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Blades Forced Response Analysis With Friction Dampers

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A multiharmonic frequency domain analysis combined with a Craig-Bampton component mode synthesis is presented to compute the dry friction damped forced response of blades. The accuracy of the analysis is established, for a cantilever beam with a dry friction damper attached, by comparison with experimental results and time domain analysis. The method has then been applied to a model fan blade damped by a blade to ground damper.

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Introduction

Friction dampers are widely used in axial turbomachinery components to reduce vibration amplitudes and therefore to increase high frequency fatigue life of blades and disks. The analysis of such systems is difficult due to the nonlinear nature of the friction law. Nevertheless, from an industrial point of view, it becomes necessary to be able to conduct analysis in order to optimize friction dampers.

Numerical integration methods, although very effective to handle multiple sticking and complicated damping laws, lead to prohibitive computational effort if parametric studies and frequency response curves are to be obtained within the design process. If one is only interested in the stationary behaviour under periodic loading, frequency domain methods are more suited.

The harmonic balance method (Nayfeh and Mook [1]) has been applied to study friction damping in turbine blades by Griffin [2]. Although computationally efficient, this method assumes the response to be limited to a single harmonic approximation. The results are adequate for continuous slip motion, but not for stick-slip motion where higher harmonics become important.

Multiharmonics methods have been introduced by various researchers. Lau and Cheung [3], Lau et al. [4, 5], proposed an incremental harmonic balance (IHB) method to solve nonlinear periodic and quasi periodic vibrations of continuous systems. Pierre, Ferri and Dowell [6] extended the IHB method to analyse the steady state response of systems with dry friction. Ferri [7] showed the equivalence between the IHB method and the Galerkin/Newton-Raphson (GNR) method proposed by Urabe [8] to analyse non linear periodic systems. More recently, several authors as Ling and Wu [9], Cameron and Griffin [10], Cardona et al. [11], improved the numerical efficiency of the IHB/GNR method by the introduction of the FFT algorithm.

However, concerning the application of these methods to industrial cases, with many degrees of freedom, very few work has been published.

The purpose of this paper is to apply the GNR method with FFT algorithm to predict the forced response of turbomachinery blades with friction dampers in an efficient way. The size reduction of the system to be solved has been performed in two steps, first by a Craig-Bampton component mode synthesis and second by a condensation technique in the frequency domain on the non linear degrees of freedom only. The accuracy of the method is discussed and the results are compared with experiments

conducted on a cantilever beam with a dry friction damper attached. Finally, the application of the method to a model fan blade is presented.

Theory

Finite element models of blades usually comprise a great number of degrees of freedom (dof), thus to simulate forced response with friction dampers, one has first to tackle the problem of model reduction. Dowell [12] proposed a component mode analysis with the modes of the structure without dampers. In order to handle systems of smaller size, we prefer to use a Craig-Bampton component mode synthesis, where the retained dof include those where frictional forces are applied. In this paper, those dof are called nonlinear dof. The set of retained dof may be restricted to the only non linear ones, but may include others where one wishes to know the displacement without using a restitution procedure. The influence on the results of the retained dof choice will be discussed later. The condensed mass and stiffness matrices are given directly by the finite element code after component mode synthesis.

The forced response with friction is computed with a GNR method using FFT algorithm. The system of equations of motion can be represented by:

$$L(x(t)) - f(t) - g(x, \dot{x}) = 0 \quad (1)$$

where $L(\cdot)$ is the differential operator representing the linear term, $f(t)$ is the external forcing term, $g(x, \dot{x})$ represents the nonlinear friction force and $x(t)$ is the system response. All are vector functions.

It has been shown by Korkmaz et al [13], among others, that for a friction damper, stick-slip mechanism is the most efficient in reducing amplitude. As a classical friction law is able to reproduce stick-slip well, it has been used throughout this work. Thus when sliding occurs between the two dof i and j , the friction force acting on the dof i is:

$$g^i(x, \dot{x}^i) = -g_d \operatorname{sgn} \dot{x}^i \quad \text{if } \dot{x}^i \neq 0$$

where $\dot{x}^i = \dot{x}^i - \dot{x}^j$, and g_d is given by a coefficient of friction times normal force to the sliding surfaces.

For numerical purposes, the friction law g has been approximated by:

$$\begin{aligned} g^i(\dot{x}^i) &= -g_d \frac{1}{\lambda} \dot{x}^i \quad \text{if } |\dot{x}^i| \leq \lambda \\ &= -g_d \operatorname{sgn} \dot{x}^i \quad \text{if } |\dot{x}^i| > \lambda \end{aligned}$$

where λ is a numerical parameter.

