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# Acoustic Source Localisation using Vibroacoustic Beamforming

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## Abstract

For acoustic source localisation in a fluid, sound pressure is traditionally measured using a microphone array, and acoustic beamforming is then applied to the pressure signals to localise the source. However, in some engineering applications, it is not feasible to place the microphone/hydrophone array in the fluid to localise an acoustic source. Hence, this work proposes a vibroacoustic beamforming for sound source localisation which applies the beamforming procedure to the measured vibration signals at the surface of a structure in contact with the fluid in which the acoustic source is located. To evaluate the localisation performance of the proposed method, a numerical study is conducted to localise a monopole source in a room using vibration data obtained from an elastic panel on the room wall. To simulate more realistic situation, different levels of signal to noise ratio are considered by moving the source away from the wall-mounted sensors and consequently altering the direct acoustic field (signal). Moreover, commonly used frequency domain beamforming techniques namely conventional beamforming, functional beamforming, minimum variance distortionless response and minimum power distortionless response are applied to the measured signals to localise the source. The performance of each technique is then evaluated by comparing the beamforming output obtained from vibroacoustic beamforming with that of acoustic beamforming and by quantifying the dynamic range and spatial resolution for each method. It can be concluded that vibroacoustic beamforming can be used for acoustic localisation as an alternative to the acoustic beamforming.

*Keywords:* Sound source localisation, Vibroacoustic, Conventional beamforming, Functional beamforming, MVDR, MPDR

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## 1. INTRODUCTION

The advent of "acoustic telescopes" to identify ships in foggy weather was the first application of acoustic source localisation more than a century ago. The concept was that if the device was pointed in the direction of a sound source, sounds collected at receivers would arrive at the same time and so reinforce each other when the total sound was heard. In addition to ship and vehicle detection, acoustic source localisation has been used to provide guidelines on product noise control, target selection and interference rejection for communication devices or speech recognition processing, and noise source identification for mechanical systems. In general, sources are detected in beamforming by scanning all possible source locations and calculating the likelihood of an actual source at each probable spot based on the strength of the combined microphones' output signals. Chiariotti et al. [1] reviewed different acoustic beamforming (ABF) methods for source localisation in different applications. The main concepts of beamforming, starting from the very basics and progressing onto more advanced concepts and techniques along with application were presented. They briefly reviewed the most common beamforming methods. The so-called conventional beamforming (CBF), also known as delay-and-sum beamforming, is the most straightforward and reliable beamforming algorithm. As a result, it is most commonly employed in practice. The phase delays between the emission of the sound signal at the source and the received signals at each microphone are used in this technique. This approach can be used in the time domain or the frequency domain by applying a discrete Fourier transform to the data. Because of the reduced computing time and the ability to perform a frequency analysis, the latter is extensively employed. Deconvolution algorithms [2, 3], especially the DAMAS algorithm [4], are the main methods used in recent years to reconstruct a clean map of source distributions from a dirty map via iterative deconvolution, and thus can significantly improve the spatial resolution that cannot be achieved by traditional beamforming algorithms. However, their inevitable iteration nature results in much more computational complexity than traditional beamforming. Hence, some scholars abandoned the deconvolution algorithm, returned to the traditional beamforming algorithm and proposed some new beamforming algorithms, for example, adaptive beamforming [5] and functional beamforming [6].

Two most common adaptive beamforming techniques widely used in the literature are the MVDR (Minimum Variance Distortionless Response) and MPDR (Minimum power Distortionless Response). They both minimize output power under the constraint that the assumed steering vector goes non-distorted. The power minimization in MVDR only includes the effect of noise while both noise and desired signal are taken into account in MPDR [5]. Ehrenberg et al. [7] conducted a sensitivity

32 analysis of these two distortionless beamformers. The MVDR beamformer, which employs the noise  
33 correlation matrix as its minimisation criterion, was found to be more resilient to steering errors than  
34 the MPDR beamformer, which uses the received signal correlation matrix. Furthermore, even if the  
35 noise correlation matrix is computed incorrectly due to steering errors in the interference direction,  
36 the MVDR advantage is maintained for a fairly good range of steering errors. The performance  
37 of the adaptive beamforming technique degrades considerably as the steering vector and covariance  
38 matrix become inaccurate [8]. Carlson introduced a diagonal loading approach that pulsates a constant  
39 diagonal loading compensation value to the diagonal components of the covariance matrix to lower the  
40 dispersive degree of the eigenvalues of the covariance matrix [9].

41 Dougherty [10] recently proposed the functional beamforming (FBF). Despite its simplicity, it has  
42 been found to improve array spatial resolution and dynamic range. The computing time for functional  
43 beamforming is comparable to that of a typical frequency domain beamformer and far less than that  
44 of deconvolution methods. The name "functional beamforming" comes from the algorithm's use of  
45 functions of matrices. An exponent parameter is used in the algorithm. In a functional sense, the cross  
46 spectral matrix (CSM) is raised to the power of the reciprocal of this exponent. The adjusted CSM is  
47 then used in the CBF method, and the values of the resulting beamforming map are elevated to the  
48 non-reciprocal exponent's power. Array sidelobes are essentially avoided for big values of the exponent.  
49 This boosts array design flexibility and greatly increases the system's dynamic range, allowing new  
50 sources to be uncovered. FBF was successfully employed to landing aircraft fly-over measurements  
51 using a 32 microphone array. It was shown that the outcomes of the simulation and the experiment were  
52 very similar. Functional beamforming produces superior acoustic images in terms of both the dynamic  
53 range and the resolution for both synthetic and experimental data [6]. In order to use beamforming  
54 techniques, measurements must be adequately sampled, which means that the array must have at least  
55 twice as many samples as the maximum wavenumber. This is not the case, however, with compressed  
56 beamforming, a type of beamforming based on compressive sensing that has gained popularity in recent  
57 years due to its capacity to reconstruct sparse signals from a small number of measurements using  
58 convex optimisation techniques [11]. Compressive sensing was developed in the signal processing and  
59 applied mathematics communities, and it has subsequently produced impressive results in acoustics. It  
60 has been used, for instance, to solve the acoustic source localisation problem using acoustic imaging [12],  
61 reverberant environment, acoustic response reconstruction [13], and direction-of-arrival estimate [14].

62 Most of the research on sound source localisation has been focused on acoustic beamforming and

63 applying beamforming techniques to the acoustic pressure signals. This requires that pressure sensor  
64 array is directly put in the acoustic medium to measure the sound pressure. However, there are some  
65 engineering applications where placing microphone/hydrophone array in the fluid is not possible or in  
66 some situations extremely difficult and costly. Therefore, acoustic source localisation through vibration  
67 of a structure is of practical interest. Leclere and Picard [15] introduced a methodology to localise  
68 acoustic sources from the measurement of airborne induced vibrations of a thin structure. To do this,  
69 the force analysis technique (FAT) was coupled with the ABF. The FAT was used to identify the  
70 parietal pressure field exciting the thin structure from vibration measurements and traditional ABF  
71 was then applied to the identified parietal pressure for the localisation of acoustic sources. The exper-  
72 imental implementation showed that the parietal pressure field was correctly identified and quantified  
73 from the vibration measurements. The drawback of this technique is that it requires coupling of two  
74 methods (FAT and ABF) with their own limitations. For example, in the FAT method, the derivative  
75 of the fourth order of the displacement should be estimated by finite difference of the acceleration  
76 measurements which can result in noise amplification. Further, the beamforming was based on the  
77 propagation of plane waves which may not be an accurate representation of the waves emitted by the  
78 acoustic source.

79 The concept of vibroacoustic beamforming (VABF) technique was first introduced within a PhD  
80 thesis in INSA-Lyon [16, 17] with the focus on the acoustic detection of defects or leaks (water-sodium  
81 reaction) inside a steam generator units of a nuclear reactor. They used VABF technique to detect  
82 the presence of an acoustic monopole inside an elastic cylindrical shell by measuring the external shell  
83 vibrations. A vibroacoustic model of the fluid-loaded shell was first considered for numerical analysis.  
84 The beamforming technique was then applied to radial velocities of the shell. Analysis of the results  
85 highlighted how the fluid-loaded shell influences the detection. Finally, a test in a water-filled steel  
86 pipe was achieved to experimentally validate the presented approach. One difficulty in the monitoring  
87 of steam generators for water leaking into sodium is that leak-induced vibrations can be masked by  
88 background noise. Therefore, it is important to take into account the background noise in the analysis  
89 and improve the signal-to-noise ratio (SNR) using appropriate beamforming algorithm. Kassab et  
90 al. [18] examined this with experimental investigations on a mock-up consisting of a cylindrical pipe  
91 coupled with a hydraulic circuit by two flanges. Vibration due to the background noise was controlled  
92 by changing the flow speed, and a sound emitter was placed in the pipe to replicate the source to be  
93 detected. The signals obtained by an array of accelerometers installed on the pipe were then processed

94 by beamforming. The conventional type of beamforming was studied, as well as an advanced type based  
95 on SNR maximisation (MaxSNR method) [19, 20]. It was discovered that traditional beamforming  
96 was ineffective at greater flow rates, which was due to the significant coherence of the vibratory signals  
97 created by the turbulent flow, which contradicted the standard beamforming assumption. Using the  
98 MaxSNR beamforming, on the other hand, considerable beamforming gain was achieved for various  
99 source positions.

100 Although the VABF has been implemented for leak detection in pipes, it has not been used for  
101 acoustic source localisation. This work explores the possibility of source localisation by applying a  
102 beamforming technique to the vibration signals measured at the surface of a structure in contact with  
103 a fluid domain where the sound source is located. Therefore, it eliminates the need of placing the sensor  
104 array directly in the fluid. This is an alternative method to the traditional ABF where sound pressure is  
105 measured by a microphone array. In this work, performance of the VABF is evaluated both qualitatively  
106 and quantitatively by comparing its source map, dynamic range and spatial resolution with those  
107 obtained by the ABF at different frequencies. Moreover, in order to have a more realistic numerical  
108 analysis, source localisation is conducted in a noisy environment with different levels of SNR to examine  
109 the effectiveness of the VABF. Furthermore, different frequency domain beamforming techniques are  
110 applied to the measured signals to localise the source. Performance of each beamforming technique is  
111 also demonstrated and the key parameters of the system which is influencing the localisation process  
112 are discussed.

113

## 114 **2. Frequency Domain Acoustic and Vibroacoustic Beamforming Methods**

115 Figure 1 shows a schematic diagram for source localisation using the acoustic and vibroacoustic  
116 beamforming methods. In the former, a flush-mounted microphone array is considered which is the  
117 standard configuration for the ABF as shown in Figure 1(a). In this case, the beamforming treatments  
118 are performed on the pressure signals measured by the microphones. It is assumed that mounting  
119 structure is rigid. Even if the mounting structure was not rigid, since in this work the ABF is conducted  
120 in air, the interaction between the acoustic medium and the elastic structure would be weak and can  
121 be neglected. Therefore, vibrations of mounting structure have a negligible effect on the pressure  
122 measured by the microphones. In the VABF, vibration of the structure due to the acoustic source  
123 and noise field excitations is measured by an accelerometer array, and a beamforming technique is

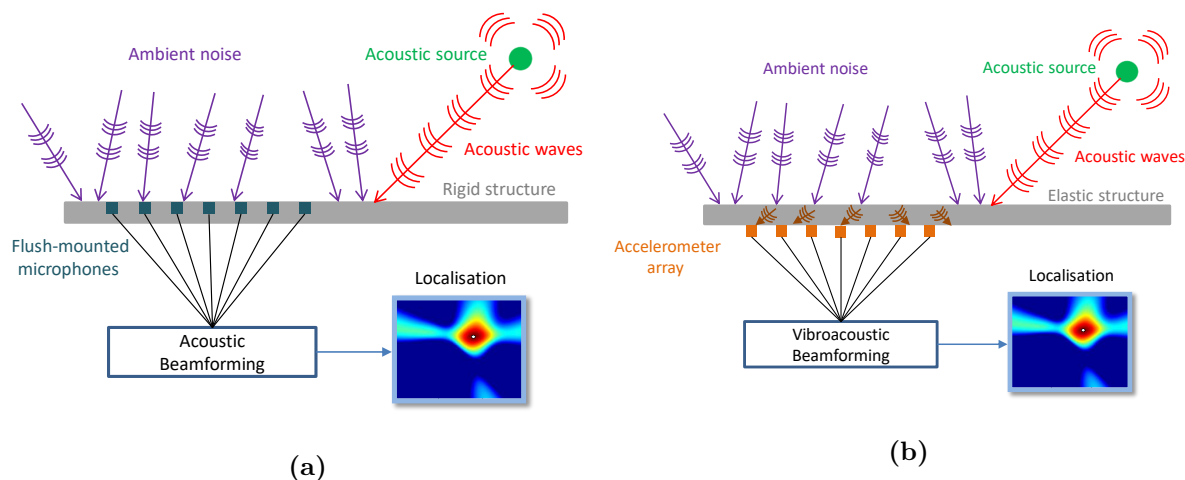


Figure 1: Illustration of an acoustic source localisation in the presence of background noise using (a) a microphone array (acoustic beamforming) and (b) an accelerometer array (vibroacoustic beamforming).

