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# Sensitivity study of a cumulative distribution functions-based model allowing to account for the anisotropy in electrical steel

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In a previous work, a model based on the use of Gaussians and allowing to account for both the non-linear and anisotropic behavior of non oriented and grain oriented electrical steel along any magnetization direction was proposed. One of its relevant advantages lies in the very limited number of experimental data that are required for it to be fully defined. However, a balance between the number of experimental data and the accuracy has to be found. This communication deals with a sensitivity analysis of the parameters defining the model, with an emphasis on the kind of experimental data that has to be chosen in order to lead to the most accurate results.

**Index Terms**—Magnetic anisotropy, non oriented electrical steel, grain oriented electrical steel, first magnetization curves modeling, cumulative distribution functions.

## I. INTRODUCTION

The accuracy of the results obtained from the simulation of an electromagnetic device, whether it is analytic or numerical, depends on numerous criteria. Among those criteria, the quality of the model used to account for magnetic behavioral laws of magnetic materials, such as electrical steels, is a key point. In the past decades, several models have been proposed in the literature, each one allowing to account for one or several physical properties such as the non-linearity, the hysteresis or the anisotropy.

With regard to the latter, the very vast majority of simulation software simply relate the magnetic induction  $\mathbf{B}$  to the magnetic field  $\mathbf{H}$  by means of a diagonal tensor composed of magnetic permeabilities along the Rolling Direction (RD), the Transverse Direction (TD) and the Orthogonal Direction (OD) [1], which is irrelevant regarding the other magnetization directions. Since the 80s, several other approaches have been proposed to improved that point. Examples of these are the elliptical models [2], the use of a set of first magnetization curves [3] or even orientation direction function based models [4], [5]. Each of the aforementioned methods have their advantages and limitations. Actually, the two first ones suffer from a lack of accuracy whereas the two last ones suffer from the amount of experimental data required for the determination of their parameters. Recently, a model based on probability density functions (PDF) and cumulative distribution functions (CDF) has been presented [6]. It has been highlighted that it allows the accurate determination of first magnetization curves of both non oriented (NO) and grain oriented (GO) electrical steel samples along any magnetization direction from a number of experimental data limited to 3 only. The aim of this

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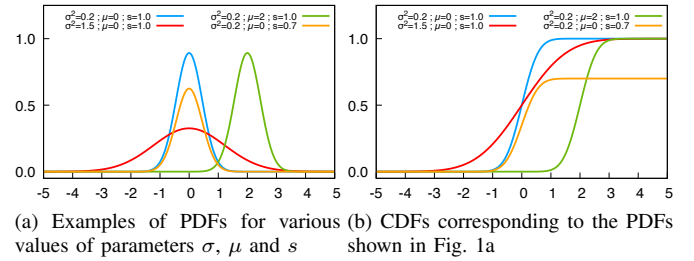


Fig. 1. Influence of parameter  $\sigma$ ,  $\mu$  and  $s$  on the shape of PDFs and their corresponding CDFs

communication is to discuss the influence of the parameters defining this model on its accuracy. In particular, the choice of the best set of experimental data that are required for the determination of those parameters is addressed.

## II. THEORETICAL BACKGROUND

Fig. 1a shows examples of PDFs, defined by (2a), for various values of the mean, the variance and a scale factor denoted  $\mu$ ,  $\sigma$  and  $s$  respectively. The associated CDFs, defined by (2b) where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (1)$$

is the error function [7], are shown in Fig. 1b. As pointed out by Robertson and France [8] in their works on isothermal remanent magnetization acquired by mineral assemblages, the shape of a CDF clearly shows high similarities with the one of a magnetization curve.

$$f(x) = \frac{s}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (2a)$$

$$F(x) = \int_{-\infty}^x f(z)dz = \frac{s}{2} \left[ 1 + \text{erf} \left( \frac{x-\mu}{\sigma\sqrt{2}} \right) \right] \quad (2b)$$

Then, after some physical considerations and a few mathematical developments it is possible to use (2b) to express the magnetization value of a given magnetic material. Nevertheless, as pointed out in [9] and [6], using a sum of PDFs, and thus a sum of CDFs, is required in order to reach a satisfactory accuracy when the method is applied to electrical steels, which leads to the following expression of the magnetization  $M$ :

$$M(H) = \sum_{i=1}^N \frac{s_i}{2} \left[ \operatorname{erf} \left( \frac{H - \mu_i}{\sigma_i \sqrt{2}} \right) - \operatorname{erf} \left( \frac{-\mu_i}{\sigma_i \sqrt{2}} \right) \right] \quad (3)$$

where  $N$  is the number of CDFs. Finally, (3) can be enhanced by making  $\mu$ ,  $\sigma$  and  $s$  dependant of  $\theta$ , where  $\theta$  is the angle between  $\mathbf{H}$  and the RD of the considered electrical steel sample. Then, a magnetic constitutive law accounting for the anisotropy is obtained:

$$B(H, \theta) = \mu_0 [H + M(H, \theta)] = \mu_0 \left[ H + \sum_{i=1}^N \frac{s_i(\theta)}{2} \left[ \operatorname{erf} \left( \frac{H - \mu_i(\theta)}{\sigma_i(\theta) \sqrt{2}} \right) - \operatorname{erf} \left( \frac{-\mu_i(\theta)}{\sigma_i(\theta) \sqrt{2}} \right) \right] \right] \quad (4)$$

