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► **To cite this version:**

Raphaël Pile, Guillaume Parent, Emile Devillers, Thomas Henneron, Yvonnick Le Menach, et al.. Application limits of the airgap Maxwell tensor. The Heighteenth Biennial IEEE Conference on Electromagnetic Field Computation (CEFC 2018), Oct 2018, Hangzhou, China. pp.1. hal-04114199

HAL Id: hal-04114199

<https://hal.science/hal-04114199>

Submitted on 5 Jun 2023

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Application Limits of the Airgap Maxwell Tensor

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Abstract—In an electrical machine, the Maxwell Tensor is widely used to compute global forces or local pressure along a surface in the air. This communication proposes to highlight the limits of the method with an academic case of slotless stator and rotor. In particular an analytic demonstration shows the existence of coefficients depending on the geometry and the wavenumber between the application of the Maxwell Tensor in the air-gap and the stator magnetic pressure.

Index Terms—Maxwell Tensor, Magnetic pressure, Electrical machines, Magneto-mechanical, Vibration.

I. INTRODUCTION

The Maxwell Tensor (MT) is widely used to compute local surface pressure and integrated forces on a given surface surrounding a body. Nonetheless the location of this surface in the studied domain has an important impact on the results [1]. In the vibro-acoustic study of electrical machines, such phenomenon can have an important impact on high wave-number's amplitude. The theoretic magnetic pressure is exactly the application of the MT at the air-ferromagnetic interface for linear isotropic media [2]. However the application of the MT at the interface is source of numerical errors [3] such that the application of the MT in the air-gap is of great interest. Then this communication proposes to highlight the limits of the application of MT in the air-gap compared to the theoretic one. The deviation between the applications of Maxwell Tensor are quantified with coefficients depending on the geometry and the wavenumber. To this purpose, the magnetic problem and the main application of MT will be presented. Then the analytical magnetic potential will be solve leading to an analytic expression of the MT in the air-gap. The last steps will be to compare the air-gap expression with theoretic magnetic pressure in order to conclude on the limits of the airgap MT.

II. PROBLEM DEFINITION

A. Magnetic problem

Understanding the sources of MT pressure variations is a difficult task because of numerous artifacts that can be produced by the numerical simulation of electrical machines such as slotting effect, sharp geometries, interference between the wave-numbers, *etc.* To avoid these artifacts, an academic slotless machine in Fig.1 is studied. In order to have only one magnetic wave-number, the magnetic potential z-component A_z is imposed at radius R_{ag} such that $\forall \theta \in [0, 2\pi]$:

$$A_z(R_{ag}, \theta) = \beta \sin(n\theta), \quad n \in \mathbb{N}^* \quad (1)$$

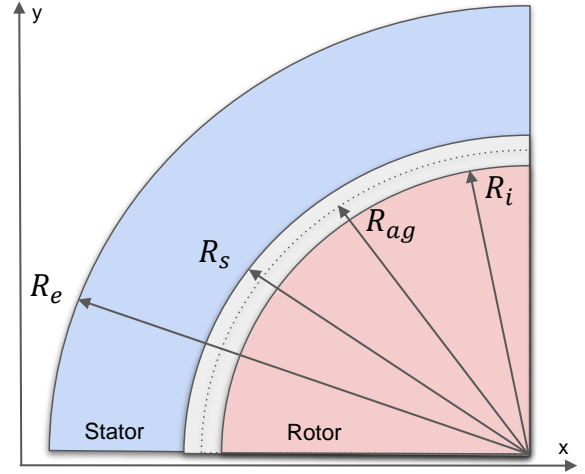


Fig. 1. Slotless electrical machine used to compare magnetic pressure on the stator and in the air-gap

The second boundary condition on the stator external yoke R_e is imposed as:

$$A_z(R_e, \theta) = 0 \quad (2)$$

Finally, the conservation of normal magnetic flux and tangential magnetic field comes directly from Maxwell equations and can be expressed with the magnetic potential at the interface between the air (permeability μ_0) and ferromagnetic media (relative permeability μ_r):

$$\forall \theta \in [0, 2\pi] \left\{ \begin{array}{l} A_z(R_s^-, \theta) = A_z(R_s^+, \theta) \\ \frac{1}{\mu_0} \frac{\partial A_z}{\partial r}(R_s^-, \theta) = \frac{1}{\mu_r \mu_0} \frac{\partial A_z}{\partial r}(R_s^+, \theta) \end{array} \right. \quad (3)$$

Then the 2D linear potential vector is solved.

