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# Isogeometric mesh refinement through Non-conforming $C^0$ Isogeometric Mortaring

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**An original mesh refinement of Isogeometric model through  $C^0$  isogeometric mortaring is proposed in this paper to address patch refinement propagation occurring when handling multipatch geometries, a particularly relevant side-effect in electromagnetic problems. The efficiency of the proposed method is demonstrated on an academic problem.**

*Index Terms*—Isogeometric Analysis, Mortar, Refinement

## I. INTRODUCTION

**I**SOGEOMETRIC analysis (IGA) is a numerical method created in 2005 [1], to overcome the Finite Element Method (FEM) geometric deterioration arising during the meshing process. When designing a Computer Aided Design (CAD) model, geometries are largely described with generalized B-Splines called NURBS (rational combination of piece-wise polynomials), with exact precision, rather than FEM-suitable mesh geometries directly.

In FEM, this exact representation is projected onto triangles/quadrangles, with a loss of information (curvature, normals, ...), and shape functions are defined on the mesh, completely separating the initial geometry and the functional space. In contrast, IGA uses the polynomials that generated the geometry as shape functions. In addition, as the NURBS are piece-wise polynomials, the support of each individual polynomial can be used to define a parametric mesh on the model, on which numerical integration can be performed with standard integration routines (quadrature, collocation, ...). The geometry is then carried during the numerical resolution, and preserved.

On simple geometries, a single NURBS is enough for a full model description, and refining the parametric mesh is easy, but on more complex geometries, a multipatch approach is needed, consisting in the description of the model with multiple NURBS. This approach is often used even for simply connected domains, leading to continuity issues at patch interfaces. For Electromagnetic applications, ensuring the  $C^0$  continuity of the quantity of interest at patch interfaces is enough in general, and it can be done in the strong sense, by modifying the piecewise polynomials that have support at the interface, but force the parametric mesh to be conformal (i.e, the parametric mesh must match) at the interface. This side effect is very problematic: To generate a  $n$ -dimensional NURBS, a tensor product of NURBS of degree  $n - 1$  is commonly used. This approach is greatly appreciated in IGA as vectorization along the the  $n - 1$  dimensions is possible, greatly improving computation times. But forcing the parametric mesh to match on the interfaces propagates the mesh refinement from one patch to another, leading to an unwanted increasing number of degrees of freedom for the overall problem. This is especially

true for electromagnetic problems, where the regions of interest can be local such as air gaps in electric machines. Solutions are emerging to avoid the refinement propagation problem and preserve a strong  $C^0$  continuity: T-Spline, Hierarchical splines, ... as presented in [2], but they all loose the tensor product vectorization aspect, as they modify the parametric model representation.

To solve this problem, a Mortar approach can be used, to weakly enforce  $C^0$  continuity (i.e to work with non-conformal parametric meshes at patche interfaces). The tensor product structure will then be preserved per-patch, and vectorisation will be possible, at the cost of a coupling matrix at each patch interface. In this paper, we will briefly introduce Isogeometric analysis, and the mesh refinement propagation issue, we will then explain the Isogeometric mortar method and its application to patchwise mesh refinement, and finally present numerical results to assess the efficiency of the proposed solution.

## II. ISOGEOMETRIC ANALYSIS

### A. NURBS

Non-Uniform Rational B-Spline (NURBS) curves are generated as a sum of  $n$  rational basis functions  $(R_{i,p})_{i=1,\dots,n}$ , of degree  $p$ , and  $n$  control points  $P_i$ :

$$C(\xi) = \sum_{i=1}^n R_{i,p}(\xi) \mathbf{P}_i \quad (1)$$

The rational basis functions are constructed as a rational sum of weighted elements of a Spline basis  $w_j(N_{j,p})_{j=1,\dots,n}$ :

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi)w_i}{\sum_{j=1}^n N_{j,p}(\xi)w_j} \quad (2)$$

And the Spline basis is constructed with a knot vector  $\Xi = [\xi_1, \dots, \xi_m]$  with  $0 = \xi_1 \leq \xi_i \leq \xi_{i+1} \leq \xi_m = 1, \forall i \in \llbracket 1, m \rrbracket$  the  $n$  elements of the spline basis functions of degree  $p$ ,  $(N_{j,p})_{j=1,\dots,n}$  are given by the recurrence formula:

$$N_{j,0}(\xi) = \begin{cases} 1, & \text{if } \xi_j \leq \xi < \xi_{j+1} \\ 0, & \text{otherwise.} \end{cases}$$

$$N_{j,p}(\xi) = \frac{\xi - \xi_j}{\xi_{j+p} - \xi_j} N_j^{p-1}(\xi) + \frac{\xi_{j+p+1} - \xi}{\xi_{j+p+1} - \xi_{j+1}} N_{j+1}^{p-1}(\xi). \quad (3)$$

Tensor products of NURBS Curve can then be used to model 2D and 3D geometries, and any NURBS curve can be refined to refine the 2D/3D geometry as illustrated in figure 1.

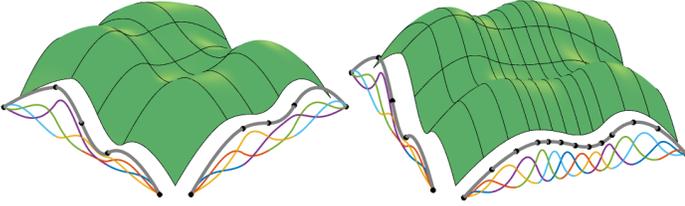


Fig. 1. **Left:** Tensor product of two NURBS Curves (Grey), their associated polynomial basis, to assemble the NURBS Surface (Green). **Right:** Refinement of one of the NURBS Curves via Knot vector insertion, and the resulting tensor surface.

