ms) for model IGFx23 (resp. GGJx22). The predicted SYO appears more
 389 clear starting from ≈ 1940 (as previously noted by Gillet et al., 2019), with for both
 390 models an amplitude of about 0.08 ms, similar to the observed one. At periods shorter
 391 than ~ 5 years, the predicted LOD shows more power than the observed LOD cleaned
 392 from the AAM contribution (i.e., after 1948).

393 The coherence spectrum analysis presented in Fig. 10 provides additional information
 394 regarding the phase of the observed and predicted signals. For both flow models a strong
 395 coherence is found for the EYO over the whole time-span (apart from the edges effects),
 396 The associated phase presents a negative off-set, fairly constant and always less than $\pi/4$.
 397 Both models also show a high coherence around 6 years. There the phase shift is close
 398 to 0 for the model IGFx23 over the time-span 1930-2010. The period range presenting
 399 high coherence is less tightened (in particular in the 1960-70's) for the model GGJx22,
 400 for which a positive phase shift (less than $\pi/4$) appears from ~ 1980 .

401 It is worth notice that prior to 1940, a significant coherence is found at a period about
 402 5 years, with a relatively weak phase shift. It corresponds to a signal of amplitude less
 403 than 0.03 ms (see Fig. 9). This may suggest either changes in the periods of core signals,
 404 or decadal modulations of the signals originating from the core. In any case, at such
 405 short periods an influence from AAM cannot be disregarded.

406 3.4.2. Oscillations in the period range 2–5 years over 1948–2021

407 Motivated by the coherence found at periods shorter than 6 years at early epochs, we
 408 performed a dedicated coherence analysis over the more recent era, starting from 1948
 409 onward, when atmospheric reanalyses are available, allowing to clean LOD series from the
 410 AAM contribution. Coherent phases are found around 5 and 3.5 years periods between
 411 the IGFx23 model and the observations, with a phase shift close to zero (Fig. 12(a)).

