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# Design of an Autonomous Ride-sharing Service through a Graph Embedding Integrated Dial-a-ride Problem: Application to the Last-mile Transit in Lyon City

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1 **Design of an Autonomous Ride-sharing Service through a Graph Embedding Integrated**  
2 **Dial-a-ride Problem: Application to the Last-mile Transit in Lyon City**

3

4

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29 **AUTHOR CONTRIBUTIONS**

30 *The authors confirm contribution to the paper as follows: study conception and design: OR, NC, JPN;*

31 *data collection: OR; analysis and interpretation of results: OR, NC, JPN; draft manuscript preparation:*

32 *OR. All authors reviewed the results and approved the final version of the manuscript.*

1 **ABSTRACT**

2 Autonomous vehicles are anticipated to revolutionize ride-sharing services and subsequently enhance the  
3 public transportation systems through a first-last mile transit service. Within this context, a fleet of  
4 autonomous vehicles can be modeled as a Dial-a-Ride-Problem with certain features. In this study, we  
5 propose a holistic solving approach for this problem, which combines the mixed-integer linear  
6 programming formulation with a novel graph dimension reduction method based on the graph embedding  
7 framework. This latter method is effective since accounting for heterogeneous travel demands of the  
8 covered territory tends to increase the size of the routing graph drastically, thus rendering exact solving of  
9 small instances computationally infeasible. An application is provided for the real transport demand of the  
10 industrial district of “*Vallée de la Chimie*” in Lyon city, France. Instances involving more than 50  
11 transport requests, and 10 vehicles could be easily solved. Results suggest that this method generates  
12 routes of reduced nodes with a lower vehicle kilometers traveled compared to the constrained K-means  
13 based reduction. Reductions in terms of GHG emissions are estimated to be around 75% lesser than the  
14 private vehicle mode in our applied service. A sensitivity analysis is also provided.

15  
16 **Keywords:** Shared Autonomous Mobility, First-Last Mile Service, Vehicle Routing, Node Embeddings.

1 **1. INTRODUCTION**

2

3 The urban expansion in most cities around the world followed an automobile-oriented pattern, leading to  
4 the emergence of several low-density metropolitan areas, thus increasing the overall infrastructure costs  
5 for public transportation (PT) (Sinha, 2003 [35]). Since then and due to this reason, PT systems have  
6 been relegated to a secondary role behind private car use in urban mobility. The PT cover for low-density  
7 areas and commercial and industrial districts outside the city is limited. Shared mobility, which is the  
8 shared use of vehicles, bicycles, and other means of transport to access PT networks, started to appear to  
9 be a viable solution to this issue, known as the first- and last-mile commute. The concept of shared  
10 mobility is not novel in itself, and could be traced back to the 1990s where it was popular, and even to the  
11 year of 1948 for the first car-sharing program (Shaheen & Chan, 2016 [36]). Yet, the major shift in shared  
12 mobility is still expected by many to happen with the advent of shared autonomous vehicles (SAV).  
13 Several AV manufacturers and service transportation provider companies have heavily invested and  
14 adopted specific ride-sharing plans, e.g., Waymo in Phoenix area (Waymo, 2020 [41]), Jaguar Land  
15 Rover (Jaguar Land Rover, 2020 [20]), and eventually Amazon, a fresh new competitor in this market  
16 share (Amazon, 2020 [2]).

17

18 The factors making SAV an attractive transit mode are mainly economic, for both sides of customers and  
19 PT and transportation network entities. Due to lower operational and investment costs over fixed routes,  
20 transportation companies could propose services at fair prices with a higher frequency. Higher customer  
21 satisfaction and time reductions may be equally obtained compared with other transit modes such as park-  
22 and-ride and active modes. Another argument in favor of SAV lies in the environmental sustainability of  
23 this technology, which is mainly due to the application of electric vehicles in SAV services. Autonomous  
24 fleets could reduce GHG emissions by up to 73% compared to conventional taxi fleets (Bauer et al., 2018  
25 [3]). It was shown that each SAV could replace around 11, respectively from 5 to 10 privately owned  
26 vehicles, with an increase of 10%, resp. from 6 to 89% travel distance via agent-based simulations of the  
27 city of Austin, Texas (Fagnant & Kockelman, 2014 [10]), resp. Lisbon (International Transport Forum,  
28 2015 [19]). The existence of empty trips justifies this latter increase. All these favorable reports led to a  
29 resurgence of small-size experimentations of SAV services in Europe, the U.S., and globally. Just in Lyon  
30 city in France, where the case study of this paper is located, there are currently three experimentations:  
31 Navya shuttle buses in the Confluence area, around Groupama stadium, and Mia buses in the Meyzieu-  
32 Décine suburb area. Therefore, there is a great need of efficient design models for the short-time  
33 deployment of SAVs, especially to handle the first-last mile transit issue (see Hyland & Mahmassani  
34 (2017) [18] for a taxonomy of SAV fleet management problem classes to accompany this current mobility  
35 shift).

36

37 A critical component of the design of an SAV service is the vehicle assignment or the process of  
38 matching customer requests to vehicles. Most SAV approaches rely on rule-based assignment methods,  
39 e.g., Gurumurthy & Kockelman (2018) [16]. Optimization models are almost not used here. This is  
40 because SAV studies are mostly on-demand systems, that are dynamic systems that require solving  
41 approaches that are computationally cost-efficient and easy to implement (Narayanan et al., 2020 [30]).  
42 On the other hand, SAV models that are reservation-based, or in other terms static systems that could be  
43 solved beforehand, are relying in general on optimization. However, they are quite few in number, e.g.,  
44 Levin (2017) [22], Ma et al. (2017) [24]. Both Levin (2017) and Ma et al. (2017) proposed a linear  
45 programming model to modelize an SAV fleet. In this study, we propose using the famous routing  
46 problem of the Dial-a-Ride-Problem (DARP), which is derived from the classical Vehicle Routing  
47 Problem (VRP) with additional constraints to account for pickup and drop-off requests. VRP and DARP  
48 are both computationally intractable and can be pinned down as mixed-integer linear programming  
49 models. According to Ho et al. (2018) [17], the largest instance solved to optimality for the basic DARP  
50 with time windows is up to 8 vehicles and 96 requests, which is quite small for practical deployment of an  
51 SAV service. Therefore, a mechanism has to be found in order to reduce the dimension of the routing

2

3

1 graph. To address this research gap, we develop a new reduction method based on the graph embedding  
2 framework, specifically around the *node2vec* algorithmic framework (Grover and Leskovec, 2016 [14]),  
3 which will be seen later. The problem we solve is a reservation-based SAV service.

4  
5 A second objective is to design an SAV service as close as it would work in a real environment. First, this  
6 was done by the choice of an application territory. We choose an industrial district in Lyon city (France),  
7 called “Vallée de la Chimie” (VC). Although VC is one of the biggest chemical and petrochemical parks  
8 in Europe, it is poorly served by public transport systems. It presents a concrete application of the last-  
9 mile transit. This study focuses on a classical commute problem between the city center and suburbs  
10 where two transportation alternatives compete: highway and rapid transit system. Second, we aim to  
11 generate realistic traffic demand accounting for intermodality, i.e., using multiple transport modes on the  
12 same trip. The traditional four-step travel models could only produce demand matrices for separate modes  
13 (McNally, 2007 [26]). We have relied on a Land Use and Transport Interactions (LUTI) model-based tool  
14 called OPTIREL to account for intermodality, without using agent-based simulations. OPTIREL has been  
15 previously applied to design new subway lines in Paris among other projects. Therefore, the problem we  
16 solve is tested on VC for the commute problem with a local rapid transit system.

17  
18 The remainder of the paper is organized as follows. A brief literature review on the integration of PT-  
19 SAV system, the DARP problem, and the graph embedding framework is presented in the next section.  
20 Section 3 introduces the optimization model and the solving approach, while Section 4 describes the  
21 study case and the generation of traffic demand. In Section 5, we experimentally evaluate the benefit of  
22 the designed last-mile service, before concluding on some final thoughts.

