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FLUID-STRUCTURE INTERACTION ANALYSIS FOR CYCLICALLY SYMMETRIC BOUNDED SYSTEMS

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ABSTRACT

Various symmetric variational formulations for fluid structure interaction in bounded medium are investigated through the additional framework of systems possessing rotationally periodic and symmetric properties.

The case of scalar field and vector-valued field are examined in details in order to apply the boundary transmission conditions on the edge of the sector of computation which are expressed in the complex plane. Numerical finite element results are then presented.

INTRODUCTION

It is proposed to investigate the problem of modal analysis of a compressible linear fluid-structure system which possesses rotationally periodic and symmetric geometric and mechanical properties.

Those problems occur in aerospace engineering for instance for liquid and solid propellant launch vehicles, for tanks of satellites with baffles and also in many other the industrial areas (vibrations of blades in a fluid environment).

One of the most current and elementary fluid-structure system possessing rotation and symmetry invariance concerns axisymmetric structures : the problem is simplified through a circumferential wave number n which corresponds to the classical Fourier decomposition.

The admissible class in which the eigensolution is searched splits into a direct sum of infinite subspaces :

$$\mathcal{E} = \mathcal{E}_0 \oplus \mathcal{E}_1 \oplus \dots \oplus \mathcal{E}_m \oplus \dots$$

\mathcal{E}_0 corresponds to axisymmetric modes, \mathcal{E}_1 corresponds to flexural modes, $\mathcal{E}_2 \dots$ corresponds to breathing modes.

Leaving away the elementary case of symmetry with respect to one or two orthogonal plane of symmetry, there remain the situation of cyclically symmetric systems : for instance a cylinder of revolution with a polygonal (regular) cross section or an axisymmetric structure with periodic baffles among many other cases (Fig. 1).

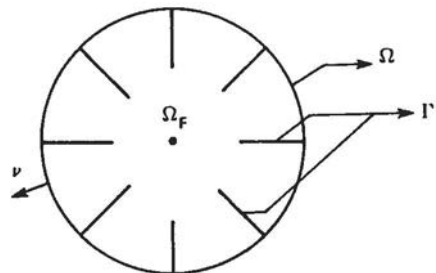


Fig. 1 - Cross section of an axisymmetric system perturbed by periodic baffles.

We are mainly interested in modal analysis of such systems. Two completely different physical phenomenon occur by considering the wave length of the mode :

- if there is a great number of periodic sectors, i.e. if the dimension of the sector is small compared to a characteristic length of the entire structure, then smearing methods or more recent so-called homogenization procedure can be applied. The entire system is replaced by a system of revolution with an anisotropic or orthotropic "equivalent" stress-strain law (1-5).

Let us quote, within the same framework of low-frequency analysis of large repetitive framelike structures, other methodologies replacing a discrete repetitive model by a continuous one (6-7).

- Alternatively, if the dimension of the sector is of order of magnitude of the entire structure, homogenization procedures are not suitable and here lies the problem of cyclically symmetric systems. Many works have been carried on that subject from the point of view of standard displacement matrix analysis of structures (8-12), with special reference to acoustics in (12). Some investigations have been carried in (13-14) on more physical basis concepts. More recently, as initiated in reference (8), the utilization of group theory concepts (15-16) together with variational formulation of the problem, previous any discretization procedure have initiated basic research on simple models in (17), (with an extension to static problems (and transient) in (18) through discrete Fourier decomposition). The group formalism has then been carefully analyzed in (19).

In the present paper, we shall in a first step recall this procedure in the scalar field case for rotationally periodic plus symmetry properties of a structure before entering the vector field case.

In a second step, we shall apply the preceding transmission conditions on a sector to fluid-structure interaction problems which constitute an area of growing importance (20-21-22). The utilization of various symmetric variational formulations introduced originally in (23-25) will then be applied.

Numerical results on a cylinder filled with liquid (polygonal cross-section) will then show the accuracy of the methodology, using the ASTRONEF-code (30).

In conclusion, some extensions and open problems will be emphasized.

SCALAR-FIELD PROBLEM - TRANSMISSION CONDITIONS ON THE EDGES OF THE SECTOR

1. ROTATIONS

Let us consider an irreducible sector of a structure (domain Ω) presenting a rotationally periodic invariance of angle α .

