



**HAL**  
open science

## On the use of carrier phase jumps to reduce some PWM switching effects

Jean-Francois Brudny, Fabrice Morganti, J. Lecointe, Guillaume Parent

### ► To cite this version:

Jean-Francois Brudny, Fabrice Morganti, J. Lecointe, Guillaume Parent. On the use of carrier phase jumps to reduce some PWM switching effects. 40th Annual Conference on IEEE Industrial Electronics Society, Oct 2014, Dallas, TX, United States. pp.762–768, 10.1109/IECON.2014.7048586 . hal-04114216

**HAL Id: hal-04114216**

**<https://hal.science/hal-04114216>**

Submitted on 26 Oct 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# On the use of carrier phase jumps to reduce some PWM switching effects

Jean-François Brudny, Fabrice Morganti, Jean-Philippe Lecoïnte, Guillaume Parent  
Univ. Lille Nord de France, F-59000 Lille, France - UArtois, LSEE, F-62400 Béthune, France  
jphilippe.lecoïnte@univ-artois.fr

**Abstract**—This paper is based on a method recently developed by the authors to characterize analytically, without switching angle determination, the voltage harmonic three-phase systems generated by the switching of PWM inverters. On this basis, it is possible to define control strategies that modify the spectral content of the output signals. The presented developments are focused on the carrier phase jump which aims to act on two harmonics simultaneously by controlling the effects on the other spectral components.

**Keywords**—PWM supply, switching harmonic control, AC variable-speed drives

## I. INTRODUCTION

The use of Pulse Width-Modulated (PWM) inverters is widespread for several decades in the field of variable speed AC motor drives. One of the main arguments, in addition to easy implementation, concerns the possibility to reject at high frequencies the voltage Switching Three-Phase Harmonic Systems (STPHS). This leads naturally to minimize their effects considering the inductive behavior presented by the machine at these frequencies. The beneficial aspects are real for the torque harmonics insofar as the concerned mechanical resonances vary at low frequencies of a few tens to a few hundred Hz, amplifying the effects of some components. However, other phenomena, particularly important, occur. The latter concern the losses, especially the iron losses [1, 2], the noise of magnetic origin [3, 4] but also premature aging of the electrical insulation system [5, 6]. To get away from these effects, the simplest solution consists in adding a filter connected to the inverter outputs. However, because of their cost and their size, they are not always compatible with the industrial constraints. This has led research teams to propose solutions to minimize some of these effects. These solutions are divided into three categories. The first consists in acting on the architecture of the power electronic converter so as to minimize the amplitude of the voltage variation during a switching and this has led, for example, to the multilevel converters [7]. The second concerns the machine itself by changing its structure, as for example, by equipping it with specific dampers [8, 9] to reduce the air gap flux density harmonics produced by the switching. The third approach focuses on an adaptation of the control strategy.

The developments presented in this paper are in the third category, with the objective of reducing the noise of magnetic origin. There are many references [10, 11, 12, 13] about the reduction of this noise, with appropriate PWM strategies. The

originality of the presented developments is based on the suppression of a force component whose effects are particularly important if it excites a stator mechanical resonance which can intervene at frequencies of a few kHz. Let us assume that this force component is generated by a given balanced STPHS in the output signals. The principle consists, using 3 distinct carriers, in removing this STPHS by making homopolar an intermediate STPHS who produces it. This principle has been already presented [14]. It consists in determining the STPHS phase angles by associating to the classic Sinus - Triangle ( $S\Delta$ ) PWM a Sinus - Sinus (SS) PWM. It has also been shown in this paper that the suppression is accompanied by the appearance of certain harmonics which may call into question, in some cases, the proposed principle. To minimize the negative effects of this strategy, a technique called "carrier-phase jump" has been developed. The goal is to remove alternately two STPHSs from the output signals exploiting the additional degree of freedom provided by this procedure.

As the size of this paper is limited and given the complexity of the phenomena, it is difficult to expose both aspects simultaneously: control strategy and noise reduction. Thus the developments will be limited to the first point. The first part recalls the analytical method used to determine the PWM output voltage characteristics without switching instant determination. Then, this procedure is applied to remove some STPHSs with the analysis of the induced secondary effects. The principle of the carrier phase jump is presented in the third part where some experimental results, about voltage spectral contents, allow concluding on the effectiveness of the proposed control strategy. Let us point out that upper indexes "s" for sinusoidal carrier and " $\Delta$ " for triangular one will be used to distinguish the variables according to the carrier waveforms. No index means that the relations are valuable whatever the case. All experiments are made by connecting to the inverter outputs a 3-phase, 50Hz, 15kW, 2 pole pair cage induction machine star connected operating at no load.

## II. CONTROL STRATEGY

Fig. 1a presents the three-phase PWM connected to a balanced load which is represented by its equivalent star connected circuit. In order to suppress a STPHS from the  $v_q$  output signals ( $q=1, 2$  or  $3$ ), it suffices to make homopolar the intermediate STPHS included in the  $w_q$  variables. This

principle emphasizes that the main quantity for the control law concerns the phases of the STPHSs.

Let us denote  $v_q^{ref}$  and  $v_{cq}^\Delta$  the phase  $q$  sinusoidal reference and triangular carrier signals, whose angular frequencies are respectively  $\omega$  (frequency  $f$ , period  $T$ ) and  $m\omega$  (frequency  $f_{PWM} = mf$ );  $m$  is defined as the modulation index. For a classical PWM,  $v_q^\Delta$  is the same whatever  $q$ . In order to easily determine the  $v_q$  component phase sequences, a  $v_{cq}^s$  sinusoidal carrier is substituted to  $v_{cq}^\Delta$ , these two signals presenting the same peak values as shown in Fig. 1b.