Isogeometric Analysis then consists in the use of such geometries using the spline basis illustrated in figure 1 as Galerkin projection functions, and, by construction, guarantee up to a  $C^{p-1}$  global continuity during the resolution.

### B. Refinement propagation

As illustrated in the previous section, the refinement of a NURBS surface (patch) obtained through tensor products propagates across the whole patch.

When considering multiple patches, strong  $C^0$  continuity at interfaces can be achieved only if the shape functions are identical along the patch interfaces, which imposes the same level of refinement on every adjacent patches. Figure 2 illustrates this on a specific problem: a FEM-BEM modeling of a 3-dimensional actuator. Although the region of interest is the air gap between Patches 1,2 and 1,6, the refinement propagates, and the increase in DoFs outside of the region of interest is substantial (patches 3,4,5).

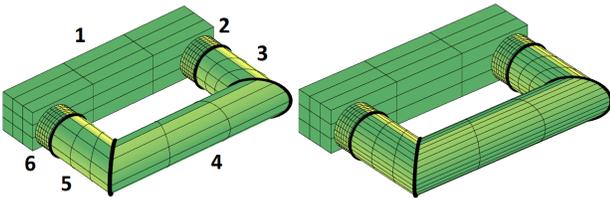


Fig. 2. **Left:** Invalid  $C^0$  patch gluing for a 3D actuator geometry. Patches 2,3 and 5,6 are not compatible. **Right:** Minimal patch refinement required for the  $C^0$  patch gluing, increasing the degree of freedom outside of the region of interest

### III. ISOGEOMETRIC MORTARING

To overcome this drawback, imposing weakly the  $C^0$  continuity across patch interface is possible using a Mortar method. Given a domain  $\Omega$ , and two patches  $\Omega_1, \Omega_2$  such that  $\Omega_1 \cap \Omega_2 \neq \emptyset$  and  $\Omega_1 \cup \Omega_2 = \Omega$ , we consider a very general problem: Find  $u$  in  $\mathcal{H}^1(\Omega)$  such that :

$$a(u, v) = \mathcal{L}(v), \quad \forall v \in \mathcal{H}^1(\Omega) \quad (4)$$

Where  $a$  is continuous on  $(\mathcal{H}^1 \times \mathcal{H}^1)$ , and bilinear, and  $L$  is a linear form. The Mortar method consist in transforming 4 into a saddle point problem, and the problem reads:

Find  $u \in \mathcal{H}^1$  and  $\lambda \in \mathcal{M}$  such that

$$a(u, v) + \int_{\Omega_1 \cap \Omega_2} \lambda (v|_{\Omega_1} - v|_{\Omega_2}) = \mathcal{L}(v) \quad \forall v \in \mathcal{H}^1(\Omega) \quad (5)$$

$$\int_{\Omega_1 \cap \Omega_2} \tau (u|_{\Omega_1} - u|_{\Omega_2}) = 0 \quad \forall \tau \in \mathcal{M}(\Omega) \quad (6)$$

Where  $\mathcal{M}$  is a set of Lagrange multipliers, constructed by picking "arbitrarily" one of the two patch as the slave patch (often the most refined one), and modifying the rational basis at the interface (and especially at the edges of the interface)  $\Omega_1 \cap \Omega_2$  as described in [3]. If  $\Omega_2$  is chosen as the slave patch, integrating  $\int_{\Omega_1 \cap \Omega_2} \lambda v|_{\Omega_1}$  and  $\int_{\Omega_1 \cap \Omega_2} \tau u|_{\Omega_1}$  is complicated as both quantities are expressed with rational basis that don't share their support. More information will be provided in the extended paper as this integration is a key point to allow non-conforming patches integration and non conforming interfaces (more complicated) as illustrated in 3

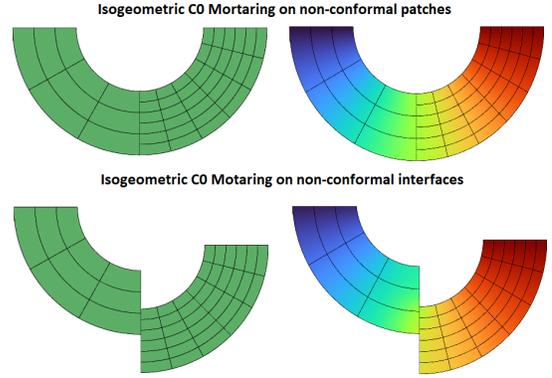
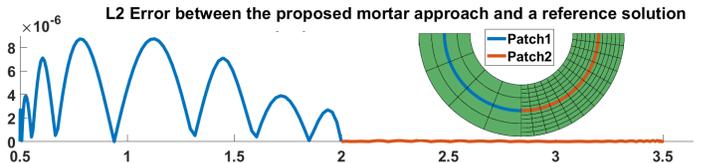


Fig. 3. Solution of a Laplace equation using a Mortar approach on non conforming patches and non conforming interfaces.

### IV. VALIDATION AND RESULTS

The efficiency of the approach is demonstrated on the non-conformal patches geometry presented in figure 3, where an L2 Error is computed between the proposed approach and a reference solution:



An application of this method to the FEM-BEM problem mentioned in 2 will be presented in the extended paper as Boundary Element Method benefit hugely from a reduction in degrees of freedom, aswell as more details regarding the numerical integration of the mortar integrals.

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