124 then applied to the vibration data to localise the acoustic source as shown in Figure 1(b). Contrary  
 125 to the ABF where pressure signals are measured by a microphone array, measured accelerations in  
 126 the VABF depend on the dynamic behaviour of the mounting structure. In this work, it is assumed  
 127 that the acoustic source to be localised produces a harmonic deterministic signal whereas the noise is  
 128 considered to be random and stochastic. Moreover, it should be mentioned that for the purpose of  
 129 a fair comparison between the ABF and VABF, the number of sensors, their positions and the array  
 130 pattern are the same in both the ABF and VABF to ensure any differences in the beamforming outputs  
 131 obtained from these two methods are attributed to their performances and not due to the variation in  
 132 the system setup and configuration.

133 In the next sub-section, four widely used beamforming techniques namely conventional, functional,  
 134 MVDR and MPDR beamformings are herein employed to evaluate the performance of the proposed  
 135 VABF against the ABF. These techniques can be applied to both the ABF and VABF. For the sake  
 136 of brevity, they are presented only for the vibroacoustic beamforming in which the array signals  
 137 correspond to the structural accelerations.

138

## 139 2.1. Beamforming Techniques

### 140 2.1.1. Conventional Beamforming

141 Let us define the CSM of the signals measured by the accelerometers at angular frequency  $\omega$  by  
 142  $\mathbf{\Gamma}(\omega)$ . For an array of  $n$  accelerometers,  $\mathbf{\Gamma}$  is an  $n \times n$  square matrix where the entry in the  $i$ -th row

143 and the  $j$ -th column corresponds to the cross spectrum density (CSD) function of the accelerations  
 144 between  $i$ -th and  $j$ -th sensors ( $S_{ij}(\omega)$ ) at frequency  $\omega$ :

$$\mathbf{\Gamma}(\omega) = [S_{ij}(\omega)], \quad \text{where } i, j = 1, 2, \dots, n, \quad (1)$$

145 assuming that the signals induced by the acoustic source are independent from those generated by the  
 146 background noise, matrix  $\mathbf{\Gamma}(\omega)$  can be expressed as follows,

$$\mathbf{\Gamma}(\omega) = \mathbf{\Gamma}_s(\omega) + \mathbf{\Gamma}_n(\omega), \quad (2)$$

147 where  $\mathbf{\Gamma}_s(\omega)$  is the CSM of the accelerations only due to the monopole source excitation and  $\mathbf{\Gamma}_n(\omega)$   
 148 is the CSM of the accelerations produced entirely by the background noise. The spatial filter of  
 149 beamforming is characterized by so-called weight vector  $\mathbf{W}$ , which is defined for each focus point  $u$  in  
 150 the detection space. The beamforming output at location  $u$  is defined by [19]

$$b_u(\omega) = \mathbf{W}_u^\dagger(\omega)\mathbf{\Gamma}(\omega)\mathbf{W}_u(\omega), \quad (3)$$

151 where the superscript  $\dagger$  denotes the Hermitian transpose.

152 It can be proved that the conventional beamforming output becomes maximum for the actual  
 153 source location when the weight vector is equal to the well-known steering vector  $\mathbf{F}_u$  defined by [17].

$$\mathbf{W}_u(\omega) = \mathbf{F}_u(\omega) = \frac{\mathbf{T}(\omega)}{\|\mathbf{T}(\omega)\|}, \quad (4)$$

154 where  $\|\cdot\|$  represents the Euclidean norm and  $\mathbf{T}(\omega) = [T_{ui}(\omega)]$  is a vector containing the transfer func-  
 155 tions for a given focus point  $u$ . In other word, the vector component  $T_{ui}(\omega)$  is basically the acceleration  
 156 at the  $i$ -th sensor due to the unit source at location  $u$ . These transfer functions can be obtained ex-  
 157 perimentally or by using analytical/numerical models which allow calculating the accelerations of the  
 158 structure when the system is excited by a monopole source in the acoustic domain. For further details  
 159 on steering vector selection, see Ref. [21], which studied four distinct steering vector formulations  
 160 from the literature. None of the formulas were found to give both the correct location and source  
 161 strength. The various steering vector formulations were demonstrated to be not equally well suited for  
 162 three-dimensional applications. The steering vector in Eq. (4) is one of two recommended formulations  
 163 identified in Ref. [21] that allows for accurate source location identification at the cost of a negligible



164 error in the estimated source strength.

### 165 2.1.2. Functional Beamforming

166 One disadvantage of the CBF is that its source map includes not only the mainlobe (the peak in  
 167 the source map where the actual source is situated), but also some sidelobes that occur at different  
 168 locations from the actual source. This is because the computed source distribution is a convolution  
 169 of the real source distribution and the array's specific response function. Despite the fact that these  
 170 sidelobes have a lower amplitude than the mainlobe, it is possible that they obscure the source map  
 171 and make source localisation difficult. The adoption of an improved beamforming algorithm known as  
 172 functional beamforming is one way to lower the high sidelobe level [10]. In this method, the CSM can  
 173 be expressed as its eigenvalue decomposition

$$\mathbf{\Gamma}(\omega) = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^\dagger, \quad (5)$$

174 where  $\mathbf{U}$  is a unitary matrix whose columns are the eigenvectors of  $\mathbf{\Gamma}$ , and  $\mathbf{\Sigma}$  is a diagonal matrix  
 175 whose diagonal elements are the eigenvalues of  $\mathbf{\Gamma}$ . It should be noted that since  $\mathbf{\Gamma}$  is a Hermitian  
 176 matrix,  $\mathbf{\Sigma}$  has only real entries. The output of functional beamforming is then given by [22]

$$b_u(\omega) = \left( \mathbf{F}_u^\dagger \mathbf{\Gamma}^{\frac{1}{\alpha}} \mathbf{F}_u \right)^\alpha = \left( \mathbf{F}_u^\dagger \mathbf{U} \mathbf{\Sigma}^{\frac{1}{\alpha}} \mathbf{U}^\dagger \mathbf{F}_u \right)^\alpha, \quad (6)$$

177 where parameter  $\alpha$  is a real number greater than unity and can be set by the user. When  $\alpha = 1$ ,  
 178 FBF essentially becomes the CBF as Eqs. (3) and (6) become identical. Using the FBF, sidelobe level  
 179 or dynamic range  $\mu$  (typically defined as the difference in dB between the mainlobe and the highest  
 180 sidelobe) and the array spatial resolution  $\delta$  (usually defined as the width of the mainlobe 3 dB below its  
 181 peak) of the source map can be improved for increasing values of  $\alpha$ . Common values for  $\alpha$  range from 20  
 182 to 300. In practice, using well calibrated arrays values of 100 leads to satisfying results [6]. The presence  
 183 of numerous sidelobes usually becomes a problem, especially at high frequencies. This procedure  
 184 significantly reduces sidelobes and increases the dynamic range of the results. The computational time  
 185 required for FBF is almost identical to classical beamforming since the only addition is the effort to  
 186 perform a spectral decomposition of the CSM.

187 *2.1.3. MVDR Beamforming*

188 MVDR is a typical beamforming algorithm which minimizes the noise in the output power in  
 189 the desired direction through adjusting a weight factor. The MVDR optimization problem can be  
 190 expressed as follows

$$\mathbf{W}_u(\omega) = \operatorname{argmin} \{ \mathbf{W}_u^\dagger(\omega) \mathbf{\Gamma}_n(\omega) \mathbf{W}_u(\omega), \quad \text{s.t.} \quad \mathbf{W}_u \mathbf{F}_u = 1 \}, \quad (7)$$

191 the solution to this is given by [23]

$$\mathbf{W}_u(\omega) = \frac{\mathbf{\Gamma}_n^{-1}(\omega) \mathbf{F}_u(\omega)}{\mathbf{F}_u^\dagger(\omega) \mathbf{\Gamma}_n^{-1}(\omega) \mathbf{F}_u(\omega)}. \quad (8)$$

192 The MVDR beamformer has higher spatial resolution than a conventional beamformer and its sidelobes  
 193 are smaller and smoother. However, it requires a matrix inversion every time the noise changes which  
 194 can be computationally expensive.

195 *2.1.4. MPDR Beamforming*

196 The MVDR beamformer is widely used in applications where it is possible to measure or estimate  
 197  $\mathbf{\Gamma}_n$ . In applications where the signal is always present, the MPDR beamformer is used instead. MPDR  
 198 is chosen as the optimal solution of

$$\mathbf{W}_u(\omega) = \operatorname{argmin} \{ \mathbf{W}_u^\dagger(\omega) \mathbf{\Gamma}(\omega) \mathbf{W}_u(\omega), \quad \text{s.t.} \quad \mathbf{W}_u \mathbf{F}_u = 1 \}, \quad (9)$$

199 this is commonly referred to as Capon's method [24]. Eq. (9) has an analytical solution given by [23]

$$\mathbf{W}_u(\omega) = \frac{\mathbf{\Gamma}^{-1}(\omega) \mathbf{F}_u(\omega)}{\mathbf{F}_u^\dagger(\omega) \mathbf{\Gamma}^{-1}(\omega) \mathbf{F}_u(\omega)}. \quad (10)$$

200 Eq. (9) also differs from Eq. (7) in that the power minimization includes the effect of the desired signal  
 201 plus noise. The constraint  $\mathbf{W}_u \mathbf{F}_u = 1$  in both equations prevents the gain in the direction of the signal  
 202 from being reduced.

### 204 3. Monopole Source Localisation in a Room

205 In this section, we initially present the configuration of a case study in Section 3.1 which is used  
 206 to study the localization performance of the VABF against ABF. The vibroacoustic model which is  
 207 employed to obtain the CSM of the signal and noise as well as the steering vectors is then presented in  
 208 Section 3.2. A numerical verification based on comparisons with finite element simulations is gathered  
 209 in Section 3.3. Finally, in Section 3.4, the SNR is formulated in function of the physical characteristics  
 210 of the case study. This allows us to compare the performance of the VABF with that of the ABF in  
 211 Section 4 for different values of SNRs (i.e. for different positions of the source).

#### 212 3.1. Case Study

213 To demonstrate performance of the vibroacoustic beamforming, let us consider a case study where  
 214 a monopole source is located in a room. It is assumed all the walls are rigid except a small section  
 215 of one of the side walls. This small part is chosen to be an elastic rectangular panel with simply  
 216 supported boundary conditions along all edges which is located in a rigid baffle (the side wall) as  
 217 shown in Figure 2. A harmonic monopole source pulsating at the angular frequency  $\omega$  is the source  
 218 to be localised. The blocked pressure on the plate surface comprises two acoustic fields: the direct  
 219 acoustic field which is due to the direct sound emitted by the point source, and the reverberant field  
 220 which is due to the reflection of the direct sound by the walls. The reverberant field is assumed to  
 221 be a fully diffuse acoustic field (DAF) which can be modeled analytically.

