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# ELASTO-ACOUSTIC DAMPED VIBRATIONS FINITE ELEMENT AND MODAL REDUCTION METHODS

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## 1-INTRODUCTION

Within the framework of vibroacoustic studies of aeronautical structures, we present various aspects of the mechanical and numerical modelling of *internal* "low frequency" elasto-acoustic responses taking into account a linear damping model on the fluid-structure interface. We suppose that the *external* environment is known (prescribed forces or displacement).

We consider the linear oscillations of an elastic structure containing an inviscid fluid, submitted to a prescribed harmonic excitation.

The structure is described by a displacement field.

Concerning the fluid, various symmetric variational formulations have been analysed for conservative systems of mass-coupling and stiffness-coupling types. The fluid is described by scalar unknown fields, namely the pressure *and* the displacement potential fields, followed by a rigorous condensation of one of those variables, depending on the considered variational formulation.

The purpose of this paper is to introduce into the preceding formulations, a *frequency-dependent* damping on the fluid-structure interface. The originality of the paper lies in the substructuring method used for forced responses of dissipative systems.

We compare a direct finite element approach with a modal reduction approach using *appropriate* generalized coordinates for the fluid through a Ritz projection procedure. Due to the *frequency-dependent* impedance model, the choice of the basic conservative system delivering the Ritz vectors is carefully analysed (truncation effects). Numerical results are presented.

## 2-ELASTO-ACOUSTIC FORMULATION

### 2.1-Notations

$\Omega_S$	domain occupied by the structure
$\Omega_F$	domain occupied by the gas
$\Sigma$	gas-structure interface
$\Sigma_{ext}$	external surface of the structure
$\rho_F$	density of the gas
$\rho_S$	density of the pressure
$p$	pressure field
$\varphi$	potential displacement of the gas
$u^S$	structural displacement field
$u^F$	gas displacement field
$\sigma_{ij}$	structural stress tensor
$n^{(S)}$	external normal unit vector, relative to $\Omega_S$
$n$	external normal unit vector, relative to $\Omega_F$
$\omega$	circular frequency
$V_F$	volume of $\Omega_F$
$S_\Sigma$	area of $\Sigma$
$c$	celerity of sound in the gas
$F_i$	external prescribed force
$Z(\omega)$	acoustical impedance

### 2.2-Formulation without wall damping [1,2,3 references included]

#### • Structure :

The variational formulation of a structure submitted to :

- an external prescribed force  $F$  on  $\Sigma_{ext}$
- the action of an internal gas pressure  $p$  on  $\Sigma$  , writes :

$$\int_{\Omega_S} \sigma_{ij}(u^S) \epsilon_{ij}(\delta u^S) - \omega^2 \int_{\Omega_S} \rho_S u^S \cdot \delta u^S - \int_{\Sigma} p u^S \cdot n = \int_{\Sigma_{ext}} F_i \cdot \delta u^S \quad (1)$$

#### • Gas :

The gas is inviscid, compressible.

We have two equations :

$$p = -\rho_F \omega^2 \operatorname{div}(u^F) \quad (2a)$$

$$\nabla p = \rho_F \omega^2 u^F \quad (2b)$$

(The first equation is the constitutive law for a barotropic fluid.)

\* We decompose  $p$  as :  $p = P + p_S$

- $P$  is a dynamic pressure and satisfies the equation :  $\int_{\Omega_F} P = 0$

- $p_S$  is a static pressure , constant over  $\Omega_F$

On one side we have :  $\int_{\Omega_F} p = \int_{\Omega_F} P + \int_{\Omega_F} p_S = V_F p_S$

and on the other side ,we have :  $\int_{\Omega_F} p = -\rho_F c^2 \int_{\Omega_F} \text{div}(u^F) = -\rho_F c^2 \int_{\Sigma} u^F \cdot n$

So we have :

$$p_S = -\frac{\rho_F c^2}{V_F} \int_{\Sigma} u^F \cdot n \quad (3)$$

\* We introduce the potential displacement field  $\varphi : u^F = \nabla\varphi$  with the uniqueness condition :  $\int_{\Omega_F} \varphi = 0$

\* We use for the gas the variables  $P, \varphi, p_S$  which are related through :

$$P + p_S = -\rho_F c^2 \Delta\varphi \quad (4)$$

$$P = \rho_F \omega^2 \varphi \quad (5)$$

$$p_S = -\frac{\rho_F c^2}{V_F} \int_{\Sigma} \frac{\partial\varphi}{\partial n} \quad (6)$$

$P$  and  $\varphi$  verify :  $\int_{\Omega_F} P = 0$  and  $\int_{\Omega_F} \varphi = 0$

