



HAL
open science

Symmetric variational formulations for the vibrations of damped structural-acoustic systems. Aerospace applications

Roger Ohayon, N. Meidinger, H. Berger

► **To cite this version:**

Roger Ohayon, N. Meidinger, H. Berger. Symmetric variational formulations for the vibrations of damped structural-acoustic systems. Aerospace applications. Proc. AIAA/ASME 28th Structures, Structural Dynamics Conference, Apr 1987, Monterey, United States. <10.2514/6.1987-817>. <hal-04114393>

HAL Id: hal-04114393

<https://hal.science/hal-04114393v1>

Submitted on 21 Mar 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

SYMMETRIC VARIATIONAL FORMULATIONS FOR THE VIBRATION OF DAMPED STRUCTURAL-ACOUSTIC SYSTEMS. AEROSPACE APPLICATIONS

R. Ohayon*, N. Meidinger**, H. Berger***

Office National d'Etudes et de Recherches Aérospatiales
BP 72. F - 92322 Châtillon Cedex, France

Abstract

We consider the linear vibrations of a structure coupled to an internal gas and subjected to a prescribed diffuse noise. The considered damping effects arise from an impedance boundary condition written on the fluid-structure interface, together with the classical structural one.

Starting from the local equations of the coupled system, the numerical analysis is based on an appropriate symmetric variational formulation leading to a symmetric matrix system, through a finite element discretization. The forced vibration problem is then solved using a Ritz-Galerkin projection on the structural-acoustic modes of the associated conservative system.

Numerical applications are presented in the case of a launch vehicle shroud model. The results present, for each considered substructure of the shroud, the dynamic responses expressed in terms of power spectral density and root mean square values.

Nomenclature

U (resp. U_g)	structural displacement field (resp. gas displacement field)
p (resp. ϕ)	pressure (resp. displacement potential) field in the gas
ρ (resp. ρ_g)	mass structural density of the structure (resp. of the gas)
σ_{ij} (resp. ϵ_{ij})	stress (resp. strain) tensor in the structure
c	velocity of sound in the gas
ω	circular frequency
$Z(\omega)$	fluid-structure boundary impedance
α, β	structural Rayleigh damping coefficients in the structure
P.S.D. (resp. R.M.S.)	Power Spectral Density (resp. Root Mean Square)

I. Introduction

This investigation is concerned with the economic and accurate computational methods in order to quantify in terms of power spectral density and root mean square values the linear vibrations of a coupled system constituted by a compressible gas contained into an elastic enclosure subjected to a prescribed diffuse noise.

In the present paper, we do not consider any damping arising from the external environment (acoustic external radiation). We are here concerned by the vibrations of such coupled system into a reverberant chamber. More specifically, the bounded structural-acoustic system will be considered as subjected to a prescribed diffuse noise.

The forced damped vibration problem is solved, through an appropriate symmetric variational formulation by using a Ritz-Galerkin projection on the modes of the structural-acoustic associated conservative system.

This fluid-structure problem has been the subject of many studies in litterature⁽¹⁻⁹⁾, we refer to the reference (7) for application of component mode procedure⁽⁸⁾ applied to a coupled system described by non symmetric matrix system. This last point has created a puzzle for many fluid-structure investigators⁽⁹⁻¹⁰⁾. The reference (11) uses the velocity potential field into the gas domain as unknown and obtains a symmetric coupled matrix system of "non conservative" type (in fact, gyroscopic). The non symmetry of the operators forming the coupled matrix system describing the structural-acoustic model occurs for conservative fluid-structure bounded medium, i.e. for an elastic undamped envelope coupled with an internal inviscid compressible gas. This strange fact is due to the choice of the state variables describing the system (ref. (12-15)).

The structure being usually described by a displacement field U , one can choose for the fluid either a displacement field U_g or a pressure field p (stress type variable).

When using a displacement type variable for the gas, there is no problem of symmetry in the final coupled matrix system, but a major drawback of the formulation comes from the existence of an infinite number of spurious zero eigenvalue frequencies connected with the irrotationality condition :

$$\text{curl } U_g = 0 \text{ in the gas domain}$$

On the contrary, this formulation is very convenient for one-dimensional problem such as pipes⁽¹⁶⁾.

A way to circumvent this irrotationality condition is to describe the gas by a scalar field

* Senior Staff Scientist
** Research Scientist
*** Senior Staff Scientist

namely the pressure perturbation at any point of the fluid domain.