If we assume the solution to be:

$$x(t) = \frac{\omega}{\pi} \left(\frac{1}{2} X_{c0} + \sum_{n=1}^N X_{cn} \cos n\omega t + X_{sn} \sin n\omega t \right) \quad (2)$$

the Galerkin procedure leads to the spectral form of (1) (see references 9, 10 and 11, for more details):

$$\Lambda \cdot X - F - G(X) = 0 \quad (3)$$

where the matrix Λ and the vector F are easily obtained from L and f . \mathbf{X} is the vector given by:

$$\mathbf{X}^T = (X_{c0}^T, X_{c1}^T, X_{s1}^T, \dots, X_{cN}^T, X_{sN}^T)$$

The vector $\mathbf{G}(\mathbf{X})$ is given by:

$$\mathbf{G}^T(\mathbf{X}) = (G_{c0}^T, G_{c1}^T, G_{s1}^T, \dots, G_{cN}^T, G_{sN}^T)$$

where

$$G_{cn}(X) = \int_0^{2\pi/\omega} \cos(n\omega t) g(x) dt$$

and

$$G_{sn}(X) = \int_0^{2\pi/\omega} \sin(n\omega t) g(x) dt$$

The solution vector \mathbf{X} of Eq. (3) is found by an iterative Newton-Raphson procedure. If we write the residual vector $\mathbf{R}(\mathbf{X})$ as:

$$\mathbf{R}(\mathbf{X}) = \Lambda \cdot \mathbf{X} - F - G(\mathbf{X})$$

the iterative system to be solved is:

$$J(X) \cdot \Delta X = -R(X) \quad (4)$$

where ΔX is the increment of the displacement vector \mathbf{X} to be found and $J(X)$ is the jacobian matrix:

$$J(X) = \frac{\partial R}{\partial X} = \Lambda - \frac{\partial G}{\partial X}$$

In order to calculate $G(X)$ and $\partial G/\partial X$ by numerical FFT, the functions are discretised with NP points per period ($NP > 2N$, N = number of harmonics):

$$\Delta t = \frac{2\pi}{\omega NP}$$

$$x_k = x(k\Delta t) \quad k \text{ from } 0 \text{ to } NP-1$$

$$\dot{x}_k = \dot{x}(k\Delta t)$$

$$\begin{aligned} g_k^i(\dot{x}_k^{ij}) &= -g_d \frac{1}{\lambda} \dot{x}_k^{ij} \quad \text{if } |\dot{x}_k^{ij}| \leq \lambda \\ &= -g_d \operatorname{sgn} \dot{x}_k^{ij} \quad \text{if } |\dot{x}_k^{ij}| > \lambda \end{aligned}$$

Each component i of $G_{cn}(X)$ and $G_{sn}(X)$ is approximated by:

$$\begin{aligned} G_{cn}^i(X) &= \sum_{k=0}^{NP-1} \cos\left(\frac{2\pi}{NP} nk\right) g_k^i(\dot{x}_k^{ij}) \\ G_{sn}^i(X) &= \sum_{k=0}^{NP-1} \sin\left(\frac{2\pi}{NP} nk\right) g_k^i(\dot{x}_k^{ij}) \end{aligned}$$

These terms are zero if the dof i is not affected by friction.

The matrix $\partial G/\partial X$ is composed of $(2N+1) \times (2N+1)$ block matrices of dimensions $p \times p$ where p is the number of dof of the initial temporal system.

$\partial G_{cn}/\partial X_{cm}$ is the $p \times p$ block corresponding to the line cn and to the column cm . The other generic blocks are: $\partial G_{sn}/\partial X_{sm}$, $\partial G_{sn}/\partial X_{cm}$, $\partial G_{cn}/\partial X_{sm}$.

