



**HAL**  
open science

# Comparison of Stabilization Methods for Finite Element Method in the Context of Space Charges

Raphaël Pile, Guillaume Parent

► **To cite this version:**

Raphaël Pile, Guillaume Parent. Comparison of Stabilization Methods for Finite Element Method in the Context of Space Charges. 24th International Conference on the Computation of Electromagnetic Fields (COMPUMAG), May 2023, Kyoto, Japan. hal-04114184

**HAL Id: hal-04114184**

**<https://hal.science/hal-04114184v1>**

Submitted on 5 Jun 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Comparison of Stabilization Methods for Finite Element Method in the Context of Space Charges.

Raphaël Pile and Guillaume Parent

Univ. Artois, UR 4025, Laboratoire Systemes Electrotechniques et Environnement (LSEE), F-62400 Bethune, France.

Partial discharges have recently become an issue for electrical machines longevity. Thus, there is an interest in studying the phenomenon through numerical simulation. This task is rather complex since it requires to couple Finite Element Method (FEM) for the electrostatic field with a numerical simulation of the charge density flow using hydrodynamics model, which is known to be unstable with FEM. Thus, this communication proposes to compare the main stabilization methods with an analytical case dedicated to space charge context. It suggests that a logarithmic formulation of the problem can accurately ensure the positivity of the density as opposed to the common stabilizations methods. However, the conditions of convergence remain to be established and it comes with a significant additional computational cost.

*Index Terms*—Partial discharges, Finite element analysis, Numerical stability, Electrohydrodynamics.

## I. INTRODUCTION

**T**ECHNOLOGICAL developments in electric motors and actuators over the last decade have led to increase voltage amplitude and switching frequency. These new constraints lead to the appearance, in the inter-coil insulation of the windings, of partial discharges that are particularly harmful to the longevity of the insulating layer [1]. In electrical engineering, the Paschen empirical law is commonly used to determine the Partial Discharges Inception Voltage (PDIV). The local evolution and properties of partial discharges remain misunderstood in the electrical engineering literature. Thus, the numerical simulation of these phenomena has recently gain interest.

Most recent simulation models of partial discharges in the literature follow the hydrodynamic approximation, where a set of transport equations for the charged particles in the gas are coupled with Poisson's equation for the electric field [2]. The Finite Element Method (FEM) is more adapted to the Poisson's equation, thus it is widespread in electrical engineering field. Moreover, it enables to easily use high order space function with unstructured meshes. However, the natural choice for the transport problem would be the Finite Volume Method (FVM) or Finite Difference Method (FDM). Indeed, these methods are more adapted to the convective part of the transport equation.

In such a case, the FEM methods is often privileged because using the same method for both physics considerably ease the coupling. Nevertheless, the FEM for the convective part of the transport equations has a major stability drawback. Thus it requires stabilization methods such as Streamline Upwinding Petrov-Galerkin (SUPG), Taylor-Galerkin (TG) [3]. Nevertheless, it is unclear why these classical stabilization methods are not used in the context of space charges. This communication proposes to clarify this point. For this purpose, a 1D validation case is proposed, derived from the Davies test case [4]. The results are discussed in term of stability, accuracy and computation time.

## II. CONVECTION-DIFFUSION SIMULATION

### A. Galerkin Variational Formulation

In general form, transport equation is expressed as [5]

$$\frac{\partial u}{\partial t}(\mathbf{x}, t) + \nabla \cdot (u(\mathbf{x}, t) \cdot \mathbf{v}(\mathbf{x}, t) - \mathbf{D}\nabla u(\mathbf{x}, t)) = s(\mathbf{x}, t) \quad (1)$$

with the position  $\mathbf{x}$  in the domain, time  $t \in [0, T]$ , velocity field  $\mathbf{v}$ , diffusion tensor  $\mathbf{D}$ , and source/sink function  $s$ . In the following,  $\mathbf{x}$  and  $t$  are omitted for ease of reading. The convective term  $\nabla \cdot (u(\mathbf{x}, t) \cdot \mathbf{v}(\mathbf{x}, t))$  is the critical part of this study. The compact form

$$\frac{\partial u}{\partial t} + \mathcal{L}(u) = s, \quad (2)$$

where  $\mathcal{L}$  is a linear differential operator.