23

## 24 **2. LITERATURE REVIEW**

25

26 In this section, we briefly review the literature of interest in this paper within the following three  
27 subsections.

28

### 29 **Integration of PT-SAV systems**

30 Several recent studies are starting to explore the benefits of integrating SAV services with existing PT  
31 systems. The most commonly adopted approach in this regard is agent-based simulations, whether it  
32 concerns the integration, e.g., Shen et al. (2018) [37], Pinto et al. (2020) [31], or merely the intersection  
33 of PT-SAV systems, e.g., Fagnant & Kockelman (2014) [10]. Liang et al. (2016) [23] proposed an  
34 optimization model for an automated taxi service serving the last mile transit of a train system. Their  
35 results are shown for a train station in Delft and include comparisons with human-driven taxis. Shen et al.  
36 (2018) [37] simulate several scenarios of integrating bus lines with SAVs for the first-mile connectivity  
37 during the morning peak hours. Their results based on the Singapore PT show significant savings if low-  
38 demand buses are substituted by SAVs, savings that go up to 860 passenger car unit-kilometers. Pinto et  
39 al. (2020) [31] proposed a bi-level optimization framework to simultaneously modelize the joint transit  
40 network parameters and the SAV mobility service. The lower-level problem is a dynamic combined mode  
41 choice – traveler assignment problem, while the upper-level is a modified transit network frequency  
42 setting problem. Their results are given for the Chicago metropolitan area. A more detailed review of  
43 overall SAV services is provided by Narayanan et al. (2020) [30].

44

### 45 **Dial-a-Ride problems**

46 Designing vehicle routes and schedules for collective people transportation such that each user request  
47 has a pickup and a drop-off point locations are referred to as Dial-a-Ride systems, as in early versions of  
48 this service, customers have to phone their requests. From a modeling perspective, DARP belongs to the  
49 family of routing problems. It is a variant of the vehicle routing problem (VRP), precisely the capacitated  
50 VRP with pickup and delivery and Time Windows, with a hands-on transporting passengers rather than  
51 goods. Early solving approaches include Psaraftis (1980) [33], who solved the single-vehicle DARP using

1 dynamic programming, and Jaw et al. (1986) [21], who developed one of the first heuristics to the multi-  
 2 vehicle DARP. For the exact approaches, Branch-and-Bound and their derived algorithms are the most  
 3 applied, e.g., Ropke et al. (2007) [34], Gschwind & Irnich (2014) [15]. Concerning metaheuristics, those  
 4 based on local search, especially those combining local search elements with other metaheuristics  
 5 currently constitute state-of-the-art solvers, e.g. Masmoudi et al. (2016) [25]. Applications of DARPs are  
 6 overall real-life problems with a diverse range of backgrounds, starting from the standard application to  
 7 the elderly and disabled people transportation to health care services, to the current emerging applications  
 8 in public transportation and shared mobility (Ho et al. 2018 [17], Mourad et al. 2019 [28]). For a more  
 9 comprehensive review of the problem classes, solving algorithms, and the applications of the DARP, see  
 10 Cordeau & Laporte (2007) [8], Molenbruch et al. (2017) [29], and Ho et al. (2018) [17].

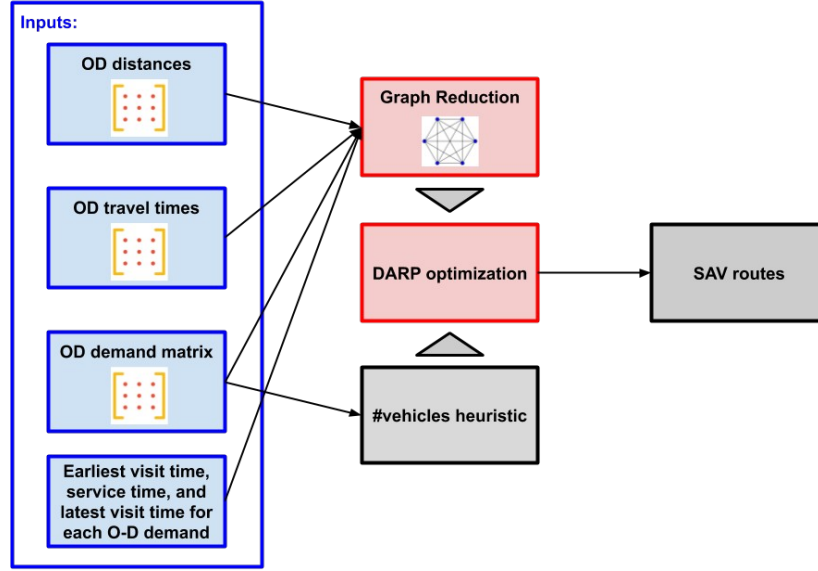
## 11 **Graph embeddings**

12 Graph embedding is a powerful method to represent a graph into a low-dimensional vector space by  
 13 preserving as much as possible its topological structure. It addresses the research question of efficient  
 14 graph analytics, since traditional methods suffer from the curse of dimensionality and space cost.  
 15 Essential graph analytic tasks, such as graph classification, node clustering, and link prediction, become  
 16 scalable to large instances. Representation learning and visualization is another research problem tackled  
 17 by graph embeddings. There are three main categories of graph embeddings: probabilistic, matrix  
 18 factorization-based, and deep learning-based methods (see Goyal & Ferrara, 2018 [13] for a survey on the  
 19 topic). Probabilistic models learn the embedding of graphs through random walks. The samplings given  
 20 by those walks can capture the neighborhood structure of nodes, connectivities, and other graph  
 21 properties such as node centrality. Compared with other probabilistic models like *DeepWalk* (Perozzi et  
 22 al., 2014 [32]) and *LINE* (Tang et al., 2015 [38]), the probabilistic model of *node2vec* (Grover &  
 23 Leskovec, 2016 [14]) is shown to perform better. This is why we chose *node2vec* implementation for the  
 24 graph embedding in the current study. The random walk exploration of this algorithm interpolates  
 25 between breadth-first (BFS) and depth-first (DFS) searches in order to construct a more informative  
 26 embedding.

## 27 **3. PROPOSED METHOD**

28  
 29 Figure 1 illustrates the flowchart of the proposed approach. It is composed of two main distinct blocks:  
 30 optimization according to the DARP formulation, and the graph reduction mechanism. Given a set of  
 31 stations in the studied area, the method's inputs are the origin-destination (OD) matrix of the traffic  
 32 demand, i.e., the number of customers traveling between any two stations, the duration of travel and the  
 33 traveled distance between all couples of stations. The earliest visit time, service time, and latest visit time  
 34 for each O-D couple of the demand matrix are also input data. Routing problems in their general  
 35 formulation are defined on an underlying graph where the set of vertices corresponds to the depot and  
 36 points to visit, and the set of arcs weighted with costs to travel, generally distances or travel times,  
 37 represent shortest paths between stations. On this defined routing graph, each point has to be visited only  
 38 once by precisely one vehicle. Since each initial station could be a pickup and a drop-off point to all the  
 39 remaining stations, and taking into account all traffic demand inputs of the OD matrix of the area, the  
 40 dimension of the routing graph to be solved can quickly increase. If  $n$  is the total number of stations, the  
 41 graph dimension could attain a maximum of  $n(n-1)$  points to visit without counting the depots. We  
 42 assume that the vehicle's capacity is greater or equal to every entry of the OD demand matrix. This will  
 43 prevent us from duplicating the same O-D couple of stations in the routing graph to account for the total  
 44 demand. The graph reduction mechanism we propose aims to detect groups of pickup and drop-off couple  
 45 points to be merged, and output a reduced graph that can be easily solvable by a mixed-integer  
 46 programming solver. Note that clustering and community detection algorithms could be used as well in  
 47  
 48

1 the step of forming groups<sup>1</sup>. We use the constrained K-means clustering method proposed Bradley et al.  
 2 (2000) [5] for comparison matter. Our method is overall a model-based heuristic around a central  
 3 component of mathematical programming. More details of the two main parts of our model is presented  
 4 in the following.