The non symmetry of the coupled matrix system comes from the fact that one have to couple two domains, one being described by displacement variable and the other by pressure (stress type) variable. The resulting matrix system has the following form :

$$\left\{ \begin{matrix} -\omega^2 & \begin{bmatrix} M & 0 \\ C & A \end{bmatrix} \end{matrix} \right\} + \left\{ \begin{matrix} K & -C^T \\ 0 & N \end{matrix} \right\} \begin{bmatrix} U \\ P \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Let us remark that the utilization of boundary integral procedure⁽¹⁷⁾ which are very efficient for external problems and also for internal problems when the fluid is incompressible⁽¹⁸⁾ leads to non linear eigenvalue problem with respect to ω^2 for compressible fluid.

We have therefore proposed in references (12-15) various symmetric variational formulations leading to symmetric matrix systems ; some of them have been applied to specific problems⁽¹⁹⁾. We have also recently extended some formulations to transient problems⁽²⁰⁾ and the reference (21) exhibits a matrix scaling procedure in order to analyze the appropriate eigenvalue solvers for such situations.

The component mode methodology has also been investigated in reference (12) as in references (4-7) but we must stress the fact that before applying such Ritz projections, one has to exhibit appropriate symmetric variational principles⁽¹²⁾ specially when the coupled system is conservative or damped through classical assumptions; on the contrary the investigators or references (4-7) have based their analysis on non symmetric systems which are not directly connected with the physical conservative aspects of the interaction. Nevertheless the analysis of references (4-7) is correct and interesting from pure computational point of view.

The key of the symmetrization procedure is to use additional variable in the fluid for instance, namely the pressure p and the displacement potential ϕ (or the reaction forces in the structure⁽²⁰⁾).

In the framework of such symmetric formulation, the main feature of this paper consists in taking into account an acoustic damping effect based on a boundary impedance $Z(\omega)$ on the fluid-structure interface.

In order to overcome the difficult problem of boundary layer description through viscous fluid equations, we introduce the following approximate dissipative model.

In a thin layer between the inviscid fluid interior domain and the fluid-structure interface, we introduce a damping behaviour through a constitutive law of "instantaneous viscoelastic" type such a dashpot which relates the pressure to the jump of the normal velocity. Of course, in the exact dissipative model, this jump is zero together with the tangential velocity, but in the approximate dissipative model, as we introduce a

damping into a third intermediate medium of infinitely small thickness, we must introduce such discontinuity.

The resulting surface constitutive law has the following form :

$$p = -j\omega Z(\omega) \llbracket u \cdot n_s \rrbracket \quad (1)$$

fluid-structure interface

and allows the dissipative model to tends continuously to the conservative one when $Z(\omega)$ tends toward infinity ((1) will be considered as a penalty condition into a variational formulation). This concept is currently used in acoustic community in order to avoid the difficulty of Navier-Stokes boundary layer problems (unsteady case).

In a first step, we shall, for sake of physical interpretation, take into account the preceding damping effect into a displacement formulation of the structural-acoustic problem. As said in the beginning of this introduction, this formulation is not convenient from numerical point of view.

In a second step, we shall reconsider the formulation with scalar unknown fields for the gas, leading to symmetric matrix system.

In a third step, we shall consider the forced vibration problem (deterministic prescribed forces or random diffuse noise).

In a fourth step, numerical results applied to a model of the shroud of a launch vehicle (ARIANE 4) are presented in order to illustrate the proposed numerical methodology.

II. Local equations of the conservative system

The figure 1 presents a schematic diagram of the gas-structure system.

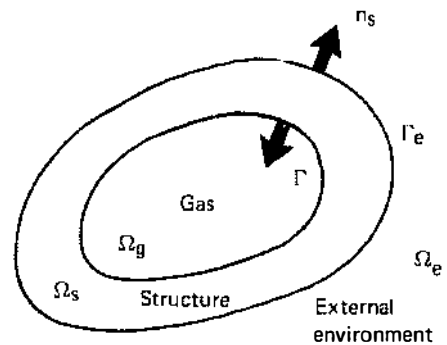


Fig. 1 - Schematic fluid-structure system

Using the displacement field variables (U, U_g) , the local equations governing the conservative vibration problem are :

- the dynamic equilibrium equation in the structure :

$$\sigma_{ij}(U)_{,j} + \rho\omega^2 U_i = 0 \quad \text{in } \Omega_s \quad (2)$$