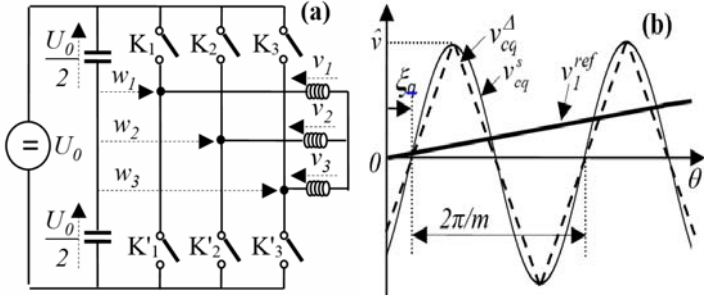


Fig. 1: Three-phase PWM inverter (a) and control signals (b)

The analytical study is done, first, considering that the  $r$  adjusting coefficient is equal to the unit. Considering a time referential tied to  $v_1^{ref}$ ,  $v_q^{ref}$  and  $v_{cq}^s$  are expressed as:

$$\left. \begin{aligned} v_q^{ref} &= \hat{v} \sin(\theta - \phi_q) \\ v_{cq}^s &= \hat{v} \sin[m(\theta - \xi_q)] \end{aligned} \right\} \quad (1)$$

where:  $\theta = \omega t$ ,  $\phi_q = (q-1)2\pi/3$  and  $\xi_q$  a phase  $q$  adjustable carrier phase difference.

The control strategy is similar as for a classical  $\Delta$  PWM: for  $v_q^{ref} > v_{cq}^s$ , the  $K_q$  switch is closed and:  $w_q = U_0/2$ ; for  $v_q^{ref} < v_{cq}^s$ ,  $K'_q$  is closed and:  $w_q = -U_0/2$ . Mathematical developments based on these inequalities lead to consider 2 groups of harmonics, characterized by their ranks  $k_1$  and  $k_2$ , to define the  $w_q^s$  spectral content. Let us denote  $n_1$  and  $n_2$  positive or null integers. Introducing the quantities:

$$\left. \begin{aligned} N^+ &= 1 + n_1 + n_2 \\ N^- &= n_1 - n_2 \end{aligned} \right\} \quad (2)$$

$k_1$  and  $k_2$  can be written:

$$\left. \begin{aligned} k_1 &= mN^+ + N^- \\ k_2 &= mN^- + N^+ \end{aligned} \right\} \quad (3)$$

If  $k_1$  is a positive integer,  $k_2$  can be positive or negative. From a practical point of view, especially for the carrier phase jump,  $w_q^s$  will be defined equation (4).

$$w_q^s = \sum_k w_{q(k)}^s = \sum_{k_1} w_{q(k_1)}^s + \sum_{k_2} w_{q(k_2)}^s \quad (4)$$

with:

$$\left. \begin{aligned} w_{q(k_1)}^s &= \hat{W}_{(k_1)}^s \sin\left[k_1|\theta - N^-\phi_q - mN^+\xi_q - (1+n_1)\pi\right] \\ w_{q(k_2)}^s &= \hat{W}_{(k_2)}^s \sin\left[k_2|\theta - N^+\phi_q - mN^-\xi_q - n_1\pi\right] \text{ for } k_2 > 0 \\ w_{q(k_2)}^s &= \hat{W}_{(k_2)}^s \sin\left[k_2|\theta + N^+\phi_q + mN^-\xi_q - (1+n_1)\pi\right] \text{ for } k_2 < 0 \end{aligned} \right\} \quad (5)$$

The amplitudes of these quantities, considering that  $W_0 = 4U_0/\pi^2$ , are given by:

$$\hat{W}_{(k_1 \text{ or } k_2)}^s = W_0 / [(2n_1 + 1)(2n_2 + 1)] \quad (6)$$

These expressions show that a  $k$  harmonic rank ( $k > 0$ ) is only generated by a term of one of the two groups. The fundamental component, corresponding to  $k=1$ , results from the  $k_2$  group for  $n_1 = n_2 = 0$ .

These STPHSs are defined by their phase sequences: Clockwise ( $C$ ), Anticlockwise ( $A$ ) and Homopolar ( $H$ ). As for the considered load:  $v_q = (2w_q - w_{q+1} - w_{q+2})/3$ , it is possible to consider a similar relationship for each  $k$  harmonic. As for ( $C$ ) and ( $A$ ) systems:  $\sum_q w_{q(k)} = 0$ , it can be deduced that:

$$\left. \begin{aligned} v_{q(k)} &= w_{q(k)} \text{ for } (C) \text{ and } (A) \text{ systems} \\ v_{q(k)} &= 0 \text{ for } (H) \text{ systems} \end{aligned} \right\} \quad (7)$$

It is important to note that these developments are valid whatever the PWM modulation: synchronous or asynchronous. Then, they show that a  $k$  ( $H$ )  $w_{q(k)}$  STPHS disappears from the  $v_q$  spectral content, so that it is possible to assimilate this converter to a filter which suppresses the ( $H$ ) STPHSs. It is this property that is exploited in this study.

It appears also that this SS analytical model has considerable advantages because it makes it possible to determine the  $w_q^s$  (or  $v_q^s$ ) spectral contents and the harmonic phase sequences, without calculating the switching instants.