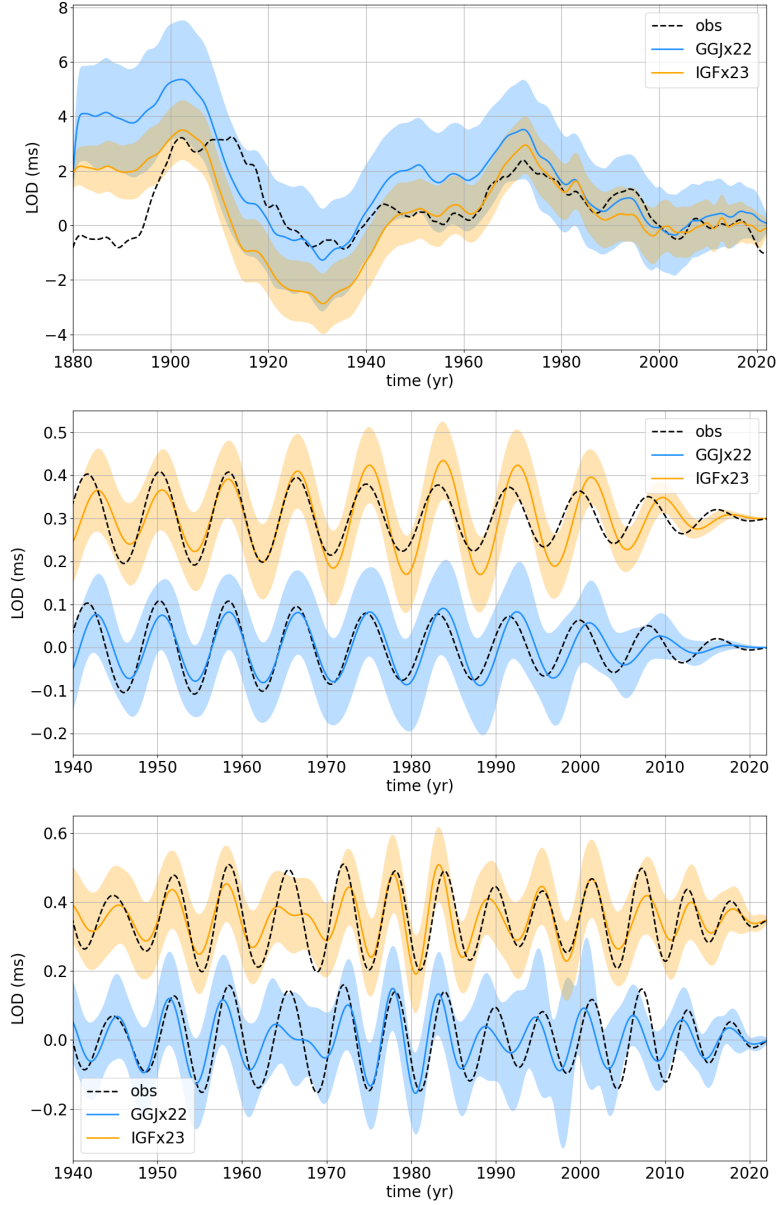


Figure 8: Observed C04 LOD variations (dashed black) and their prediction for the two core flow models IGF_x23 (orange) and GGJ_x22 (blue) – see text for details. Observations were corrected for zonal tides, and atmospheric angular momentum using NCEP model after 1948 only. top: low-pass at periods $T \geq 2$ years. middle: band-pass for periods $7.5 \leq T \leq 9.5$ years. bottom: band-pass for periods $4.5 \leq T \leq 7.5$ years. We applied causal Butterworth filters of order 2. Shaded areas represent the $\pm 1\sigma$ standard deviation within the ensemble of inverted flow models.

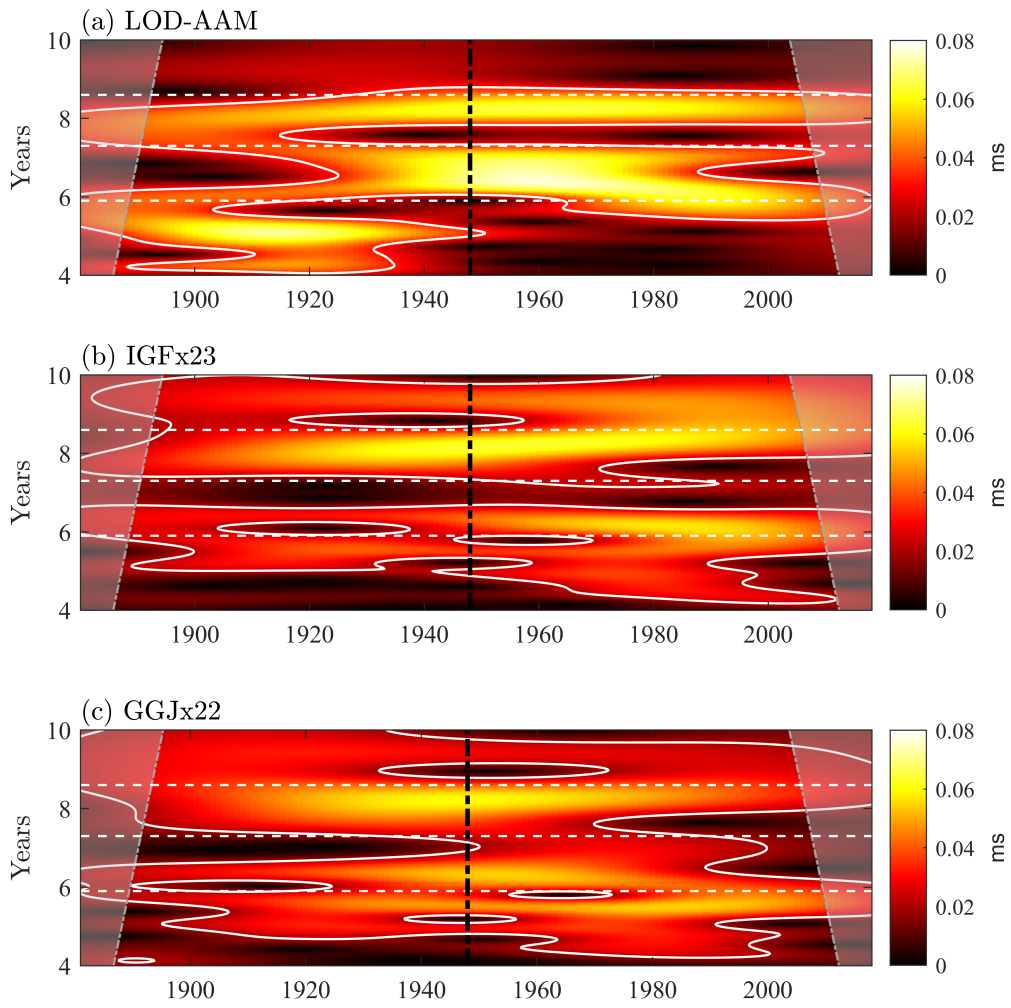


Figure 9: Wavelet analysis for the observed and predicted LOD variations between 1880 and 2021. AAM was corrected only after 1948 (vertical black dashed line) using NCEP model. Scalograms for the observed LOD (a), and the predicted LOD for the IGFx23 model (b) and the GGJx22 model (c). Horizontal white dashed lines correspond to the 5.9, 7.3 and 8.5-year periods. Thin white contour lines correspond to the 95% confidence levels.