- the boundary condition :

$$\sigma_{ij}(U) n_{js} = 0 \quad \text{on } \Gamma_e \quad (3)$$

- the dynamic equilibrium equation and the constitutive law in the gas :

$$-\text{grad } p + \rho_g \omega^2 U_g = 0 \quad \text{in } \Omega_g \quad (4)$$

$$p = -\rho_g c^2 \text{div } U_g \quad \text{in } \Omega_g \quad (5)$$

- the fluid-structure interface conditions :

$$U_g \cdot n_s = U \cdot n_s \quad \text{on } \Gamma \quad (6)$$

$$\sigma_{ij} n_{js} = -p n_s \quad (7)$$

III. Local equations of the weakly dissipative model

The dynamic equation (2) is modified as follows in order to take into account a classical Rayleigh damping :

$$\begin{aligned} \sigma_{ij}(U)_{,j} + j\omega [\alpha \sigma_{ij,j}(U) - \beta \rho U_i] \\ + \rho \omega^2 U_i = 0 \end{aligned} \quad (8)$$

As said in the introduction, we consider the following associated weakly-dissipative model : we introduce a thin layer in the fluid-structure vicinity with a damped constitutive law. The equation (6) is therefore replaced by :

$$p = -j\omega Z(\omega) [U_g \cdot n_s - U \cdot n_s] \quad (9)$$

with a great value of Z ((6) is replaced by a penalty condition).

The equations of this approximate dissipative model, which tends continuously to the conservative one as $Z \rightarrow \infty$, are : (8), (3), (4), (5), (9), (7).

IV. Variational symmetric formulation of the weakly-dissipative coupled system

As said in the introduction, we shall use a mixed formulation using the triplet $(U ; p \text{ and } \phi)$ where ϕ is the potential displacement of the fluid in order to have a symmetric formulation (of mass-coupling type). Let us introduce : $X = (U, p, \phi)$ and :

$$k(X, X) = \frac{1}{2} \int_{\Omega} \sigma_{ij}(U) \epsilon_{ij}(U) + \frac{1}{2\rho_g c^2} \int_{\Omega_g} p^2 \quad (10)$$

$$m(X, X) = -\frac{1}{2} \int_{\Omega_g} \rho_g |\nabla \phi|^2 + \frac{1}{c^2} \int_{\Omega_g} p \phi \quad (11)$$

$$+ \rho_g \int_{\Gamma} \phi U \cdot n + \frac{1}{2} \int_{\Omega} \rho |U|^2$$

$$a_s(X, X) = \frac{\alpha}{2} \int_{\Omega} \sigma_{ij}(U) \epsilon_{ij}(U) + \frac{\beta}{2} \int_{\Omega} \rho |U|^2 \quad (12)$$

$$a_g(X, X) = \frac{\rho_g^2 \omega^2}{2Z} \int_{\Gamma} \phi^2 \quad (13)$$

where
 $k(X, X)$ represents the elastic energy of the structure and the gas
 $m(X, X)$ represents (for the solution) the kinetic energy of the structure and the gas
 $a_s(X, X)$ represents the damping in the structure
 $a_g(X, X)$ represents the surface damping on the fluid-structure interface.

Let us denote by (F, X) the work done by prescribed forces.

The variational symmetric formulation writes :

The functional :

$$\begin{aligned} L_{\omega}(X) = k(X, X) + j\omega [a_s(X, X) + a_g(X, X)] \\ - \omega^2 m(X, X) - (F, X) \end{aligned} \quad (14)$$

is stationary on an admissible space V which contains the essential condition on U (clamped structure) or on ϕ if the gas is in contact with an external environment at rest ($\phi = 0$ on a prescribed surface).

Let us note that one could derive many other formulations as shown in ref. (20).

V. Ritz-Galerkin projection on the conservative system

The discretization of (14) in a finite dimension space leads to the following finite element matrix system :

$$\begin{aligned} -\omega^2 M U + j \omega [\alpha K U + \beta M U] - \omega^2 [M U + C \phi] = F \\ -\omega^2 C_g \phi + K_g p = 0 \\ -\omega^2 [C^T U + C_g^T p + M_g \phi] + j \omega \left[\frac{\rho_g^2 \omega^2}{Z(\omega)} D_g \phi \right] = 0 \end{aligned} \quad (15)$$

A rigorous static condensation of the ϕ unknowns (except one if the cavity is closed, i.e. if M_g is singular of rank $(N - 1)$) as a function of (U, p) is possible.