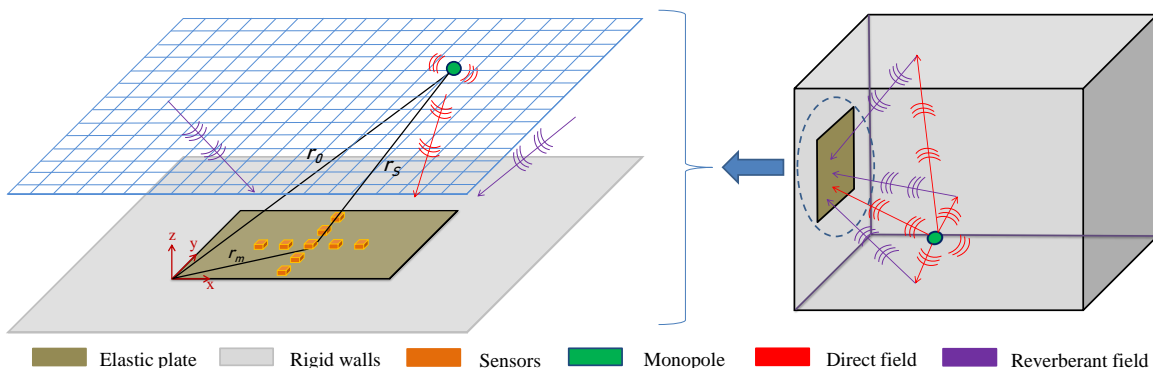


Figure 2: Schematic diagram showing an accelerometer array on a baffled panel under monopole source excitation; beamforming detection space (left), and the baffled panel on the side wall of a rigid room (right).

### 3.2. Vibroacoustic Beamforming Simulation

The vibroacoustic beamforming outlined in section 2 could be applied to any type of structure. As Eqs. (2) and (3) indicate, the CSD functions of the accelerations between sensors  $i$ -th and  $j$ -th,  $S_{ij}$ , and the transfer functions  $T_{ui}(\omega)$  between each sensor and the focus point in the detection space need to be known to calculate the beamforming output. The elements of the CSM can be expressed as the sum of the CSD functions of the accelerations due to the source ( $S_{ij}^s$ ) and noise ( $S_{ij}^n$ ) excitations as follows

$$S_{ij}(\omega) = S_{ij}^s(\omega) + S_{ij}^n(\omega), \quad (11)$$

and

$$S_{ij}^s(\omega) = a(\mathbf{x}_i, \mathbf{x}_q, \omega) a^*(\mathbf{x}_j, \mathbf{x}_q, \omega). \quad (12)$$

where  $a(\mathbf{x}_i, \mathbf{x}_q, \omega)$  is the acceleration of the structure at  $i$ -th sensor due to the monopole source excitation at location  $q$  (actual source location). The transfer function is then given by

$$T_{ui}(\omega) = a(\mathbf{x}_i, \mathbf{x}_u, \omega), \quad (13)$$

where  $a(\mathbf{x}_i, \mathbf{x}_u, \omega)$  is the acceleration of the structure at  $i$ -th sensor due to the monopole source excitation of unit intensity at location  $u$  (focus point in the detection space). \* indicates complex conjugate. On the other hand, the CSD of the response due to the random noise can be written as [25]

$$S_{ij}^n(\omega) = \frac{1}{4\pi^2} \int_{\infty} H_a^*(\mathbf{x}_i, \mathbf{k}, \omega) \phi_{pp}(\mathbf{k}, \omega) H_a(\mathbf{x}_j, \mathbf{k}, \omega) d\mathbf{k}, \quad (14)$$

where  $H_a(\mathbf{x}_i, \mathbf{k}, \omega)$  is the sensitivity function of the structure acceleration excited by a unit wall plane wave.  $\phi_{pp}(\mathbf{k}, \omega)$  is the wavenumber-domain CSD of the wall pressure field due to the noise, and  $\mathbf{k}$  is the wavevector with components  $k_x$  and  $k_y$  in the  $x$ - and  $y$ - directions, respectively.

For the baffled panel with simply supported boundary conditions along its edges, the panel acceleration excited by a monopole source is given by [25]

$$a(\mathbf{x}_i, \mathbf{x}_q, \omega) = -\omega^2 \sum_{m=1}^M \sum_{n=1}^N \frac{F_{mn}(\mathbf{x}_q) \varphi_{mn}(\mathbf{x}_i)}{\Omega(\omega_{mn}^2 - \omega^2 + j\eta\omega\omega_{mn})}, \quad (15)$$

where  $M$  and  $N$  are the cut-off modal orders in the  $x$ - and  $y$ - directions, respectively.  $j = \sqrt{-1}$

is imaginary unit and  $\Omega = \rho_s h L_x L_y / 4$  is the modal mass.  $\eta$  is the damping loss factor,  $\rho_s$  is panel density,  $h$  is panel thickness and  $L_x$ ,  $L_y$  are the panel length and width in the  $x$ - and  $y$ - directions, respectively. The modal frequencies are given by

$$\omega_{mn} = \sqrt{\frac{D}{\rho_s h} \left( \left( \frac{m\pi}{L_x} \right)^2 + \left( \frac{n\pi}{L_y} \right)^2 \right)}, \quad (16)$$

where  $D = Eh^3/(12(1-\nu^2))$  is the flexural rigidity,  $E$  is the Young's modulus and  $\nu$  is Poisson's ratio.  $\varphi_{mn}(\mathbf{x})$  are the panel mode shapes given by

$$\varphi_{mn}(\mathbf{x}) = \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right), \quad (17)$$

and

$$F_{mn}(\mathbf{x}_q) = \int_{\Omega} p(\mathbf{x}_q, \mathbf{x}', \omega) \varphi_{mn}(\mathbf{x}') d\mathbf{x}', \quad (18)$$

where  $\Omega$  is the integration domain over the panel surface and  $p(\mathbf{x}_q, \mathbf{x}', \omega)$  is the acoustic wall pressure generated by a monopole source given by [26]

$$p(\mathbf{x}_q, \mathbf{x}', \omega) = \frac{j\rho_0\omega Q_v}{2\pi r_s} e^{-jk_0 r_s}, \quad r_s = |\mathbf{x}_q - \mathbf{x}'|, \quad (19)$$

227 where  $Q_v$  is the source strength,  $\rho_0$  is the fluid density,  $k_0 = \omega/c_0$  is the acoustic wavenumber and  
 228  $c_0$  is speed of sound in the fluid. Eq. (18) can be numerically computed using rectangular integration  
 229 method. Once the plate accelerations are computed by Eq. (15), the CSM of signal and transfer  
 230 functions can be obtained using Eqs. (12) and (13), respectively.

To be able to conduct the beamforming procedure, one also needs to calculate the CSM of noise as expressed by Eq. (14). This requires that the sensitivity function  $H_a(\mathbf{x}, \mathbf{k}, \omega)$  and the CSD of noise to be known. The sensitivity function for a simply supported rectangular plate is given by [27]

$$H_a(\mathbf{x}, \mathbf{k}, \omega) = -\omega^2 \sum_{m=1}^M \sum_{n=1}^N \frac{\psi_{mn}(\mathbf{k}) \varphi_{mn}(\mathbf{x})}{\Omega(\omega_{mn}^2 - \omega^2 + j\eta\omega\omega_{mn})}, \quad (20)$$

where the modal forces  $\psi_{mn}$  are calculated by integration over the panel surface as follows

$$\psi_{mn}(\mathbf{k}) = \int_{\Omega} \varphi_{mn}(\mathbf{x}) e^{-j(k_x x + k_y y)} d\mathbf{x} = I_m^x(k_x) I_n^y(k_y), \quad (21)$$

231 and

$$\{I_s^r(k_r)|(r, s) = (x, m) \vee (y, n)\} = \left\{ \begin{array}{ll} \left( \frac{s\pi}{L_r} \right) \frac{(-1)^s e^{-j(k_r L_r)} - 1}{k_r^2 - \left( \frac{s\pi}{L_r} \right)^2}, & k_r \neq \frac{s\pi}{L_r} \\ \frac{1}{2}jL_r, & \text{otherwise} \end{array} \right\}. \quad (22)$$

Where  $\vee$  is a logical connective which means 'or'. The CSD of the stochastic noise field can be expressed in terms of the auto spectrum density (ASD) function  $S_{pp}^{\text{diffuse}}(\omega)$  and the normalized CSD function of the stochastic field  $\tilde{\phi}_{pp}(\mathbf{k}, \omega)$  as follows [28, 29]

$$\phi_{pp}(\mathbf{k}, \omega) = S_{pp}^{\text{diffuse}}(\omega) \tilde{\phi}_{pp}(\mathbf{k}, \omega), \quad (23)$$

232 the normalised CSD function of the DAF in the wavenumber-frequency space is given by [30].

$$\tilde{\phi}_{pp}(k_x, k_y, \omega) = \left\{ \begin{array}{ll} \frac{2\pi}{k_0 \sqrt{k_0^2 - k_x^2 - k_y^2}}, & k_0^2 > k_x^2 + k_y^2 \\ 0, & k_0^2 \leq k_x^2 + k_y^2 \end{array} \right\}. \quad (24)$$

233 To compute the CSM of noise, Eq. (14) can now be evaluated numerically using the rectangular  
 234 integration method. For the DAF excitation, since the normalized CSD function  $\tilde{\phi}_{pp}(k_x, k_y, \omega)$  is null  
 235 for the wavenumbers larger than the acoustic wavenumber, the wavenumber domain of integration in  
 236 Eq. (14) is basically inside the acoustic circle defined by  $\Omega_a = \{\mathbf{k} \in \mathbb{R}^2, |\mathbf{k}| < k_0\}$ . Finally, Eq. (3) can  
 237 be used to calculate the beamforming output and perform source localisation.

### 238 3.3. Model Verification

To verify the mathematical models of the panel under monopole and DAF excitations, the spatial average of the ASD of the panel acceleration under each excitation is analytically computed and is compared with those obtained using the finite element method (FEM) in COMSOL as shown in Figures 3 and 4, respectively. The spatial average of the ASD of the panel acceleration is given by [25]

$$\langle a^2 \rangle = \frac{1}{A_p} \int_{\Omega} S_{aa}(\mathbf{x}, \omega) d\mathbf{x}, \quad (25)$$

where  $A_p$  is the surface area of the panel, for the monopole excitation Eq. (25) can be written as follows

$$\langle a^2 \rangle = \frac{\omega^4}{4} \sum_{m=1}^M \sum_{n=1}^N \left| \frac{F_{mn}(\mathbf{x}_q)}{\Omega(\omega_{mn}^2 - \omega^2 + j\eta\omega\omega_{mn})} \right|^2, \quad (26)$$

and for the DAF excitation it can be expressed as [25]

$$\langle a^2 \rangle = \frac{\omega^4}{16\pi^2} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{m=1}^M \sum_{n=1}^N \left| \frac{\psi_{mn}(k_x^i, k_y^j)}{\Omega(\omega_{mn}^2 - \omega^2 + j\eta\omega\omega_{mn})} \right|^2 \phi_{pp}(k_x^i, k_y^j, \omega) \delta k_x \delta k_y, \quad (27)$$

239 where  $\delta k_x$  and  $\delta k_y$  are the wavenumber resolutions in the  $k_x$  and  $k_y$  directions, respectively.  $N_x$ ,  $N_y$   
 240 are the number of points considered for sampling the wavenumber domain. For numerical analysis,  
 241 the dimensions and material properties of the panel are listed in Table 1. The fluid density and speed  
 of sound were set to 1.225 kg/m<sup>3</sup> and 343 m/s, respectively. It should be mentioned that the standard

Table 1. Dimensions and material properties of the panel

Parameter	Value
Young's modulus, $E$ (GPa)	70
Poisson's ratio, $\nu$	0.3
Mass density, $\rho_s$ (kg/m <sup>3</sup> )	2700
Length, $L_x$ (mm)	480
Width, $L_y$ (mm)	420
Thickness, $h_s$ (mm)	3.2
Damping loss factor, $\eta$	0.01

242

243 FEM is an element-based method which can be used for analysing a structure under deterministic  
 244 excitation. It can be directly used for monopole excitation as it is deterministic. However, the DAF  
 245 excitation is random. Hence, to be able to simulate the plate response under the DAF excitation, the  
 246 diffuse sound field is defined as a sum of 100 uncorrelated plane waves moving in random directions.  
 247 The response of the plate to 30 realizations of these deterministic excitation forces are then computed  
 248 and the final response is obtained by calculating the ensemble average over all the different realizations  
 249 at each frequency [27, 28]. For this verification, the monopole is located at  $(x,y,z)=(0.240$  m, 0.227 m,  
 250 0.017 m) and a unity ASD for the DAF in both models are assumed.