5 **Figure 1. Flowchart showing the method's main modules.**

6  
7  
8 **Mathematical formulation**

9 In this study, we build on Cordeau (2006) [7] three-index formulation  $x_{ij}^k$  of the DARP by considering a  
 10 new objective function and adding new variables. The points to visit are partitioned into two sets: the set  
 11 of pickup points  $P$  and the set of drop-off points  $D$ , such that each demand request of the OD demand  
 12 matrix has a pickup point  $i \in P$ , a corresponding delivery point which will be denoted  $deliv(i) \in D$ , a  
 13 maximum riding time  $L_i$ , and a maximum waiting time  $M_i$ . Each point  $i \in P \cup D$  has a service time  $ser_i$   
 14 , the earliest time to visit  $tw_i^{early}$ , a latest time to visit  $tw_i^{late}$ , and a demand load denoted  $req_i$ , such that for  
 15  $i \in P, req_i \geq 0$  and  $req_{deliv(i)} = -req_i$ .

16  
 17  $K$  is the set of vehicles. Each vehicle  $k \in K$  has a capacity  $Q_k \in \mathbb{N}$ , GHG emissions up to  $E_k$  by  
 18 kilometer, starts from the depot denoted  $s_k$  and ends at the depot denoted  $e_k$ . Let  $D_{start} = \{s_k : k \in K\}$  be  
 19 the set of all start depots and  $D_{end} = \{e_k : k \in K\}$  the set of all end depots. For all  $j \in D_{start} \cup D_{end}$ , the  
 20 values  $serv_j = req_j = 0$  are fixed. We associate different start and end depots to each vehicle to make the  
 21 model more general. However, all these depots may correspond to the same geographical location, which  
 22 coincide in the case of our application to the transit station of the PT network.

23  
 24 The underlying routing graph  $G = (V, A)$  has a vertex set equal to  $V = P \cup D \cup D_{start} \cup D_{end}$ . The arc  
 25 set is defined as  $A = \{(i, j) : (i \in D_{start}, j \in P) \vee (i \in D, j \in D_{end}) \vee (i, j \in P \cup D, i \neq j, i \neq deliv(j))\}$ ,

2 <sup>1</sup> This is different from cluster and route methods, which first apply a clustering algorithm to the input visiting points, and  
 3 second generate optimal routes for each cluster, e.g., Chen et al. (2020) [6].

1 and the weights of the graph are given by  $c_{ij}$  and  $d_{ij}$ , which denote respectively the duration to travel and  
 2 the distance from  $i$  to  $j$ .

3  
 4 The decision variable  $x_{ij}^k$  indicates whether the arc  $(i, j)$  is traversed by vehicle  $k \in K$  or not,  $r_i$  and  $w_i$   
 5 gives respectively the riding time and the waiting time for the request associated with pickup  $i \in P$ ,  $t_i^k$  is  
 6 the arrival time at point  $i$  using vehicle  $k$ , and  $q_i^k$  is the load of the vehicle  $k$  when leaving the point  $i$ .

7  
 8 In addition to minimizing total travel times for all vehicles, the degree of customer dissatisfaction in the  
 9 sense of Psaraftis (1980) [33] is also minimized plus to the greenhouse gas emissions due to the transport  
 10 of passengers. Plausible values of  $E_k$  are drawn from the life-cycle assessment (LCA) conducted by  
 11 Gawron et al. (2019) [12].  $(\omega_1, \omega_2, \omega_3)$  gives the weights between the three quantities in the objective  
 12 function. The model is as follows.

$$15 \text{ Min } \omega_1 \sum_{k \in K} \sum_{i, j \in V} c_{ij} x_{ij}^k + \omega_2 \sum_{i \in P} (r_i + w_i) + \omega_3 \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} E_k d_{ij} x_{ij}^k \quad (1)$$

16  
 17  
 18 Subject to:

$$19 \sum_{i \in P} x_{s_i}^k = \sum_{i \in D} x_{i_e}^k = 1 \quad (k \in K), \quad (2)$$

$$20 \sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1 \quad (i \in P), \quad (3)$$

$$21 \sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{deliv(i)j}^k = 0 \quad (i \in P, k \in K), \quad (4)$$

$$22 \sum_{j \in V} x_{ji}^k - \sum_{j \in V} x_{ij}^k = 0 \quad (i \in P \cup D, k \in K), \quad (5)$$

$$23 t_j^k \geq (t_i^k + ser_i + c_{ij}) x_{ij}^k \quad (i, j \in V, k \in K), \quad (6)$$

$$24 t_{deliv(i)}^k \geq t_i^k + ser_i + c_{idiv(i)} \quad (i \in P, k \in K), \quad (7)$$

$$25 tw_i^{early} \leq t_i^k \leq tw_i^{late} \quad (i \in P \cup D, k \in K), \quad (8)$$

$$26 q_j^k \geq (q_i^k + req_i) x_{ij}^k \quad (i, j \in V, k \in K), \quad (9)$$

$$27 r_i \geq t_{deliv(i)}^k - (t_i^k + ser_i) \quad (i \in P, k \in K), \quad (10)$$

$$28 w_i \geq t_i^k - tw_i^{early} \quad (i \in P, k \in K), \quad (11)$$

$$29 c_{idiv(i)} \leq r_i \leq L_i \quad (i \in P), \quad (12)$$

$$30 0 \leq w_i \leq M_i \quad (i \in P), \quad (13)$$

$$31 0 \leq q_i^k \leq Q_k \quad (i \in V, k \in K), \quad (14)$$

$$32 x_{ij}^k = 0 \vee 1 \quad (i, j \in V, k \in K). \quad (15)$$

33  
 34 The routing constraints (2)-(5) ensure that each vehicle starts and ends at its corresponding depot points in  
 35 constraint (2), each request is answered in constraint (3), the same vehicle is used for pickup and drop-off  
 36 in constraint (4), and the flow conservation in constraint (5). Constraint (6) tracks the service time, while  
 37 constraint (7) ensures that pickup points are visited before their delivery points, and time window  
 38 constraints are given in (8). Constraint (9) tracks the load of vehicles, which is needed later to compute  
 39 the vehicle occupancy rate (VOR) indicator. Constraints (10) and (11) are respectively relative to the  
 40 riding time and the waiting time for each request. Constraints (12)-(15) represent the binary and the  
 41 bounding restrictions for the decision variables. Note that the current mixed-integer programming solvers



1 can efficiently handle indicator constraints as in (6) and (9). Other useful constraints to add concern the  
 2 battery management aspect, which includes the recharge time, the battery energy consumption and  
 3 detours to recharge stations as in Bongiovanni et al. (2019) [4]. Recent simulations (Vosooghi et al., 2020  
 4 [40]) suggest that a best strategy for SAV could be deploying batteries with more charging space to avoid  
 5 charging the battery within the rush hours of the morning and the evening peaks, otherwise alternating  
 6 charging and routing could drastically increase the passenger kilometer traveled. For this reason and for  
 7 the time being, we suppose that battery charging operations are set outside the routing process, and that  
 8 vehicles are set to be initially charged before transporting customers.