### III. CHOICE OF EXPERIMENTAL DATA

In [6], the model given by (4) was developed and tested on 4 different grades of electrical steel (2 NO and 2 GO). It was highlighted that the three parameters  $\mu$ ,  $\sigma$  and  $s$  follow a polynomial trendline whatever the steel grade. It was also pointed out that the degree of those polynomials is equal to 2. Hence, it was possible to fully determine the parameters of (4) by fitting to experimental magnetization curves along 3 different magnetization angles  $\theta$  only, which is a great improvement compared to [3]–[5] which require a lot of experimental data to be accurate. However, the influence of the set of 3 values of  $\theta$  to use for the determination of  $\mu$ ,  $\sigma$  and  $s$  was not addressed in [6]. This is the aim of this communication.

Let's denote those three values by  $\theta_{\min}$ ,  $\theta_0$  and  $\theta_{\max}$ . Due to the periodicity properties of the anisotropy in an electrical steel it appears logical to set  $\theta_{\min} = 0^\circ$  and  $\theta_{\max} = 90^\circ$ . There is no doubt, however, that the choice of  $\theta_0$  has a major influence on the accuracy of the model. Figs. 2a, 2b, 2c and 2d show first magnetization curves along various magnetization directions –  $\theta = 15^\circ$ ,  $\theta = 45^\circ$ ,  $\theta = 60^\circ$  and  $\theta = 75^\circ$  respectively – obtained from (4) when parameters  $\mu$ ,  $\sigma$  and  $s$  are determined from 3 different experimental curves such that  $\theta_{\min} = 0^\circ$ ,  $\theta_{\max} = 90^\circ$  and  $\theta_0 \in \{15^\circ; 45^\circ; 60^\circ; 75^\circ\}$ . For the sake of comparison, the first magnetization curves obtained experimentally along the considered  $\theta$  angle are also plotted in those figures. As previously stated, Fig. 2 clearly highlights the importance of the attention that has to be given to the choice of  $\theta_0$ . It is worth noting that using  $\theta_0 = 15^\circ$  or  $\theta_0 = 75^\circ$  leads to a poor accuracy. As a matter of fact, they only match the experimental curve in the case where  $\theta = \theta_0$ . On the contrary, the optimal value of  $\theta_0$  can be placed between  $\theta_0 = 45^\circ$  and  $\theta_0 = 60^\circ$ , with a preference to a value close to the latter. As a matter of fact, Fig. 2 clearly shows that using the first magnetization curve obtained from (4) with  $\theta_0 = 60^\circ$  matches the experimental

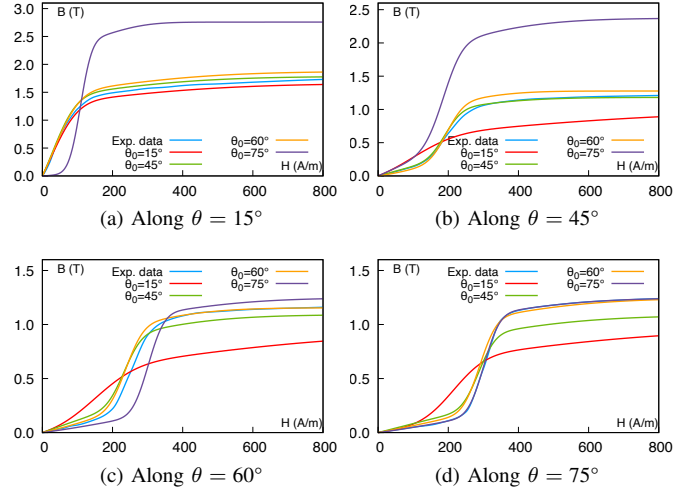


Fig. 2. First magnetization curves obtained experimentally and from the model governed by (4) along various magnetization directions  $\theta$ .

curve whatever the value of  $\theta$ . Such a value can be directly linked to the emergence of the hard direction at  $\theta = 54.7^\circ$ , which is a typical behavior of magnetic anisotropic behavior of electrical steel.

### IV. OUTLOOKS

As highlighted in section III, the choice of the value of  $\theta_0$  has a great influence on the accuracy of the model. Other parameters, such as the number of PDFs  $N$  in 4 or the degree of the polynomial defining the parameters  $\mu$ ,  $\sigma$  and  $s$  will be addressed in the extended paper.

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