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial A_z}{\partial r} \right) + \frac{\partial^2 A_z}{\partial \theta^2} = 0, \quad \forall \theta \in [0, 2\pi], \quad \forall r \in [0, R_e] \quad (4)$$

This partial derivative problem can be solved analytically according to [4]. Thus the magnetic potential, flux and field can be analytically computed in the studied domain and it leads to the expression (19) in linear isotropic ferromagnetic media. The next section provides the calculation steps which leads to the analytical expression of magnetic potential, flux and field.

B. Maxwell-Tensor

The theoretic application of the MT for the 2D linear computation of magnetic pressure P requires to apply (5) on

a surface which exactly corresponds to the interface between the air and a ferromagnetic: media [2][5].

$$P = \frac{1}{2} \left(\frac{1}{\mu_0} - \frac{1}{\mu_r} \right) (B_n^2 - B_t^2) \quad (5)$$

where B_n and B_t are the magnetic fluxes locally normal (resp. tangential) to the interface. Note that this expression should leads to the same results on both side of the interface since B_n and B_t are continuous. The first major hypothesis is to neglect ferromagnetic permeability contribution $\frac{1}{\mu_0} \gg \frac{1}{\mu_r}$ such that (5) is reduced to:

$$P = \frac{1}{2\mu_0} (B_n^2 - B_t^2) \quad (6)$$

The last common hypothesis is to consider $B_n \gg B_t$ close to an interface between air and ferromagnetic media. It leads to the simplified MT:

$$P = \frac{1}{2\mu_0} B_n^2 \quad (7)$$

Supposing the two last hypothesis, the application of the MT at the interface remains source of numerical errors [3] Since (6) and (7) does not depend on ferromagnetic properties, it can be observed in recent publications that the MT is nearly always applied in the air-gap and this method can leads to accurate results for vibro-acoustics studies. This paper aims to identify the limits of such approximation of the MT. Since the paper focuses on a slotless case, and for more clarity, the previous B_n and B_t will be respectively assimilate the their projection B_r and B_θ in the polar referential.

III. ANALYTIC MAGNETIC POTENTIAL

A solution exists and is unique for the previous system [6]. Then a method consists to state a function A_z and to check if it fulfills the boundary conditions: a solution similar to [4] is searched for the upper air-gap $\forall \theta \in [0, 2\pi]$ and $\forall r \in [R_a, R_s]$:

$$A_z(r, \theta) = \left(\gamma_n \frac{E_n(r, R_s)}{E_n(R_a, R_s)} + \alpha_n \frac{E_n(R_a, r)}{E_n(R_a, R_s)} \right) \sin(n\theta) \quad (8)$$

with E_n a polynomial function defined by:

$$E_n : (x, y) \in \mathbb{R}^2 \rightarrow \left(\frac{x}{y} \right)^n + \left(\frac{y}{x} \right)^n \quad (9)$$

In the same way, for the stator ring $\forall \theta \in [0, 2\pi]$ and $\forall r \in [R_s, R_e]$:

$$A_z(r, \theta) = \left(\zeta_n \frac{E_n(r, R_e)}{E_n(R_s, R_e)} + \chi_n \frac{E_n(R_s, r)}{E_n(R_s, R_e)} \right) \sin(n\theta) \quad (10)$$

Under this form, the vector A_z is satisfying the Poisson's equation 4. Then the coefficient γ_n , α_n , ζ_n and χ_n have to be determined in order to satisfy the boundary conditions. If a correct set of these coefficients is found, then A_z would be the unique solution of the problem. First, satisfying the boundary condition (1) in the air-gap leads to:

$$\gamma_n = \beta \quad (11)$$

The second boundary condition (2) leads to:

$$\chi_n = 0 \quad (12)$$

Next the interface conditions (3) can be used to determine α_n and ζ_n :

$$\alpha_n = \zeta_n \quad (13)$$

$$\frac{(\beta \frac{\partial E_n}{\partial r}(r=R_s, R_s) + \alpha_n \frac{\partial E_n}{\partial r}(R_a, r=R_s))}{E_n(R_a, R_s)} = \frac{\zeta_n \frac{\partial E_n}{\partial r}(r=R_s, R_y)}{\mu_r E_n(R_s, R_y)} \quad (14)$$

