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## Determination of Flux Tube Portions by Adjunction of Electric or Magnetic Multivalued Equipotential Lines

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Abstract—In 2D electrostatic and magnetostatic finite element problems, portions of flux tubes are defined by field lines together with their orthogonal lines. These two sets of lines can be obtained as the isovalue lines of scalar and vector potentials. One set is related to a multivalued potential that, when made single-valued via the definition of cuts, undergoes discontinuities, resulting in lines interruptions at the cuts. A method is proposed for obtaining continuous isovalue lines for the multivalued potentials. This then helps in locating selected portions of flux tubes and quantifying their equivalent properties, i.e., capacitances or reluctances, to be used in equivalent circuits.

Index Terms-Finite element analysis, field lines, flux tubes

### I. INTRODUCTION

An electromagnetic problem is usually solved through either a scalar or a vector potential formulation. In 2D and in case of a total potential, the associated field is either parallel or perpendicular to its isovalue lines, which makes their determination a key point for the characterization of flux tubes portions. For a given problem, obtaining both sets of lines requires the use of dual formulations. Whereas the commonly used formulations (with electric scalar potential in electrostatics and magnetic vector potential in magnetostatics) involve single-valued potentials, their dual formulations (with electric vector potential and magnetic scalar potential, respectively) involve multivalued potentials, *i.e.*, presenting discontinuities through some cut lines making their support domain simply connected [1]. The aim of this work is the development of a method allowing to adjunct the parts of the isovalue lines at the cuts in order to be able to exploit them as some boundaries of flux tubes portions.

### II. MULTIVALUED POTENTIALS

Electrostatic and magnetostatic problems are defined in a domain  $\Omega = \Omega_c \cup \Omega_c^C$  where  $\Omega_c$  is the support of charges or currents, respectively. In the multiply connected domain  $\Omega_c^C$  made simply connected by introduction of cuts, the divergence-free electric flux density d can be expressed as the curl of an electric vector potential u — in 2D, u is perpendicular to the studied plane, with  $u = u e_z$  — and the curl-free magnetic field h as the gradient of a magnetic scalar potential  $\phi$ :

$$\boldsymbol{d} = \operatorname{curl} \boldsymbol{u} = \operatorname{grad} \boldsymbol{u} \times \boldsymbol{e}_z, \, \boldsymbol{h} = -\operatorname{grad} \boldsymbol{\phi}$$
 (1)

Potentials u and  $\phi$  present discontinuities through the cuts that are related to the total electric charges Q and currents I, respectively, in all the portions of  $\Omega_c$ . The iso-u lines represent the lateral boundaries of electric flux tubes whereas the iso- $\phi$  are the gates of the magnetic flux tubes. Due to the discontinuities of u and  $\phi$  at the cuts, the flux tubes crossing some cuts are not bounded by two single isovalue lines. Indeed, for a fixed isovalue from a part of the boundary  $\partial \Omega_c$  of  $\Omega_c$ , the corresponding isoline can end at a cut (Fig. 1a). In that case, the potential on the other side of the cut has to be shifted by the related discontinuity level in order to obtain a continuous line through the cut (Fig. 1b).



Fig. 1. Iso-u lines for a two-conductor electrostatic problem (a cut line joins the conductors)

Once the adjuncted isovalue lines of the multivalued potential are obtained (Fig. 1b), they can be combined with the isovalue lines of the single valued potential (e.g., electric scalar potential v) calculated from the dual formulation in order to characterize all the boundaries of any flux tube portion (Fig. 2).



Fig. 2. Electric flux tubes boundaries (iso-u and iso-v lines)

In the extended paper, details about the formulations and the implementation of the method will be given with applications to both electrostatic and magnetostatic problems to determine equivalent circuits.

### REFERENCES

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