FEM consists in solving the physics equations by using a variational formulation of the problem. Let us consider  $H_0^s$ , a Sobolev space with null value on the boundary. Then, a variational formulation problem of (1) can be written:

$$\text{Find } u \in H, \text{ with } w \in H \text{ such that} \quad (3)$$

$$a(u, w) - \langle s, w \rangle = 0$$

where

$$a(u, w) = \left\langle \frac{\partial u}{\partial t}, w \right\rangle - \langle u \cdot \mathbf{v}, \nabla w \rangle + \langle \mathbf{D}\nabla u, \nabla w \rangle. \quad (4)$$

However, FEM encounters difficulties in dealing with the instabilities of convective problems. It can be shown that the FEM discretization introduces a purely numerical (unstable) diffusion [6]. Several options are discussed in the following.

### B. Main Galerkin Stabilization Methods

#### 1) Streamline Upwinding Petrov-Galerkin

An idea is to stabilize this numerical diffusion by adding an artificial diffusion term in the formulation. The SUPG method consists in adding this term in the direction of flow [6]. It can be performed by modifying the test function. The problem (3) becomes:

$$a(u, w + \tau \mathbf{v} \cdot \nabla w) - \langle s, w + \tau \mathbf{v} \cdot \nabla w \rangle = 0 \quad (5)$$

where  $\tau$  is an arbitrary stabilization parameter, often evaluated as  $\tau = h/(2|v|)$  with  $h$  the element size [6].

### 2) Taylor-Galerkin

In the TG stabilization, the time discretization plays a pivotal role. Instead of manipulating the Galerkin space discretization, the time-stepping method is chosen so as to stabilize it in natural way, perhaps, under a certain time step restriction. TG algorithms do not contain free parameters and are directly applicable to multidimensional transport problems [6].

The idea is to use a Taylor expansion to create the time discretization scheme. For example, the second order approximation gives

$$u(t + \Delta t) \approx u(t) + \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2} \frac{\partial^2 u}{\partial t^2}. \quad (6)$$

The time derivatives can be expressed as

$$\frac{\partial u}{\partial t} = -\mathcal{L}(u) + s \quad (7)$$

$$\frac{\partial^2 u}{\partial t^2} = -\mathcal{L}\left(\frac{\partial u}{\partial t}\right) + \frac{\partial s}{\partial t} = \mathcal{L}(\mathcal{L}(u) - s) + \frac{\partial s}{\partial t} \quad (8)$$

such that (6) becomes a time discretization scheme [3], [6]

$$\frac{u^{n+1} - u^n}{\Delta t} + \mathcal{L}(u^n) - \frac{\Delta t}{2|v|} \mathcal{L}(\mathcal{L}(u^{n+\theta}) - s) = s + \frac{\partial s}{\partial t}. \quad (9)$$

It is assumed that the derivative of the source/sink term can be computed independently (numerically or analytically). The parameter  $\theta$  corresponds to the time discretization scheme. Then the variational formulation of this problem is [3], [6]

$$a(u, w) - \frac{\Delta t}{2|v|} \langle \mathcal{L}(u) - s, \mathcal{L}^*(w) \rangle - \langle s + \frac{\partial s}{\partial t}, w \rangle = 0, \quad (10)$$

where  $\mathcal{L}^*$  is the adjoint operator of  $\mathcal{L}$ .

### C. Logarithmic Variational Formulation

Another method consists in a change of the unknown in (1) by using  $u = \exp(U)$  instead of  $u$  such that [7]

$$\exp(U) \frac{\partial U}{\partial t} + \nabla(\exp(U) \cdot \mathbf{v} - \mathbf{D} \exp(U) \nabla U) = s. \quad (11)$$

Then, the corresponding logarithmic variational formulation (LOG) is build the same way. The main issue is that the variational problem is not linear anymore. Thus, it requires to iteratively evaluate  $\exp(U)$ . By construction, the positivity of the solution is ensured.