*Remark : For axisymmetric structures , for all the circular wave numbers  $n \geq 1$  ,  $p_S = 0$  and therefore does not appear in the equations.*

\* At the interface  $\Sigma$  :

We have the fluid-structure interaction relations :

$$\sigma_{ij} n_j^{(S)} = p \cdot n_i \quad (7)$$

$$u^S \cdot n = u^F \cdot n \quad (8)$$

These relations write :

$$\sigma_{ij} n_j^{(S)} = P \cdot n_i + p_S \cdot n_i = \rho_F \omega^2 \varphi \cdot n_i + p_S \cdot n_i \quad (9)$$

$$u^S \cdot n = \frac{\partial\varphi}{\partial n} = \frac{1}{\rho_F \omega^2} \frac{\partial P}{\partial n} \quad (10)$$

• Variational formulation :

From these equations , we obtain two symmetrical variational formulations :

$\forall (\delta u^S, \delta P, \delta\varphi, \delta p_S)$  with  $\delta\varphi \in \{\varphi / \int_{\Omega_F} \varphi = 0\}$ , and  $\delta P \in \{P / \int_{\Omega_F} P = 0\}$ ,

• *Stiffness coupling*

$$* \int_{\Omega_S} \sigma_{ij}(u^S) \epsilon_{ij}(\delta u^S) - \int_{\Sigma} P \delta u^S \cdot n - p_S \int_{\Sigma} \delta u^S \cdot n - \omega^2 \int_{\Omega_S} \rho_S u^S \cdot \delta u^S = \int_{\Sigma_{ext}} F \cdot \delta u^S$$

$$* - \int_{\Sigma} \delta P u^S \cdot n - \int_{\Omega_F} \frac{1}{\rho_F c^2} P \cdot \delta P + \int_{\Omega_F} \nabla\varphi \cdot \nabla\delta P = 0$$

$$* \int_{\Omega_F} \nabla \delta \varphi \cdot \nabla P - \omega^2 \int_{\Omega_F} \rho_F \nabla \varphi \cdot \nabla \delta \varphi = 0$$

$$* - \delta p_S \int_{\Sigma} u^S \cdot n - \frac{V_F}{\rho_F c^2} p_S \cdot \delta p_S = 0$$

• **Mass coupling**

$$* \int_{\Omega_S} \sigma_{ij}(u^S) \epsilon_{ij}(\delta u^S) - p_S \int_{\Sigma} \delta u^S \cdot n - \omega^2 \left\{ \int_{\Omega_S} \rho_S u^S \cdot \delta u^S + \int_{\Sigma} \rho_F \varphi \delta u^S \cdot n \right\} = \int_{\Sigma_{ext}} F \cdot \delta u^S$$

$$* \int_{\Omega_F} \frac{1}{\rho_F c^2} \delta P \cdot P - \omega^2 \int_{\Omega_F} \frac{1}{c^2} \varphi \cdot \delta P = 0$$

$$* - \omega^2 \left\{ \int_{\Sigma} \rho_F \delta \varphi u^S \cdot n + \int_{\Omega_F} \frac{1}{c^2} P \cdot \delta \varphi - \int_{\Omega_F} \rho_F \nabla \varphi \cdot \nabla \delta \varphi \right\} = 0$$

$$* - \delta p_S \int_{\Sigma} u^S \cdot n - \frac{V_F}{\rho_F c^2} p_S \cdot \delta p_S = 0$$

### 2.3-Formulation with wall damping

The originality of the paper lies in the analysis of fluid-structure systems , *using a wall-damping model* , through direct finite element methods or through substructuring analysis (as done for conservative systems in [1,3]).

• **Wall damping**

The following fluid-structure interface modelling is introduced : between the fluid and the structure,we consider a third medium,without thickness. We relax the continuity constraint at the interface  $u_S \cdot n = u_F \cdot n$  through a regularization/penalization procedure.