We have previously shown (17-18) that if φ denotes a scalar state variable in the system, the problem is treated in a complex plane by introducing the companion variable ψ which is the complexification of the admissible vector variable \vec{u} in which belongs φ :

$$u = \varphi + j \psi \quad (1)$$

The transmission boundary conditions which must be applied on the edge of the sector are (Fig. 2):

$$\begin{cases} u(B(\theta+\alpha)) = \exp(jm\alpha) u(A(\theta)) \\ m = 0, 1, \dots, (m-1) \end{cases} \quad (2)$$

m being the number of sectors which reconstitutes the entire structure.

This result is classical in the framework of irreducible representations of rotation group (15-16), the "characters" of the representations being $\exp(jn\alpha)$, $n \in \mathbb{N}$ (integer set). One can find this result also from the considerations of (16).

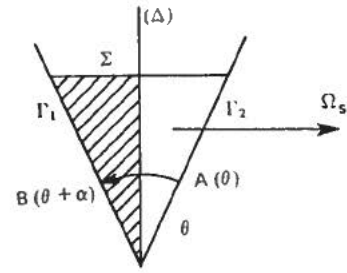


Fig. 2 - A sector plus symmetry.

If \vec{v} denotes the external unit normal to the irreducible sector Ω , then:

$$\frac{\partial u}{\partial \vec{v}}(B) = - \exp(jm\alpha) \frac{\partial u}{\partial \vec{v}}(A) \quad (3)$$

The following properties hold:

1) m even

We have:

- 2 real representations of dimension 1. (the modes are not degenerated, i.e. the frequencies are not multiple)

- $(\frac{m}{2} - 1)$ real representations of dimension 2.

2) m odd

We have:

- 1 real representation of dimension 1.

- $(\frac{m-1}{2})$ real representations of dimension 2.

Remark: The number n (integer) in the relation (2) represents the wave-number of the phenomenon (like the n (circumferential wave number) of the classical Fourier series).

2. ROTATIONS PLUS SYMMETRY

If the entire structure has also symmetry with respect to (Δ) -type axis (Fig. 2), then the sector Ω can be reducible to an irreducible sector (Fig. 2, dashed line): Ω' .

The modes can be then classified into symmetric and antisymmetric with respect to the axis (Δ) .

If we consider that φ is symmetric with respect to (Δ) , the relation (2) shows that ψ is then anti-symmetric with respect to (Δ) , i.e.:

$$\begin{cases} \frac{\partial \varphi}{\partial \vec{v}} = 0 & \text{on } (\Delta) \end{cases} \quad (4)$$

$$\begin{cases} \psi = 0 & \text{on } (\Delta) \end{cases} \quad (5)$$

The transmission conditions (2) then became written only on one edge of the half-sector $\Omega'(\Gamma_1)$:

$$\varphi \exp \frac{m\alpha}{2} + \psi = 0 \quad |_{\Gamma_1} \quad (6)$$

If \vec{v} is still the external unit normal to Ω' , we have then:

$$\frac{\partial \psi}{\partial \nu} - \frac{\partial \psi}{\partial \nu} \tan \frac{m\alpha}{2} = 0 \quad (7)$$

Remark : In order to recover the mode shapes on the entire structure, one has to use again the relation (2) :

$$\begin{cases} \psi(\theta + \alpha) = \psi(\theta) \cos m\alpha - \psi'(\theta) \sin m\alpha \\ \psi'(\theta + \alpha) = \psi'(\theta) \sin m\alpha + \psi(\theta) \cos m\alpha \end{cases} \quad (8)$$

3. RECALL OF THE PROCEDURE FOR A SCALAR FIELD IN THE CASE OF ROTATION PLUS SYMMETRY

Let us solve the wave equation (c = velocity of sound) :

$$\Delta \psi + \frac{\omega^2}{c^2} \psi = 0 \quad \text{with } \psi = 0 \text{ boundary condition on } \Sigma \text{ (Fig. 2).}$$

We introduce a complexification of the space by the companion mode ψ such that :

$$\Delta \psi + \frac{\omega^2}{c^2} \psi = 0 \quad \text{with } \psi = 0 \text{ on } \Sigma.$$

The admissible vector space is :

$$\begin{aligned} \mathcal{E}_\alpha &= \{ \psi, \psi \in \text{appropriate mathematical vector spaces} / \\ &\psi = 0 \text{ on } \Sigma, \psi = 0 \text{ on } \Gamma_1; \psi = 0 \text{ on } \Gamma_2, \\ &\psi \tan \frac{m\alpha}{2} + \psi = 0 \text{ on } \Gamma_1 \} \end{aligned} \quad (9)$$

The variational formulation of the problem is FOR EACH n :