Given that this relatively interesting properties, it was necessary to analyze the validity limits of this model by comparing the results with those obtained by considering a  $\Delta$  PWM. To do that, both numerical and experimental investigations were carried out. The main properties that have been established [14] for a synchronous modulation can be summarized as follows:

- 缀耀耀 modeling gives identical results concerning the phases and, consequently, the STPHS phase sequences, either for a sinusoidal carrier or a triangular one,

TABLE I: ANALYTICAL RANKS AND PHASE SEQUENCES OF  $w_q^{s \text{ or } \Delta}$  HARMONICS

		$k_1$ group								
$n_1$	0		1		2		3			
$n_2$	0	$m$ (H)	SS : 100	$2m+1$ (C)	SS : 33.33	$3m+2$ (A)	SS : 20	$4m+3$ (H)	SS : 14.28	
			SΔ : 59.97		SΔ : 18.04		SΔ : 6.16		SΔ : 0.88	
	1	$2m-1$ (A)	SS : 33.33	$3m$ (H)	SS : 11.11	$4m+1$ (C)	SS : 6.66	$5m+2$ (A)	SS : 4.76	
			SΔ : 18.04		SΔ : 11.23		SΔ : 6.71		SΔ : 3.79	
	2	$3m-2$ (C)	SS : 20	$4m-1$ (A)	SS : 6.66	$5m$ (H)	SS : 4	$6m+1$ (C)	SS : 2.85	
			SΔ : 6.16		SΔ : 6.71		SΔ : 5.16		SΔ : 3.71	
	3	$4m-3$ (H)	SS : 14.28	$5m-2$ (C)	SS : 4.76	$6m-1$ (A)	SS : 2.85	$7m$ (H)	SS : 2.04	
			SΔ : 0.88		SΔ : 3.79		SΔ : 3.71		SΔ : 3.09	
			$k_2$ group							
	$n_1$	0		1		2		3		
	$n_2$	0	1 (C)	SS : 100	$m+2$ (A)	SS : 33.33	$2m+3$ (H)	SS : 20	$3m+4$ (C)	SS : 14.28
				SA : 100		SA : 31.81		SA : 21.24		SA : 15.73
1		$-m+2$ (C)	SS : 33.33	3 (H)	SS : 11.11	$m+4$ (C)	SS : 6.66	$2m+5$ (A)	SS : 4.76	
			SΔ : 31.81		SΔ : 0.01		SΔ : 1.78		SΔ : 3.32	
2		$-2m+3$ (H)	SS : 20	$-m+4$ (A)	SS : 6.66	5 (A)	SS : 4	$m+6$ (H)	SS : 2.85	
			SΔ : 21.24		SΔ : 1.78		SΔ : 0		SΔ : 0.04	
3		$-3m+4$ (A)	SS : 14.28	$-2m+5$ (C)	SS : 4.76	$-m+6$ (H)	SS : 2.85	7 (C)	SS : 2.04	
			SΔ : 15.73		SΔ : 3.32		SΔ : 0.04		SΔ : 0	

- In return, the model leads to characterize the STPHS amplitudes with a relatively large inaccuracy while the hierarchy between the different components is maintained,
- The previous conclusions are independent of the  $r$  value, which leads to a remarkable result: the phases are independent of the  $r$  value as discussed in [15].

Table 1 shows, in relative values, the main  $w_q$  spectral components of a three-phase SS and SΔ PWMs considering a synchronous modulation with  $r=1$ . The relative magnitudes are defined by:

$$\delta \left| \hat{W}_{(k)}^{s \text{ or } \Delta} \right| = 100 \left| \hat{W}_{(k)}^{s \text{ or } \Delta} \right| / \hat{W}_{(1)}^{s \text{ or } \Delta} \quad (8)$$

where:  $\hat{W}_{(1)}^s = W_0$ ,  $\hat{W}_{(1)}^A = U_0/2$  and:  $\hat{W}_{(1)}^s / \hat{W}_{(1)}^A = 0.81$ .

The values relating to SS are determined from the analytical expression (6), while those relating to SΔ result from a numerical simulation and an experimental validation (which concerns only the (C) and (A) systems insofar only the  $v_{q(k)}^A$  signals can be measured) made for  $m=55$ .

### III. SIMPLE VOLTAGE HARMONIC CONTROL

The following developments present only the method to simply remove a  $v_q^s$  STPHS. As the goal is to remove alternatively two STPHSs,  $x$  and  $y$  denote the ranks of the suppressed STPHSs. As, in certain cases, that method leads to generate unbalances on the phases, these quantities will be defined using  $q$ . As a result, the new STPHSs will be characterized by the variables  $w_{q(k)}^{s(x \text{ or } y)}$  or  $v_{q(k)}^{s(x \text{ or } y)}$ . To illustrate this method, only 2 harmonics of the  $k_2$  group,

which contributes mainly on the definition of the spectral content [14], are suppressed. The method is exactly the same for modifying the harmonics of the  $k_1$  group.

#### A. Suppression of the harmonic of $m+2$ rank

Let us suppress the harmonic of  $x=m+2$  rank of the  $k_2$  group with  $k_2 > 0$ .  $w_{q(k_2)}^s$  is defined by (5) with  $n_1=1$  and  $n_2=0$ . That leads to:  $w_{q(k_2)}^s = \hat{W}_{(k_2)}^s \sin[k_2|\theta + \phi_q - m\xi_q - \pi]$ . To make this system (H), the condition (9) must be satisfied:

$$m\xi_q^{(x)} = (\phi_q - C^{(x)}) \quad (9)$$

where  $C^{(x)}$  is a constant value. Thus the modified systems, deduced from (5), can be expressed by (10) where  $\delta_q^1 = (1+2n_1)\phi_q$ :

$$\left. \begin{aligned} w_{q(k_1)}^{s(x)} &= \hat{W}_{(k_1)}^s \sin[k_1|\theta - \delta_q^1 + N^+ C^{(x)} - (1+n_1)\pi] \\ w_{q(k_2)}^{s(x)} &= \hat{W}_{(k_2)}^s \sin[k_2|\theta - \delta_q^1 + N^- C^{(x)} - n_1\pi] \text{ for } k_2 > 0 \\ w_{q(k_2)}^{s(x)} &= \hat{W}_{(k_2)}^s \sin[k_2|\theta + \delta_q^1 - N^- C^{(x)} - (1+n_1)\pi] \text{ for } k_2 < 0 \end{aligned} \right\} (10)$$