The equation (8) is replaced by the equation (11) :

$$p = j\omega Z(\omega)(u^S \cdot n - u^F \cdot n) \quad (11)$$

(the pressure is proportionnal to the jump of normal velocity at the interface)

The equation (10) is replaced by the equation (12) :

$$\frac{\partial \varphi}{\partial n} = u^S \cdot n - \frac{j}{\omega Z(\omega)} p_S - \frac{j\rho_F \omega}{Z(\omega)} \varphi \quad (12)$$

$$\text{or} \quad \frac{\partial P}{\partial n} = \rho_F \omega^2 \frac{\partial \varphi}{\partial n} = \rho_F \omega^2 u^S \cdot n - \frac{j\rho_F \omega}{Z(\omega)} p_S - \frac{j\rho_F \omega}{Z(\omega)} P \quad (13)$$

• **Variational formulations**

The two variational formulations are modified by the introduction :

- of the wall damping
- of Rayleigh damping for the structure at the discretized level

$\forall (\delta u^S, \delta P, \delta \varphi, \delta p_S)$  with  $\delta \varphi \in \{\varphi / \int_{\Omega_F} \varphi = 0\}$ , and  $\delta P \in \{P / \int_{\Omega_F} P = 0\}$ ,

• *Stiffness coupling*

$$* \int_{\Omega_S} \sigma_{ij}(u^S) \epsilon_{ij}(\delta u^S) - \int_{\Sigma} P \delta u^S \cdot n - p_S \int_{\Sigma} \delta u^S \cdot n - \omega^2 \int_{\Omega_S} \rho_S u^S \cdot \delta u^S = \int_{\Sigma_{ext}} F \cdot \delta u^S$$

$$* - \int_{\Sigma} \delta P u^S \cdot n - \int_{\Omega_F} \frac{1}{\rho_F c^2} P \cdot \delta P + \int_{\Omega_F} \nabla \varphi \cdot \nabla \delta P + \frac{j}{\omega Z(\omega)} \int_{\Sigma} P \cdot \delta P + \frac{j}{\omega Z(\omega)} p_S \int_{\Sigma} \delta P = 0$$

$$* \int_{\Omega_F} \nabla \delta \varphi \cdot \nabla P - \omega^2 \int_{\Omega_F} \rho_F \nabla \varphi \cdot \nabla \delta \varphi = 0$$

$$* - \delta p_S \int_{\Sigma} u^S \cdot n - \frac{V_F}{\rho_F c^2} p_S \cdot \delta p_S + \frac{j}{\omega Z(\omega)} \delta p_S \int_{\Sigma} P + \frac{j S_{\Sigma}}{\omega Z(\omega)} \delta p_S \cdot p_S = 0$$

• *Mass coupling*

$$* \int_{\Omega_S} \sigma_{ij}(u^S) \epsilon_{ij}(\delta u^S) - p_S \int_{\Sigma} \delta u^S \cdot n - \omega^2 \left\{ \int_{\Omega_S} \rho_S u^S \cdot \delta u^S + \int_{\Sigma} \rho_F \varphi \delta u^S \cdot n \right\} = \int_{\Sigma_{ext}} F \cdot \delta u^S$$

$$* \int_{\Omega_F} \frac{1}{\rho_F c^2} \delta P \cdot P - \omega^2 \int_{\Omega_F} \frac{1}{c^2} \varphi \cdot \delta P = 0$$

$$* \frac{j \rho_F \omega^3}{Z(\omega)} \int_{\Sigma} \varphi \cdot \delta \varphi + \frac{j \rho_F \omega}{Z(\omega)} p_S \int_{\Sigma} \delta \varphi$$

$$- \omega^2 \left\{ \rho_F \int_{\Sigma} \delta \varphi u^S \cdot n + \frac{1}{c^2} \int_{\Omega_F} P \delta \varphi - \rho_F \int_{\Omega_F} \nabla \varphi \cdot \nabla \delta \varphi \right\} = 0$$

$$* - \delta p_S \int_{\Sigma} u^S \cdot n - \frac{V_F}{\rho_F c^2} p_S \cdot \delta p_S + \frac{j \rho_F \omega}{Z(\omega)} \delta p_S \int_{\Sigma} \varphi + \frac{j S_{\Sigma}}{\omega Z(\omega)} \delta p_S \cdot p_S = 0$$

## 2.4-Matrix system

The discretization of the preceding formulations lead to the following matrices which are used in the matrix equation : (with a damping matrix  $\mathbf{D}$ , where  $\mathbf{D}_S$  denotes the usual Rayleigh structural damping )

$$(\mathbf{K} + j\mathbf{D}(\omega) - \omega^2\mathbf{M})\mathbf{X} = \mathbf{F}$$

$$\mathbf{X}^t = (u^S \mid P \quad \varphi \quad p_S) \quad \mathbf{F} = (F \mid 0 \quad 0 \quad 0)$$

• *Stiffness coupling*

$\mathbf{K}_S$	$-C_2$	0	$-C_1$
$-C_2^t$	$\frac{-M_F}{\rho_F}$	$\mathbf{K}_{FO}$	0
0	$\mathbf{K}_{FO}^t$	0	0
$-C_1^t$	0	0	$-\frac{V_F}{\rho_F c^2}$