Find $\omega^2 \in \mathbb{R}_+$ and $(\psi, \psi) \in \mathcal{E}_\alpha$ such that :

$$\begin{cases} \int_{\Omega'_\alpha} \text{grad } \psi \cdot \text{grad } \delta \psi + \int_{\Omega'_\alpha} \text{grad } \psi \cdot \text{grad } \delta \psi \\ - \frac{\omega^2}{c^2} \left[\int_{\Omega'_\alpha} \psi \delta \psi + \int_{\Omega'_\alpha} \psi \delta \psi \right] = 0, \\ \forall \delta \psi, \delta \psi \in \mathcal{E}_\alpha \end{cases} \quad (10)$$

For each n varying from 0 to $\frac{m}{2}$ (m even) or $\frac{m-1}{2}$ (m odd), we have an eigenvalue problem to solve in the irreducible cell Ω'_α :

$$\left[\begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} - \frac{\omega^2}{c^2} \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \right] \begin{bmatrix} \psi \\ \psi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (11)$$

plus transmission conditions which connect ψ and ψ along Γ_1 (here his the parameter n (integer)).

From the practical point of view, we must double the variables to construct the system (11) on Ω'_α and then we must utilize constraints between variable :

$$\psi \tan \frac{m\alpha}{2} + \psi = 0 \quad \text{on } \Gamma_1, \text{ this for each given } n.$$

VECTOR-FIELD PROBLEM - TRANSMISSION CONDITIONS ON THE EDGES OF THE SECTOR

Let us introduce a change of notation. In the vector-field case, we have three components of the displacement field $\mathbf{U} (\mu_1, \mu_2, \mu_3)$.

In order to derive easily the transmission conditions (2), we shall consider cylindrical coordinates for the components of $\mathbf{U} (\mu_1, \mu_2, \mu_3)$.

The companion mode (complexification of the space) is denoted $\mathbf{V} (v_r, v_\theta, v_z)$.

Let us choose, as in the scalar-field case, \mathbf{U} symmetric and \mathbf{V} anti-symmetric with to (Δ) .

We have then :

$$\begin{bmatrix} \mu_r(B(A)) \\ \mu_\theta(B(A)) \end{bmatrix} = \begin{bmatrix} \cos m\alpha & -\sin m\alpha \\ \sin m\alpha & \cos m\alpha \end{bmatrix} \begin{bmatrix} \mu_r(A) \\ \mu_\theta(A) \end{bmatrix} \quad (12)$$

The same for μ_z, ν_z .

which leads to :

$$\begin{cases} \mu_r \tan \frac{m\alpha}{2} + \nu_r = 0 & \text{on } \Gamma_1 \\ \mu_\theta \tan \frac{m\alpha}{2} + \nu_\theta = 0 & \text{on } \Gamma_1 \end{cases} \quad (13)$$

But for μ_θ and ν_θ , one finds :

$$\mu_\theta - \tan \frac{m\alpha}{2} \nu_\theta = 0 \quad (14)$$

$$\text{with : } \begin{cases} \mu_\theta = 0 \\ \nu_r = \nu_\theta = 0 \end{cases} \quad \text{on } (\Delta) \quad (15)$$

Remark : If m (number of sectors) is even, then the cases $n = 0$ and $n = \frac{m}{2}$ are particular decoupled situations (the frequency of the whole structure is not multiple). The case $n = 0$ is the "axisymmetric" mode shape.

FLUID-STRUCTURE INTERACTION SITUATION

Let us consider the case of figure (1) : structure with elastic baffles containing a fluid (inviscid, linear, compressible).

1. ADDED STIFFNESS OPERATOR - STIFFNESS COUPLING

Let us recall that usually, a symmetric formulation of the vibrations of a fluid in an elastic container is established by using the displacement field \mathbf{U} in the structure, and the couple p and ψ in the fluid (p is the pressure, ψ is the displacement potential) (23-25), leading to mass coupling terms only. We shall here use a stiffness coupling only, with the same variables as follows :