#### B. Suppression of the harmonic of $m-2$ rank

The method to suppress the  $y=m-2$  harmonic rank of the  $k_2$  group with  $k_2 < 0$  defined by (5) is similar to the previous one. This harmonic is defined by  $n_1=0$  and  $n_2=1$ . That leads to satisfy the equality (11) where  $C^{(y)}$  is a constant value:

$$m\xi_q^{(y)} = -(\phi_q - C^{(y)}) \quad (11)$$

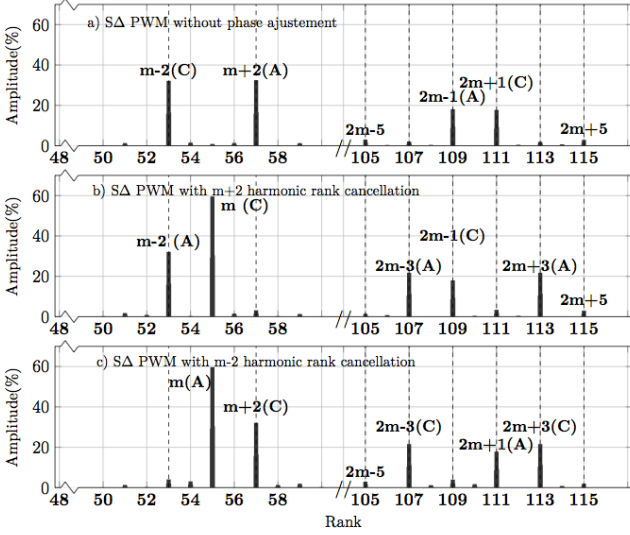


Fig. 2. Relative experimental  $v_q^A$  spectra resulting from  $m+2$  (b) and  $m-2$  (c) rank harmonic cancellations for  $r=1$

The modified systems are characterized by the expression (12) where  $\delta_q^2 = (1+2n_2)\phi_q$ :

$$\left. \begin{aligned} w_{q(k_1)}^{s(y)} &= \hat{W}_{(k_1)}^s \sin \left[ k_1 |\theta + \delta_q^2 - N^+ C^{(y)} - (1+n_1)\pi \right] \\ w_{q(k_2)}^{s(y)} &= \hat{W}_{(k_2)}^s \sin \left[ k_2 |\theta - \delta_q^2 - N^- C^{(y)} - n_1\pi \right] \text{ for } k_2 > 0 \\ w_{q(k_2)}^{s(y)} &= \hat{W}_{(k_2)}^s \sin \left[ k_2 |\theta + \delta_q^2 + N^- C^{(y)} - (1+n_1)\pi \right] \text{ for } k_2 < 0 \end{aligned} \right\} \quad (12)$$

### C. Experimental measurements

The mathematical developments show that the method requires the use of 3 carriers in order to satisfy the expressions (9) or (11). Moreover, it is easy to observe that this control strategy does not modify the STPHSs initial amplitudes and that, in this case, all the modified STPHSs are balanced (this is not systematically the case when an harmonic of the  $k_1$  group is suppressed [14]). It appears that the suppression of an harmonic is totally independent of the values of the constants.

The Fig. 2a presents the experimental spectrum (SA PWM) of  $v_q^A$  obtained for  $r=1$ ,  $m=55$  and a single carrier. Fig. 2b and c show the spectra for controls corresponding respectively to the suppression of the harmonics of rank  $x=m+2$  and  $y=m-2$  with  $C^{(x)}=C^{(y)}=0$  so that:  $\xi_q^{(x)}=\phi_q/m$  and  $\xi_q^{(y)}=-\phi_q/m$ . The suppression is clearly shown on the spectra although the control results from a mathematical law established for a SS PWM.

The suppression of the  $m+2$  harmonic rank leads also to the cancellation of the  $2m+1$  harmonic rank and the generation of the harmonics of ranks  $m$  and  $2m\pm 3$ . Similarly, the suppression of the  $m-2$  harmonic rank causes the suppression of the  $2m-1$  harmonic rank and the

generation of harmonics of rank  $m$  and  $2m\pm 3$ .

These characteristics can easily be justified analytically as well as the changes on the phase sequences which appear also on these figures. Let us note, in accordance with what has already been stated, that the theoretical relative amplitude of  $m$  rank component (SS PWM) is 100% while, experimentally, for a SA PWM, it is only 60%.

In order to minimize the effects of harmonics generated by this control, a technique called "carrier-phase jump" has been developed. In this case, the suppression of a harmonic becomes rather a mitigation. The idea relies on the use of the additional degree of freedom provided by the constants in order to control the amplitude of the unwanted harmonics which reappear.

## IV. CARRIER PHASE JUMP

### A. Principle

The rank  $x$  harmonic is suppressed during the first  $v_q^{ref}$  period and the rank  $y$  component during the second one. That imposed transition leads to a transition on the modified STPHSs. The general expression of a  $k$  rank STPHS, is given, taking into account (10) and (12), by:

$$w_{q(k)}^{s(x \text{ or } y)} = \hat{W}_{(k)}^s \sin \left[ k\theta - \psi_{q(k)}^{s(x \text{ or } y)} \right] \quad (13)$$