The partial derivatives of the polynomial E can be expressed the following way:

$$\begin{cases} \frac{\partial E_n(r, R)}{\partial r} = \frac{n}{r} F_n(r, R) \\ \frac{\partial E_n(R, r)}{\partial r} = -\frac{n}{r} F_n(r, R) \end{cases} \quad (15)$$

with F_n a polynomial function defined by:

$$F_n : (x, y) = \left(\frac{x}{y} \right)^n + \left(\frac{y}{x} \right)^n \quad (16)$$

Then (14) leads to:

$$\alpha_n = \frac{2\beta}{F_n(R_a, R_s) + \frac{F_n(R_s, R_e) E_n(R_a, R_s)}{\mu_r E_n(R_s, R_e)}} \quad (17)$$

Thus, the unique solution of the system is entirely defined with the geometry constant and excitation's wavenumber. Note that considering infinite permeability in the ferromagnetic media leads to $\alpha_n \approx \frac{2}{F_n(R_a, R_s)}$ such that the solution in the air-gap becomes independent of the stator's thickness. The magnetic flux is derived from the magnetic potential:

$$\mathbf{B} = \mathbf{curl}(\mathbf{A}) = \frac{1}{r} \frac{\partial A_z}{\partial \theta} \mathbf{e}_r - \frac{\partial A_z}{\partial r} \mathbf{e}_\theta = B_r \mathbf{e}_r + B_\theta \mathbf{e}_\theta \quad (18)$$

With (18) and (8) radial magnetic flux and tangential magnetic flux can be analytically expressed:

$$\begin{cases} B_r(r, \theta) = \frac{n}{r} \left(\beta \frac{E_n(r, R_s)}{E_n(R_a, R_s)} + \alpha_n \frac{E_n(R_a, r)}{E_n(R_a, R_s)} \right) \cos(n\theta) \\ B_\theta(r, \theta) = \frac{n}{r} \left(\beta \frac{F_n(r, R_s)}{E_n(R_a, R_s)} - \alpha_n \frac{F_n(R_a, r)}{E_n(R_a, R_s)} \right) \sin(n\theta) \end{cases} \quad (19)$$

IV. ANALYTIC MAXWELL TENSOR STUDY

A. Simplified Maxwell Tensor with infinite permeability

In this first section, the simplified MT (7) is considered such that an air-gap magnetic pressure function is defined:

$$P : \begin{cases} [R_a, R_s] \times [0, 2\pi] & \rightarrow \mathbb{R}^+ \\ (r, \theta) & \rightarrow \frac{1}{2\mu_0} B_r^2(r, \theta) \end{cases} \quad (20)$$

As previously discussed, the magnetic pressure computed with this function is only rigorous when $r = R_s$. Then the rigorous application is compared with the air-gap one by defining a ratio function between both:

$$R : \begin{cases} [R_a, R_s] \times [0, 2\pi] & \rightarrow \mathbb{R}^+ \\ (r, \theta) & \rightarrow \frac{P(r, \theta)}{P(R_s, \theta)} \end{cases} \quad (21)$$

Using (19), (21) can be written as:

$$R(r, \theta) = \left(\frac{R_s}{r} \right)^2 \left(\frac{2E_n(r, R_s) F_n(R_a, R_s) + E_n(R_a, r)}{E_n(R_a, R_s)} \right)^2 \quad (22)$$

The advantage of comparing ratio is that the results do not depend on the angular position θ nor the excitation

amplitude β . This first result is plotted on the the Fig. 2 for several excitation wavenumber n . It can be observed that both location in the air-gap r and considered wavenumber n have a combined impact on the computed magnetic pressure. The phenomenon has an important impact for high magnetic wavenumbers. At this point, it is possible to suspect that the B_θ contribution is not negligible for high magnetic wavenumber n . The next section take up this possibility.

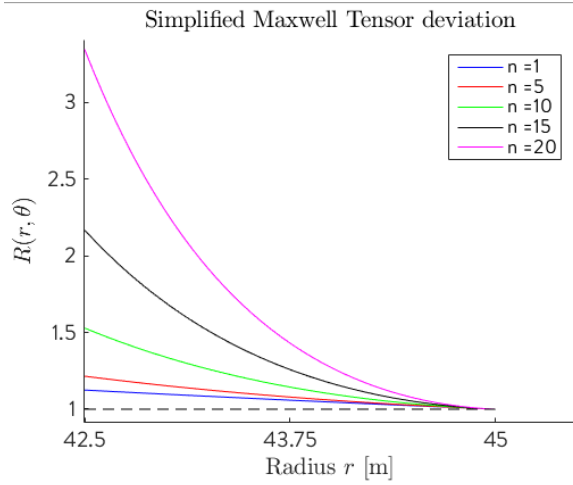


Fig. 2. Ratio between the air-gap MT and theoretic magnetic pressure as function of radius for several wavenumbers