Find $(\mathbf{U}; p \text{ and } \psi) \in \mathcal{E}$ and $\omega^2 \in \mathbb{R}_+$ such that :

$$\begin{cases} \int_{\Omega} \sigma_{ij}(\mathbf{u}) \varepsilon_{ij}(\delta \mathbf{u}) - \omega^2 \int_{\Omega} \rho \mathbf{u} \cdot \delta \mathbf{u} - \int_{\Gamma} \delta \mathbf{u} \cdot \nu p = 0 \\ \int_{\Omega_F} \text{grad } \psi \cdot \text{grad } \delta p - \frac{1}{\rho_F c^2} \int_{\Omega_F} p \delta p - \int_{\Gamma} (\mathbf{u} \cdot \nu) \delta \psi = 0 \\ \int_{\Omega_F} \text{grad } p \cdot \text{grad } \delta \psi - \omega^2 \int_{\Omega_F} \rho_F \text{grad } \psi \cdot \text{grad } \delta \psi = 0 \\ \forall (\delta \mathbf{u}; \delta p, \delta \psi) \in \mathcal{E} \end{cases} \quad (16)$$

σ_{ij} (resp. ϵ_{ij}) is the stress tensor (resp. deformation tensor), ρ_s (resp. ρ_f) is the density of the structure (resp. the fluid which occupies the domain Ω_f), Γ denotes the fluid-structure interaction surface.

The matrix discretization is :

$$\begin{bmatrix} K & D & 0 \\ D^T & K_F & E \\ 0 & E^T & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & F \end{bmatrix} \begin{bmatrix} U \\ p \\ \psi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

Remark : The elimination of p in (17) leads to a stiffness operator.

In order to apply the symmetry procedure, one has to rewrite the formulation (16) by replacing Ω by Ω_s and Γ by Γ_s (Fig. 3) :

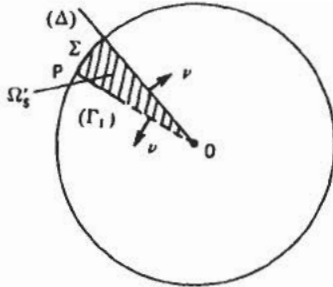


Fig. 3 - Fluid-structure irreducible sector.

Γ_s is divided into two parts (structural $\Gamma_s^{(1)}$ and fluid $\Gamma_s^{(2)}$) (Fig. 4).

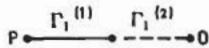


Fig. 4 Partitioning of Γ_1 .

On $\Gamma_s^{(1)}$, one applies the condition (13) and (14) (u, ν as ν is the external normal to Ω_s) is regarded as u_p and satisfies equation (14). On $\Gamma_s^{(2)}$, one applies the transmission conditions (2) on the variable ψ (not necessarily from the discretization point of view on p).

Due to the variational principle (16), it is not necessary to apply any transmission conditions on ψ on $\Gamma_s^{(1)}$.

2. FLUID-STRUCTURE SYMMETRIC FORMULATION USING ONLY p -VARIABLE FOR THE FLUID

If we introduce the new field variable R (dynamic reaction forces) in the structure (25)

$$R = \rho_s \omega^2 U \quad (18)$$

One has a symmetric formulation for the fluid-structure problem in $(U, R ; p)$ field-variables.

$$\left\{ \begin{array}{l} \omega^2 \int_{\Omega} \sigma_{ij}(U) \epsilon_{ij}(\delta U) - \omega^2 \int_{\Omega} R \cdot \delta U - \omega^2 \int_{\Gamma} p \nu \cdot \delta U = 0 \\ \int_{\Omega} \frac{1}{\rho_s} R \cdot \delta R - \omega^2 \int_{\Omega} U \cdot \delta R \\ \int_{\Omega_f} \frac{1}{\rho_f} \text{grad } p \cdot \text{grad } \delta p - \omega^2 \int_{\Omega_f} \frac{1}{\rho_f c^2} p \delta p - \omega^2 \int_{\Omega_f} U \cdot \delta p = 0 \\ \forall (\delta U, \delta R; \delta p) \in \mathcal{C}, (U, R, p) \in \mathcal{C} \end{array} \right. \quad (19)$$

From the finite element point of view, R is not necessary assembled from one element to the next one. This formulation is interesting as only p remains as a variable for the fluid (huge fluid domain plus plate or shell type structural domain).

Remark : Let us quote that "non conservative" type of symmetrization have been discussed in (27). Again here, the vector field transmission conditions (13), (14) can be applied on the variational formulation (19).

NUMERICAL FINITE ELEMENT RESULTS

A cylinder with polygonal cross-section have been computed :

- Total computation on the whole structure
- Computation on an irreducible sector (Fig. 2).

The gain of time on a UNIVAC-1110/83 computer is of a ratio 1/8. The sector was not divided again into smaller finite elements.

The computation was very efficient and it is impossible to distinguish on Figures 5, 6, 7 -which present the mode shape corresponding to a frequency of 18 Hz- the total computation and the one taking into account the symmetries.