251 It can be seen from Figures 3 and 4 that analytical results are in excellent agreement with the  
 252 FEM results particularly in lower frequencies. A small shift in frequency can be observed in the  
 253 response at higher frequencies between the two predictions. This can be attributed to the fact that the  
 254 classical plate theory (CPT) is used in the analytical model which ignores transverse shear deformation  
 255 whereas it was included in the FEM model. It is well known that CPT overestimates the higher natural

256 frequencies as it is the case for our comparison. However, this is not affecting our numerical analysis  
 257 for evaluating the beamforming performance in this work. The vertical blue lines in Figures 3 and 4  
 258 correspond to the selected frequencies for numerical investigations in the next section, which correspond  
 259 to one non-resonance frequency (1489 Hz) and three resonance frequencies (1614 Hz, 2425 Hz and 3114  
 260 Hz).

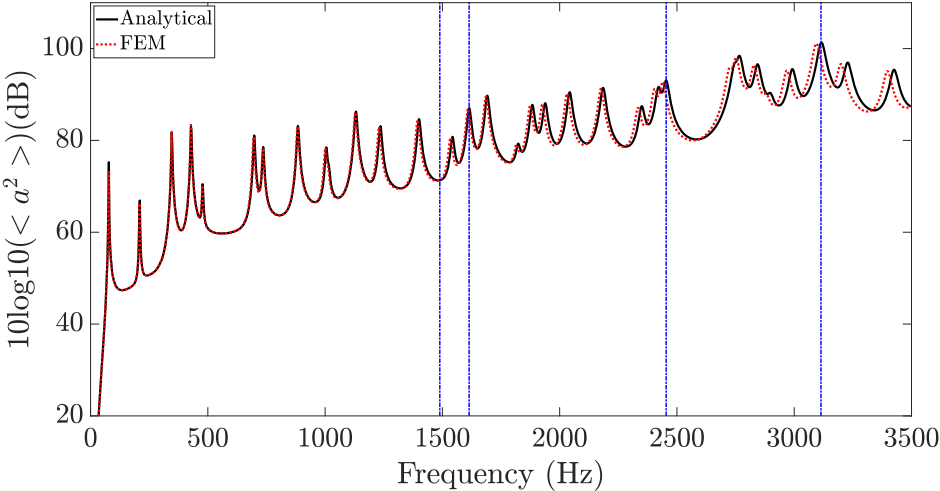


Figure 3: Predicted spatial average of the ASD of the panel acceleration due to monopole excitation (dB ref. 1 (m/s<sup>2</sup>)<sup>2</sup>).

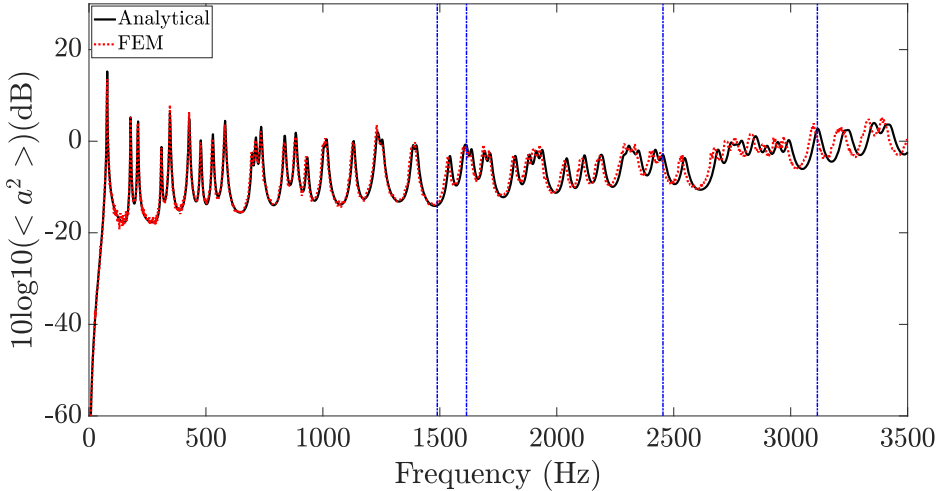


Figure 4: Predicted spatial average of the ASD of the panel acceleration due to DAF excitation (dB ref. 1 (m/s<sup>2</sup>)<sup>2</sup>).

261

262 *3.4. Signal to Noise Ratio in Function of the Source Distance from the Plate*



263 It is assumed that the sound field in the room is diffuse. One of the properties of this diffuse field is  
 264 that the sound pressure level is the same for all positions in the room, and there is no preferred sound  
 265 direction. Although this is roughly fulfilled at frequencies above the Schroeder frequency, we know  
 266 from experience that as we get close to a source, we can readily hear where the sound comes from and  
 267 the level increases as we get closer. We are in the immediate field of the source in this situation. Since  
 268 we have both direct and diffuse fields in a room, the point where both fields have equal strength is the  
 269 transition between them.

270 In order to evaluate performance of vibroacoustic beamforming, the numerical results in the next  
 271 section will be compared against the corresponding results obtained by the acoustic beamforming using  
 272 a microphone array located on a rigid wall in the room. Therefore, it is appropriate to define the SNR  
 273 based on the pressure rather than acceleration to be able to achieve such a comparison in the same  
 274 pressure field condition. Let us define the SNR as the ratio expressed in dB of the ASD of the blocked  
 275 pressure at the centre of the array due to the direct field induced by the source to be localised,  $S_{pp}^{\text{direct}}$ ,  
 276 over the ASD of the diffuse field  $S_{pp}^{\text{diffuse}}$  as follows

$$\text{SNR} = 10 \log_{10} \left( \frac{S_{pp}^{\text{direct}}(r_s, \omega)}{S_{pp}^{\text{diffuse}}(\omega)} \right), \quad (28)$$

277 to express the SNR as a function of the distance between the source and the centre of the array and  
 278 the room characteristics, one can write  $S_{pp}^{\text{direct}}$  using Eq. (19) as

$$S_{pp}^{\text{direct}}(r_s, \omega) = p(\mathbf{x}_q, \mathbf{x}', \omega) p^*(\mathbf{x}_q, \mathbf{x}', \omega) = \frac{\rho_0^2 \omega^2 Q_v^2}{4\pi^2 r_s^2}, \quad (29)$$

279 the sound power radiated by a monopole source in an unbounded space is given by [31, 32]

$$W = \frac{Q_v^2 \rho_0 \omega^2}{8\pi c_0}, \quad (30)$$

280 Eq. (29) can be then rewritten in terms of sound power as follows

$$S_{pp}^{\text{direct}} = \frac{2\rho_0 c_0 W}{\pi r_s^2}. \quad (31)$$

281 On the other hand, the ASD of the blocked pressure related to the diffuse field can be expressed as a

282 function of the sound power and the reverberation time  $TR_{60}$  using the Sabine theory [33, 34],

$$S_{pp}^{\text{diffuse}}(\omega) = \frac{50W\rho_0c_0TR_{60}}{V}, \quad (32)$$

283 where  $V$  is the room volume. Using Eqs. (31) and (32), SNR can be finally expressed as

$$\text{SNR} = 10\log_{10}\left(\frac{V}{25\pi r_s^2 TR_{60}}\right). \quad (33)$$

284

#### 285 **4. Performance Assessment of the Acoustic Beamforming versus Vibroacoustic Beam-** 286 **forming**

To evaluate performance of the proposed VABF for a source localization in a room, its beamforming outputs are compared with those obtained using the ABF. Twenty sensors (microphones/accelerometers) are used in a circular array of radius 95.5 mm which are evenly distanced by 30 mm along the circumference of the circle. To avoid spatial aliasing and adequately sample the vibration data by the sensors for a given wavelength, the distance between the sensors  $d$  must satisfy the following criterion:

$$d < \frac{1}{2}\min\{\lambda_0, \lambda_f\}, \quad (34)$$

287 where  $\lambda_0$  and  $\lambda_f$  are the acoustic wavelength and plate flexural wavelength, respectively. Clearly, for  
288 the ABF only acoustic wavelength needs to be considered in Eq. (34).

289 To simulate a realistic condition for SNR calculation, a standard reverberant room with the volume  
290 of 147 m<sup>3</sup> and the mean reverberation time of 3.1 s (averaged over the frequency range) are chosen.  
291 The SNR in the room is a function of distance between the source and the receiver (centre of the  
292 sensor array) as indicated by Eq. (33). It should be noted that to have different SNRs in this work,  
293 the source is moved away from the centre of the array in  $yz$ -plane along a line which makes 45-degree  
294 angle with  $y/z$ -axes. Three different scenarios for SNR are herein examined as follows:

- 295 • Case 1: SNR=30 dB, which is the case when the source is very close to the receiver ( $r_s=25$  mm).  
296 Therefore, the direct field is significantly stronger than reverberant field.
- 297 • Case 2: SNR=0 dB, this is the case when the source is at the certain distance from the receiver  
298 ( $r_s=777$  mm) such that both fields have the same strength.

299 • Case 3: SNR=-5 dB, which is the case when the source is further away from the receiver  
300 ( $r_s=1382$  mm) and the reverberant field is dominant.

301 It should be noted that to fully describe the excitation fields for the numerical analysis, first the  
302 location of the source ( $r_s$ ) is determined based on the given SNR using Eq. (33). The ASD of the  
303 direct pressure field is then obtained from Eq. (29) for a unit monopole source (i.e.  $Q_v=1$  m<sup>3</sup>/s).  
304 Finally, the ASD of the DAF pressure is computed from Eqs. (30) and (32), and is applied to both  
305 acoustic and vibroacoustic beamformings. It is worth noting that the performance of the beamforming  
306 treatments will be independent of the value of the source strength, as the ASD of the direct and diffuse  
307 fields are proportional to this value. Hence, only the value of the SNR is important in the numerical  
308 investigations.

309 In what follows, the source maps generated using both the ABF and VABF are presented for three  
310 different SNRs as mentioned above. In addition to source maps which are useful for visualization, to  
311 quantify the performance of each beamforming technique, spatial resolution ( $\delta$ ) and dynamic range ( $\mu$ )  
312 are computed for each case and are shown above each source map. The dynamic range is herein defined  
313 as the difference between the mainlobe and the highest sidelobe in the source map in the detection  
314 space, and the array spatial resolution is defined as the width of the mainlobe in the  $x$ -direction 3 dB  
315 below its peak. The resolution refers to the beamforming technique's capacity to resolve and separate  
316 sources that are close together. The dynamic range, also known as sidelobe level, is a measurement of  
317 the array's capacity to reject sources coming from directions where the array is not guided. Sidelobes  
318 are basically non-focus local maxima that cause misleading peaks and sources in the acoustic map.

#### 319 4.1. Case 1, SNR=30 dB

320 Figures 5 and 6 show the source maps of a monopole source for both the ABF and VABF using a  
321 circular array obtained by the CBF, FBF, MVDR and MPDR respectively at two distinct frequencies  
322 of 1489 Hz and 1614 Hz when SNR=30 dB. The actual source location is shown by a small white cross  
323 in the source map. The output of beamforming is normalised so that the mainlobe peak is at 0 dB.  
324 According to Figure 5, at the non-resonance frequency of 1489 Hz, dynamic ranges in the VABF is  
325 considerably lower than those in the ABF using CBF, FBF, MVDR, and CBF produces the worst  
326 sound map with dynamic ranges of 10 dB and 3 dB for the ABF and VABF, respectively. However,  
327 we can see that using the MPDR, both the ABF and VABF produce clear sound map with almost the  
328 same spatial resolution, and dynamic ranges of 35 dB and 32 dB, respectively.