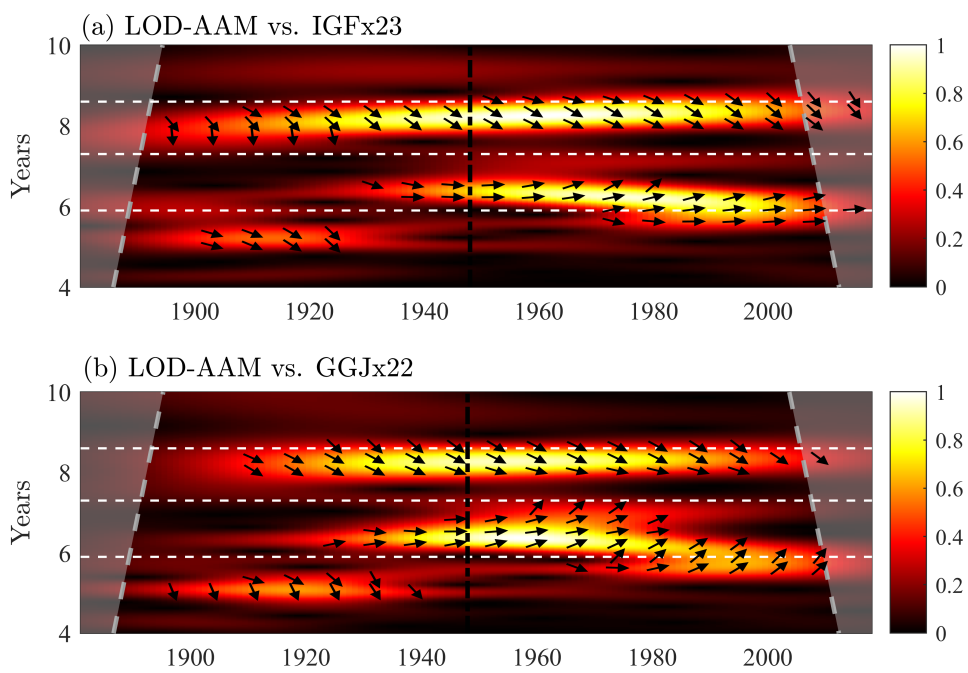


Figure 10: Wavelet coherency and phase diagrams between the observed and predicted LOD variations between 1880 and 2022, for the core flow models IGFx23 (a) and GGJx22 (b), in the period range [4,10] years. AAM was corrected after 1948 only (vertical black dashed line) using NCEP model. Horizontal dashed lines correspond to the 5.9, 7.3 and 8.5-year periods and contour lines correspond to the 95% confidence levels.

412 The former is observed over the whole time-span, while the latter has a coherence larger
 413 than 0.3 from about 1970 onward. Significant coherences, out of phase, are also visible
 414 at intermediate periods (about 4.5 years) for this model. The coherence diagram slightly
 415 differs for the alternative model GGJx22, which shows predictions approximately in phase
 416 with observations at a period around 4.25 years, starting from ~ 1995 . The amplitude of
 417 the observed (AAM cleaned) LOD at 5 years periods, about 0.02 ms, is over-estimated by
 418 both models, as illustrated in Fig. 11. Around 3 to 4 years periods, the observed signal
 419 is of the order of 0.01 ms, comparable to what is obtained for the GGJx22 model. The
 420 IGFx23 model over-estimates LOD changes at this period, in particular after ~ 1980 . It
 421 suggests possible high frequency leakage that may be smoothed out in the construction
 422 of the CHAOS-7 model (Finlay et al., 2020) used to build model GGJx22 over the past 2
 423 decades, while they would be allowed by the sequential scheme considered by Istaş et al.
 424 (2023), who built their magnetic field model simultaneously with the flow model. As a
 425 matter of fact, interannual LOD predictions from core flow models are sensitive to the
 426 considered magnetic data (or field model) from which they derive. In particular the way
 427 these handle external magnetic sources (co-estimated, cleaned a priori, or filtered..., see
 428 Lesur et al., 2022) might be key in order to properly recover rapid changes in the LOD
 429 of core origin.

430 A closer look at these short periods is available in Fig. 13 where we show the observed
 431 and predicted time series over 1962–2022, band-pass filtered at periods between 2 and
 432 4.5 years. Residual LOD changes at these time-scales, once cleaned for AAM, show an
 433 amplitude of the order of 0.05 ms. This is a tiny signal, weaker by a factor of about 4
 434 in comparison with the corrected AAM signal in the same period range (see Fig. 9 in
 435 Gross et al., 2004). Predictions from both considered flow models show fluctuations of
 436 amplitude overall comparable to that of the observed LOD residuals. One may wonder
 437 if these just reflect noise in the reconstructed flows, or if they carry some geophysical
 438 significance.