The only non-zero terms of these matrices correspond to those affected by friction. For example, the generic term i, j of the first block matrix is numerically calculated by:

$$\frac{\partial G_{cn}^i(X)}{\partial X_{cm}^j} = \sum_{k=0}^{NP-1} \cos\left(\frac{2\pi}{NP} nk\right) \frac{\partial g_k^i(\dot{x}_k^{ij})}{\partial \dot{x}_k^j}$$

The expression of the differential terms is not straightforward, it is obtained by:

$$\frac{\partial g_k^i}{\partial \dot{x}_k^j} = \frac{\partial g_k^i}{\partial \dot{x}_k^{ij}} \frac{\partial \dot{x}_k^{ij}}{\partial \dot{x}_k^j}$$

Since:

$$\begin{aligned} \dot{x}_k^{ij} &= \frac{\omega^2}{\pi} \left(\sum_{n=1}^N -n \sin\left(\frac{2\pi}{NP} nk\right) d(X_{cn}^i - X_{cn}^j) \right. \\ &\quad \left. + n \cos\left(\frac{2\pi}{NP} nk\right) d(X_{sn}^i - X_{sn}^j) \right) \end{aligned}$$

we can write:

$$\frac{\partial \dot{x}_k^{ij}}{\partial X_{cm}^j} = \frac{m\omega^2}{\pi} \left(\sin\left(\frac{2\pi}{NP} mk\right) \right).$$

So, we obtain:

$$\begin{aligned} \frac{\partial G_{cn}^i}{\partial X_{cm}^j} &= \frac{m\omega^2}{\pi} \left(\sum_{k=0}^{NP-1} \cos\left(\frac{2\pi}{NP} nk\right) \sin\left(\frac{2\pi}{NP} mk\right) \frac{\partial g_k^i}{\partial \dot{x}_k^{ij}} \right) \\ &= m\omega \left(\frac{\omega}{2\pi} \sum_{k=0}^{NP-1} \sin\left(\frac{2\pi}{NP} k(n+m)\right) \frac{\partial g_k^i}{\partial \dot{x}_k^{ij}} \right. \\ &\quad \left. - \frac{\omega}{2\pi} \sum_{k=0}^{NP-1} \sin\left(\frac{2\pi}{NP} k(n-m)\right) \frac{\partial g_k^i}{\partial \dot{x}_k^{ij}} \right) \end{aligned}$$

In practice, we begin by numerically calculating the terms $\partial g_k^i/\partial \dot{x}_k^{ij}$ for k varying from 0 to NP-1, with the formulas:

$$\begin{aligned} \frac{\partial g_k^i}{\partial \dot{x}_k^{ij}} &= -g_d \frac{1}{\lambda} \quad \text{if } |\dot{x}_k^{ij}| \leq \lambda \\ &= 0 \quad \text{if } |\dot{x}_k^{ij}| > \lambda \end{aligned}$$

The terms $\partial G_{cn}^i/\partial X_{cm}^j$ can then be calculated, for n and m varying from 1 to N , by using an FFT algorithm.

In order to reduce even more the computational time, the iterative system (4) is reduced to the non linear dof only, i.e. the dof affected by friction. The solution vector \mathbf{X} can be partitioned according to the linear and non linear dof as $X = (X_{ln}, X_{nl})$, and the system (4) can be written:

$$\begin{bmatrix} J_{ln} & J_{ln,nl} \\ J_{nl,ln} & J_{nl}(X_{nl}) \end{bmatrix} \begin{Bmatrix} \Delta X_{ln} \\ \Delta X_{nl} \end{Bmatrix} = - \begin{Bmatrix} R_{ln} \\ R_{nl} \end{Bmatrix}$$

It follows immediately that it is equivalent to solve one system of smaller size:

$$(J_{nl} - J_{nl,ln} J_{ln}^{-1} J_{ln,nl}) \Delta X_{nl} = -R_{nl} + J_{nl,ln} J_{ln}^{-1} R_{ln}$$

and to restore the linear dof by:

$$\Delta X_{ln} = J_{ln}^{-1} (R_{ln} - J_{ln,nl} \Delta X_{nl})$$

This reduction technique is specially interesting when the number of normal modes and retained linear dof of the component mode synthesis is important.

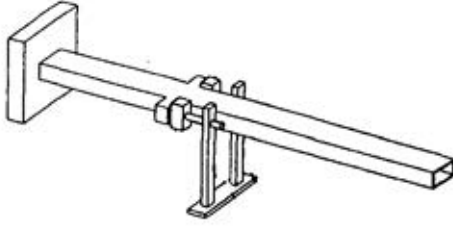


Fig. 1 Beam and dampers overview

Experiment

In order to gain a better understanding of friction damping and to have experimental data to validate numerical methods, a simple experiment has been realized for a cantilever beam excited by an electrodynamic shaker and damped by a dry friction damper. A brief description of the experimental set up and of the results will be given here. The reader is referred to Korkmaz et al. [14] for more details.