$\mathbf{M}_S$	0	0	0
0	0	0	0
0	0	$\rho_F \mathbf{K}_F$	0
0	0	0	0

$\omega \mathbf{D}_S$	0	0	0
0	$\frac{D_F}{\omega \rho_F Z(\omega)}$	0	$\frac{D_C}{\omega Z(\omega)}$
0	0	0	0
0	$\frac{D_C^t}{\omega Z(\omega)}$	0	$\frac{S_F}{\omega Z(\omega)}$

• *Mass coupling*

$\mathbf{K}_S$	0	0	$-C_1$
0	$\frac{M_F}{\rho_F}$	0	0
0	0	0	0
$-C_1^t$	0	0	$-\frac{V_F}{\rho_F c^2}$

$\mathbf{M}_S$	0	$\rho_F C_2$	0
0	0	$\mathbf{M}_{FO}$	0
$\rho_F C_2^t$	$\mathbf{M}_{FO}^t$	$-\rho_F \mathbf{K}_F$	0
0	0	0	0

$\omega \mathbf{D}_S$	0	0	0
0	0	0	0
0	0	$\frac{\rho_F \omega^3 D_F}{Z(\omega)}$	$\frac{\rho_F \omega D_C}{Z(\omega)}$
0	0	$\frac{\rho_F \omega D_C^t}{Z(\omega)}$	$\frac{S_F}{\omega Z(\omega)}$

with :

$$\mathbf{K}_F \rightarrow \int_{\Omega_F} \vec{\nabla} \varphi \cdot \vec{\nabla} (\delta \varphi)$$

$$\mathbf{K}_{FO} \rightarrow \int_{\Omega_F} \vec{\nabla} \varphi \cdot \vec{\nabla} (\delta P)$$

$$M_F \rightarrow \frac{1}{c^2} \int_{\Omega_F} \varphi \cdot \delta \varphi$$

$$M_{FO} \rightarrow \frac{1}{c^2} \int_{\Omega_F} \varphi \cdot \delta P$$

$$D_F \rightarrow \rho_F \int_{\Sigma} \varphi \cdot \delta \varphi$$

$$D_C \rightarrow \int_{\Sigma} \varphi$$

$$C_1 \rightarrow \rho_S \int_{\Sigma} \delta u^S \cdot n$$

$$C_2 \rightarrow \int_{\Sigma} P \delta u^S \cdot n$$

$$\mathbf{K}_S \rightarrow \int_{\Omega_S} \sigma_{ij}(u^S) \epsilon_{ij}(\delta u^S)$$

$$\mathbf{M}_S \rightarrow \int_{\Omega_S} \rho_S u^S \cdot \delta u^S$$

$$D_S = \alpha \mathbf{K}_S + \beta \mathbf{M}_S$$

**Remark** : Concerning the *conservative* system, we can proceed to a *rigorous condensation* of one of the two scalar fields (the pressure  $P$  or the displacement potential  $\varphi$ ). This condensation is not possible for the *dissipative* system .

## 2.5-Modal projection (Ritz-Galerkin method)

The gas variables  $P$  and  $\varphi$  are expressed in term of a linear combination of the eigenmodes  $\varphi_\alpha$ , (acoustic modes in a rigid enclosure).

$$P = \Sigma p_\alpha \varphi_\alpha \quad \varphi = \Sigma q_\alpha \varphi_\alpha$$

$\varphi_\alpha$  verifies the identity :

$$\int_{\Omega_F} \nabla \varphi_\alpha \cdot \nabla \varphi_\alpha - \frac{\omega_\alpha^2}{c^2} \int_{\Omega_F} \varphi_\alpha \cdot \varphi_\alpha = 0$$

( $\varphi_\alpha$  satisfies the condition  $\int_{\Omega_F} \varphi_\alpha = 0$ )

$\mu_\alpha$  is the generalized mass :  $\mu_\alpha = \frac{1}{c^2} \int_{\Omega_F} \varphi_\alpha^2$

$\Phi_n$  is the matrix of the  $n$  first eigenmodes  $\{\varphi_\alpha\}$

Matrices  $K_F$  and  $K_{F0}$  are replaced by the diagonal matrix  $[\omega_\alpha^2 \mu_\alpha]_{\alpha=1,n}$

Matrices  $M_F$  and  $M_{F0}$  are replaced by the diagonal matrix  $[\mu_\alpha]_{\alpha=1,n}$

Matrix  $C_2$  (resp.  $D_C, D_F$ ) is replaced by the matrix  $C_2^* = C_2 \Phi_n$