439 Despite relatively large model uncertainties, over the past two decades where geomag-
 440 netic observations are the more dense and accurate, the phase of the LOD signal is rather
 441 well recovered. This is particularly true for the model GGJx22 inverted from CHAOS-
 442 7 since 1999, that captures convincingly about more than 4 consecutive periods. The
 443 alternative model IGFx23, inverted at the recent epochs from virtual geomagnetic ob-
 444 servatory data, over-estimates the oscillations between 2010-2014, the period covered by
 445 the less accurate CryoSat-2 observations (a satellite not dedicated to magnetic records).

446 4. Summary and discussion

447 4.1. LOD oscillations in the range 6–9 years

448 Our analysis confirms the existence of two lines at 6 and 8.5 years in the observed
 449 LOD, as first put forward by Duan and Huang (2020). The isolation of these two lines is
 450 clearer once subtracted the AAM contribution. Hsu et al. (2021) propose the existence
 451 of another line at an intermediate period of 7.3 years. Around ~ 7 -yr our wavelet analysis
 452 detects oscillations in hydrological angular momentum functions with an amplitude of the
 453 order 0.01 ms (Fig. 4). This signal is likely partly cancelled out by water redistribution
 454 (see Fig. 1). A 7 yr signal may be present in LOD series cleaned for the atmospheric
 455 contribution only, although from our analysis it does not pass the 95% confidence interval

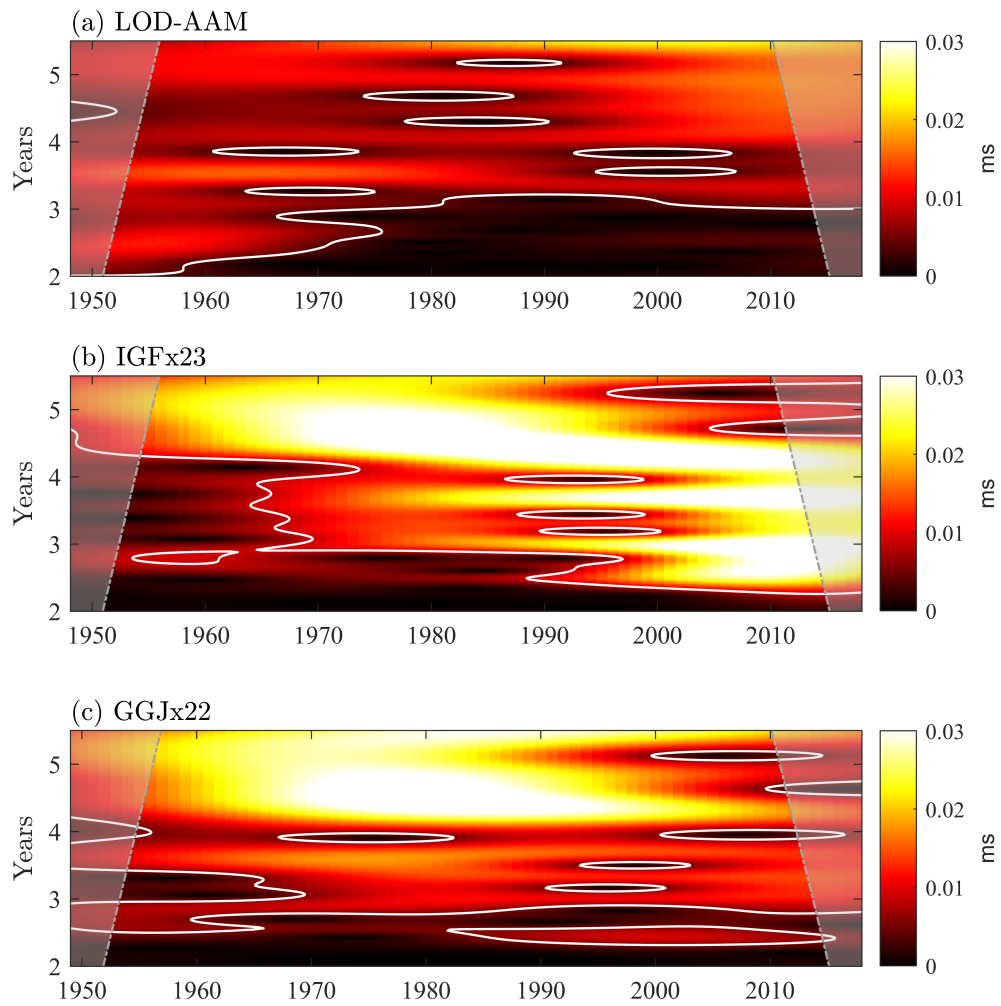


Figure 11: Wavelet analysis between observed (C04 series) and predicted LOD variations, after atmospheric angular momentum correction (using NCEP model), between 1948 and 2021. Scalograms for the observed LOD (a), and for the predicted LOD for IGFx23 (b) and GGJx22 (c) models. Contour lines correspond to the 95% confidence levels.