## III. NUMERICAL APPLICATION

The unstabilities comes from the convection terms in (3). Hence, the diffusion tensor and the source term are neglected in this validation case.

The Davies test case [4] is a very restrictive case to evaluate the accuracy of stabilizations methods: the density can effectively form a plateau, however local discontinuities in the derivative do not occur in areas of low density. Thus, it is proposed to replace the window shape by a clipped gaussian. The results after one period are presented in Fig. 1. The analytical solution is also the initial condition for all methods. The relative total error (using L2 norm) gives 39% for the standard

Galerkin method, 37% for the SUPG stabilization, 36% for the TG, and 31% for the logarithmic method. The computation time gives about 3 s/CPU for the standard Galerkin method, 12 s/CPU for the SUPG stabilization, 20 s/CPU for the TG, and 34 s/CPU for the logarithmic method.

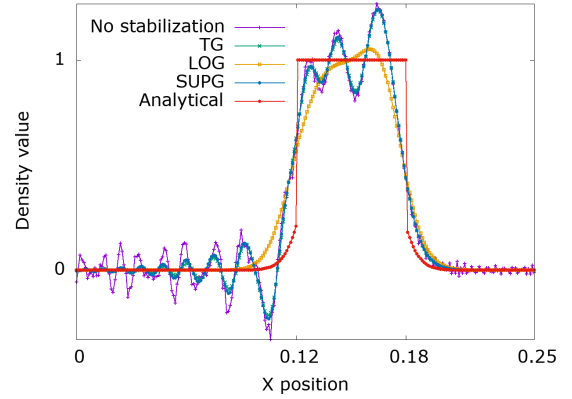


Fig. 1. Davies test: Comparison between analytical solution and numerical schemes using 1000 elements and 200 time steps.

SUPG and TG have similar accuracy. TG has the significant advantage of avoiding to evaluate stabilization parameter. The main issue is that the numerical oscillations lead to unphysical negative values for the charge density for both SUPG and TG. Even, with the proposed smoothed window case, the positivity of the solution could not be ensured. On the other hand, the logarithmic method seems to be a good candidate for the spatial charges simulation. However, it is significantly slower due to the non-linear formulation and its stability/convergence still has to be proven to the authors knowledge. For example, the method did not converge on the original Davies test case.

The obtained results validate that the FEM with logarithmic stabilization could be a viable method for partial discharge simulations. However, it comes with an added computational cost. Future research will therefore focus on the coupling with the electric field, in order to determine the computational cost associated with a change of method (FVM, Lagrangian, etc.) for the hydrodynamics.

## REFERENCES

- [1] D. Fabiani, G. Montanari, and A. Contin, "Aging acceleration of insulating materials for electrical machine windings supplied by pwm in the presence and in the absence of partial discharges," in *Proc. IEEE Int. Conf. Solid Dielectr., 7th*, Eindhoven, Netherlands, 2001, pp. 283–286.
- [2] G. E. Georghiou, A. P. Papadakis, R. Morrow, and A. C. Metaxas, "Numerical modelling of atmospheric pressure gas discharges leading to plasma production," *J. Phys. D: Appl. Phys.*, vol. 38, no. 20, pp. R303–R328, oct 2005.
- [3] R. Codina, "Comparison of some finite element methods for solving the diffusion-convection-reaction equation," *Comput. Methods Appl. Mech. Eng.*, vol. 156, no. 1-4, pp. 185–210, apr 1998.
- [4] A. J. Davies and W. Niessen, *Solution Continuity Equations in Ioniz. Plasma Growth*. Boston, MA: Springer US, 1990, pp. 197–217.
- [5] L. Liu and M. Becerra, "An Efficient Semi-Lagrangian Algorithm for Simulation of Corona Discharges: The Position-State Separation Method," *IEEE Trans. Plasma Sci.*, vol. 44, no. 11, pp. 2822–2831, nov 2016.
- [6] D. Kuzmin, "A guide to numerical methods for transport equations," Tech. Rep., 2010.
- [7] S. Singh, "Computational framework for studying charge transport in high-voltage gas-insulated systems," Ph.D. dissertation, Chalmers Tekniska Hogskola (Sweden), 2017.