9  
 10 For the heuristic to determine the number of vehicles  $|K|$  (mentioned in Figure 1), we use the following  
 11 formula for vehicles with an identical capacity  $Q=Q_k$ :

$$12 \quad \text{vehicles}(m) = \sum_{i,j \in V^2, i \neq j} \text{demand}(i,j) / (n \times Q).$$

13  
 14  $m$  is an estimate of the average number of O-D requests each vehicle is expected to answer during the  
 15 time horizon (usually  $m=3$  or 4).  
 16  
 17  
 18

### 19 **Graph reduction mechanism**

20 The steps of the graph reduction are listed in the below algorithm. At first, *node2vec* is applied to the  
 21 routing graph  $G$ , which is introduced in the previous section and is weighted by the travel durations  
 22  $(c_{ij})_{i,j \in A}$ , for the following inputs: the desired dimension size of the embedding  $d$ , the number of random  
 23 walks to launch from each node and their length, plus other parameters related to the guided exploration  
 24 strategy of the walks. Thereafter, each node of  $G$  can be represented by a feature vector of dimension  $d$ .  
 25 To obtain a similarity matrix  $S$  for the nodes of  $G$ , we apply the cosine similarity, which is defined for  
 26 any two real-valued vectors  $v_1$  and  $v_2$  as the cosine of the angle  $\theta$  between them, i.e.,  
 27  $\cos(\theta) = v_1 \cdot v_2 / (\|v_1\| \times \|v_2\|)$ . Other measures of similarity and distances could be also used at this step.  
 28 This step is important because it gives a quantification of the similarity between any two nodes of  $G$   
 29 while taking into account as much as possible the topology structure of the graph.  
 30

31 The remaining steps concern merging the points to visit. We opt for merging pick-up and drop-off (P-D)  
 32 couples two by two. This gives us more flexibility in the rate of contraction of the graph  $G$ , and allows us  
 33 to quickly compute the optimal order within the merged P-D couples. For each two P-D couples  
 34  $(p_1, \text{deliv}(p_1), p_2, \text{deliv}(p_2))$ , we compute the following average similarity using the similarity matrix  $S$

$$35 \quad \text{simil}(p_1, p_2) = \left( S(p_1, p_2) + S(\text{deliv}(p_1), \text{deliv}(p_2)) + S(p_1, \text{deliv}(p_1)) \right) / 6 \\
 36 \quad + \left( S(p_2, \text{deliv}(p_2)) + S(p_2, \text{deliv}(p_1)) + S(p_1, \text{deliv}(p_2)) \right) / 6, \quad (17)$$

37  
 38 which indicates how close the two couples are. Note that we affect a higher input parameters for BFS  
 39 over DFS in the guided search of *node2vec* to favor closer nodes within the vicinity of each node of  $G$ .  
 40 After ranking pairs of the P-D couples by their similarity, we choose the top  $r \times (|P| \cup |D|) / 2$  pairs, with  
 41  $r$  is the rate of contraction of the graph. The following step proceeds by computing the optimal order of  
 42 visiting points within each of the chosen pairs. There are only six possible orders respecting the pickup  
 43 and drop-off constraints among the total 32, which are shown in Figure 2-a. For each optimal order, we  
 44 output two aggregated nodes in the novel graph, i.e., if an optimal order is for instance  
 45  $(p_1, \text{deliv}(p_1), p_2, \text{deliv}(p_2))$ , the new nodes will correspond to  $p' = (p_1, \text{deliv}(p_1))$  and

1  $deliv(p') = (p_2, deliv(p_2))$ . The final step updates the graph weights involving the new nodes, as shown  
 2 in Figure 2-b. Distances between nodes are updated in the same manner. The graph reduction algorithm  
 3 has an overall time complexity of  $O(|V|^2)$ , since *node2vec* runs in  $O(|V|^2)$  and the number of pairs of P-  
 4 D couples processed in the merging steps is at maximum  $COMBIN(n = (|P| \cup |D|)/2, k=2) \leq |V|^2/8$ .

5  
6

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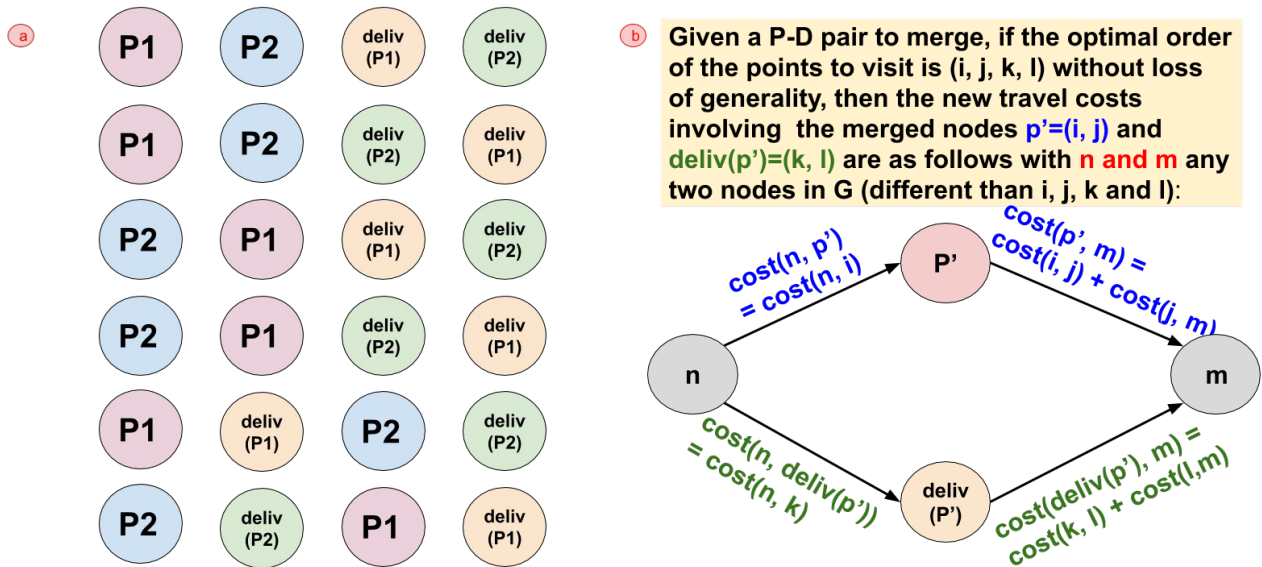
**Algorithm:** *node2vec*-based graph reduction

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**Inputs:** The routing graph  $G=(V, A)$ , the contraction rate  $r$ .

1. Apply the *node2vec* algorithm to  $G$ .
  2. Compute the similarity matrix  $S$  of  $G$  using the cosine measure on the embedding of 1.
  3. Compute for each two P-D couples  $(p_1, deliv(p_1), p_2, deliv(p_2))$  the value  $simil(p_1, p_2)$ .
  4. Select the top  $r \times (|P| \cup |D|)/2$  similar pairs of P-D couples.
  5. Compute the optimal order of points to visit within each chosen pair.
  6. Update the graph's costs for the arcs going to/coming from the newly merged nodes.
- 

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12 **Figure 2. Possible arrangements for a pair of P-D couples in (a),**  
 13 **and the updated travel costs involving merged P-D pairs in (b).**  
 14

1 **4. CASE STUDY: THE INDUSTRIAL DISTRICT OF “VALLÉE DE LA CHIMIE”, LYON CITY**

2  
3 In this section, we describe the studied territory and the process used to generate the input demand data.