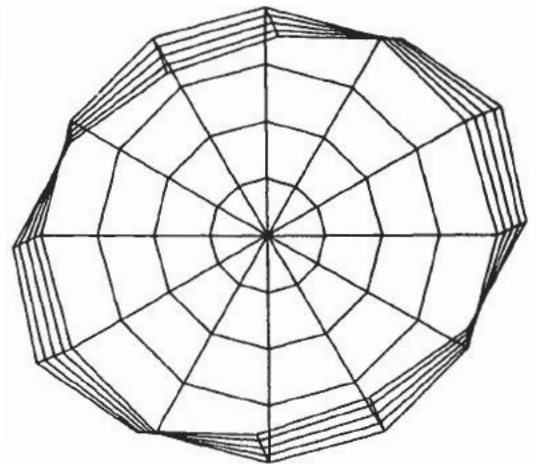


Fig. 5 - Cross section mode shape (18 Hz).

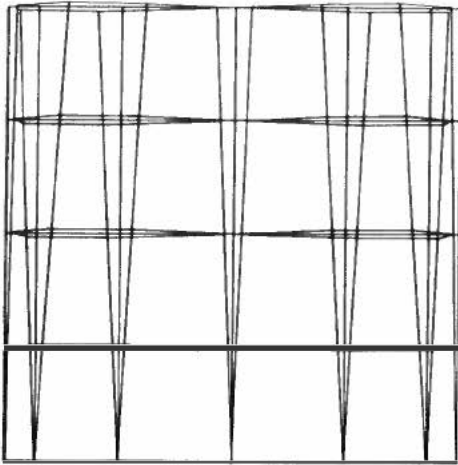


Fig. 6 - Vertical view of the mode shape (18 Hz).

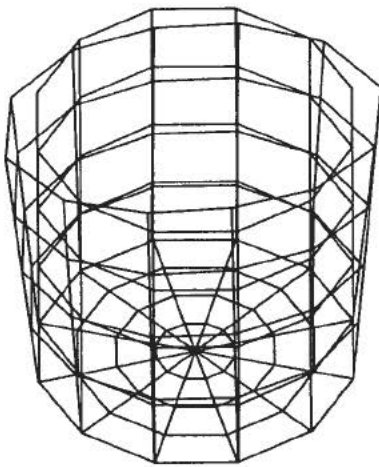


Fig. 7 - Perspective view of the mode shape (18 Hz).

The ASTRONEF code was used for those computations on the UNIVAC (Exec 8) system (30).

CONCLUSION

There remain many open problems such as taking into account damping terms in the structure.

The extension to structure-structure coupling may be carried using the results of (29). The case of infinite surrounding medium is under investigations (submarine propeller for instance), using methodologies developed for fluid-structure coupling (without symmetry concepts) in (26). Let us quote that translation repetitive properties for structures lead to Brillouin zones computation (28). This shows that the area of symmetry concepts in dynamic of continuous medium is a very prospective one.

REFERENCES

1. Babuska, I., "Homogenization approach in engineering", in Computing Methods in Applied Sciences and Engineering, Lecture Notes in Economics and Math. Systems, 134, 1976, pp. 137-153.