The Ritz-Galerkin projection on the eigenvectors of the conservative system associated to (14) or (15) leads to the following condensed matrix system :

$$[-m_i(\omega^2 - \omega_i^2) \delta_{ij} + j \omega D_{ij}(\omega)] \gamma_j = f_i \quad (16)$$

where the modal coupling is localized in the damping term D_{ij} .

$$\begin{aligned} D_{ij} = \alpha K(U_i, U_j) + \beta M(U_i, U_j) \\ + \frac{\omega^2}{Z(\omega)} \int_{\Gamma} \frac{p_i p_j}{\omega_i^2 \omega_j^2} \end{aligned}$$

where $(\omega_i$ and $\omega_j)$ represent two conservative eigenvalues the structural acoustic system ; m_i represents the corresponding generalized mass (appropriately normed).

If we have used the velocity-potential symmetric formulation of ref. (11), the damping terms would not appear alone in a $(j\omega)$ term.

The advantage of the preceding analysis is to preserve the standard character of the dissipative system as in any standard structural system.

VI. Numerical applications

Numerical applications are presented for the case of the shroud of ARIANE IV European Launch vehicle coupled with an internal gas.

Numerical applications

Let us consider the axisymmetric shroud (composite material) coupled with an internal gas (figure 2).

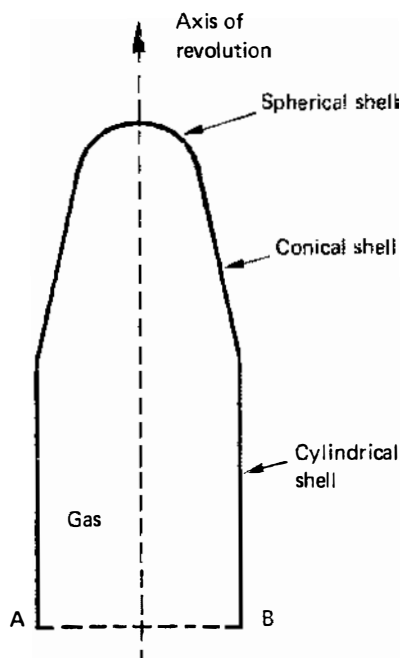


Fig. 2 - Description of the shroud

The figure 3 presents a finite element mesh which allows the computation of :

- acoustic modes in a rigid cavity with a zero pressure condition on AB (figure 2),
- in vacuo structural modes,
- structural-acoustic coupling modes through finite element symmetric variational formulations (figure 3).

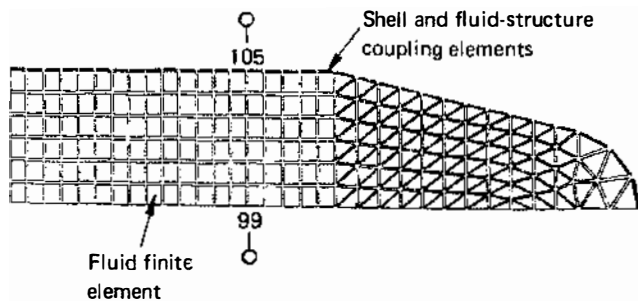


Fig. 3 - Finite element mesh of the structural-acoustic system

Table 1 - Circumferential wave number, $N = 1$

Structural-acoustic modes		Acoustic modes		Structural modes	
Number	Freq. (Hz)	Number	Freq. (Hz)	Number	Freq. (Hz)
1	24.55			1	26.07
2	53.56	1	54.23		
3	65.94	2	66.61		
4	78.46	3	79.93	2	80.98
5	83.07				
6	92.32	4	92.60		
7	105.25	5	106.20		
8	119.30	6	121.01		
9	132.87				
10	134.83	7	136.51	3	135.87
		8	148.22		
11	149.16				
12	150.43	9	153.09		
13	153.30	10	155.66	4	164.14
14	161.76				
15	166.60	11	166.72		
16	168.56	12	170.03	5	180.17
				6	192.14
				7	201.20
				8	213.30
				9	224.14
				10	236.67
				11	257.77
				12	271.91
47 modes < 300 Hz					

In order to analyze the coupling, one has to refer to the :

- acoustic mode shapes in rigid cavity (figures 4, 5) where the iso-pressure are represented. The dotted lines represent the negative pressure, the bold line the zero pressure and the line the positive pressure.
- structural in vacuo mode shapes (figures 6, 7),
- structural acoustic mode shapes (figures 8-9, 10-11).