Experimental set up. The beam, represented in Fig. 1, is a straight iron tube of 1.33 m length, with a rectangular cross section of 0.08 m by 0.04 m and a thickness of 0.002 m. Two very stiff plates are fixed to the beam at a distance of 0.318 m from the clamped end.

The design of the friction dampers is shown in Fig. 2. A friction damper is applied against each of the two plates fixed to the beam, in order to preserve symmetry. It is possible to apply a variable normal load on the contact area of each damper.

An electrodynamic shaker is attached to the beam at a distance of 1.28 m from the clamped end.

Accelerometers are placed on the beam, perpendicular to span, at the friction damper (point 2) and at the shaker (point 1) locations.

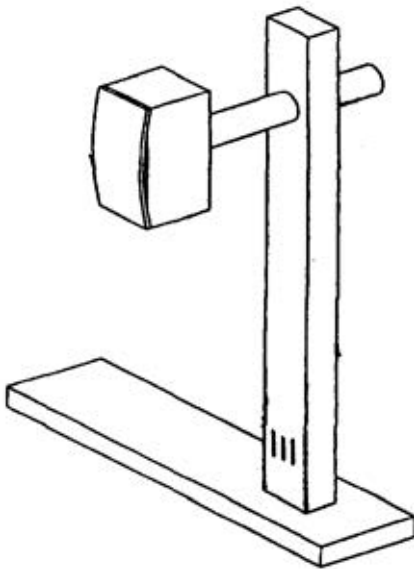


Fig. 2 Friction damper mechanism

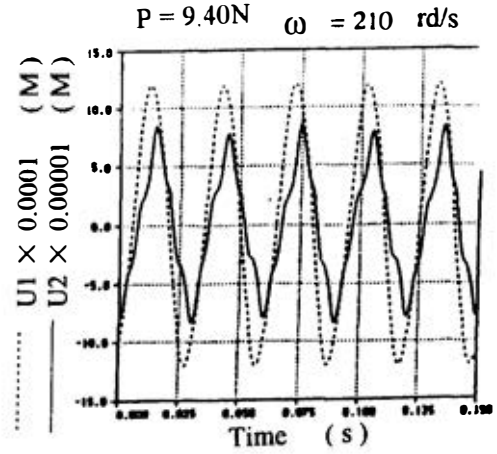


Fig. 3 Experimental displacements u_1 and u_2 for the first beam mode forced response

Measurements. A normal load FN is applied on the friction dampers. Then the beam is excited with a periodic force $P \cos(\omega t)$, whose frequency is swept over a given range. Measurements have been done for several levels of excitation force P and normal loads FN . For all measurements, the transfer functions between the excitation force P and the responses at point 1 (displacement u_1) and at point 2 (displacement u_2) have been determined.

Results. The resonant frequencies for the first mode, with the dampers sliding and sticking, are respectively 25.4 Hz and 35.1 Hz.

The temporal displacement responses $u_1(t)$ and $u_2(t)$ are shown on Fig. 3 for $FN = 466$ N, $P = 9.4$ N and $\omega = 210$ rad/s (33.4 Hz).

The transfer functions between the displacement u_1 and the excitation force P are represented in figure 4 for $FN = 372.8$ N and several levels of the excitation force P .

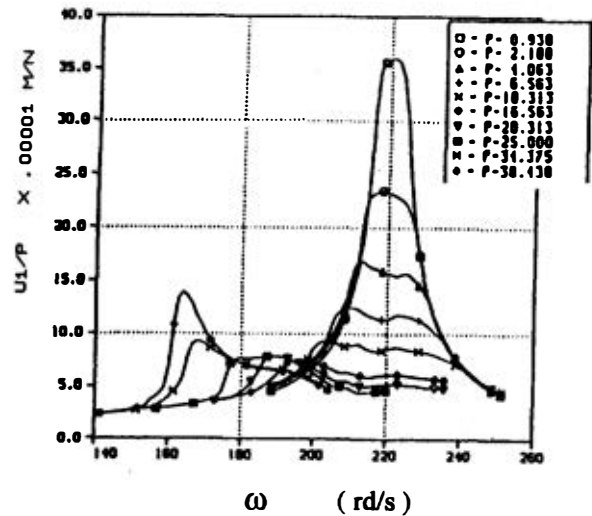


Fig. 4 Experimental frequency response functions for the first beam mode

Validation

The beam tested previously has been chosen for the validation of the computations.