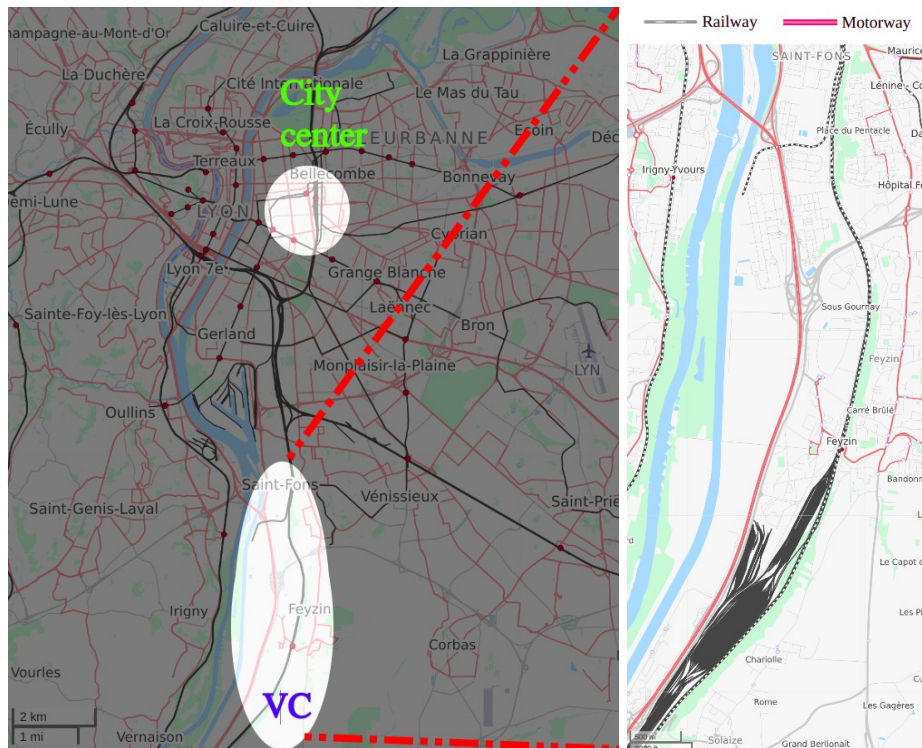
4  
5 **Territory**

6 Lyon is the second-largest urban area of France and is home to several industrial hubs. The largest of  
7 them is “Vallée de la Chimie” (VC) for approximately  $17\text{ km}^2$ . Situated about  $10\text{ km}$  away from city  
8 center, VC has currently over 1,000 companies, 6 research and development centers, mostly working in  
9 the chemical, energy and the environmental sectors, and it attracts over 20,000 jobs (Métropole de Lyon,  
10 2020 [27]).

11  
12 In terms of mobility infrastructure, a Rapid Transit System (RTS) links VC to the city center on a regular  
13 schedule (each 30 min in rush hours, otherwise once by hour). There are quite few buses connecting VC  
14 to Lyon city (in total three, two are drawn in the right map of Figure 3 in light red lines). The highway A7  
15 also crosses the territory. This artery is essential for logistic transportation and gets severely congested in  
16 the morning rush hours, as A7 is also the south entry point to the city. Commuting to work accounts for a  
17 significant part of mobility flows from/to VC. By considering the low PT supply and the highway’s high  
18 saturation, a typical last-mile transit issue occurs. A number of experimentations to overcome this issue  
19 are currently conducted in this territory: Personal Rapid Transit (PRT) from the Esprit project (Esprit,  
20 2019 [9]) and a demand-responsive transit (DRT) service from the local PT service (TCL, 2020 [39]). VC  
21 constitutes a good study example for the SAV deployment.

22  
23 This test case is finally classic and generalizable to several other mobility situations all over the world.  
24 Indeed, it corresponds to the framework of a commute problem with two main alternatives: PT and the  
25 private vehicle which experiments bottleneck.

26

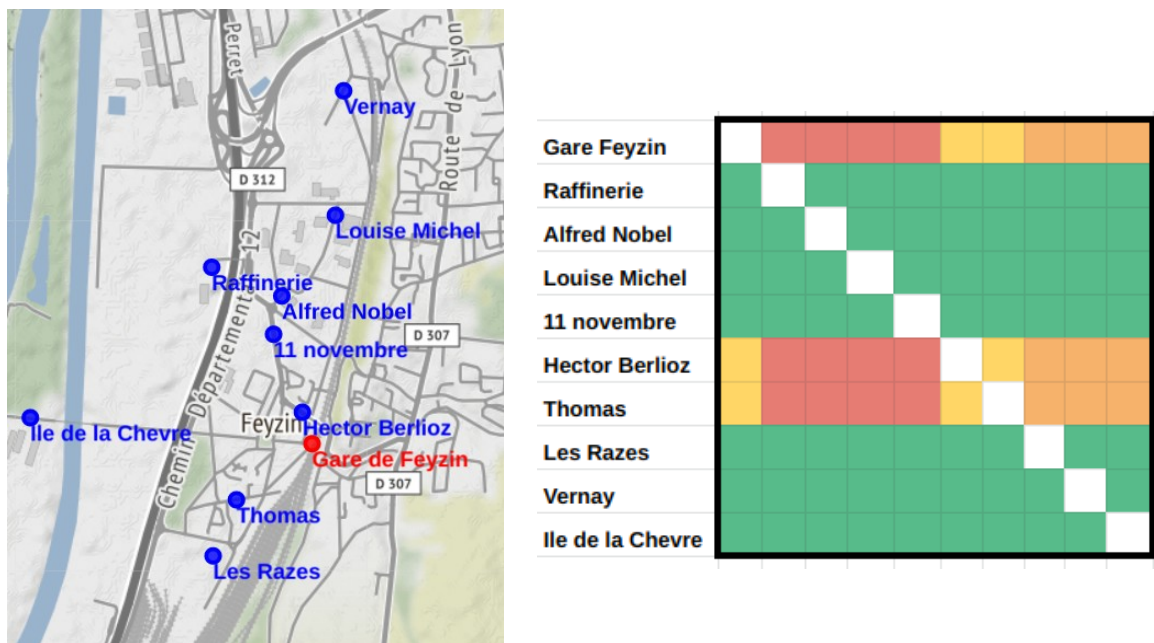


27 **Figure 3. The placement of VC within the Lyon urban area in the left map, and**  
28 **the transport network connecting VC in the right map. Source: [www.openstreetmap.org/](http://www.openstreetmap.org/).**

1 **Demand generation**

2 To generate the traffic demand, we apply a Land Use and Transport Interaction (LUTI) model. LUTI  
 3 models are spatial interaction models that integrate the socio-demographic and economic features of the  
 4 population to the transport infrastructure and mobility features, to answer questions such as, what is the  
 5 transport modal share of a middle-sized income household living in the suburb to commute to work to the  
 6 city center. With LUTI models, the cause/effect relationships between land use and transport become well  
 7 understood, and the mobility patterns are well identified. It is noteworthy that LUTI models are  
 8 extensively used for urban and transport plannings (see Acheampong et al., 2015 [1] for a review on the  
 9 topic).

10  
 11 We use the LUTI model called OPTIREL, which has the advantage to account for intermodality.  
 12 OPTIREL proceeds by decomposing the area’s space according to the transport mode used from/to the  
 13 target territory. For instance, the first zone of the studied territory is solely accessible by walking, the  
 14 second zone is larger and is accessible by bus or RTS system (from/to the territory), the third zone is even  
 15 larger. It delimits areas accessible by private vehicles combined with relay parking. In this manner, it is  
 16 possible to define mobility solutions combining several modes in the step of the modal decomposition.  
 17 The classical four-step travel models define zones according to the administrative or the population  
 18 census based spatial decomposition, which turns out to be a rigid assumption for combining several  
 19 modes in the third step of modal share. The traffic demand considered in our application corresponds to  
 20 the PT demand arriving in the territory by train during the morning peak [7h-10h]. This interval will set  
 21 our time horizon. Ten stations are defined for the planned SAV service, which correspond to stations of  
 22 the Esprit project (Esprit, 2019 [9]), and cover the north part of VC. They are shown in Figure 4. The  
 23 main transit station is “Gare Feyzin” which will coincide with the depots of all vehicles. We can notice  
 24 from our input OD demand matrix, generated by OPTIREL, and shown in Figure 4, that traffic coming  
 25 from “Gare Feyzin” is the highest. Interestingly, the traffic is also high from the stations “Hector  
 26 Berlioz” and “Thomas” which are the closest stations to the transit station. Finally, duration and distances  
 27 between stations are generated using web mapping services.