$w_{q(k)}^{s(x \text{ or } y)}$ , defined by identification, depends on  $\phi_q$  and on  $C^{(x)}$  and  $C^{(y)}$ . The Fourier series of resulting  $w_{q(k)}^{s(x,y)}$  signal requires considering an interval which corresponds to  $2T$ . The two expressions of this signal on the considered interval are:

$$\left. \begin{aligned} w_{q(k)}^{s(x)} &= \hat{W}_{(k)}^s \sin \left[ 2k(\theta - \psi_{q(k)}^{s(x)} / 2k) \right] \\ w_{q(k)}^{s(y)} &= \hat{W}_{(k)}^s \sin \left[ 2k(\theta - \psi_{q(k)}^{s(y)} / 2k) \right] \end{aligned} \right\} \quad (14)$$

Let us denote  $h_k$  the rank of the harmonic. If  $t=0$  is the instant which corresponds to the passage of  $w_{q(k)}^{s(x)}$  to  $w_{q(k)}^{s(y)}$ , it can be written:

$$w_{q(k)}^{s(x,y)} = \sum_{h_k=1}^{\infty} \hat{W}_{q(k)h_k}^{s(x,y)} \cos(h_k\theta - \beta_{q(k)h_k}^{s(x,y)}) \quad (15)$$

$$\left. \begin{aligned} \hat{W}_{q(k)h_k}^{s(x,y)} &= \hat{W}_{(k)}^s \sqrt{A_{q(k)h_k}^2 + B_{q(k)h_k}^2} \\ \text{with: } \text{tg} \beta_{q(k)h_k}^{s(x,y)} &= B_{q(k)h_k} / A_{q(k)h_k} \end{aligned} \right\} \quad (16)$$

Table 2 gives the expressions of  $A_{q(k)h_k}$  et  $B_{q(k)h_k}$  with:

$$\psi_{q(k)}^{s+} = (\psi_{q(k)}^{s(x)} + \psi_{q(k)}^{s(y)}) / 2, \quad \psi_{q(k)}^{s-} = (\psi_{q(k)}^{s(x)} - \psi_{q(k)}^{s(y)}) / 2.$$

On  $T$ , for given  $k$ , this method leads to associate to the initial harmonics some harmonic groups. The latter are centered on the  $k$  Integer (I) rank harmonic, that is called main component and which amplitude is defined by the values given in Table 2 for:  $h_k=2k$ . This  $k$  component is

TABLE 2: EXPRESSIONS OF THE SERIES FOURIER COEFFICIENTS FOR A JUMP ON THE K RANK TERMS

	$A_{q(k)h_k}$	$B_{q(k)h_k}$	$\hat{W}_{q(k)h_k}^{s(x,y)}$	$\beta_{q(k)h_k}^{s(x,y)}$
$h_k$ even $\neq 2k$	0	0	0	
$h_k$ odd	$8k \frac{\sin \psi_{q(k)}^{s+} \sin \psi_{q(k)}^{s-}}{(2k+h_k)(2k-h_k)\pi}$	$-4h_k \frac{\cos \psi_{q(k)}^{s+} \sin \psi_{q(k)}^{s-}}{(2k+h_k)(2k-h_k)\pi}$		
	$\mp \frac{2}{\eta\pi} \sin \psi_{q(k)}^{s+} \sin \psi_{q(k)}^{s-}$	$\pm \frac{2}{\eta\pi} \cos \psi_{q(k)}^{s+} \sin \psi_{q(k)}^{s-}$	$\hat{W}_{(k)}^s \frac{2}{\eta\pi}  \sin \psi_{q(k)}^{s-} $	$\psi_{q(k)}^{s+} + \pi / 2$
$h_k = 2k$	$-\sin \psi_{q(k)}^{s+} \cos \psi_{q(k)}^{s-}$	$\cos \psi_{q(k)}^{s+} \cos \psi_{q(k)}^{s-}$	$\hat{W}_{(k)}^s  \cos \psi_{q(k)}^{s-} $	$\psi_{q(k)}^{s+} + \pi / 2$

 TABLE 3. EXPRESSIONS OF  $\psi_{q(k)}^{s(x)}$  AND  $\psi_{q(k)}^{s(y)}$  AND (C), (A) AND (H) COMPONENT CHARACTERIZATION

	$\psi_{q(k)}^{s(x)}$	$\psi_{q(k)}^{s(y)}$	$\psi_{q(k)}^{s+}$	$\psi_{q(k)}^{s-}$	(C)	(A)	(H)
$m-2$	$-\phi_q - C^{(x)} + \pi$	$C^{(y)} + \pi$	$\pi - (\phi_q + C^-) / 2$	$-(\phi_q + C^+) / 2$	0	0.5	0.5
$m+2$	$-C^{(x)} + \pi$	$\phi_q + C^{(y)} + \pi$	$\pi + (\phi_q - C^-) / 2$	$-(\phi_q + C^+) / 2$	0.5	0	0.5
$m$	$\phi_q - C^{(x)} + \pi$	$-\phi_q + C^{(y)} + \pi$	$\pi - C^-$	$\phi_q - C^+ / 2$	0.5	0.5	0
$2m-1$	$\phi_q - 2C^{(x)} + \pi$	$2C^{(y)} + \pi$	$\pi - C^- + \phi_q / 2$	$-C^+ + \phi_q / 2$	0.5	0	0.5
$2m+1$	$-2C^{(x)}$	$-\phi_q + 2C^{(y)}$	$\pi - C^- + \phi_q / 2$	$-C^+ + \phi_q / 2$	0	0.5	0.5
$2m-3$	$-\phi_q - 2C^{(x)} + \pi$	$\phi_q + 2C^{(y)} + \pi$	$\pi - C^-$	$-\phi_q - C^+$	0.5	0.5	0
$2m+3$	$-\phi_q - 2C^{(x)}$	$\phi_q + 2C^{(y)}$	$-C^-$	$-\phi_q - C^+$	0.5	0.5	0

surrounded by harmonics whose ranks are only defined for  $h_k$  odd.  $h_k$  can be expressed as follows:  $h_k = 2k \pm \eta$  with  $\eta$  odd. On  $T$ , these ranks are defined by:  $k \pm \eta / 2$ . These harmonics of Non-Integer (NI) ranks are defined by the values given in the first line for  $h_k$  odd in Table 2. Insofar these harmonics present significant values only for the  $\eta$  first values, the second line for  $h_k$  odd gives the simplified expressions valid only under these conditions.