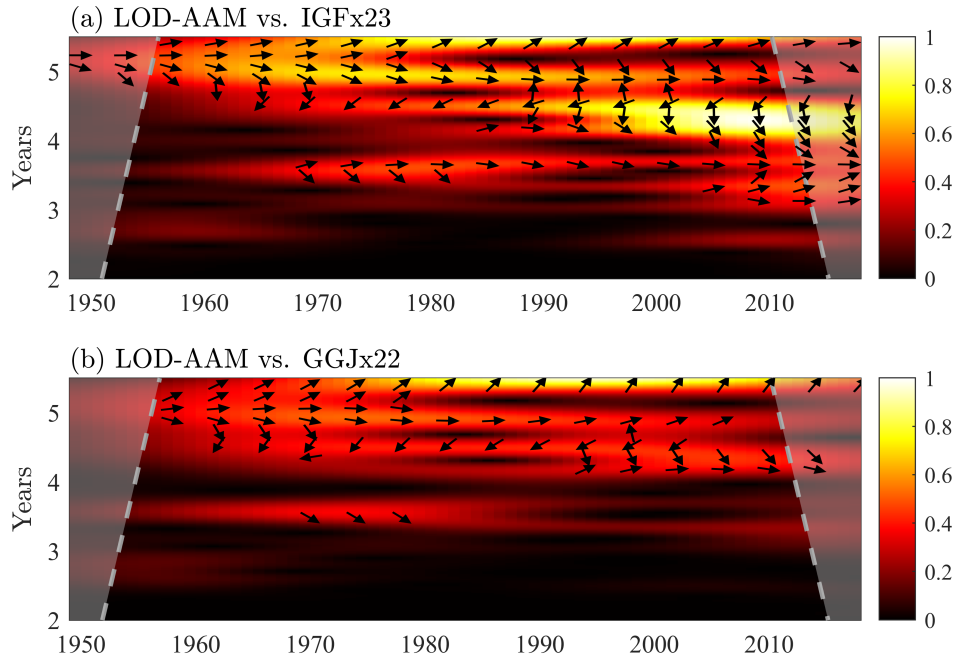


Figure 12: Wavelet coherence analysis between observed (C04 series) and predicted LOD variations, after atmospheric angular momentum correction (using NCEP model), between 1948 and 2021, for the IGFx23 (a) and GGJx22 (b) models.

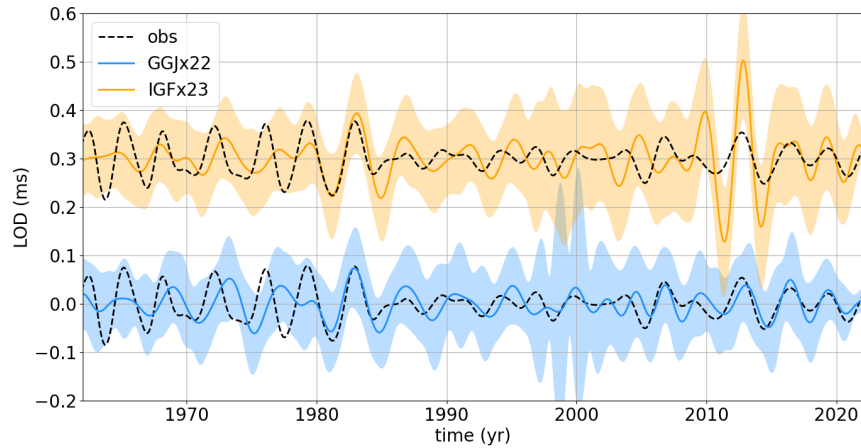


Figure 13: Observed LOD variations (dashed black, C04 series corrected from NCEP AAM) and their prediction for the two core flow models IGFx23 (orange) and GGJx22 (blue), band-pass filtered for periods $2 \leq T \leq 4.5$ years with a causal Butterworth filters of order 2. Observations were corrected for zonal tides, and atmospheric angular momentum using NCEP model.

456 (Fig. 5). Our tests with synthetic series presenting oscillators at 6 and 8.5 years, but not
457 around 7 years, show the possibility of false positives detection of this latter intermediate
458 spectral line. It is much influenced by the presence of slow LOD changes (the background
459 and a 3rd damped oscillator of decadal period in our synthetic tests). Only longer time-
460 series would help decide whether the apparent ~ 7 -yr oscillation in LOD is real or not.

461 We see no evidence either for the presence of a 7-yr line in LOD predictions from
462 core flow models, while clear predicted signals show up at periods around 6 and 8 years
463 (Fig. 9). Furthermore, the coherence analysis of LOD data with the two core flow models
464 does not exhibit any significant coherency around 7 years (Fig. 10), contrary to what is
465 seen around 6 and 8 years.