(resp.  $D_F^* = \Phi_n^t D_F \Phi_n, D_C^* = D_C \Phi_n$ )

So, the "stiffness" matrix, the "mass" matrix, and the "damping" matrix for the matrix equation become, in the  $(u^S ; p_\alpha, \varphi_\alpha, p_S)$  coordinates :

• *Stiffness coupling*

$K_S$	$-C_2^*$	0	$-C_1$
$-C_2^{*t}$	$\frac{-[\mu_\alpha]}{\rho_F}$	$[\mu_\alpha \omega_\alpha^2]$	0
0	$[\mu_\alpha \omega_\alpha^2]$	0	0
$-C_1^t$	0	0	$-\frac{V_F}{\rho_F c^2}$

$M_S$	0	0	0
0	0	0	0
0	0	$\rho_F [\mu_\alpha \omega_\alpha^2]$	0
0	0	0	0

$-\omega^2$

$\omega D_S$	0	0	0
0	$\frac{D_F^*}{\omega \rho_F Z(\omega)}$	0	$\frac{D_C^*}{\omega Z(\omega)}$
0	0	0	0
0	$\frac{D_C^{*t}}{\omega Z(\omega)}$	0	$\frac{S_F}{\omega Z(\omega)}$

$+j$



• Mass coupling

$\mathbf{K}_S$	0	0	$-C_1$
0	$\frac{ \mu_\alpha }{\rho_F}$	0	0
0	0	0	0
$-C_1^t$	0	0	$-\frac{V_F}{\rho_F c^2}$

$-\omega^2$

$\mathbf{M}_S$	0	$\rho_F C_2^*$	0
0	0	$[\mu_\alpha]$	0
$\rho_F C_2^{*t}$	$[\mu_\alpha]$	$-\rho_F [\mu_\alpha \omega_\alpha^2]$	0
0	0	0	0

$\omega \mathbf{D}_S$	0	0	0
0	0	0	0
0	0	$\frac{\rho_F \omega^3 D_F^*}{Z(\omega)}$	$\frac{\rho_F \omega D_G^*}{Z(\omega)}$
0	0	$\frac{\rho_F \omega D_G^{*t}}{Z(\omega)}$	$\frac{S_T}{\omega Z(\omega)}$

$+j$

**S-CONSERVATIVE SYSTEM**

System (S) :  $(\mathbf{K} + j\mathbf{D}(\omega) - \omega^2\mathbf{M})X = F$

Conservative system :  $(\mathbf{K} - \omega^2\mathbf{M})X = 0$

We compare eigenvalues and eigenmodes obtained by :

- structure and gas are described by finite elements (cf ¶2.2)
- the structure is described by finite elements and gas by generalized coordinates (cf ¶2.5)

We present the frequencies of the coupled fluid-structure system for a cylinder :

$$n = 0$$

	Structure : finite elements Fluid : finite elements	Structure : finite elements Fluid : generalized coord.
$f_1$	113.05	113.35
$f_2$	177.01	177.09
$f_3$	342.70	342.73
$f_4$	445.28	446.53
$f_5$	451.12	452.27
$f_6$	517.93	517.99
$f_7$	692.61	692.63

$$n = 1$$

	Structure : finite elements Fluid : finite elements	Structure : finite elements Fluid : generalized coord.
$f_1$	239.52	240.33
$f_2$	275.45	275.81
$f_3$	500.69	501.06
$f_4$	528.02	528.40
$f_5$	603.83	604.27
$f_6$	694.17	695.59
$f_7$	723.56	724.36
$f_8$	855.63	856.24
$f_9$	1008.13	1008.84
$f_{10}$	1072.96	1075.17
$f_{11}$	1176.60	1174.43

In this conservative system , we can note that the effects of modal truncature are negligible. (we gave the results with two digits after the decimal for pure numerical comparison purposes)

## 4-DISSIPATIVE SYSTEM

### 4.1- Gas in a rigid structure

#### 4.1.a- Description

In order to study the effect of the wall damping under forced responses, we study the case of a rigid structure filled with a gas, submitted to a prescribed displacement.