Let us point out that for  $k=1$ , the expressions of  $w_{q(k)}^{s(x)}$  and  $w_{q(k)}^{s(y)}$  that can be deduced from (10) and (12) are the same. Thus, the carrier phase jump has no effect on this component.

### B. Numerical applications

The expressions of  $\psi_{q(k)}^{s(x)}$  and  $\psi_{q(k)}^{s(y)}$ , grouped in Table 3, are obtained:

- by identification considering the expression (10) and (12), with  $k$  taking successively the values:  $m \pm 2$ ,  $m$ ,  $2m \pm 1$  and  $2m \pm 3$ ,
- and by characterizing the quantities  $\varepsilon_q^{(x \text{ or } y)}$  with their general expressions given by (9) and (11).

The expressions of  $\psi_{q(k)}^{s+}$  et  $\psi_{q(k)}^{s-}$ , expressed in function of  $C^+ = C^{(x)} + C^{(y)}$  and  $C^- = C^{(x)} - C^{(y)}$ , are also given in this table.

These mathematical developments have a notable interest because the  $h_k$  rank harmonic amplitudes, which differ with the value of  $q$ , only depend on  $\psi_{q(k)}^{s-}$  (see Table 2) and thus on  $C^+$ . The phases depend only on  $\psi_{q(k)}^{s+}$  and thus on  $C^-$ .

Fig. 3 shows the variations of  $|\cos \psi_{q(k)}^{s-}|$  and  $|\sin \psi_{q(k)}^{s-}|$  with  $C^+$  for the previous values of  $k$  and the three values of  $q$ . Let us point out, in order to exploit these curves, that for  $k = 2m \pm 1$  and  $2m \pm 3$  the horizontal scale must be divided by 2. The harmonic amplitudes are obtained by multiplying these quantities by the coefficients given in Table 2. Fig. 3 shows that, for a given value of  $q$ , the maximal value of the main component  $k$  corresponds to non-integer harmonic whose values are null.

The new STPHSs are all unbalanced, so they can be characterized with their components (C), (A) et (H). The study made on these component modulus, and more especially on  $|\cos \psi_{q(k)}^{s-}|$  and  $|\sin \psi_{q(k)}^{s-}|$ , shows that, for a given value of the main component  $k$ , the coefficients which intervene in their definitions (multiplying the modulus given in Table 2) are independent from  $h_k$ , as well as the values given to  $C^+$  and  $C^-$ . The numerical values of these coefficients are given also in Table 3.

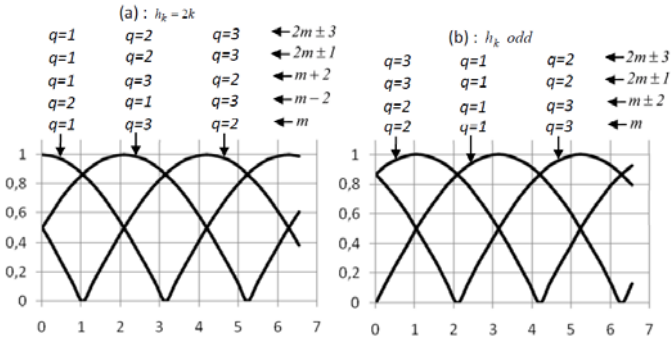


Fig. 3. Variations of  $|\cos \psi_{q(m)}^{s-}|$  (a) and  $|\sin \psi_{q(m)}^{s-}|$  (b) with  $C^+$  (rd)

First, it can be underlined that a carrier phase jump which involves in the transition an ( $H$ ) STPHS, leads to corresponding  $w_q$  STPHS which contains ( $H$ ) component. Table 3 shows that in this case, the output STPHSs are balanced. They are unbalanced for all the other cases.

The second remark concerns the NI rank harmonics. They form balanced STPHSs when they are tied to the  $m \pm 2$  and  $2m \pm 1$  rank main components, and unbalanced STPHSs for the other considered main components. These harmonics combine them as shown in Fig. 4 which considers these quantities which appear around  $m$  (Fig. 4a) and around  $2m$  (Fig. 4b). For this analysis, the number of NI rank harmonic considered is limited to the  $\eta$  first four values. It results that the characteristic predetermination of NI rank harmonics needs to associate several terms taking into account the  $\beta_{q(k)h_k}^{s(x,y)}$  quantities given in Table 2, which depend on  $\psi_{q(k)}^{s+}$ , and consequently on  $C^-$  quantities as it appears in Table 3.

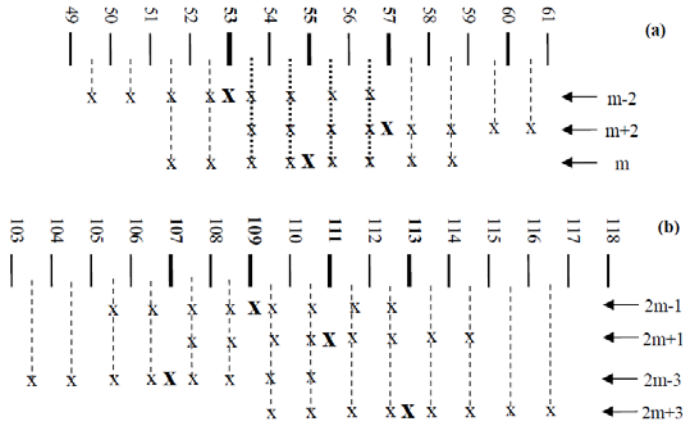


Fig. 4. NI rank harmonics: (a) around  $m$ ; (b) around  $2m$

TABLE 4.  $v_q^s$  COMPONENT RELATIVE AMPLITUDES FOR  $C^+ = 0$

	$m$ (100%)		$m \pm 2$ (33.3%)		$2m \pm 1$ (33.3%)		$2m \pm 3$ (20%)	
	I	NI	I	NI	I	NI	I	NI
$q = 1$	100	0	16.65	10.64	16.65	10.64	20	0
$q = 2$	50	55	16.65	10.64	16.65	10.64	10	11.1

### C. SS PWM: experimental validation

The goal of these applications is to validate the theoretical developments and the previous case ( $x = m + 2$  and  $y = m - 2$ ) is considered with  $m = 55$  and  $r = 1$ .