2. Bensoussan, A., Lions, J.L., Papanicolaou, G., Asymptotic analysis for periodic structure, North-Holland, 1978.
3. Duvaut, G., "Analyse fonctionnelle en mécanique des milieux continus, homogénéisation", Theoretical and Applied Mechanics, W.T. Koiter Ed, North-Holland, 1976.
4. Sanchez-Palencia, E. "Non-homogeneous media and vibration theory", Lecture Notes in Physics, Vol. 127, Springer-Verlag, 1980.
5. Ohayon, R., "Homogénéisation par développements asymptotiques mixtes - Vibrations d'un anneau hétérogène à structure périodique", Compte-Rendus Acad. Sciences Paris, Série A, 288, 1979, pp. 173-176.
6. Nayfeh, A.H., Hartle, M.S., "Dynamic characteristics of large repetitive framelike structures", J. of Applied Mechanics, Paper 84-APM-35, 1984.
7. Noor, A.K., Nemeth, M.P., "Micropolar beam models for lattice grids with rigid joints", to appear in Computer Methods in Applied Mechanics and Engineering.
8. Evensen, D.A., "Vibration analysis of multisymmetric structures", A.I.A.A. Journal, Vol. 14, 1976.
9. Thomas, D.L., "Dynamics of rotationally periodic structures", Int. J. Num. Meth. in Engng. Vol. 14, N° 1, 1979.
10. Vold, H., "Efficient implantation of the discrete Fourier transform analysis of cyclically symmetric structures", MAFELAP 1978, J.R. Whiteman Ed., Academic Press, 1979.
11. Henry, R., Ferraris, G., "Substructuring and wave propagation : an efficient technique for impeller dynamic analysis", A.S.M.E. Paper N° 83-GT-150, 1983.
12. Mao Neal, R.H., Harder, R.L., Mason, J.B., "Nastran Cyclic Symmetry Capability", Nastran Users Experience, NASA, TMX 2893, Sept. 1973, pp. 395-421.
13. Wildheim, S.J., "Excitation of rotationally periodic structures", J. of Applied Mechanics, Vol. 46, 1979, pp. 878-882.
14. Bauer, L., Reiss, E.L., "Cutoff wavenumbers and modes of hexagonal waveguides", Siam Journal of Applied Math., Vol. 35, N° 3, 1978, pp. 508-514.
15. Lyubarski, G.Y., The application of group theory in physics, Pergamon Press, 1960.
16. Serre, J.P., Représentations linéaires des groupes finis, Hermann Ed., Paris, 1978.
17. Ohayon, R., Valid, R., "Static and dynamic computational methods for cyclically symmetric structures", 3rd Int. Conf. on Num. Meth. in Engng. Sc. (Gann-Issina), Paris 14-16 March 1983, T.P. ONERA n° 1983-14.

18. Valid, R., Ohayon, R., "Static and dynamic analysis of cyclically symmetric structures", 16th Int. Congress of Theor. and Appl. Mech., Lungby, Denmark, 19-25 August 1984.
19. Bossavit, A., "Groupes, symétrie et problèmes variationnels", Mafelap Conference, London 30 April-3 May, 1984.
20. Zienkiewicz, O.C., Bettess, P., "Fluid-Structure Interaction", Ocean Structural Dynamics Symposium, Sept. 8-10, Oregon State University, Corvallis, Oregon, 1982.
21. Belytschko, T.B., Schumann, U., "Fluid-Structure Interactions in Light Water Reactor Systems", Nucl. Engng. Design., 60, 1980, pp. 173-195.
22. Liu, W.K., Belytschko, T.B., "Fluid-Structure interaction with sloshing", Proc. SMIRT-7 Int. Conf., Vol. B., Chicago, Ang. 1983.
23. Morand, H., Ohayon, R., "Substructure variational analysis of the vibrations of coupled fluid-structure systems - Finite element results", Int. J. Num. Meth. Engng., 14, 5, 1979, pp. 741-755.
24. Ohayon, R., Valid, R., "True symmetric variational formulations for fluid-structure interaction in bounded domains - Finite element results", Num. Meth. for Coupled Systems, Chapter 10, R.W. Lewis, P. Bettess, E. Hinton Eds., John-Wiley and Sons Ltd, 1984.
25. Ohayon, R., "Transient and modal analysis of bounded medium fluid-structure problems", Int. Conf. on Num. Meth. for Transient and Coupled Problems, Venice, Italy, 9-13 July 1984, T.P. ONERA N° 1984-56.
26. Nicolas-Vuillierme, B., Ohayon, R., "Fluid-structure interaction for bounded and infinite medium vibration problems. Some recent advances at ONERA", Symposium on Recent Advances on Fluid-Structure Interaction, San Antonio, Texas, June 1984, ASME-Conference - P.V.P. Vol. 78/A.M.D - Vol. 64, Eds. G.C. Everstine, Au-Yang, M.K., 1984.
27. Everstine, G.C., "A symmetric potential formulation for fluid-structure interaction", J. of Sound and Vibration, Vol. 79, N° 1, 1981, pp. 157-160.
28. Ohayon, R., "Vibration analysis of periodic structures and Brillouin zones computation", I.U.T.A.M./ICTAM Congress, Lungby, Denmark, August 1984.
29. Ohayon, R., Valid, R., "Principes variationnels symétriques couplés de type primal-dual en élastodynamique", Compte-Rendus Acad. Sciences Paris, Série II, t. 297 et 298, Nov. 1983 et 1984, T.P. ONERA n° 1984-42.
30. A.S.T.R.O.N.E.F. - Analyse de Structures à l'ONERA par éléments finis - Set of codes of Computational and Structural Mechanics Division, ONERA - 92320 - Châtillon - France.