49 **Figure 4. The locations of the ten stations considered in the study (left map), and the input OD**  
 50 **demand matrix (right plot), which is colored according to the demand intensity: green=low, yellow**  
 51 **and orange=middle, and red=high.**

1 **5. RESULTS**

2

3 In this section, we test the proposed method on the study case and show its usefulness.

4

5 **Parameters setting**

6 For solving the DARP, ILOG CPLEX 12.10 solver is used, with the termination criteria set to a relative  
 7 gap of 1%. Although the routing graph is reduced, we have noticed that the problem (1)-(15) takes  
 8 extended time to find a first solution, which is not the case for the problem (1)-(15) excluding the  
 9 precedence constraint (7). Thus, we solve the latter problem in a first step, rearrange the points to visit  
 10 for each vehicle by solving the single-vehicle DARP problem (1)-(15), and then consider this solution a  
 11 starting point for our main optimization. The effect of adding an initial good quality starting point is  
 12 beneficial in our CPLEX solving process. In the absence of precise data about the customers' time  
 13 windows, we set the constraints (8) to two time windows, each with one hour duration, and uniformly at  
 14 random partition P and D points on the two time windows, with the condition that each dropoff point has  
 15 the same time window of its pickup point. We do not consider customers waiting times  $w_i$  with this  
 16 configuration of time windows. A fixed service time  $ser_i=1$  minute is associated to each point to visit  
 17  $i \in P \cup D$ . In the baseline scenario, we set the capacity of vehicles to  
 18  $Q=25 > 20 = \max_{i,j \in V^2, i \neq j} demand(i,j)$ , the number of vehicles to  $vehicles(m=3)=4$ , the maximum  
 19 riding time to  $L=15$  minutes, and the weights of the objective function to  $(\omega_1, \omega_2, \omega_3)=(1,1,0.1)$ . The  
 20 GHG emission rate is chosen to be equal to  $E_k=0.171 \text{ Kg CO}_2 - eq/km$ , which corresponds to the short-  
 21 range electric autonomous taxi scenarios in Gawron et al. (2019) [12]. For the input parameters of  
 22  $node2vec$ , the dimension size is set to 30, while the walk length is equal to 6 for 1000 random walks  
 23 launched from each node. Since  $node2vec$  is a stochastic method, we launch this method multiple times,  
 24 exactly 50 times for each configuration of  $Q$  and  $L$ , then choose the instance with the minimal inside  
 25 travel time. This latter value is given by the summation of travel times inside the reduced nodes. For  
 26 instance, the inside travel time of the graph of Figure 2-b is equal to  $c_{ij}+c_{kl}$ . The calculation is done for  
 27 each configuration of  $Q$  and  $L$  as the possible arrangements of P-D couples could be less than six (Figure  
 28 2-a). All experiments are performed on an Intel(R) Core(TM) i7-8750H CPU @ 2.20GHz machine with  
 29 32 GB RAM memory.

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32 **Constrained K-means**

33 The constrained K-means clustering proposed by Bradley et al. (2000) [5] extends the traditional  
 34 algorithm ensuring that each cluster contains at least a minimum number of elements. The problem of  
 35 minimum cost flow optimization is used to solve the new formulation. Similarly, it is possible to add a  
 36 maximum size constraint to clusters. In order to have a fair comparison with  $node2vec$ , we associate to  
 37 each pickup node a value similar to the average of the expression (17) :

38

$$39 \quad cost_{avg}(p) = \sum \left( c_{pq} + c_{deliv(p)deliv(q)} + c_{pdeliv(p)} + c_{qdeliv(q)} + c_{pdeliv(q)} + c_{qdeliv(p)} \right) / 12$$

$$40 \quad + \sum \left( c_{qp} + c_{deliv(q)deliv(p)} + c_{deliv(p)p} + c_{deliv(q)q} + c_{deliv(q)p} + c_{deliv(p)q} \right) / 12.$$

41

42 This expression has twelve terms for each  $q \in P$  to account for the asymmetry of the travel times  $c_{ij}$ .  
 43 Clusters are generated with a maximum size of two and a minimum size of one, for the clustering with the  
 44 best inside travel time among 50 runs.

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1 **Indicators**

2 Two main performance measures are used: the vehicle kilometers traveled (VKT), and the vehicle  
 3 occupancy rate (VOR). Two additional measures are extracted from them. VKT indicates the total  
 4 distance traveled by all vehicles of  $K$  to satisfy the demand, and is equal to,

$$5 \quad VKT(x) = \sum_{k \in K} \sum_{i, j \in V^2, i \neq j} d_{ij} x_{i,j}^k.$$

6 We also use a correlated measure to VKT, which is the total GHG emissions generated for all trips to  
 7 provide an order of magnitude of SAV emissions :  $GHG(x) = E_k VKT(x)$ . To get a glimpse of the GHG  
 8 environmental gain of using SAV, which is previously established in the literature, we show the gap:

$$9 \quad GHG_{gain}(x) = \frac{GHG_{pv}(demand) - GHG(x)}{GHG_{pv}(demand)},$$

10 with  $GHG_{pv}(demand)$  represents GHG emissions estimated for the input demand and the personal  
 11 vehicle mode.

12

13 The third measure VOR is an average of the load rate of all vehicles at each step of the routing, and is  
 14 equal to,

$$15 \quad VOR(x, q) = \frac{1}{|K|} \frac{1}{|P| \cup |D|} \sum_{k \in K} \sum_{i, j \in P \cup D, i \neq j} x_{ji}^k \frac{q_i^k}{Q}.$$

16 VOR can also be seen as an index of comfort of the mobility service. The last measure, the zero  
 17 occupancy rate (ZOR) is equal to the percentage of trips between any two stations which have an  
 18 occupancy rate equal to zero, and is given by,

$$19 \quad ZOR(x, q) = \frac{1}{|K|} \sum_{k \in K} \sum_{i, j \in P \cup D, i \neq j, q_i^k = 0} \frac{x_{ji}^k}{(|P| \cup |D|) - 1}.$$

20 All measures are computed on the original non-reduced problem.

21

22

23 **Baseline scenario**

24 Accounting for the total demand of VC leads to solving a problem of 54 points to visit<sup>2</sup> plus the dummy  
 25 start and end vehicle depots, which is a computationally hard problem for  $k > 2$  vehicles. The graph  
 26 reduction mechanism makes this optimization more manageable. CPLEX solver could attain a solution  
 27 for the baseline scenario within a relative MIP gap of 23% in 0.28, 277.17, and 3609.62 seconds when the  
 28 graph contraction rate is respectively 50%, 45%, and 40%. In the rest of the study, we set the contraction  
 29 rate of the routing graph at  $r = 50\%$ . Figure 5 illustrates the SAV routes obtained for the baseline  
 30 scenario. This scenario has a total traveled distance of  $VKT = 65.46$  (respectively  $74.44 km$ ) for an  
 31 average occupation rate of  $VOR = 42.1\%$  (resp.  $31.6\%$ ), and a total emissions of  $GHG = 11.12$  (resp.  
 32  $12.73 Kg CO_2 - eq$ ) for *node2vec* (resp. constrained K-means) based node reduction. By drawing on the  
 33 estimates of the vehicle occupancy rate and the GHG emissions of private vehicles computed for the  
 34 Lyon urban area (François et al, 2017 [11]), we obtain :

$$35 \quad GHG_{pv}(demand) = \sum_{i, j \in V^2, i \neq j} demand(i, j) \times d_{ij} \frac{175 g CO_2 - eq / Km}{1.33 person / car} = 43.8 Kg CO_2 - eq.$$

36 Therefore, compared to the private vehicle mode, the SAV service leads to a reduction of 74.61% of  
 37 GHG emissions for the territory of VC when relying on the *node2vec* reduction.

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2 2 The number of points to visit for the full OD demand matrix of Figure 4 is 180 points. We have only 54 points in our input  
 3 data, since some O-D couples have a null demand.