- First, the constants are adjusted such as:  $C^{(x)} = C^{(y)} = 0$ , leading to  $C^+ = 0$ . The analytical developments lead to the  $v_q^{s(x,y)}$  component relative amplitudes in % given in Table 4. The quantities for NI rank harmonics are defined for  $\eta = 1$ . Let us precise that only 2 lines,  $q = 1$  and  $q = 2$ , appear in this Table. This particularity results from the fact that, for the considered cases, the  $q = 2$  and  $3$  components have the same amplitudes. When the numerical values for  $q = 1$  et  $2$  are equal, the STPHSs are balanced.

The  $v_q^{s(x,y)}$  spectra corresponding to this SS PWM are presented in Fig. 5. Let us consider the main components. It can be noted a very good correspondence between the calculated values and the values measured on the spectra.

Concerning the NI rank harmonics, let us consider the  $m + 0.5$  rank harmonic which results of the combination of 3 terms:

- the harmonic relating to  $k = m$  for  $\eta = 1$ , which relative amplitude is 0% for  $q = 1$  and 55% for  $q = 2$  and  $3$ ,
- the harmonic relating to  $k = m + 2$  for  $\eta = 2$ , which relative amplitude is 5.32% for the 3 phases,
- the harmonic relating to  $k = m - 2$  for  $\eta = 3$ , which relative amplitude is 3.55% for the 3 phases.

The approach is exactly the same to characterize the  $m - 0.5$  rank harmonic. Without doing rigorous calculus, it can be estimated that the components of  $m \pm 0.5$  ranks have an amplitude of a few percents for  $q = 1$  and an amplitude between 50 and 60% for  $q = 2$  and  $3$ . These quantities correspond to those measured on the spectra. A similar approach for the other NI harmonics leads to similar conclusions.

- The constants are adjusted such as:  $C^{(x)} = 0$  et  $C^{(y)} = \pi$ , leading to  $C^+ = \pi$ . The analytical developments lead to the  $v_q^{s(x,y)}$  component relative amplitudes given in Table 5 where only the harmonics around the  $m$  rank are considered because, considering the remark about the periodicity of the Fig. 3 curves, the spectral content around  $2m$  is the same than the content obtained for  $C^+ = 0$ . The spectral contents shown in Fig. 6 concern only the harmonics around  $m$ . The comparison between the predetermined and the measured values, shows the validity of the analytical method.

### D. SA PWM: experimental validation

The spectra measured for a SA PWM have the same properties when the constants are changed; only the amplitudes are modified. For example, the spectra shown in Figs. 7 corresponds to the spectra given in Fig. 6 SS PWM.



## References

- [1] S. Khomfoi, V. Kinnares, P. Viriya. "Influence of PWM characteristics on the core losses due to harmonic voltages in PWM fed induction motors", *Pow. Eng. Society Winter Meeting*, Vol.1, 2000, pp.365-369.
- [2] A. Boglietti and A. Cavagnino, "Iron loss prediction with PWM supply: An overview of proposed methods from an engineering application point of view," *Electric Power Systems Research* 80, no. 9 pp.1121-1127, Mar. 2010.
- [3] S. Iida, Y. Okuma, S. Masukawa, Miyairi, Shota, B.K. Bose. "Study on magnetic noise caused by harmonics in output voltages of PWM inverter," *IEEE Trans. on Ind. Elec.*, Vol.38, N° 3, pp.180-186, 1991.
- [4] P. Pellerey, G. Favennec, V. Lanfranchi, G. Friedrich, "Active reduction of electrical machines magnetic noise by the control of low frequency current harmonics," *IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society*, October 2012, pp.1654-1659.
- [5] V. Mihaila, S. Duchesne, D. Roger, "A simulation method to predict the turn-to-turn voltage spikes in a PWM fed motor winding," *IEEE Trans. on Dielec. and Elec. Insul.*, Vol. 18, N°5, 2011, pp. 1609- 1615.
- [6] A. Cavallini, D. Fabiani, G.C. Montanari, "Power Electronics and Electrical Insulation Systems – Part 2 : Life Modeling for Insulation Design," *DIES, IEEE Elec. Insul. Mag.*, Vol. 26; N°4, 2013, pp. 33-39.
- [7] M. Malinowski, K. Gopakumar, J. Rodriguez, M.A. Pérez "A Survey on Cascaded Multilevel Inverters," *IEEE Trans. on Ind. Elec.*, Vol. 57, N° 7, July 2010 pp.2197-2206.
- [8] J-F. Brudny, J-Ph. Lecoïnte. "Rotor Design for Reducing The Switching Magnetic Noise of AC Electrical Machine Variable-Speed Drives". *IEEE Trans. on Ind. Elec.*, Vol. 58, November 2011, pp. 5112-5120.
- [9] M. Madarres, A. Vahedi, M.R. Ghazanchaei, « Investigations on Dynamic and Steady State Performance of Axial Flux Hysteresis motors Considering Rotor configurations". International Revirw of Electrical Engineering, Vol. 5, N° 1, ISSN. 1827-6660, February, 2010.
- [10] E.R.C. da Silva, E.C. dos Santos, C.B. Jacobina, "Pulsewidth Modulation Strategies". *IEEE Ind. Electr. mag*, 5.2, 2011, p. 37–45.
- [11] D.-J. Kim, J.-W. Jung, J.-P. Hong, K.-J. Kim, C.-J. Park, "A Study on the Design Process of Noise Reduction in Induction Motors," *IEEE Trans. on Magnetics*, Vol.48, N° 11, November 2012, pp. 4638-4641.
- [12] T.G. Habetler and D.M. Divan. "Acoustic noise reduction in sinusoidal PWM drives using a randomly modulated carrier". *IEEE Trans. on Power Electronics*, Vol. 6, July 1991, pp. 356–363.
- [13] G.A. Covic and J.T. Boys. "Noise quieting with random PWM ac drives". *Electric Power App, IEE Proc.*, Vol. 145, 1998, pp. 1–10.
- [14] J.F. Brudny, T.Szkudlapski, F.Morganti, J-Ph. Lecoïnte, Method for controlling the PWM switching: Application to magnetic noise reduction, Accepted en avril 2014 pour *IEEE Trans. on Ind. Elec.*. On line
- [15] S.R. Bowes, "New sinusoidal pulsewidth-modulated inverter," *Electrical Engineers, Proceedings of the Institution of*, Vol.122, N° 11, November 1975, pp.1279-1285.