Validation has been conducted as follows:

- Creation of the Finite Element Model.
- Creation of 6 reduced models (1 or 2 retained dof, 1, 3 or 10 normal modes), which have been compared to the Finite Element Model with respect to eigen-frequencies and mode shapes predictions.
- Analysis of the multiharmonic, friction damped, forced response. The influence of the reduction technique and the number of harmonics has been studied.
- Analysis of the temporal, friction damped, forced response.

Finite Element Model. A finite element model of the beam with 10 beam elements has been done. Additional lumped parameters have been added to represent the mass (2 kg) and inertia ($8.5 \times 10^{-4} \text{ kg.m}^2$) of the added plates and the moving mass of the shaker (0.133 kg). The stiffness of the friction dampers is equal to $2.4 \times 10^7 \text{ N/m}$. The material properties of the beam are: Young's modulus $E = 2.0 \times 10^{11} \text{ Pa}$, Poisson's ratio $\nu = 0.3$ and mass density $\rho = 7800 \text{ kg/m}^3$. The sliding and sticking eigen-frequencies of the model have been found to be respectively 25.4 Hz and 35.9 Hz. The agreement with the experimental results is acceptable. Structural damping for the first mode was estimated to be $\xi = 1.95$ per cent when the damper is sticking (35.9 Hz) and $\xi = 2.55$ per cent when the damper is sliding (25.4 Hz). Rayleigh damping was then assumed in the form $\alpha K + \beta M$ where $\alpha = 2.6 \times 10^{-5}$ and $\beta = 7.49$.

Component Mode Synthesis. The choice of the retained dof is an important step in the model reduction. The smallest set of retained dof is composed of those where friction dampers apply. In our beam case, this set is reduced to a single translation dof and is called set1. For large structures, to avoid a lengthy displacement restitution procedure, one may wish to retain additional dof where the displacement is to be known. For our beam, we add to set1 the displacement at the shaker location, to form the set2.

We performed several component mode syntheses (CMS), with 1, 3 and 10 normal modes (NM), for set1 and set2.

The 6 reduced models obtained, have been compared to the initial Finite Element Model for two eigenvalue analyses. One with a spring of value equal to $2.4 \times 10^7 \text{ N/m}$ attached at the damper location and the other with the displacement at the same location free to move. These configurations are called hereafter respectively fixed and free boundary conditions. The eigen-frequencies obtained with the various models are presented on table 1. The modal displacement ratio $(u_2/u_3) \times 100$ for these models is reported on table 2, where u_2 and u_3 are respectively the displacements perpendicular to span at the damper location and at the free end of the beam.

To analyse the first beam mode forced response, one may think that the 1 NM models are pertinent. The 10 NM set1

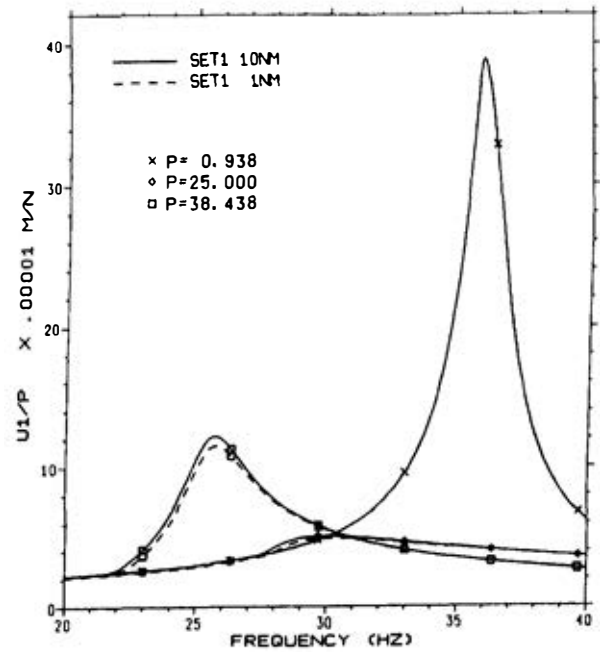


Fig. 5 Model influence on the first beam mode forced response

model will be used as a reference model in the remaining part of this work.

Harmonic Analysis. We chose to simulate forced response with friction damping for the first mode. This mode has been chosen because we have experimental results. The results obtained with the reduced models presented previously, will be compared with one another, in order to evaluate the effectiveness of the component mode synthesis technique, combined with friction modeling. Then the influence of the number of harmonics in the response will be studied.