Fig. 4 - 11th acoustic mode (N=1)



Fig. 5 - 12th acoustic mode (N=1)



Fig. 6 - 4th structural mode (N=1)



Fig. 7 - 5th structural mode (N=1)



Fig. 8 - 14th structural-acoustic mode (N=1)
structural displacement mode shape

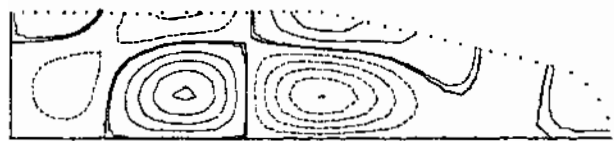


Fig. 9 - 14th structural-acoustic mode (N=1)
iso-pressure mode shape



Fig. 10 - 15th structural-acoustic mode (N=1)
structural displacement mode shape



Fig. 11 - 15th structural-acoustic mode (N=1)
iso-pressure mode shape

The figure 12 illustrates the following conclusions (for a particular circumferential wave number $N = 1$) :

1) The structural-acoustic mode $N^\circ 14$ contains without any change the structural mode $N^\circ 5$ and perturbs the shape of the acoustic mode $N^\circ 11$.

2) The structural-acoustic mode $N^\circ 15$ contains without any change the acoustic mode $n^\circ 12$ but is perturbed by the presence of the structural in vacuo modes $N^\circ 4-5$.

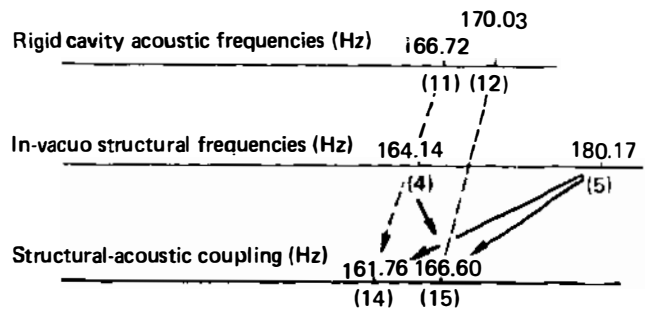


Fig. 12 - Coupling frequencies extracted from the table 1

This means that for some modes, the acoustic has an effect on the structure and that for some other modes, it is the contrary.

Let us now consider the shroud of ARIANE IV excited by a diffuse noise. The figures 13 to 24 describe substructure type responses of the shroud for the circumferential wave number $N = 0$ in the following range of frequency : [22.4 Hz, 178.3 Hz]. These responses are expressed in terms of power spectral density values (P.S.D.) and root mean square values (R.M.S.). The acoustic boundary condition on AB (figure 2) is here that of a rigid wall.

In the above frequency range, two vacuo structural modes only put into action. These modes, located respectively at frequencies 108,8 Hz and 189.3 Hz, are the fundamental mode and the second one coinciding with a local shape of the shroud's cylindrical shell.

The involved substructures are the cylindrical conical and spherical shells of the shroud (figure 2), the cavity volume contained in this one and moreover two acoustic microphones located at the finite element mesh nodes 99 and 10j (figure 3).

A shell type substructure response is given by the arithmetical average of the energy acceleration values in relation to the substructure finite element mesh nodes, while the cavity acoustic response is given by the spatial average of the acoustic energy values.