The variational formulation writes :

$$\int_{\Omega_F} \nabla \varphi \cdot \nabla \delta \varphi + j \rho_F \frac{\omega}{Z(\omega)} \int_{\Sigma} \varphi \cdot \delta \varphi - \frac{\omega^2}{c^2} \int_{\Omega_F} \varphi \cdot \delta \varphi = \int_{\Sigma} u^S \cdot n \delta \varphi$$

Its discretized form is :  $(\mathbf{K}_F + j \frac{\omega}{Z(\omega)} \mathbf{D}_F - \omega^2 \mathbf{M}_F) \varphi = U$

#### 4.1.b- Forced response

##### 4.1.b.α- *PROJECTION BASES (Ritz-Galerkin method)*

\*We can solve the finite element system directly, which is expensive

\*We can project the system on the modal truncated base  $\Phi_n$  (eigenvectors of  $\mathbf{K}_F - \omega^2 \mathbf{M}_F$ )

$Z(\omega)$  is complex; we note  $\frac{1}{Z(\omega)} = R(\omega) - I(\omega)$  where  $R(\omega)$  and  $I(\omega)$  are real (and positive in our case). We give values of the acoustic impedance  $Z(\omega)$  figure 1.

So, we can write this system :  $(\mathbf{K}_F + \omega I(\omega) \mathbf{D}_F + j \omega R(\omega) \mathbf{D}_F - \omega^2 \mathbf{M}_F) \varphi = U$

We can note :  $\mathbf{K}'_F = \mathbf{K}_F + \omega I(\omega) \mathbf{D}_F$  with  $\omega I(\omega) \simeq \text{constant}$  for a range of  $\omega$

The wall damping has two effects :

-equivalent damping due to the real part of  $Z(\omega)$

-added stiffness due to the imaginary part of  $Z(\omega)$

We note  $\Phi'_n = \{\varphi'_\alpha\}_{\alpha=1,n}$  with  $\varphi'_\alpha$  eigenvector of  $(\mathbf{K}'_F - \omega^2 \mathbf{M}_F) \varphi' = 0$

We project the system on the truncated base  $\Phi'_n$ .

##### 4.1.b.β- *RESULTS*

We present numerical results figure 2.

\* We can observe a gap (around 10%) between resonant frequencies and eigenfrequencies of  $\Phi_n$ . These resonant frequencies are closed to eigenfrequencies of  $\Phi'_n$ . This means that the effect of added stiffness, due to the imaginary part of  $\frac{1}{Z(\omega)}$ , is not negligible.

\*We obtain satisfactory results with the projection on  $\Phi_n$  if we take enough eigenvectors. The effect of modal projection is not negligible in this case. We obtain better results with the projection on  $\Phi'_n$ , but this base is less convenient from computational point of view .

### 4.2- Gas in an elastic structure

The system can be solved in two different ways :

-the system is projected on a truncated base of elasto-acoustic modes.

-the gas variables  $P$  and  $\varphi$  are projected on the acoustic eigenmodes, like in ¶2.5. The structural displacement field  $u_S$  is projected on structural eigenmodes (this point will not be discussed here). These projections lead to a smaller system which can be solved directly.

We use this second method (numerical results are presented figure 3) :

The same gap between resonant frequencies and eigenfrequencies can be observed (effect of added stiffness of wall damping).

The number of acoustic eigenmodes should be carefully examined.

## 5-CONCLUSION

We have presented numerical experiments of modal truncation, which prove the efficiency and the precision of the method used.

For elasto-acoustic modes, modal projection for the fluid leads to a smaller system, easier to compute, and does not modify the results.

Concerning forced responses, because of the effect of *added stiffness* of the wall damping, the modal projection base must be carefully chosen. The effects of modal truncation are not negligible. In spite of this, this method gives good results.

Mathematical analysis in order to evaluate the error could be interesting.

Among the open problems, let us mention the proof of the convergence of modal reduction and also adaptative mesh refinement for such problems.

## REFERENCES

- [1] H.J.P. Morand and R. Ohayon, "Finite element method applied to the prediction of the vibrations of liquid-propelled launch vehicles", ASME Conference, Hawaii, July 23-27, 1989, T.P. ONERA 1989 - 92, (1989). (references included)
- [2] R. Ohayon, "Alternative variational formulations for static and modal analysis of structures containing fluids", ASME Conference, Hawaii, July 23-27, 1989, T.P. ONERA 1989 - 93, (1989). (references included)
- [3] H.J.P. Morand, "Modal methods for analysis of the vibrations of structures coupled with fluids", ASME Conference, Hawaii, July 23-27, (1989). (references included)
- [4] See also alternative approaches undergoing at FFA-Sweden (P.Göranson), and at CIRA-Italy (S.De Rosa, M.Cavaliere, G.Pezzullo, A.Concilio).