We analyse the beam in the configuration that leads to the results presented on Fig. 4 for the excitation force levels P of 0.938 N, 25 N and 38.438 N. When we simulate the forced response with a single harmonic, all models give equivalent results. The values of $u_1/P \times 10^5$ over the frequency range of 20 Hz to 40 Hz are plotted on Fig. 5 and the maxima are reported in table 3. On Fig. 5, the results of all models superimposed the reference model (10 NM set 1) results, except for the 1 NM set 1 model. In fact, we saw that this model does not represent perfectly the modal displacement at the damper location for the free and fixed boundary conditions (see Table 2). In the follow-

Table 1 Eigenfrequencies (Hz); Mode 1—damper location free or fixed

		free	fixed
Full FEM		25.40	35.91
	1NM	25.40	35.91
CMS set 1	3NM	25.40	35.91
	10NM	25.40	35.91
CMS set 2	1NM	25.40	35.92
	3NM	25.40	35.92
	10NM	25.40	35.92

Table 2 Displacement ratio $u_2/u_3 \times 100$; Mode 1—damper location free or fixed

		free	fixed
Full FEM		8.982	1.440
	1NM	8.970	1.439
CMS set 1	3NM	8.982	1.440
	10NM	8.982	1.440
CMS set 2	1NM	8.981	1.439
	3NM	8.982	1.440
	10NM	8.982	1.440

Table 3 Model influence on the first beam mode forced response

Max($u_1/P \times 10^5$) m/N over 20Hz-40Hz				
P	CMS set 1			CMS set 2
	1 NM	3NM	10NM	1NM
0.938N	38.2	38.2	38.2	38.5
25.0N	4.9	5.1	5.1	5.1
38.438N	11.4	12.2	12.2	12.1

ing part of this work, the 1 NM set 2 model will be used to study the first mode forced response.

We can say that component modes synthesis is an efficient way to reduce the initial Finite Element model for studying forced response with dry friction. Nevertheless it is necessary to check the reduced models ability to represent the eigen frequencies and mode shapes of the initial model for the free and fixed boundary conditions. We find that, for our case studies, small size models are sufficient.

The influence of the harmonics number has been studied for the first mode. The results are presented on Table 4 and on Figs. 6 and 7. We see that the number of harmonics has a great influence on the response when stick-slip occurs. We find for $P = 25$ N a 35 percent difference on the maximum response amplitude when computing the response with a single harmonic or with seven harmonics.

Finally the frequency response functions have been computed for all experimental excitation force levels with seven or nine harmonics. These curves are presented Fig. 8 and are to be compared with those of experimental results shown in Fig. 4. We can say that the results agree quite well.

The computational time (CPU time) is much more dependent on the system size to be solved than on the harmonics number. We find, as an average, that the resolution of a system with 3 equations needs 30 percent of the CPU time required for 11 equations, whereas a one harmonic analysis needs 70 percent of the CPU time required for a seven harmonics analysis.

Temporal Analysis. Temporal analyses have been conducted with the initial Finite Element model. The steady state responses have been compared with the experimental and multi-harmonics (GNR) method results. The computed values are presented on Table 5.

We can first note that the GNR method fits very well with Time Integration method except for low excitation forces. However these low excitation responses do not correspond to the highest efficiency of friction dampers and are of minor interest from a design point of view. The correlation with experimental results is acceptable and the discrepancies are due to modelisation, not to numerical methods.

Application

The method is now applied to a model fan blade, damped by a blade to ground damper and excited by a concentrated periodic force. The blade root is clamped and the first flexion

Table 4 Influence of the harmonics number on the first beam mode forced response

Max($u_1/P \times 10^5$) m/N over 20Hz-40Hz			
P	0.938N	25.0N	38.438N
1H	38.46	5.07	12.11
3H	38.46	7.57	13.67
5H	38.46	7.24	14.83
7H	38.46	6.84	14.95

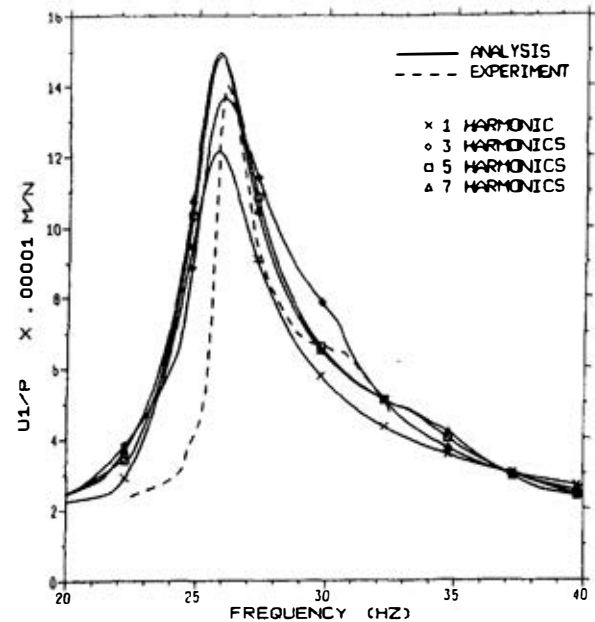


Fig. 6 Influence of the harmonics number on the first beam mode forced response $P = 38.438$ N