B. Maxwell Tensor with infinite permeability

Now the MT with tangential magnetic contribution (6) is considered such that a new air-gap magnetic pressure function is defined:

$$P : \begin{cases} [R_a, R_s] \times [0, 2\pi] & \rightarrow \mathbb{R}^+ \\ (r, \theta) & \rightarrow \frac{1}{2\mu_0} (B_r^2(r, \theta) - B_\theta^2(r, \theta)) \end{cases} \quad (23)$$

The definition of R (21) stay the same, but the comparison is not straightforward. Indeed, since B_r depends on $\cos(n\theta)$ and B_θ depends on $\sin(n\theta)$, the ratio R depends on θ too. In order to better understand the air-gap evolution of the MT, a Fourier decomposition must be performed:

$$P(r, \theta) = P_0(r) + P_{2n}(r) \cos(2n\theta) \quad (24)$$

Introducing the amplitude of magnetic flux as follow:

$$\begin{cases} B_r(r, \theta) = \underline{B}_r(r) \cos(n\theta) \\ B_\theta(r, \theta) = \underline{B}_\theta(r) \sin(n\theta) \end{cases} \quad (25)$$

Then the MT magnetic pressure can be decomposed under the form:

$$P(r, \theta) = \frac{B_r^2(r) - B_\theta^2(r)}{2} + \frac{B_r^2(r) + B_\theta^2(r)}{2} \cos(2n\theta) \quad (26)$$

This is a classical result: a magnetic wavenumber n in the air-gap is recomposed into pressure wavenumbers 0 and $2n$. In order to avoid θ dependency, the amplitude of each harmonics

are going to be compared independently. It means two new ratio functions can be defined:

$$R_0 : \begin{cases} [R_a, R_s] & \rightarrow \mathbb{R} \\ r & \rightarrow \frac{B_r^2(r) - B_\theta^2(r)}{B_r^2(R_s) - B_\theta^2(R_s)} \end{cases} \quad (27)$$

$$R_{2n} : \begin{cases} [R_a, R_s] & \rightarrow \mathbb{R} \\ r & \rightarrow \frac{B_r^2(r) + B_\theta^2(r)}{B_r^2(R_s) + B_\theta^2(R_s)} \end{cases} \quad (28)$$

The next step is to introduce (19) into (27) in order to get the analytic expression of each function R_0 and R_{2n} . After a some calculations which are left to the reader, the expressions of these two functions simplify remarkably well as follows:

$$R_0(r) = \frac{R_s^2}{r^2} \quad (29)$$

$$R_{2n} = \frac{R_s^2}{r^2} \frac{F_{2n}(r, R_s)}{2} \quad (30)$$

Then combining (24) with (29) and (30), the relation between the air-gap MT and the theoretic application at position $r = R_s$ can be established:

$$P(r, \theta) = \frac{R_s^2}{r^2} \left(P_0(R_s) + P_{2n}(R_s) \frac{F_{2n}(r, R_s)}{2} \cos(2n\theta) \right) \quad (31)$$

The differences between theoretic pressure and air-gap pressure lie in the two coefficients of (31). It depends on the geometry, the radius of application and the wavenumber. Note that it does not depend on θ and β . Fig. 3 shows how these coefficients can affect magnetic pressure harmonics for $r \in [R_{ag}, R_s]$.

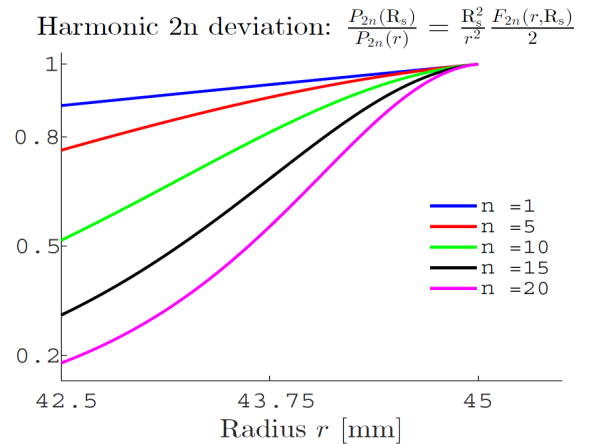


Fig. 3. Comparing stator and air-gap Maxwell Tensor magnetic pressure for the harmonic $2n$

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