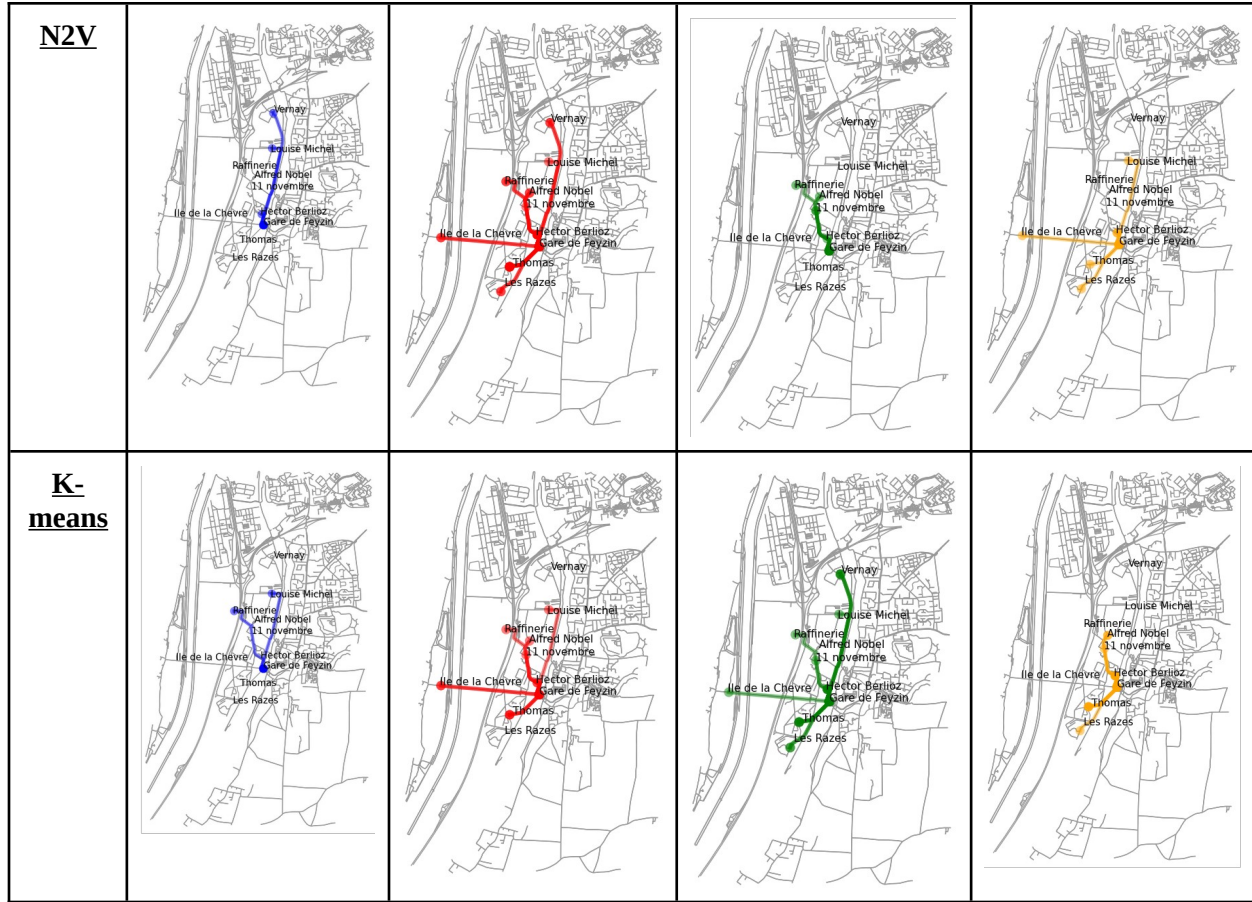


Figure 5. SAV routes for the baseline scenario with the graph embedding reduction (first row) and the K-means based reduction (second row).

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### Sensitivity analysis

7 Figure 6 shows the sensitivity analysis performed on the baseline scenario, by varying the fleet size  $|K|$ ,  
 8 the maximum riding time  $L$ , and the vehicle's capacity  $Q$ . For a number of vehicles equal to 1, 5, and 10,  
 9 the total traveled distance VKT slowly decreases to be resp. 67.70, 64.53, and 59.88 km for the *node2vec*  
 10 reduction. K-means clustering generates tours in the same decreasing trend, but with higher VKT, which  
 11 is an indication of a lower performance to find best pairs of P-D couples to merge. While  $|K|$  is  
 12 multiplied by 10, VKT of *node2vec* is divided by 1.13. Having more vehicles actually allows more  
 13 flexibility for the routing optimization to produce efficient assignments of vehicles to the transport  
 14 requests. VKT is decreasing in function of  $Q$ , which is expected since larger capacities imply smaller  
 15 tours. *Node2vec* is again much efficient than K-means reduction in this regard. VKT is overall stable  
 16 under varying  $L$ . As some values of  $L$  and  $Q$  may constraint the number of possible arrangements of  
 17 Figure 2-a, an oscillatory effect when varying  $L$  and  $Q$  could be seen for VKT and VOR index as well.  
 18  $GHG_{gain}$  plots on the other hand show the substantial cut in terms of GHG emissions if SAV are deployed  
 19 instead of private vehicles, especially for the *node2vec* method. The reduction corresponds to resp.  
 20 73.57% and 76.62% of GHG emissions if the fleet size is equal to resp. 1 and 10. Concerning the vehicle  
 21 occupation rate, we notice that this rate is not affected much by the size  $|K|$  and the riding time  $L$ , and  
 22 oscillates around the mean value of 35%. The vehicle's capacity  $Q$ , on the other hand, has some influence  
 23 on VOR. VOR decreases from 38.5 (resp. 43.3%) to 24.9 (resp. 22.7%) when the capacity increases from

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1 20 to 50 for *node2vec* (resp. K-means) reduction. This particular point should be carefully considered by  
2 the designers of SAV services to achieve a fair tradeoff between users' comfort and the costs for an  
3 additional capacity of automated vehicles and buses. As for empty trips, ZOR is solely impacted by the  
4 size  $|K|$ . This index decreases for an increase of  $|K|$ , otherwise it has a constant value around 10%.

5  
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## 7 **6. CONCLUSION**

8

9 In this study, we propose a model-based heuristic design for a shared autonomous vehicle (SAV) service  
10 by combining the formulation of the Dial-a-Ride-Problem (DARP) and a graph reduction mechanism.  
11 This latter procedure is based on the graph embedding framework *node2vec*, and allows us to merge  
12 similar couples of pickup and drop-off points, and hence solving instances with a large number of O-D  
13 requests and fleet size. A study case analysis is provided for an industrial district of the Lyon urban area  
14 (France), wherein the traffic demand is generated by a LUTI model. In the paper, we address in a classical  
15 commute problem between city center and suburbs where two transportation alternatives compete:  
16 highway and rapid transit system (RTS). The SAV service is deployed as a last-mile transit for the RTS.  
17 Our results suggest that *node2vec* is more efficient for node reduction than the constrained K-means (in  
18 terms of vehicle traveled distance), in addition around 75% reduction of GHG emissions are gained by  
19 SAV service when compared to the private vehicle choice.

20

21 The limitations of the study mainly concerns the maximum reduction rate, which is currently 50% as  
22 reduction is done by merging couples of pickup and drop-off pairs, and the specificity of our application  
23 case. One natural improvement is to enlarge the reduction mechanism to three pickup and drop-off pairs.  
24 We also think that integrating the current approach with the LUTI model would generate precise  
25 partitions of the mode share for the studied territory and population, which would in return help to fix the  
26 adequate parameters of the SAV service, i.e., load capacity, fleet size, maximum riding time, in a cause-  
27 and-effect loop.

28

29

## 30 **ACKNOWLEDGEMENTS**

31 This work is supported by "Lyon Urban School" (ANR-17-CONV-0004) of Université de Lyon, within  
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33 Research Agency (ANR). We would like to warmly thank Guy Bourgeois for making the data of  
34 OPTIREL available.