mode is studied. The concentrated force can represent a shaker excitation in a component test. If aerodynamic excitation forces were computed as described in reference 15, they would have to be condensed on the components of the

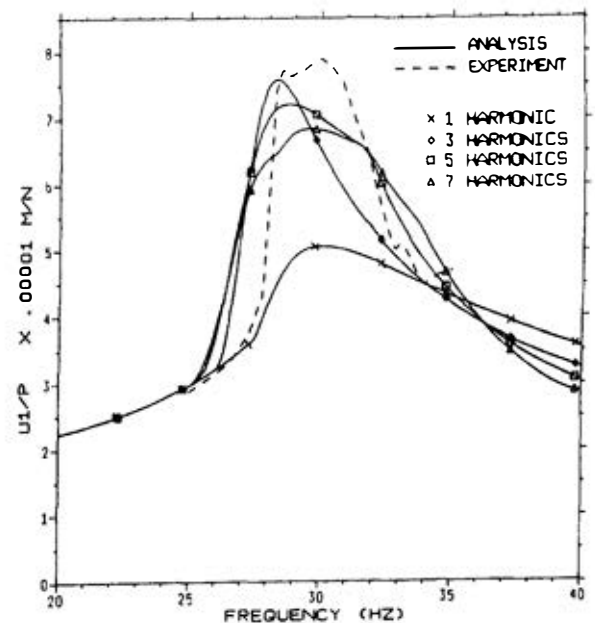


Fig. 7 Influence of the harmonics number on the first beam mode forced response $P = 25.0$ N

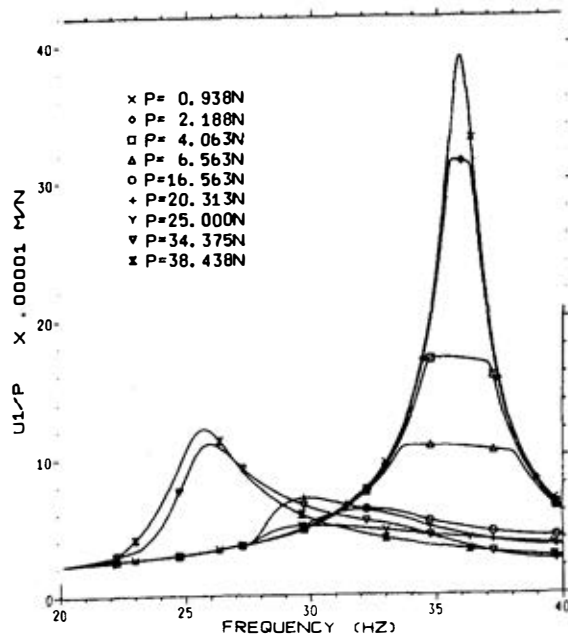


Fig. 8 Numerical frequency response function for the first beam mode

reduced system of equations and the method would apply in the same way.

Model. The finite element mesh of the blade with its frictional platform is shown in Fig. 9. The model contains 632 elements and 2045 nodes. The airfoil was modelled using shell version of the solid elements while the rest of the structure was modelled with solid elements. The reduced model of the system is obtained, as mentioned earlier, using Craig-Bampton mode synthesis method with 6 NM and retaining the dof of only 3 nodes at the following locations: the blade tip, the shaker and the friction damper (see Fig. 9). Local coordinate system had been introduced at the blade tip and at the friction damper location. The dry friction damper has a stiffness of 2.4×10^7 N/m and a mass of 0.1 kg. The Coulomb friction coefficient equals 0.66. The typical values of α and β related to Rayleigh damping are $\alpha = 1.62 \cdot 10^{-5}$ and $\beta = 0.593$, corresponding to $\xi = 0.3$ per cent.