466 The nice fit of the amplitude and phase for the SYO and EYO favor a source in the
467 outer core dynamics. It could naturally occur due to the propagation in the fluid core
468 of axisymmetric torsional Alfvén waves (Gillet et al., 2010, 2017) or of quasi-geostrophic
469 magneto-Coriolis waves (Gillet et al., 2022; Istas et al., 2023). These indeed potentially
470 contain an axisymmetric component that may carry angular momentum. They could thus
471 trigger LOD changes over a wide range of frequencies. Such waves have been detected
472 in numerical simulations at parameters approaching Earth’s conditions (Aubert et al.,
473 2022). However, contrary to torsional waves, their ability to efficiently couple with the
474 mantle so as to generate detectable angular momentum changes has not yet been proven.

475 4.2. *On possible LOD signals from the core at periods shorter than 5 years*

476 Cleaning for atmospheric contributions much reduces the variance of the LOD signal
477 at periods shorter than 4.5 years (Gross et al., 2004; Duan et al., 2015). The contribution
478 of surface water redistribution (combining sea level changes, hydrology and oceans) to
479 LOD changes is much less than that of the atmosphere at interannual time-scales. Given
480 the limits of the magnetic models on short interannual time-scales (e.g., Ropp et al.,
481 2020), one may wonder if LOD signals associated with core motions are detectable on
482 periods as short as 3 to 4 years. Our wavelet analysis shows two ranges with coherent
483 oscillations around 3.5 and 5 years periods. Their amplitude, of the order of 0.01 ms, is
484 much less than the atmospheric signal that has been removed to the LOD data. We thus
485 cannot yet entirely disregard at this stage a remaining influence from surface envelopes.
486 However, such an amplitude is coherent with tendency of core motions for a weaker energy
487 towards high frequencies (Gillet et al., 2015; Aubert, 2018). Furthermore, the predicted
488 signal correctly predicts the phase (and amplitude, depending on the considered core flow
489 reanalysis) over more than four contiguous oscillations when magnetic data are the more
490 dense and accurate (the past two decades). Here again, one may invoke higher harmonics
491 of torsional modes (see Fig. 8 in Gillet et al., 2017) and/or of magneto-Coriolis modes.
492 Our findings call for further research both regarding the cleaning of external LOD sources,
493 as well as the reanalysis of core motions toward short periods.

494 **Acknowledgments**

495 We thank Richard Holme and an anonymous reviewer for their comments and sug-
496 gestions which have improved our paper. Discussions with Dominique Jault and Ingo
497 Wardinski have enriched this work. This study is part of the DYNOCORE project
498 which benefits from the CNES financial support through the TOSCA program. This

499 work has been funded by ESA in the framework of EO Science for Society, through con-
500 tract 4000127193/19/NL/IA (Swarm + 4D Deep Earth: Core). It has also been partially
501 supported by the French Space Agency (CNES) in the context of the *Swarm* mission of
502 ESA. NG is part of Labex OSUG@2020 (ANR10 LABX56).

503 Data availability

504 Effective angular momentum functions were downloaded from
505 <https://www.iers.org/IERS/EN/DataProducts/GeophysicalFluidsData/geoFluids.html>
506 also available from <https://isdc.gfz-potsdam.de/esmdata/> (for series starting in 1976).
507 LOD data were obtained from the Earth orientation center at [https://hpiers.obspm.fr/eop-](https://hpiers.obspm.fr/eop-pc/index.php)
508 [pc/index.php](https://hpiers.obspm.fr/eop-pc/index.php). Flow models are available at <https://geodyn.univ-grenoble-alpes.fr/>

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640 Appendix A. Supplementary information

641 Hundred random realizations of the synthetic LOD series, as described in Section 2.2.1,
642 have been carried out. We illustrate here the influence of the high-pass filtering with a
643 cut-off period of 10 yr (Fig. A.1). We also show the CWT when there is no long-period
644 background in the synthetic time-series, for a single oscillator of period 6 years, or two
645 oscillators of periods 6 and 8.5 years (Fig. A.2). Two other realizations for the synthetic
646 case with two oscillators and a background plus noise are also provided (Fig. A.3).