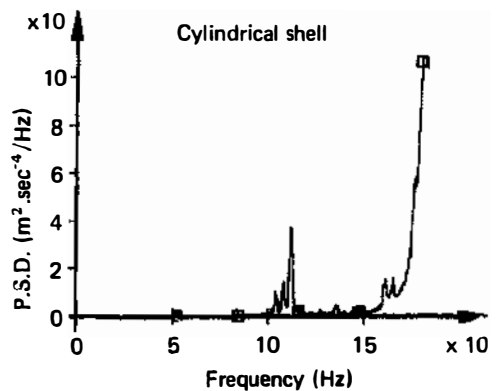


Fig. 13

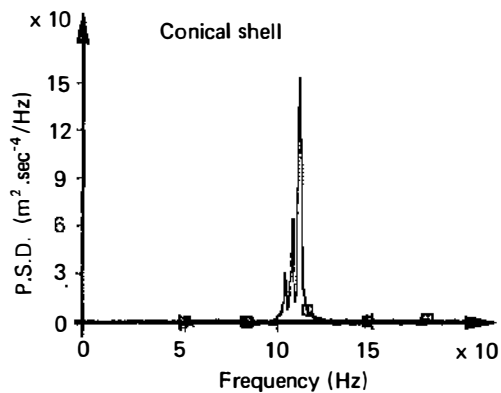


Fig. 14

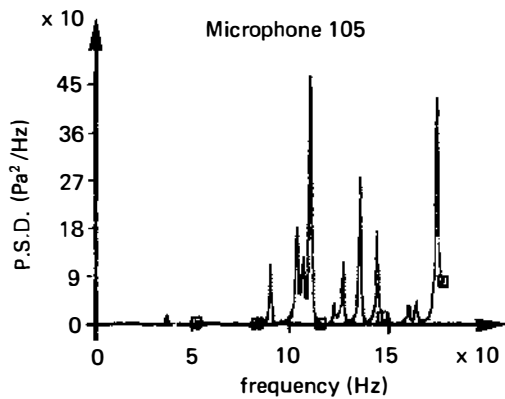


Fig. 18

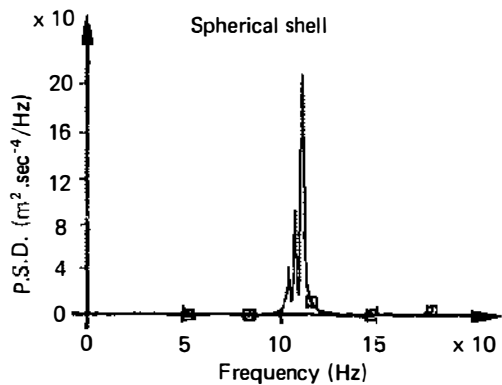


Fig. 15

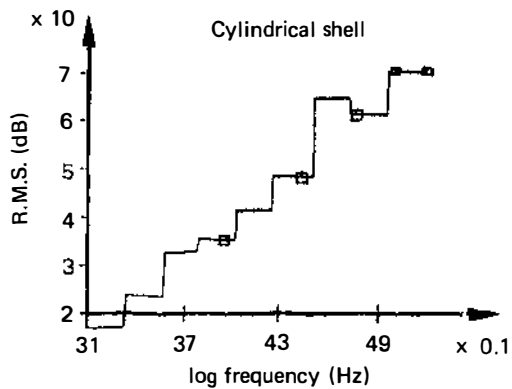


Fig. 19

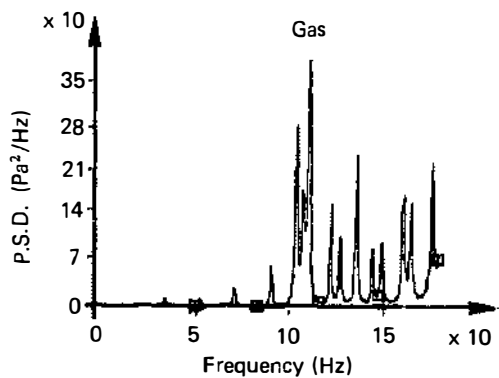


Fig. 16

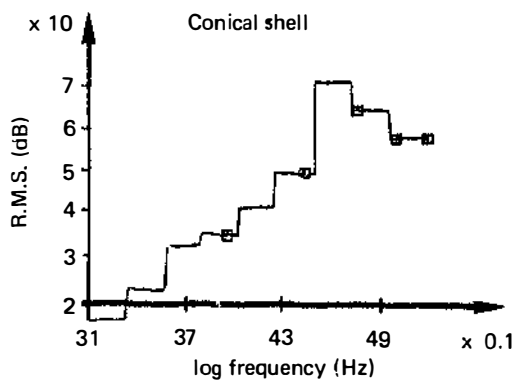


Fig. 20

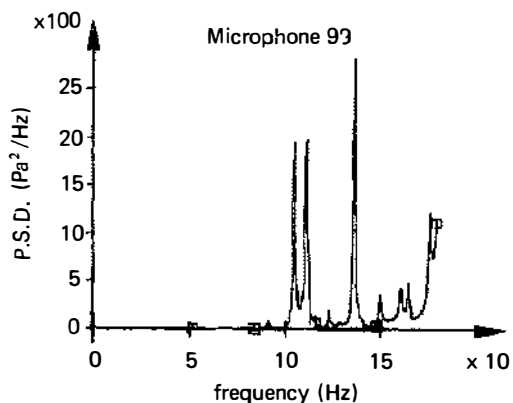


Fig. 17

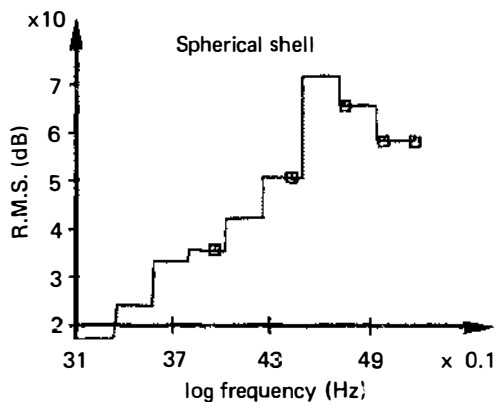


Fig. 21

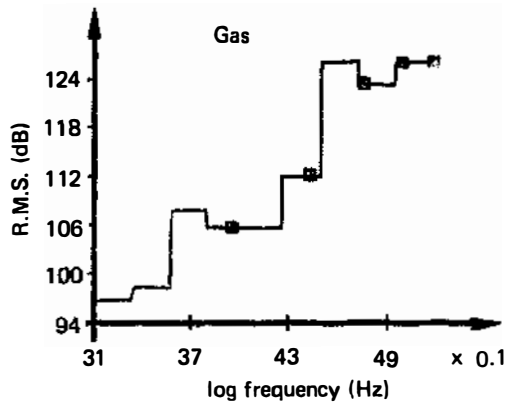


Fig. 22

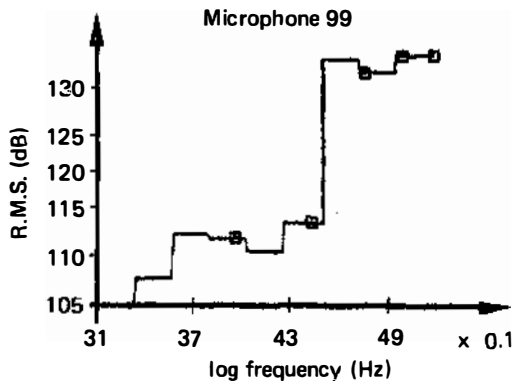


Fig. 23

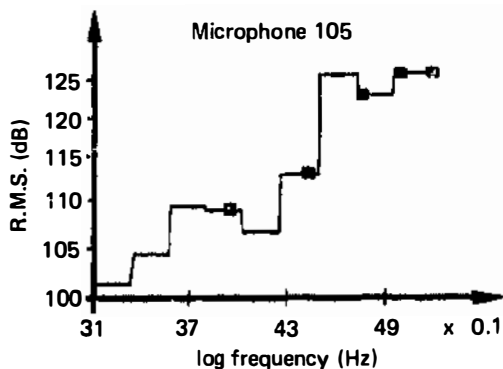


Fig. 24

The acoustic power spectral density (figure 16) increases the most in the frequency range [100 Hz, 110 Hz], in which the fundamental mode of the shroud lies. This one excites much more the conical and spherical shells than the cylindrical one. Out of this frequency range, the cavity acoustic response reveals that the structural acoustic modes coming from the rigid cavity acoustic modes participate in a significantly way to the interior noise.

More generally, the comparison between the different response diagrams shows directly the substructure to substructure contribution and allows to conclude that such a response description framework is able to economically and accurately predict the interior noise sources and the hierarchy of the vibration transmission paths.

Conclusion

The present paper has shown numerical aerospace applications of a structural-acoustic damped model, numerically treated by using a symmetric appropriate variational formulation.

A Ritz-Galerkin projection on the associated conservative system is used through a finite element discretization.

Further developments including experimental correlations⁽²³⁾, rigorous coupling with an external environment (radiation phenomenon) are now undertaken. Preliminary results on equipment bay of ARIANE IV has already shown accurate correlations between the numerical results and the tests.

Acknowledgements

This study is sponsored by C.N.E.S. (Centre National d'Etudes Spatiales - Direction des Lanceurs - Evry - France). The authors thank Mr. Morand and Mr. Borello (C.N.E.S), Mr. Bourguin and Mr. Tretout (ONERA) for valuable discussions concerning the present analysis.