Results. Figure 10 shows, for a fixed excitation force, the displacement amplitude at the blade tip as a function of excitation frequency for different normal forces (from 0 to F1) applied at the frictional damper. A single harmonic approximation for the response has been considered in these computations. We note that for the normal forces F1 and 0, the frequencies at which the amplitudes are maximum correspond to the first eigen frequencies of the blade, for fixed and free conditions at damper location. Numerical results correlate qualitatively well with

Table 5 Analysis and experiment comparison for the first beam mode forced response

Max($u_1/P \times 10^5$) m/N over 20Hz-40Hz			
P	GNR	Time Integ.	Exp.
0.938N	38.46	36.70	36.10
25.0N	6.84	6.84	7.89
34.375N	11.45	11.49	9.31
38.438N	14.95	14.90	14.00

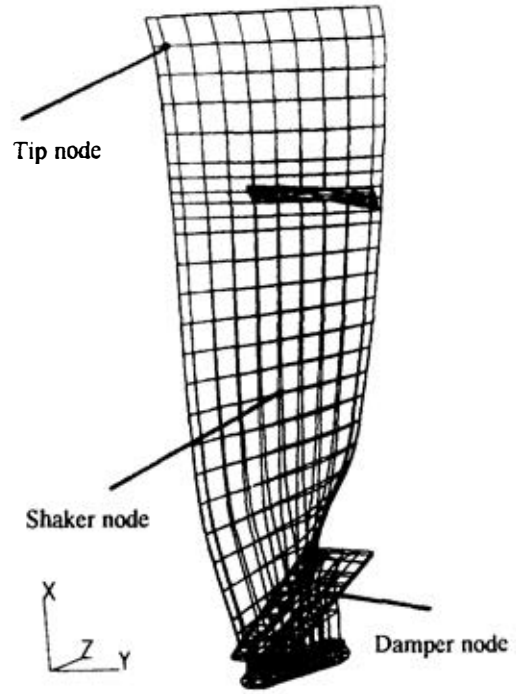


Fig. 9 Fan blade FEM

component tests in progress. Detailed comparison of results will be reported elsewhere.

Throughout this work, the general purpose Finite Element software code SAMCEF was used to obtain the reduced models

Displacement at tip

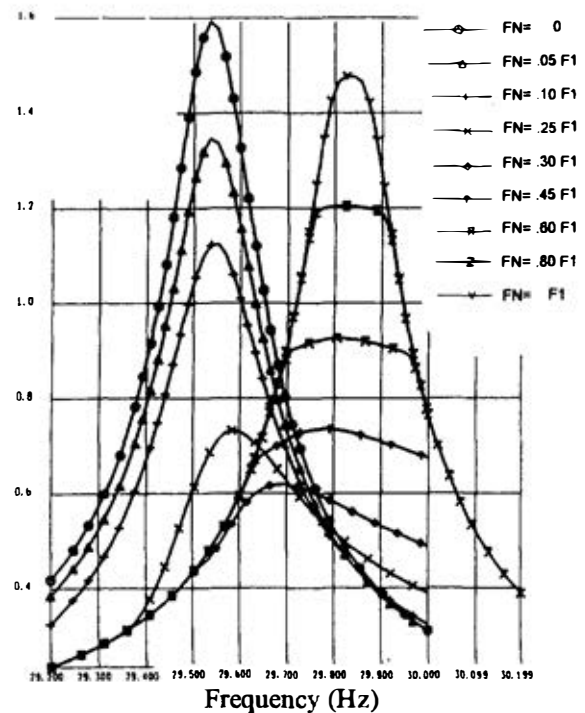


Fig. 10 Numerical frequency response function for the first blade mode

using component modes synthesis and to perform the eigen-frequencies analysis as well as the nonlinear temporal computations.

Conclusions

A numerical method to study the forced response of blades with friction dampers has been presented. This method combines a multiharmonic analysis with a Craig-Bampton component mode synthesis and is easy to use for industrial analysis as presented here for a fan blade. It allows damper optimization within the design process.

The accuracy of the method has been tested on a cantilever beam. It has been shown that, with component mode synthesis, small size reduced models are able to represent properly the structure dynamics for friction damping analysis. Higher harmonics of the response greatly influence the result of the GNR method, specially when stick-slip occurs. A 35 percent difference has been found, in one case, on the displacement amplitude between a one and a seven harmonics analysis. As the highest effectiveness of a friction damper is obtained when stick-slip is present, multiharmonic analysis is of great interest if one wants to optimize dampers. In most of the useful configurations, the analysis with seven harmonics fits reasonably well with temporal analysis and with experimental results.

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