329 A similar study is conducted in Figure 6 where a resonance frequency of 1614 Hz is considered.  
330 As the two considered frequencies are close to each other, the beamforming outputs obtained using  
331 the ABF are very similar as can be seen in Figures 5 and 6. However, unlike the ABF in which a  
332 microphone array is used, in the VABF vibration sensors are employed. Therefore, performance of the  
333 VABF significantly depends on the dynamic behavior of the structure. It is expected that at resonance  
334 frequencies, the VABF performs poorly compared with the ABF due to the strong coherence of the  
335 vibratory field, even though the wall pressure field induced by the noise is weakly coherent [19]. This  
336 is indeed the case as shown in Figure 7 where the coherence of the vibratory field between all 20  
337 sensors are plotted at 1489 Hz and 1614 Hz. At 1489 Hz, the coherence between any two different  
338 accelerometers is low as indicated by blue color, whilst a strong coherence can be observed at 1614 Hz  
339 mostly in yellow/orange color spread all over the map. This is why the source localisation is not  
340 feasible using the CBF, FBF and it is very difficult via the MVDR due to the low dynamic ranges of  
341 0 dB, 1 dB and 8 dB, respectively. However, the source is easily localised using the VABF via the  
342 MPDR technique with a high dynamic range of 28 dB. It is evident from Figures 5 and 6 that if MPDR  
343 is used, both the ABF and VABF are able to localise the source with a high dynamic range at both  
344 resonance and non-resonance frequencies when the signal (direct acoustic field) is considerably higher  
345 than noise (reverberant acoustic field).

346

347

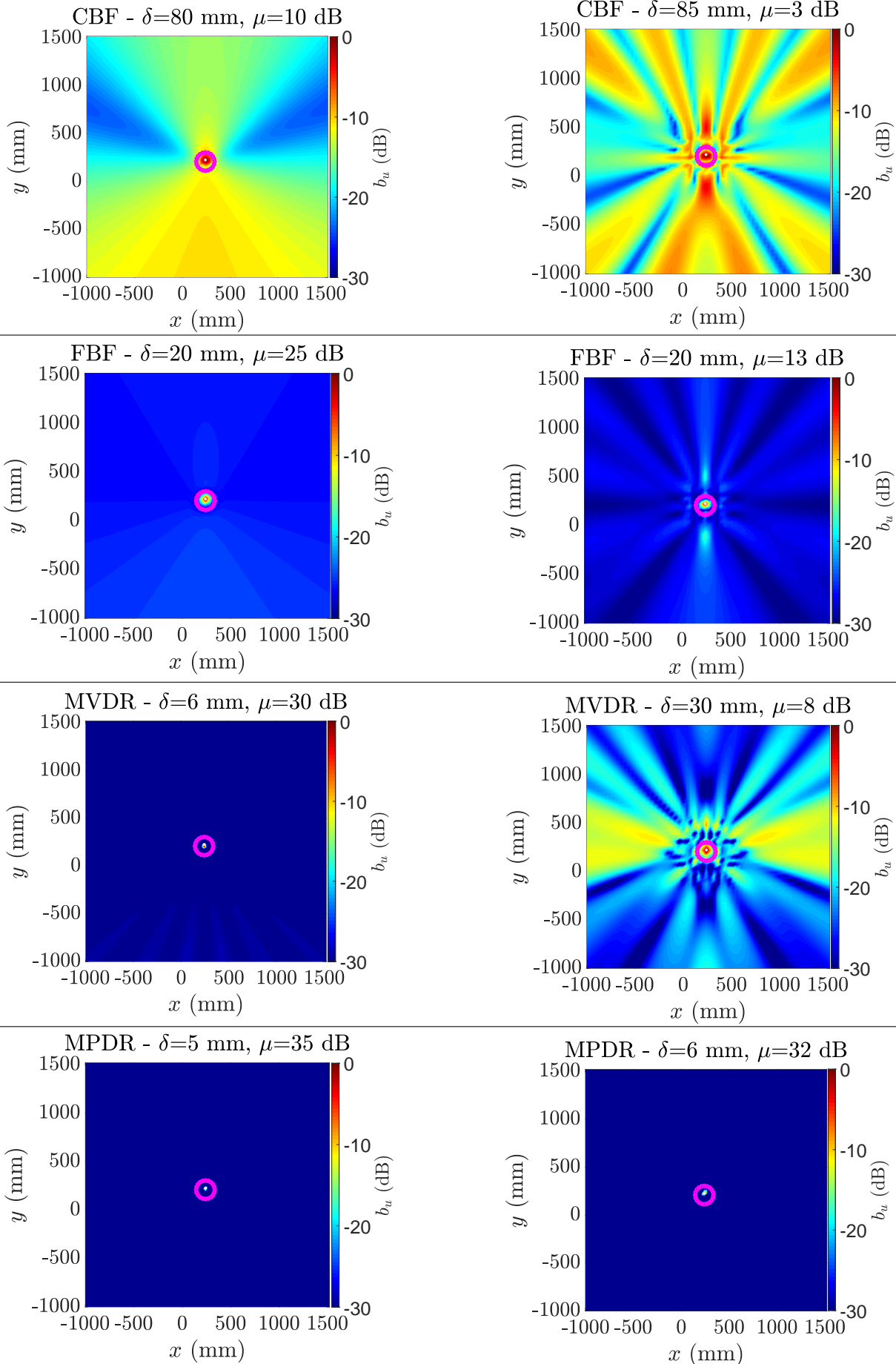


Figure 5. Sound map of a monopole source at 1489 Hz with SNR=30 using (a) acoustic beamforming (left column) and (b) vibroacoustic beamforming (right column) obtained by CBF, FBF, MVDR and MPDR.

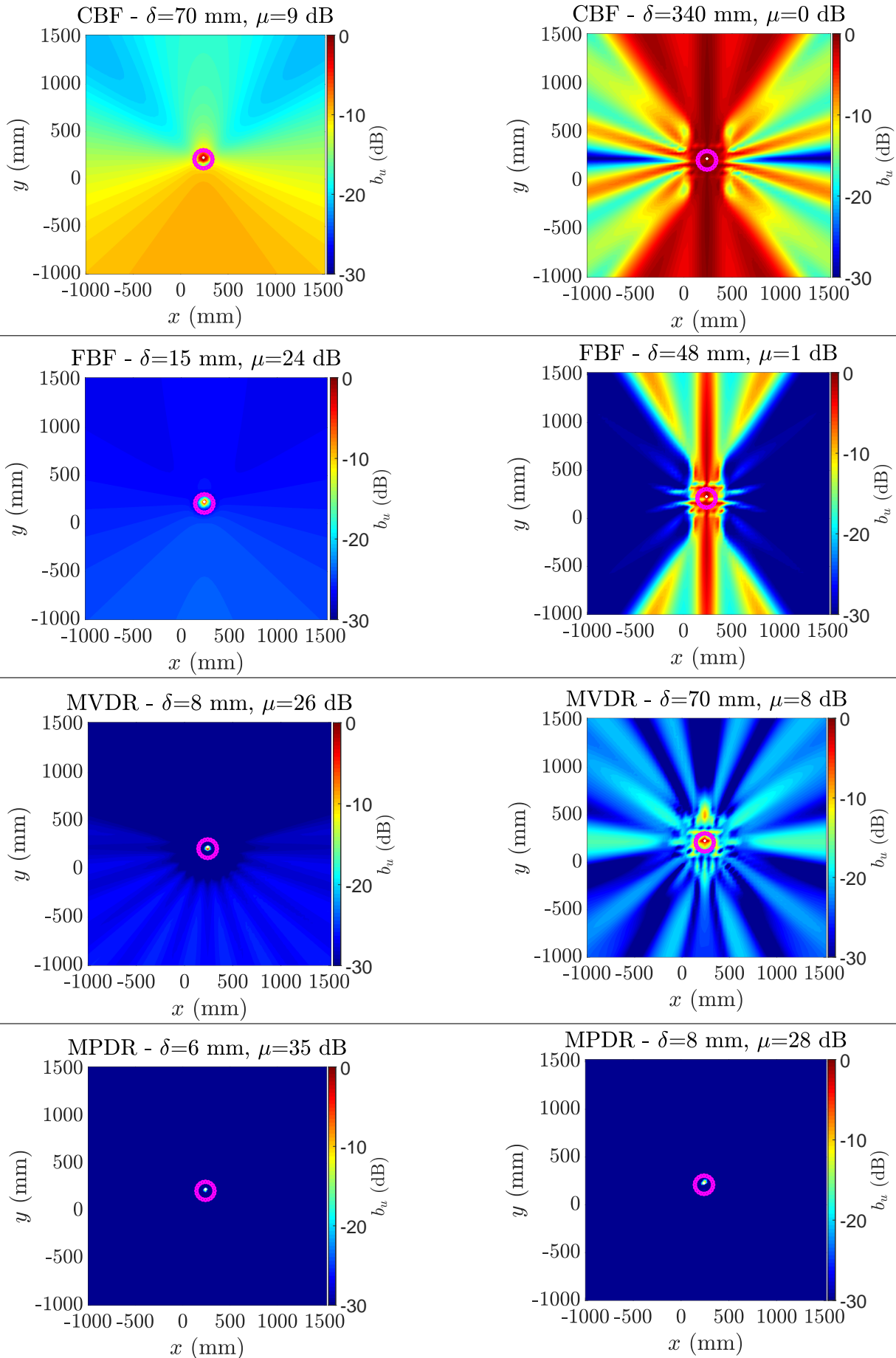


Figure 6. Sound map of a monopole source at 1614 Hz with SNR=30 using (a) acoustic beamforming (left column) and (b) vibroacoustic beamforming (right column) obtained by CBF, FBF, MVDR and MPDR.

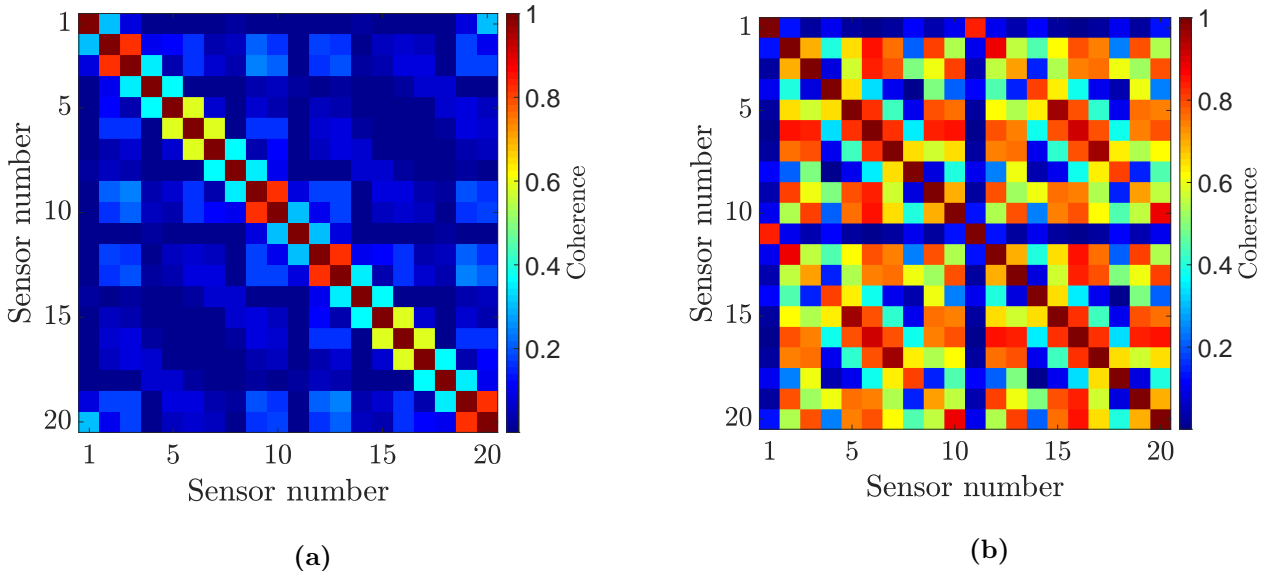


Figure 7. Coherence between 20 accelerometers in the circular array for SNR=30 at frequencies of (a) 1489 Hz and (b) 1614 Hz.

348 4.2. Case 2 (SNR=0 dB) and Case 3 (SNR=-5 dB)

349 The studies in Figures 5 and 6 are now repeated for the case when SNR=0 dB as shown in Figures 8  
 350 and 9. Figure 8 reveals that at non-resonance frequency of 1489 Hz both the ABF and VABF fail  
 351 to localise the source when using the CBF or FBF due to very low dynamic range. Although the  
 352 FBF proved to be able to suppress the sidelobes and improve the sound map quality for SNR=30 dB  
 353 at 1489 Hz as shown in Figure 5, when the noise level is as high as the signal level, its performance  
 354 deteriorates. The MPDR continues to provide the most clear sound map in both the ABF and VABF  
 355 with the dynamic ranges of 8 dB and 7 dB, respectively. As shown in Figure 8, the MPDR in the  
 356 VABF generates almost the same dynamic range as that in the ABF. However, the spatial resolution  
 357 achieved by the VABF is nearly half of that obtained in the ABF.