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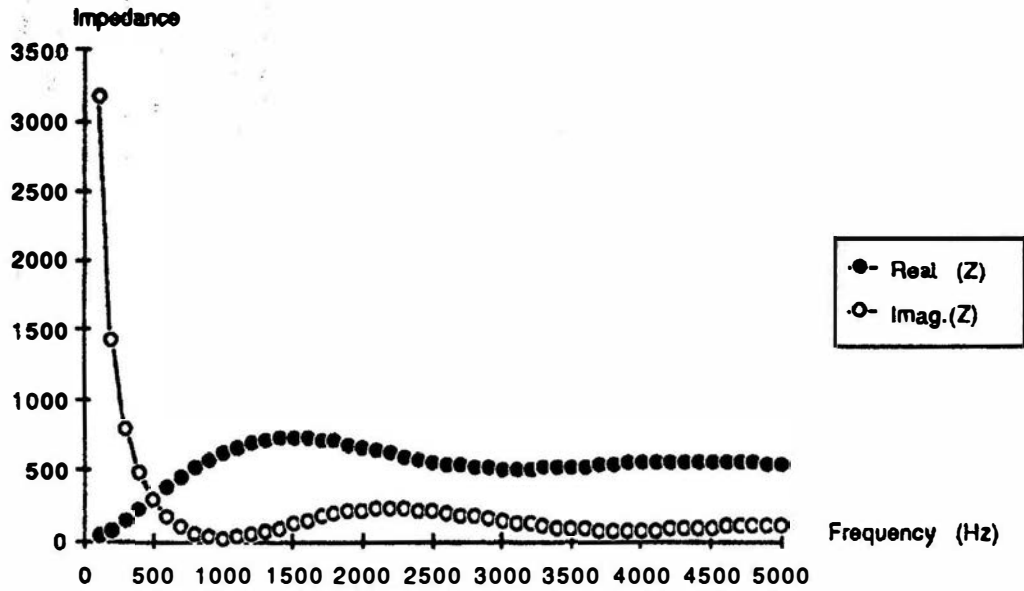


Fig. 1 - WALL DAMPING : ACOUSTIC IMPEDANCE

# FORCED RESPONSE - GAS IN A RIGID STRUCTURE

Mean absolute value of pressure (pa)