References

1. Gladwell, G.M.L., A variational formulation of damped acousto-structural problems. *Journal of Sound and Vibration*, Vol. 3, 1966, P. 233.
2. Craggs, A. and Stead, G., Sound transmission between enclosures. A study using plates and acoustic finite elements. *Acustica*, Vol. 35, 1976, p. 89.
3. Dowell, E.H., *Acoustoelasticity*. Princeton University AMS Report 1280, 1976.
4. Wolf, J.A. Jr., Modal synthesis for combined structural - Acoustic systems. *AIAA Journal of Sound and Vibration*, Vol. 80, N° 2, January 1982, p. 247.
5. Nefske, D.J., Wolf, J.A. Jr. and Howell, L.J. : Structural-acoustic finite element analysis of the automobile passenger compartment : a review of current practice. *Journal of Sound and Vibration*, Vol. 80, N° 2, January 1982, p. 247.
6. Petyt, M. and Lim, S.P., Finite element analysis of the noise inside a mechanically excited cylinder. *International Journal for Numerical Methods in Engineering*, Vol. 13, 1978, p. 109.
7. Sung, S.H. and Nefske, D.J., Component mode synthesis of a vehicle structural-acoustic system model. *AIAA Journal*, Vol. 24, N° 6, June 1986, p. 1021.
8. Craig, R.R. Jr., A review of time-domain and frequency-domain component mode synthesis method. In *Combined Experimental/Analytical Modeling of Dynamic Structural Systems*, Ed. by D.R. Martinez and A.K. Miller, AMD, Vol. 67, 1985, p. 1.

9. Zienkiewicz, O.C. and Newton, R.E., Coupled vibration of structure submerged in a compressible fluid. Int. Symp. on Finite Element Techniques. Institut für Statik und Dynamik der Luft und Raumfahrt Konstruktionen, University of Stuttgart, Germany, 1969.
10. Mac-Neal, R.H., Citerly, R. and Chargin, M., A symmetric modal formulation of fluid-structure interaction. ASME paper 80-C2/PVP, 1980.
11. Everstine, G.C., A symmetric potential formulation for fluid-structure interaction. Journal of Sound and Vibration, Vol. 79, 1981, p. 157.
12. Morand, H. and Ohayon, R., Symmetric variational formulations for the elastoacoustic vibration problem. Finite element results. Proc. 2nd Int. Symp. on Finite Element Method. in Flow Problems, ICCAD Series N° 2/76. T.P. ONERA 1976-32, June 1976.
13. Morand, H. and Ohayon, R., Substructure variational analysis of the vibrations of coupled fluid-structure systems. Finite element results. Int. Journal for Numerical Methods in Engineering, Vol. 14, N° 5, 1979, p. 741.
14. Ohayon, R. and Valid, R., True symmetric variational formulations for fluid-structure interaction in bounded domains. Finite element results. Chap. 10 in Numerical Methods in Coupled Systems, Ed. by R.W. Lewis, P. Bettess and E. Hinton, John Wiley and Sons, 1984.
15. Nicolas-Vullierme, B. and Ohayon, R., Fluid-structure interaction for bounded and infinite medium vibration problems - Some recent advances at ONERA. In Advances in Fluid-Structure Interactions - 1984, Ed. by G.C. Everstine and M.K. Au-Yang, PVP, Vol. 78, AMD, Vol. 64, 1984.
16. Ohayon, R., Variational analysis of a slender fluid-structure system - the elasto-acoustic beam - A new symmetric formulation. Int. Journal for Num. Meth. in Eng., Vol. 22, 1986, p. 637.
17. Cruse, T.A. and Rizzo, F.J. (Eds), Boundary integral equation method: computational applications in applied mechanics. Proc. Am. Soc. Mech. Eng., Special Publication, Vol. 11, 1975.
18. Geers, T.L. and DeRuntz, J.A., Added mass computation by the boundary integral method. Int. Journal for Num. Meth. in Eng., Vol. 12, 1978, p. 531.
19. Geradin, M., Roberts, G. and Huck, A., Eigenvalue analysis and transient response of fluid-structure interaction problems. Engineering Computations Journal, Vol. 1, 1984, p. 151.
20. Ohayon, R., Transient and modal analysis of bounded medium fluid-structure problem. Proc. Int. Conf. on Num. Meth. for Transient and Coupled Problems, R.W. Lewis, P. Bettess, E. Hinton Eds., Pineridge Press, Swansea, U.K., 1984, T.P. ONERA 1984-56.
21. Felippa, C.A., Symmetrization of the contained compressible fluid vibration eigenproblem. Communications in Applied Num. Meth. Journal, Vol. 1, 1985, p. 24.
22. Borello, G., Réponses basses fréquences de structures axisymétriques en bruit diffus. Proc. Int. Conf. on Spacecraft Structures, C.N.E.S. - Toulouse, 3-6 December 1985, ESA Report SP-238.