358 Let us now consider the resonance frequency of 1614 Hz in Figure 9 for SNR=0 dB. It can be  
 359 observed that at this frequency, both the ABF and VABF perform poorly in particular using the CBF  
 360 or FBF due to low dynamic range. The dynamic range in the ABF using MVDR is 7 dB which is  
 361 the same as that in the VABF. It can be seen from both Figures 8 and 9 that for all beamforming  
 362 techniques except the CBF and FBF, spatial resolutions in the VABF is significantly smaller than  
 363 those in the ABF when SNR=0 dB. At 1614 Hz, both beamforming methods have the same dynamic  
 364 range of 7 dB using the MVDR as shown in Figure 8. The dynamic range of the MPDR-based VABF is  
 365 3 dB higher than that of the corresponding output in the ABF, and the VABF spatial resolution is less  
 366 than half of that in the ABF. Hence, localisation is slightly easier using the VABF at this frequency.  
 367 So far, it has been demonstrated that the MPDR produces the best beamforming output for source  
 368 localisation. Therefore, we only use this technique for all the subsequent numerical investigations.

369 Since in the VABF vibration data is collected at the surface of the structure, a question that  
370 might arise is that how the structural damping will affect the VABF performance when the excitation  
371 frequencies coincide with resonance frequencies of the structure especially at low SNRs. Hence, in the  
372 subsequent section the effect of damping on the source localisation is investigated.

#### 373 4.2.1. *Effect of Structural Damping on the Vibroacoustic Beamforming* 375

376 It is well known that structural damping has a prominent role in the structure response when it is  
377 subjected to an excitation at a frequency that is close to a natural frequency. Exactly at resonance,  
378 the vibration amplitude becomes substantially large and the vibratory field exhibits strong coherence  
379 as demonstrated in Figure 7. The actual amplitude at resonance is controlled solely by the amount of  
380 damping. Hence, in this section the effect of damping on the source localisation is investigated. Figure  
381 10 presents the effect of structural damping on the dynamic range and spatial resolution using the  
382 MPDR for the circular array at four distinct frequencies of 1489 Hz, 1614 Hz, 2454 Hz and 3114 Hz  
383 when SNR=0 dB. The marker symbols correspond to the results for the VABF, while the horizontal  
384 lines represents corresponding results for the ABF as they are independent of structural damping.

385 It is evident from Figure 10(a) that dynamic range in the VABF can be improved by increasing  
386 structural damping at all considered frequencies except at 1614 Hz where the dynamic range slightly  
387 decreases as structural damping increases. For  $\eta \geq 0.1$  the VABF achieves a dynamic range higher  
388 than that of the ABF at 1489 Hz, 1614 Hz and 2454 Hz. However, at 3114 Hz, to enhance the VABF  
389 dynamic range to the same level as that in the ABF, a damping loss factor of 0.4 is required. It should  
390 be noted that the overall dynamic range is low for both methods when SNR=0 dB, for example, by  
391 having a loss factor of 0.3, one could achieve a similar dynamic range (1 dB difference) for both methods  
392 at 3114 Hz. Moreover, by further increasing the loss factor, the dynamic range converges to a certain  
393 value for higher loss factors at all frequencies. It is also observed that the effect of damping loss factor  
394 on the dynamic range is greater at higher resonance frequencies. For example, increasing damping  
395 loss factor from 0.02 to 0.14 has almost doubled the dynamic range at frequency of 2454 Hz as shown  
396 in Figure 10(a). This behavior is attributed to the coherence reduction of the vibratory field due to  
397 the high damping loss factor. Figure 11 presents the coherence between different accelerometers for  
398 two different loss factors of 0.01 and 0.3 at 2454 Hz. It can be seen that the coherence is considerably  
399 reduced by increasing the loss factor from 0.01 to 0.3.

400 Figure 10(b) shows the spatial resolution as a function of damping loss factor. This figure indicates  
401 that the spatial resolution obtained from the VABF is considerably smaller than that in the ABF.



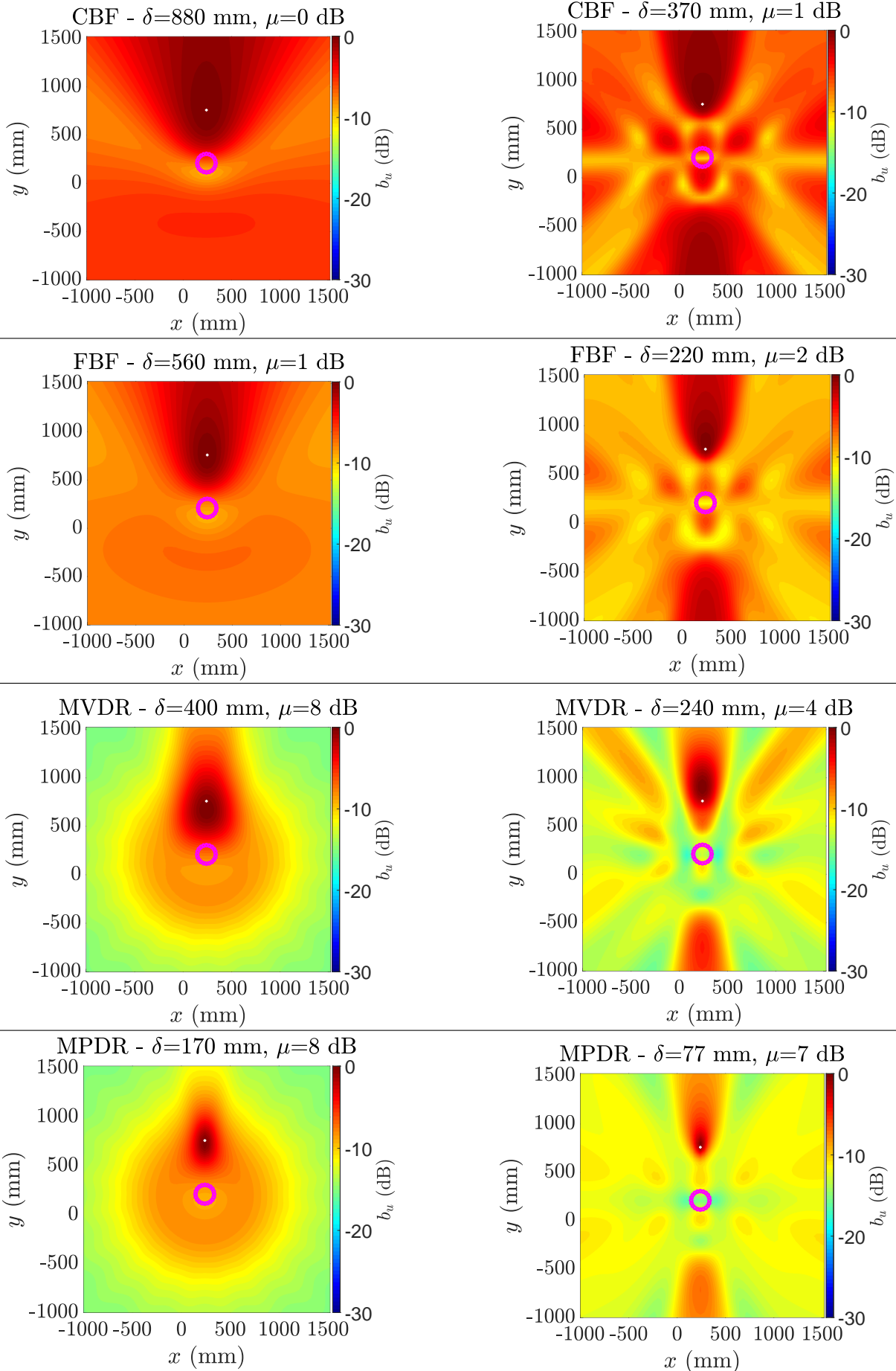


Figure 8. Sound map of a monopole source at 1489 Hz with SNR=0 using (a) acoustic beamforming (left column) and (b) vibroacoustic beamforming (right column) obtained by CBF, FBF, MVDR and MPDR.

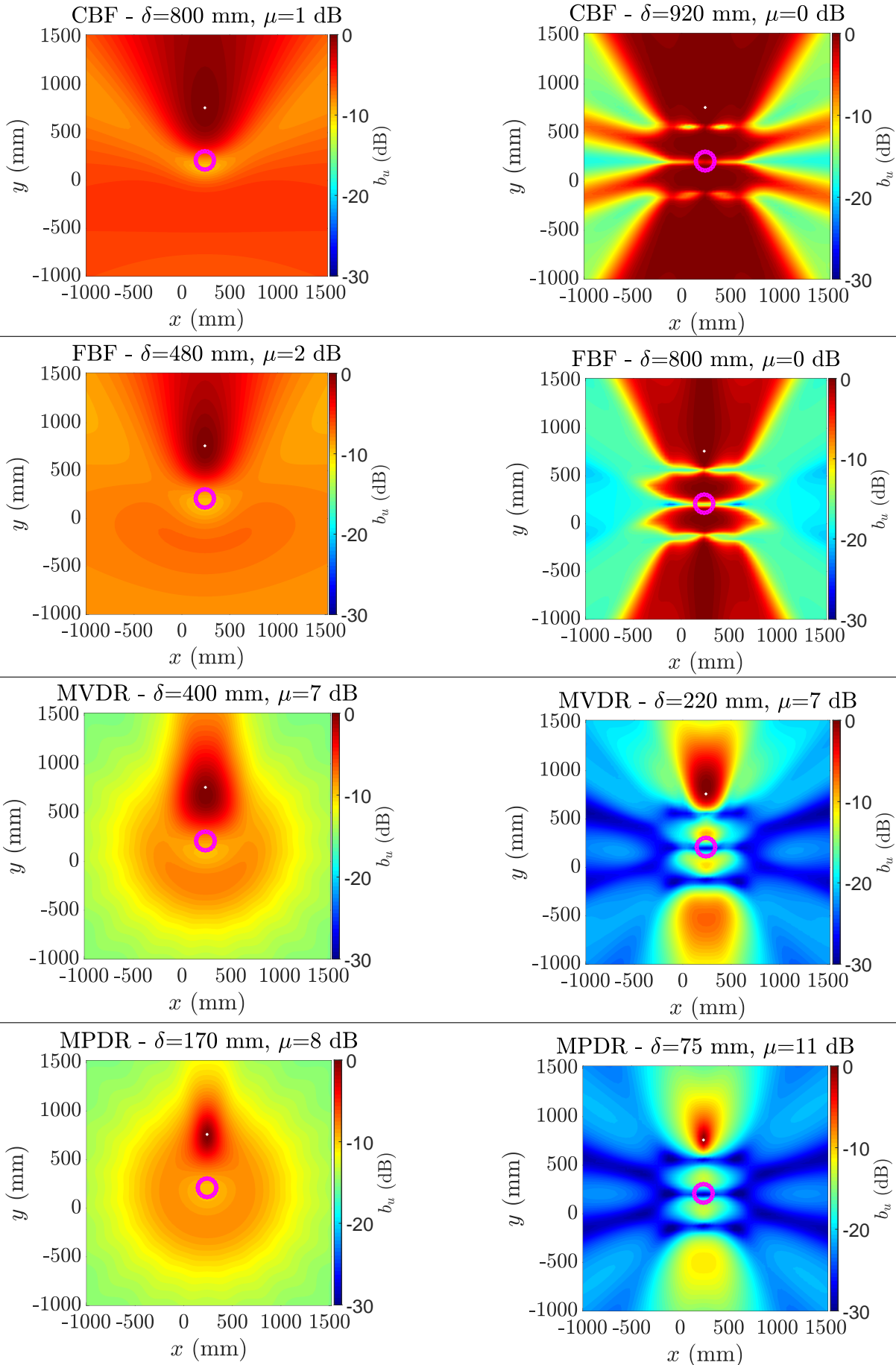


Figure 9. Sound map of a monopole source at 1614 Hz with SNR=0 using (a) acoustic beamforming (left column) and (b) vibroacoustic beamforming (right column) obtained by CBF, FBF, MVDR and MPDR.