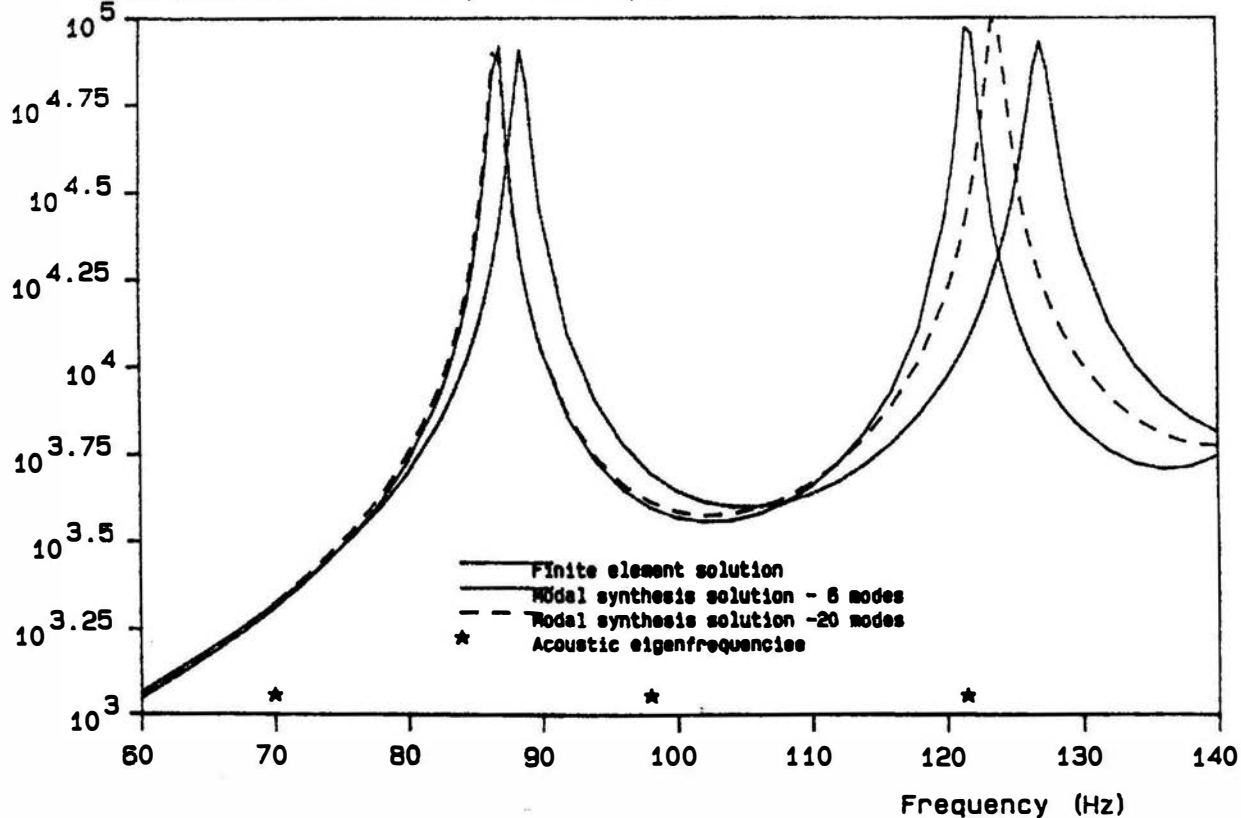
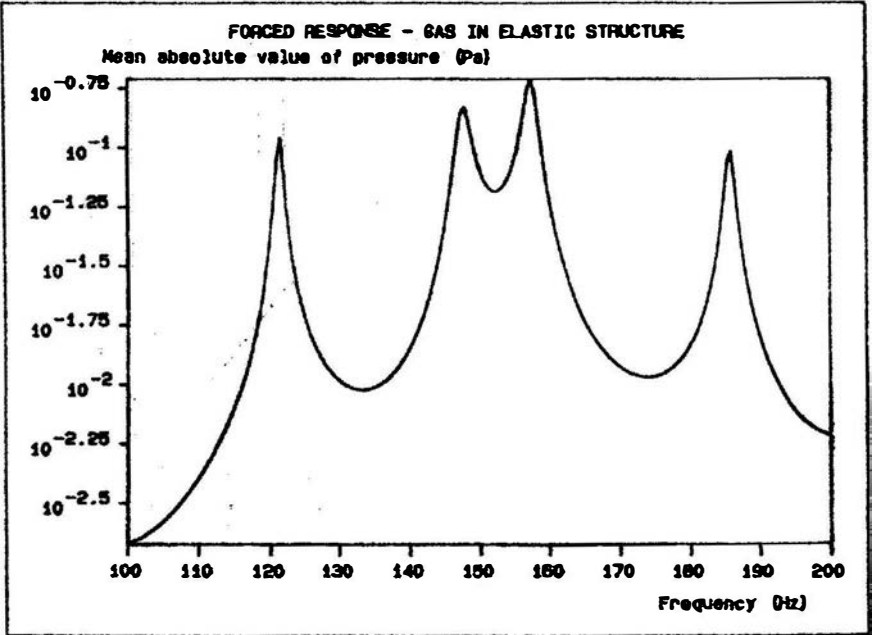
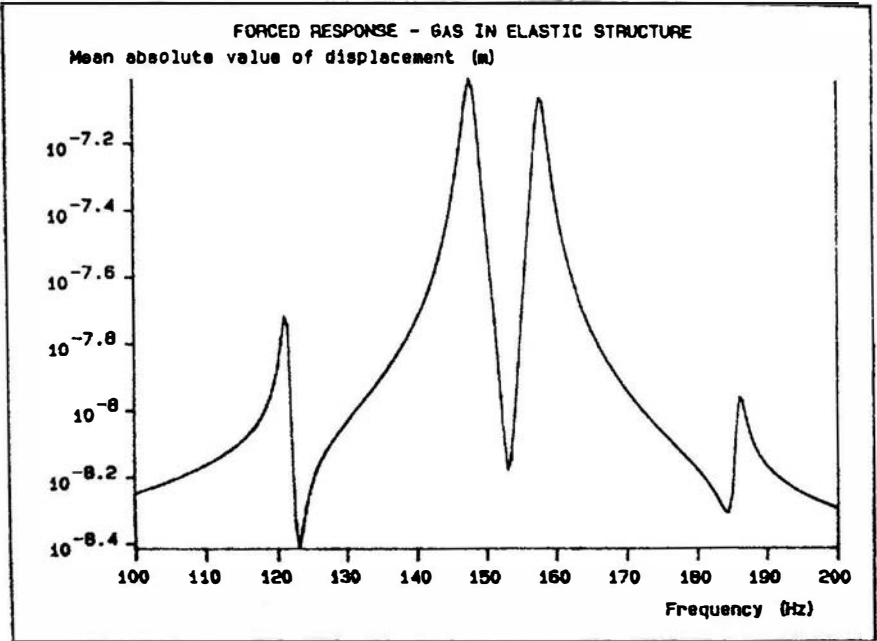


Fig. 2 - FORCED RESPONSE - GAS IN A RIGID STRUCTURE



**Fig. 3 - FORCED RESPONSE - GAS IN AN ELASTIC STRUCTURE**