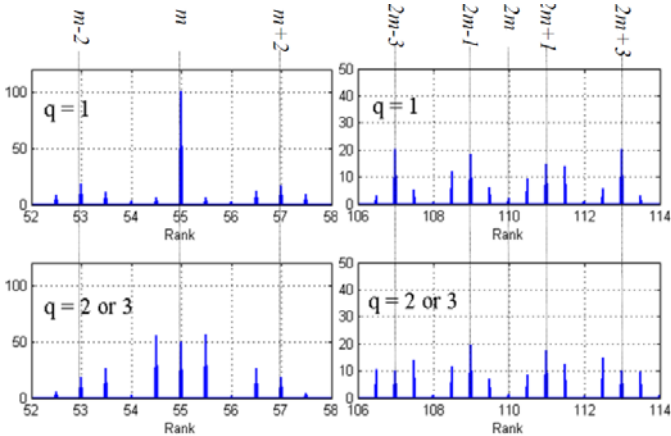


Fig. 5.  $v_1^s$  and  $v_2^s$  spectra around  $m$  and  $2m$  for a SS PWM and  $C^+ = 0$

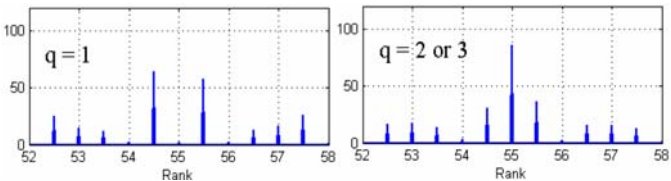


Fig. 6.  $v_1^s$  and  $v_2^s$  spectra around  $m$  for a SS PWM and  $C^+ = \pi$

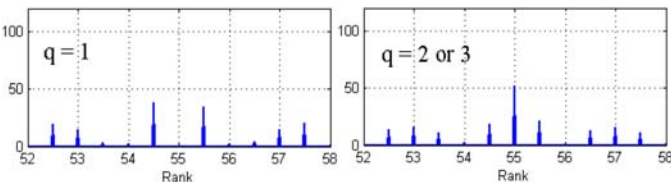


Fig. 7.  $v_1^A$  and  $v_2^A$  spectra around  $m$  for a SA PWM and  $C^+ = \pi$

TABLE 5.  $v_q^s$  COMPONENT RELATIVE AMPLITUDES FOR  $C^+ = \pi$

	$m$ (100%)		$m \pm 2$ (33.3%)	
	I	NI	I	NI
$q = 1$	0	64	16.65	10.64
$q = 2$	86,6	32	16.65	10.64

## V. CONCLUSION

In this paper, the authors use a model that they have developed. It allows controlling the STPHSs of the output SA PWM in order to reduce some undesirable effects generated by the switching. Many controls can be applied; the carrier phase jump is presented in this paper. This method which consists in acting simultaneously on several harmonics, leads, in term of spectral content, on similar results as the spread-spectrum with the generation of harmonics of non-integer ranks.

A fundamental aspect of the method concerns the simple and fast determination of the consequences due to the control on all the components of the spectrum, what is generally not developed in the litterature. From an application perspective, the goal is to control the noise of magnetic origin produced by AC machines supplied by PWM converters.

**Jean-François Brudny** (M'92-SM'04) received the Ph. D and the D. Sc degrees from the Lille University, France, in 1984 and 1991. In 1992 he joined the University of Artois, Béthune, as a full Professor. He is the head of the Electromechanical Systems and Environment Research Laboratory

**Fabrice Morganti** PhD born in 1965. He received the PhD degree in 1994 from The Lille-I University, France . He is Associate Professor at the Artois University and he joined the institute LSEE (Electrical Systems and Environment Research Laboratory), France.

**Jean-Philippe Lecoïnte** (M'04) received the MSc degree in Electrical Engineering from Lille University in 2000. He received the PhD degree and the DSc degree from the University of Artois, France, in 2003 and 2012. He is currently Full Professor in LSEE, University of Artois.

**Guillaume Parent** received the Ph.D. degree in electrical engineering jointly from the Lille University, Lille, France, and the Liège University, Liège, Belgium, in 2008. He is currently an Associate Professor of electrical engineering at the University of Artois.