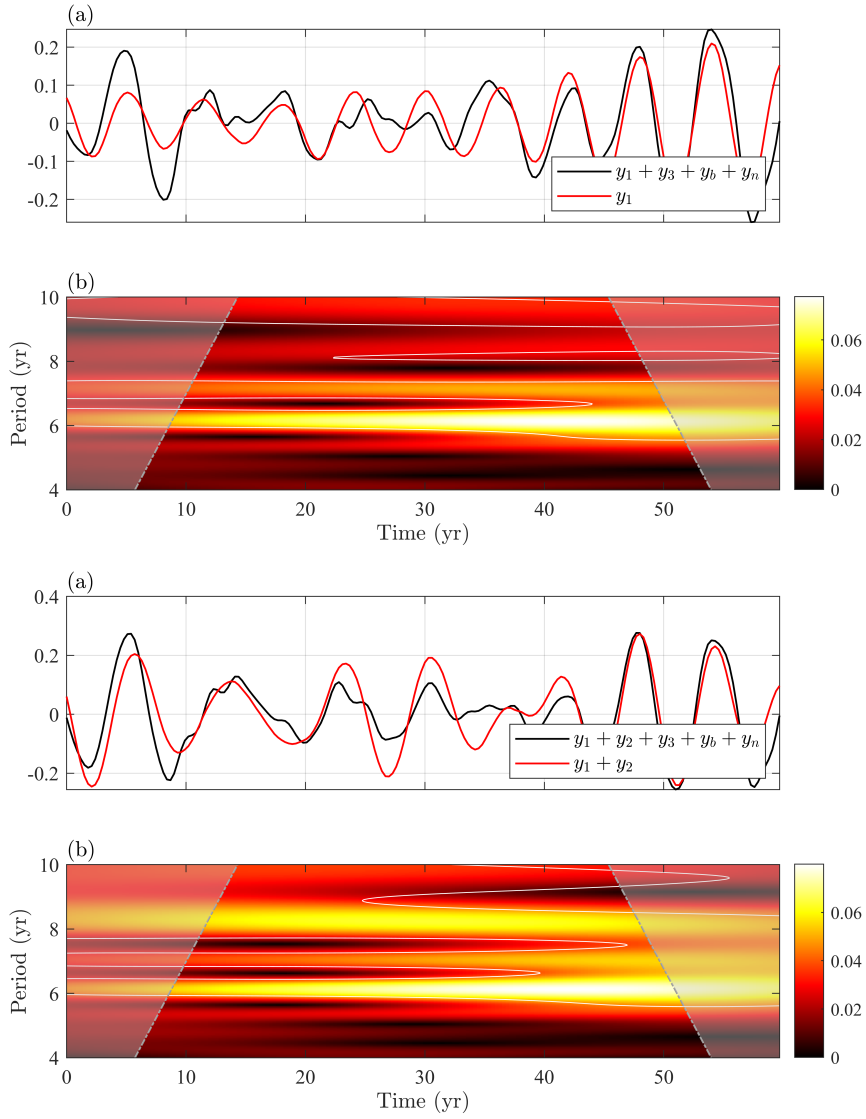


Figure A.1: High-pass filtered synthetic signal composed of an auto-regressive stochastic background (y_b), a colored noise (y_n), and various oscillators. Top: two damped oscillators at 6 and 17 years periods, with $Q = 20\pi$ and $Q = \pi$ (y_1 and y_3). Bottom: three damped oscillators at 6, 8.5 and 17 years periods, with $Q = 20\pi$, 20π and π respectively (y_1 , y_2 and y_3). (a) Time series, (b) Continuous Wavelet Transform.