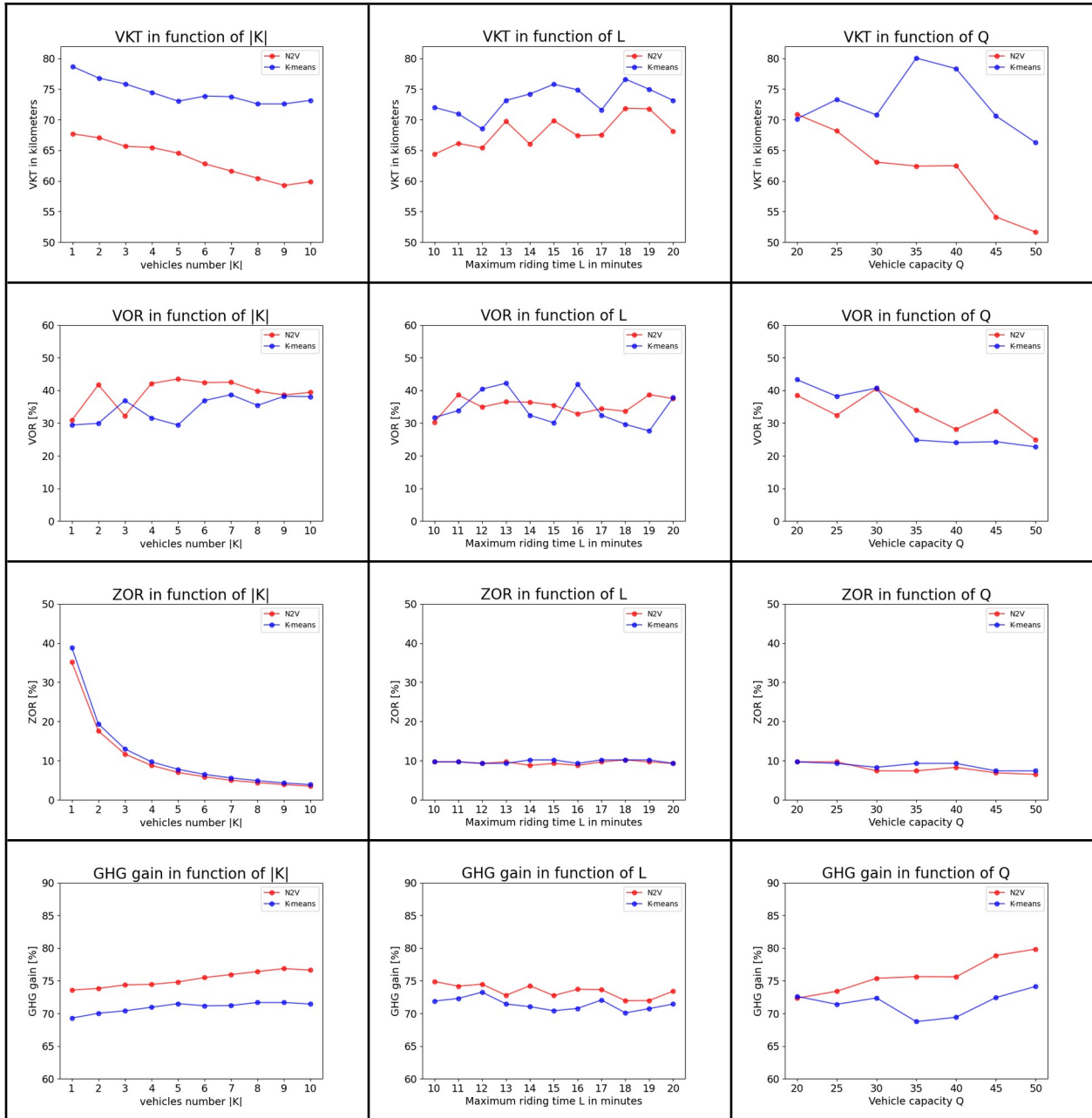
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**Figure 6. Sensitivity analysis on the baseline scenario of the fleet size (first column), the clients maximum riding time (second column) and the vehicles' capacity (third column) for the indicators VKT (fist row), VOR (second row), ZOR (third row) and GHG gain (fourth row).**

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3

**Table 1. Numerical values of the sensitivity analysis (plotted in Figure 6).**

Fixed parameters	Variable parameter	VKT [Km]		VOR [%]		ZOR [%]		GHG [Kg CO2-eq]	
		N2V	K-means	N2V	K-means	N2V	K-means	N2V	K-means
Q = 25 L = 15 min	K  = 1	<b>67.7</b>	78.7	<b>30.8</b>	29.4	<b>35.2</b>	38.9	<b>11.6</b>	13.45
	K  = 2	<b>67.0</b>	76.8	<b>41.7</b>	29.9	<b>17.6</b>	19.4	<b>11.5</b>	13.13
	K  = 3	<b>65.6</b>	75.8	<b>32.2</b>	36.9	<b>11.7</b>	13.0	<b>11.2</b>	12.97
	K  = 4	<b>65.5</b>	74.4	<b>42.1</b>	31.6	<b>8.8</b>	9.7	<b>11.2</b>	12.73
	K  = 5	<b>64.5</b>	73.0	<b>43.5</b>	29.4	<b>7.0</b>	7.8	<b>11.0</b>	12.49
	K  = 6	<b>62.8</b>	73.9	<b>42.4</b>	36.9	<b>5.9</b>	6.5	<b>10.7</b>	12.63
	K  = 7	<b>61.6</b>	73.7	<b>42.5</b>	38.7	<b>5.0</b>	5.6	<b>10.5</b>	12.61
	K  = 8	<b>60.4</b>	72.6	<b>39.8</b>	35.4	<b>4.4</b>	4.9	<b>10.3</b>	12.41
	K  = 9	<b>59.3</b>	72.6	<b>38.6</b>	38.2	<b>3.9</b>	4.3	<b>10.1</b>	12.41
	K  = 10	<b>59.9</b>	73.2	<b>39.4</b>	38.1	<b>3.5</b>	3.9	<b>10.2</b>	12.51
K  = 4 Q = 25	L = 10min	<b>64.3</b>	72.0	<b>30.3</b>	31.7	<b>9.7</b>	9.7	<b>11.0</b>	12.31
	L = 11min	<b>66.1</b>	70.9	<b>38.6</b>	33.8	<b>9.7</b>	9.7	<b>11.3</b>	12.1
	L = 12min	<b>65.4</b>	68.5	<b>34.9</b>	40.4	<b>9.3</b>	9.3	<b>11.2</b>	11.7
	L = 13min	<b>69.7</b>	73.2	<b>36.5</b>	42.2	<b>9.7</b>	9.3	<b>11.9</b>	12.5
	L = 14min	<b>66.0</b>	74.2	<b>36.4</b>	32.4	<b>8.8</b>	10.2	<b>11.3</b>	12.7
	L = 15min	<b>69.8</b>	75.8	<b>35.5</b>	30.1	<b>9.3</b>	10.2	<b>11.9</b>	13.0
	L = 16min	<b>67.4</b>	74.9	<b>32.8</b>	41.9	<b>8.8</b>	9.3	<b>11.5</b>	12.8
	L = 17min	<b>67.5</b>	71.6	<b>34.4</b>	32.4	<b>9.7</b>	10.2	<b>11.5</b>	12.2
	L = 18min	<b>71.9</b>	76.6	<b>33.6</b>	29.6	<b>10.2</b>	10.2	<b>12.3</b>	13.1
	L = 19min	<b>71.8</b>	74.9	<b>38.7</b>	27.6	<b>9.7</b>	10.2	<b>12.3</b>	12.8
L = 20min	<b>68.1</b>	73.2	<b>37.5</b>	37.8	<b>9.3</b>	9.3	<b>11.6</b>	12.5	
K  = 4 L = 15 min	Q = 20	<b>70.9</b>	70.1	<b>38.5</b>	43.3	<b>9.7</b>	9.7	<b>12.1</b>	11.0
	Q = 25	<b>68.2</b>	73.3	<b>32.4</b>	38.2	<b>9.7</b>	9.3	<b>11.7</b>	12.5
	Q = 30	<b>63.1</b>	70.8	<b>40.5</b>	40.7	<b>7.4</b>	8.3	<b>10.8</b>	12.1
	Q = 35	<b>62.4</b>	80.1	<b>34.0</b>	24.8	<b>7.4</b>	9.3	<b>10.7</b>	13.7
	Q = 40	<b>62.5</b>	78.3	<b>28.1</b>	24.0	<b>8.3</b>	9.3	<b>10.7</b>	13.4
	Q = 45	<b>54.1</b>	70.6	<b>33.6</b>	24.3	<b>6.9</b>	7.4	<b>9.3</b>	12.1
	Q = 50	<b>51.6</b>	66.2	<b>24.9</b>	22.7	<b>6.5</b>	7.4	<b>8.8</b>	11.3

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