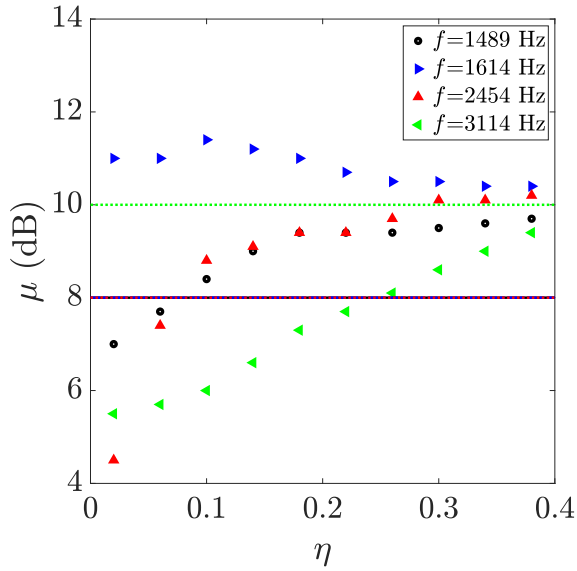
402 This was also demonstrated in Figures 8 and 9. Moreover, increasing structural damping adversely  
403 affect the spatial resolution. However, regardless of the damping value, the VABF spatial resolution  
404 is always smaller than that in the ABF. The spatial resolution is also converging to a certain value as  
405 damping increases.

406 To further examine the performance of the VABF for low SNRs and to visualize the source map  
407 of the VABF for higher structural damping, the VABF outputs at frequencies of 1489 Hz, 1614 Hz,  
408 2454 Hz and 3114 Hz with  $\eta = 0.3$  and SNR=0 dB or -5 dB are compared with corresponding outputs  
409 using the ABF in Figures 12 and 13, respectively. Generally, a more clear sound map can be generated  
410 using the VABF compared with the map obtained using the ABF for both SNRs at most considered  
411 frequencies. When SNR=0 dB, the dynamic ranges and spatial resolutions of the ABF are respectively  
412 7-10 dB and 125-170 mm, whilst using the VABF corresponding values are 9-10 dB and 80-96 mm.  
413 This behavior holds true for SNR=-5 dB where the dynamic ranges of the VABF are very similar  
414 to those of the ABF with significantly smaller spatial resolutions. For SNR=-5 dB that noise level  
415 is higher than the direct acoustic field, both beamforming methods fail to localise the source due to  
416 the low dynamic range and the presence of pronounced sidelobe contamination in the sound map at  
417 all considered frequencies. According to Figures 12 and 13, by choosing an appropriate damping loss  
418 factor, one can acquire a similar performance in source localisation using the VABF compared with  
419 that of the ABF at low SNRs.

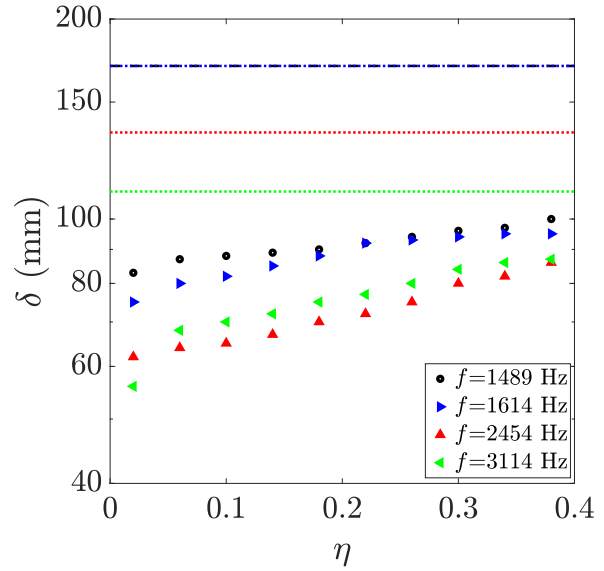
420

#### 422 4.3. *Effect of Array Pattern on the Vibroacoustic Beamforming*

421 Figure 14 presents the source map of a monopole source using three different array patterns namely  
423 circular, cross and random patterns at frequency of 3114 Hz using the ABF and VABF for SNR=-5 dB.  
424 The damping loss factor is set to  $\eta = 0.3$ . In all three cases, 20 sensors are employed and the criterion  
425 in Eq. (34) is respected. The size of the array is kept almost the same for different patterns. It is  
426 apparent from this figure that the circular and random arrays provide the same dynamic range for the  
427 VABF but the cross array has slightly lower dynamic range. The best beamforming output for the  
428 ABF is generated using the random array. Although using different array pattern slightly changes the  
429 beamforming performance at different frequencies, based on the data in Figure 14 both beamforming  
430 methods exhibit roughly similar performance using different array pattern. It should be noted that  
431 although a circular array was used for numerical analysis and evaluation of the VABF performance in  
432 the previous sections, the same conclusions can be drawn using other array patterns.  
433

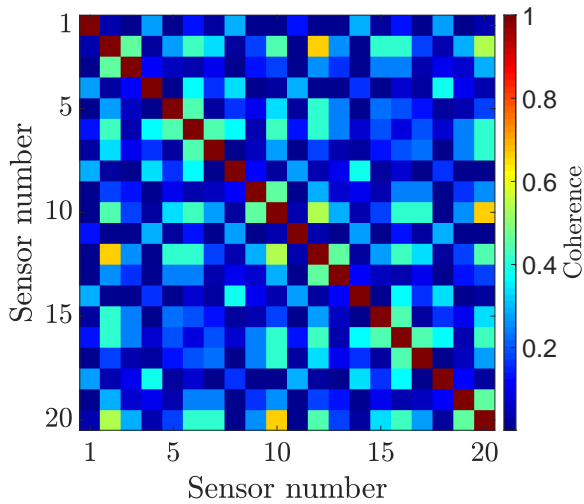


(a)

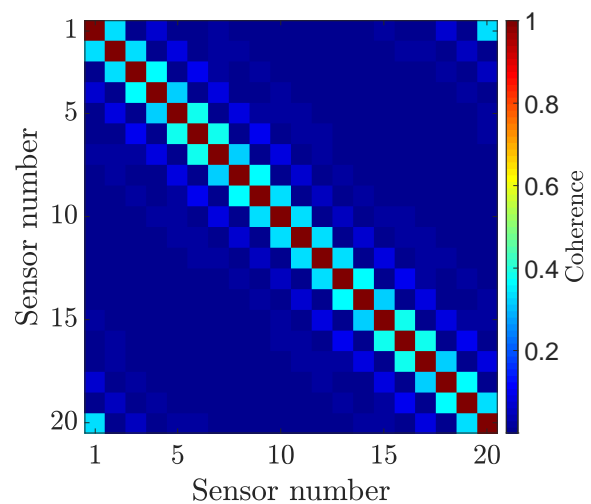


(b)

Figure 10. Effect of structural damping on (a) dynamic range and (b) spatial resolution of the vibroacoustic beamforming for a circular array using MPDR at four distinct frequencies of 1489 Hz, 1614 Hz, 2454 Hz and 3114 Hz; the horizontal lines represent the corresponding dynamic range and spatial resolution using the acoustic beamforming.



(a)



(b)

Figure 11. Coherence between 20 accelerometers in the circular array for SNR=0 at frequency of 2454 Hz (a)  $\eta=0.01$  and (b)  $\eta=0.3$ .

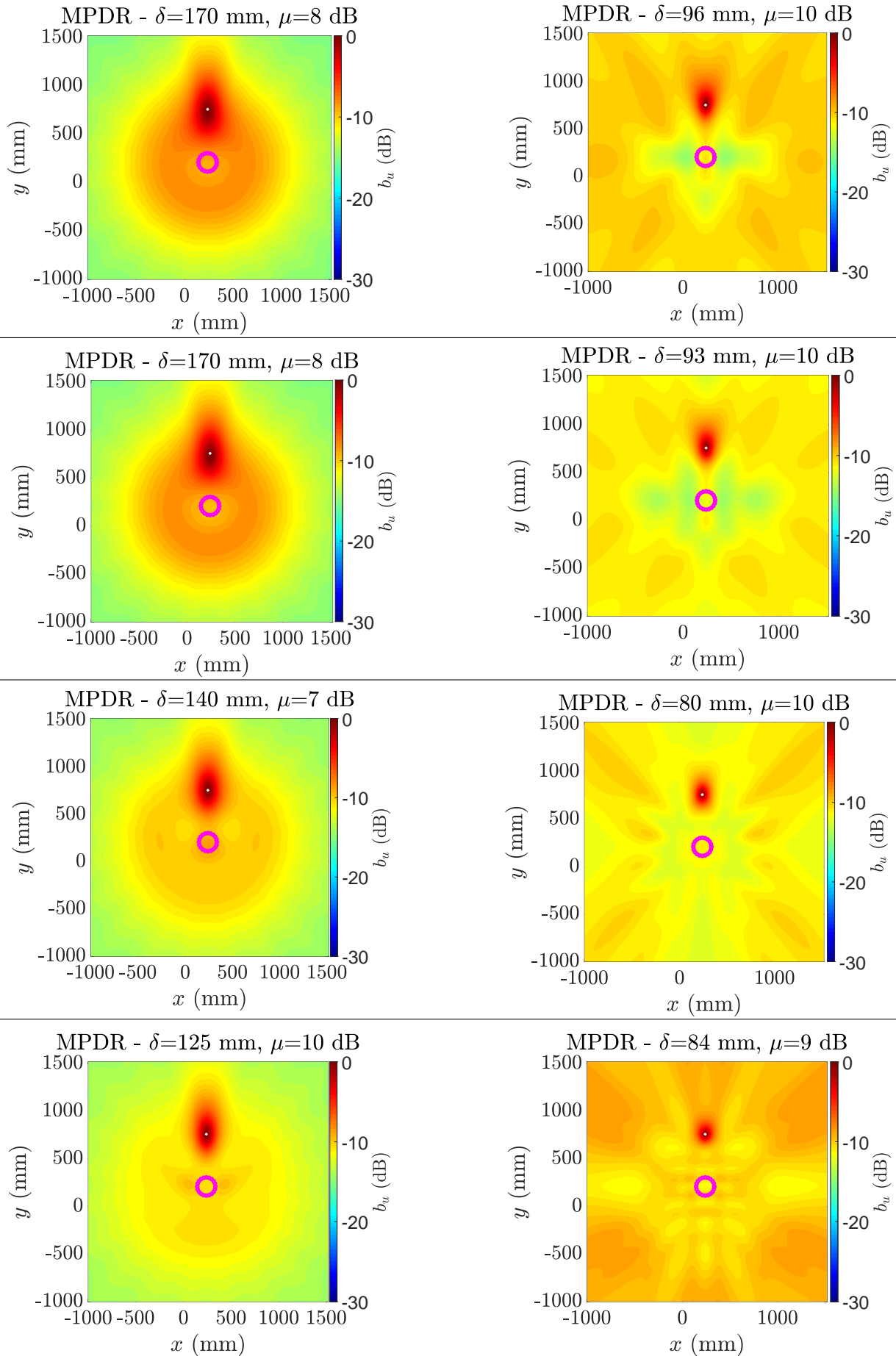


Figure 12. Sound map of a monopole source at 1489 Hz, 1614 Hz, 2454 Hz and 3114 Hz (1<sup>st</sup>-4<sup>th</sup> rows, respectively) with SNR=0 using (a) acoustic beamforming (left column) and (b) vibroacoustic beamforming (right column) with a loss factor of 0.3 obtained by MPDR.