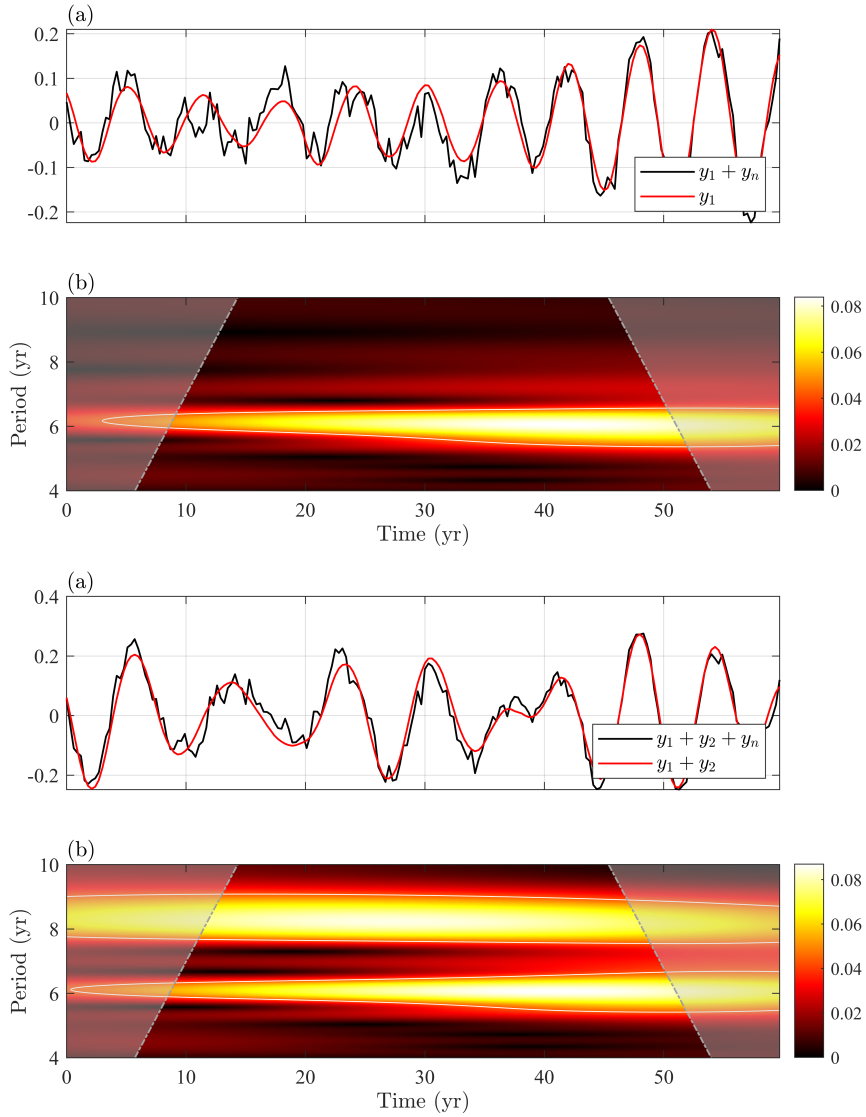


Figure A.2: Synthetic signals in the absence of a background, embedded in colored noise (y_n). Top: one damped oscillator of period 6 years, with $Q = 20\pi$ (y_1). Bottom: two damped oscillators of periods 6 and 8.5 years, both with $Q = 20\pi$ (y_1 and y_2). (a) Time series, (b) Continuous Wavelet Transform.

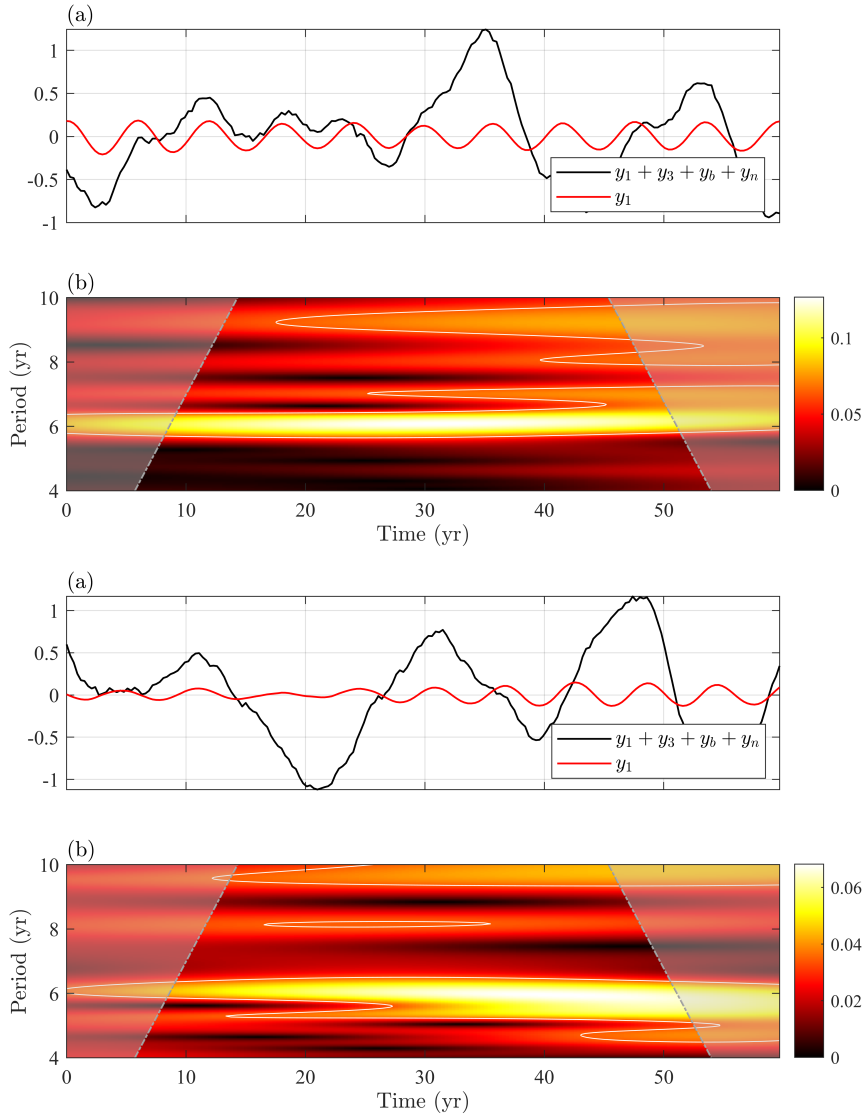


Figure A.3: Two other realizations of a synthetic signal composed of an auto-regressive stochastic background (y_b) plus two damped oscillators at 6 and 17-yr periods, with $Q = 20\pi$ and $Q = \pi$ (y_1 and y_3) embedded in colored noise (y_n). (a) Time series, (b) Continuous Wavelet Transform.