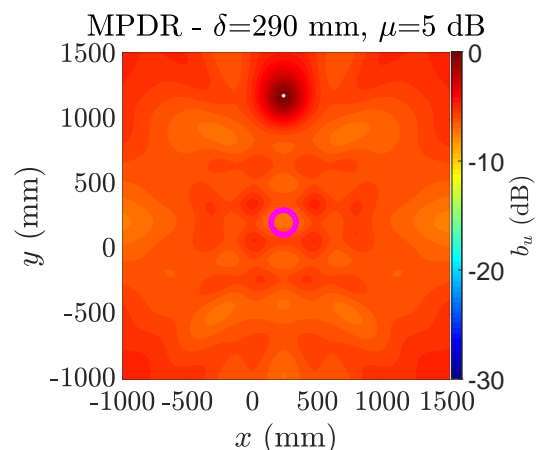
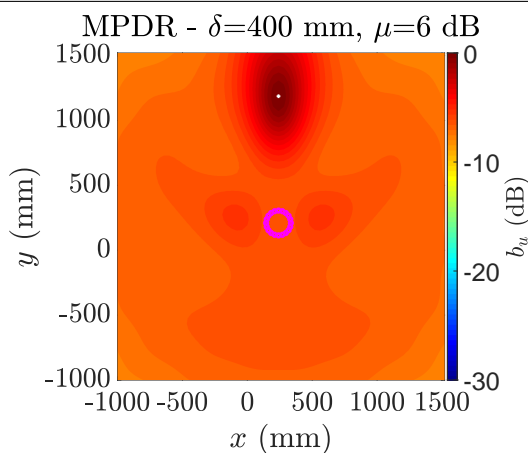
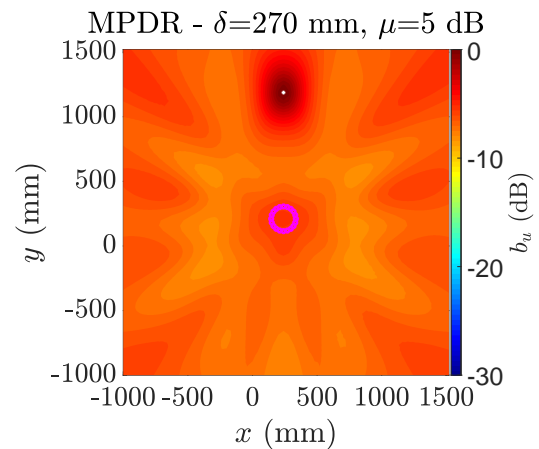
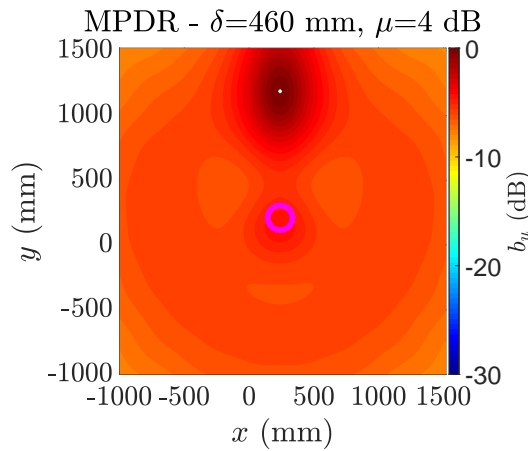
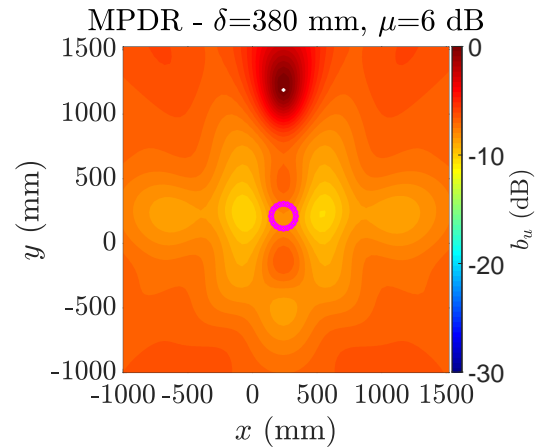
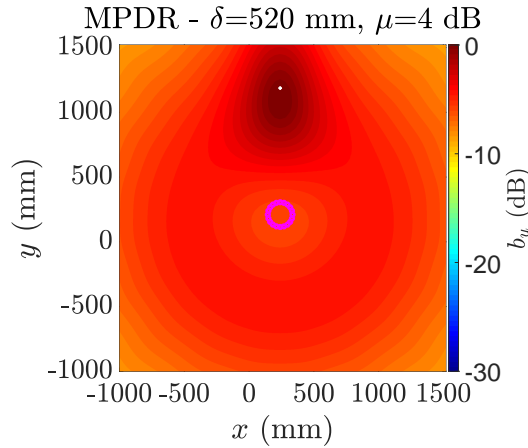
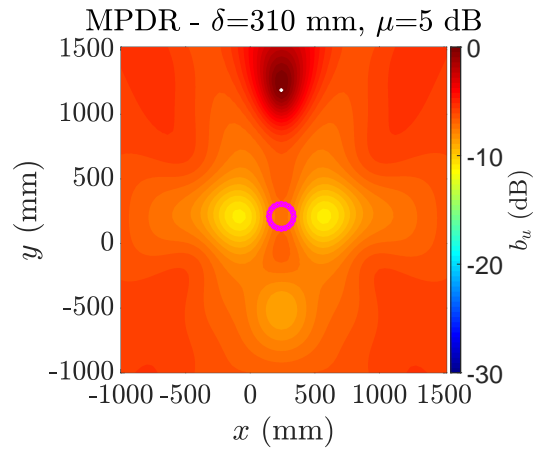
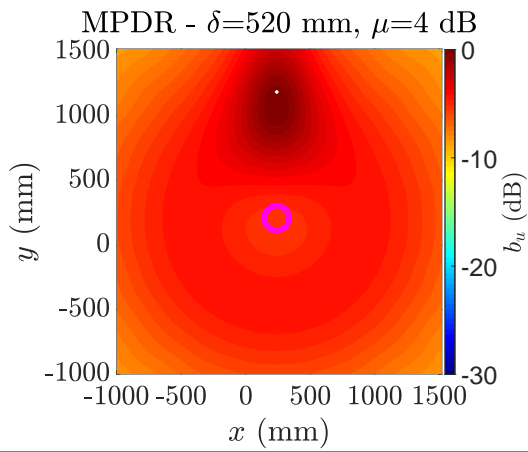


Figure 13. Sound map of a monopole source at 1489 Hz, 1614 Hz, 2454 Hz and 3114 Hz (1<sup>st</sup>-4<sup>th</sup> rows, respectively) with SNR=-5 using (a) acoustic beamforming (left column) and (b) vibroacoustic beamforming (right column) with a loss factor of 0.3 obtained by MPDR.

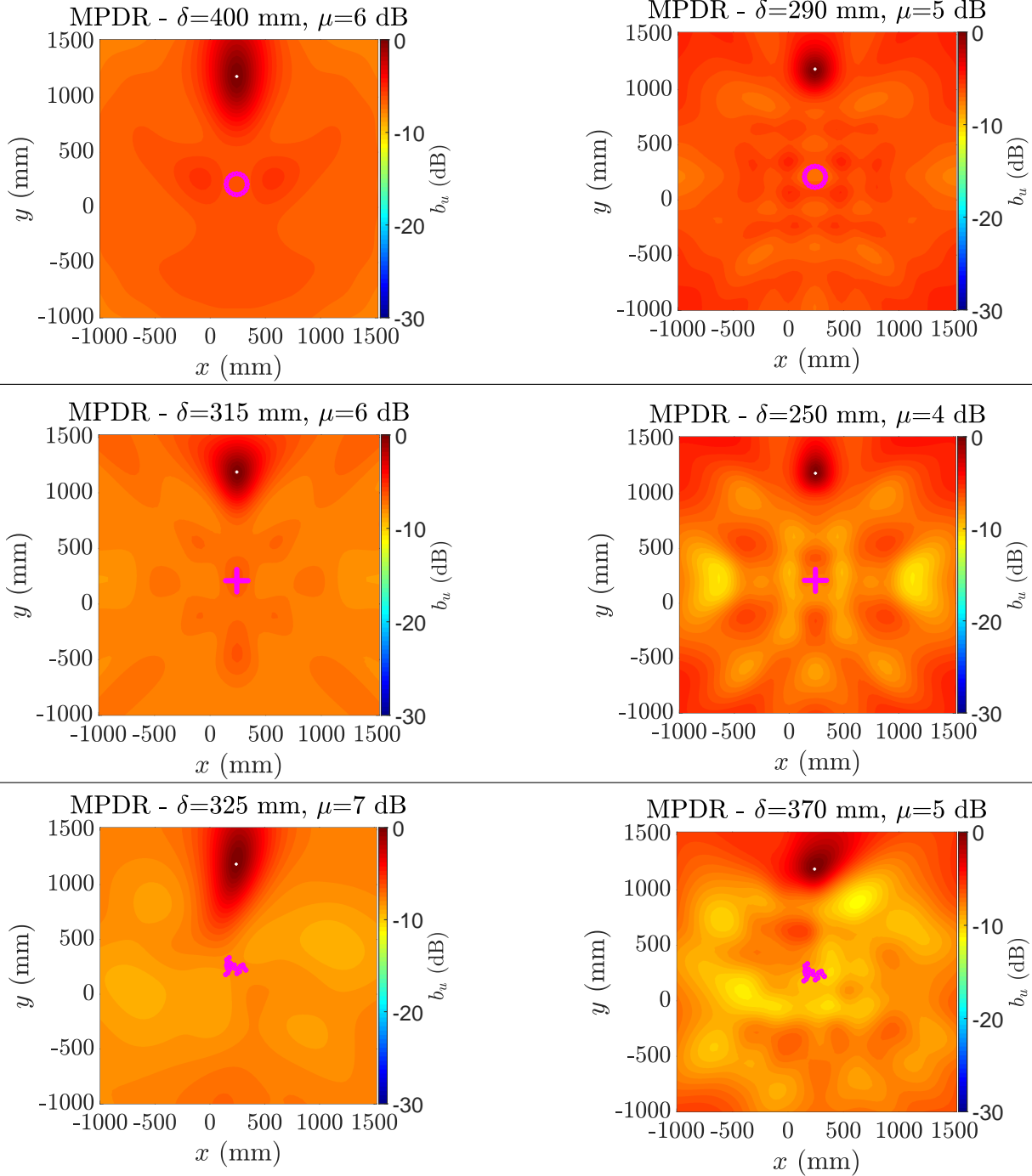


Figure 14. Sound map of a monopole source at 3114 Hz with SNR=-5 obtained by circular, cross and random arrays using (a) acoustic beamforming (left column) (b) vibroacoustic beamforming (right column) with a loss factor of 0.3 obtained by MPDR.

## 435 5. Conclusions

436 A vibroacoustic beamforming was proposed for localising an acoustic source using vibration data.  
437 This approach is an alternative to the traditional acoustic beamforming where a microphone/hydrophone  
438 array has to be placed in the acoustic medium and beamforming algorithm is then applied to measured  
439 pressure signal. VABF can be useful when it is not feasible to put the sensor array in the fluid. As  
440 the beamforming techniques are applied to the vibration data, the sensor array could be outside the  
441 targeted acoustic medium and can be mounted on the structure that is in contact with the fluid in  
442 which a source needs to be localised. The proposed method was applied to source localisation in a  
443 room. Performance of the VABF was examined by employing CBF, FBF, MVDR and MPDR and  
444 comparing the predicted results with corresponding data obtained from the ABF prediction. It was  
445 demonstrated that the MPDR produces the highest dynamic range and the clearest source map in  
446 both the ABF and VABF at all considered frequencies.

447 It was also shown that when SNR is high, the MPDR-based VABF can be applied to the vibration  
448 response from a structure with a small damping to achieve similar beamforming output for source lo-  
449 calisation to that of the MPDR-based ABF. This does not hold true for source localisation when SNR  
450 is zero or negative as the source map becomes contaminated with sidelobes and the VABF underper-  
451 forms. However, it was confirmed that this underperformance can be compensated by increasing the  
452 structural damping, which results in reducing the coherence of the vibratory field and consequently  
453 enhances the dynamic range in the VABF to the same level or even higher than that in the ABF.  
454 Moreover, whilst increasing the damping loss factor also widens the spatial resolution in the VABF, it  
455 always remains below the corresponding value in the ABF. Hence, it can be concluded that by applying  
456 the VABF to the vibration data from a structure with moderate damping, a similar performance in  
457 source localisation to that in the ABF can be accomplished at any SNR. It is worth noting that in  
458 order to optimise the performance of the VABF for low SNRs in the frequency range of interest, the  
459 proposed numerical study can be employed at the design stage of the mounting structure (plate) to  
460 estimate the minimum value of the damping loss factor (eventually in function of the frequency) which  
461 is needed to obtain the targeted performance. Accordingly, viscoelastic layers can then be produced  
462 and glued on the surface of the structure to reach the required damping.



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