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FEM-BEM modeling of nonlinear magnetolectric effects in heterogeneous composite structures

A. Urdaneta-Calzadilla ¹, N. Galopin ¹, I. Niyonzima ¹, O. Chadebec ¹, G. Meunier ¹, B. Bannwarth ¹

¹ Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab, F-38000 Grenoble, France

This paper proposes a mathematical model for 3D nonlinear magnetolectric effects in heterogeneous composite structures. Through the coupling of the Finite Element Method (FEM) with the Boundary Element Method (BEM), only the active material is explicitly considered, and thus a single mesh is used to support all phenomena. A mixed formulation is used to model the magnetic phenomena, a vector potential formulation in the volume and a scalar potential formulation in the free space domain. Material laws for the magnetostrictive composite phase are derived from partial derivatives of a scalar invariant's formulation of the Helmholtz free energy. The coupled problem is solved by iteratively solving single-physics problems, and the full algorithm is applied to the modeling of a test case.

Index Terms—Boundary-element methods, Finite element analysis, Helmholtz free energy, Magnetolectric effects, Magnetostriction.

I. INTRODUCTION

ELECTROMAGNETIC coupling in the form of induction takes place for non-static fields. Devices whose purpose is to exchange electric to magnetic energy or vice-versa, usually rely on this phenomenon. Electromagnetic interaction can otherwise be achieved by magnetolectric composites, which is based on mechanical exchanges through the combination of piezoelectric and magnetostrictive materials which have, respectively, strong electromechanical and magnetomechanical couplings. [1]. Numerical modeling of these phenomena is usually performed through the FEM. Nevertheless, to correctly account for the decay of magnetic fields at infinity, the FEM needs a big enough free space domain to be explicitly considered and meshed. This implies a large number of degrees of freedom, notably with regard to the quantity of active materials and their distance from the sources. This can be avoided by setting up a coupling between the BEM and the FEM for the magnetic part, and FEM for the electric and mechanical parts.

Contrary to piezoelectricity, which in most situations can be accurately described by linear relations, magnetostriction is strongly nonlinear. Many approaches have been taken to take account of this non-linearity. We implemented the description of magnetostriction in terms of scalar invariants of the Helmholtz free energy [2].

We first present the considered material laws, then the used formulations and finally some results for the chosen test case.

II. CONSTITUTIVE LAWS

The considered behavioral law for the piezoelectric phase is given by the two linear coupled relationships (1),

$$\begin{aligned} \mathbf{T}(\mathbf{S}, \mathbf{E}) &= \mathbf{C}^E : \mathbf{S} - \mathbf{e}^t \cdot \mathbf{E} \\ \mathbf{D}(\mathbf{S}, \mathbf{E}) &= \mathbf{e} : \mathbf{S} + \boldsymbol{\varepsilon}^S \cdot \mathbf{E} \end{aligned} \quad (1)$$

with \mathbf{D} the electric displacement field, \mathbf{E} the electric field, \mathbf{T} the Cauchy stress tensor and \mathbf{S} the linear strain tensor.

Inside the magnetostrictive phase, the Helmholtz free energy ψ can be expressed in the case of isotropic materials as a polynomial expression on its invariants [2]:

$$\psi = \frac{1}{2} \lambda I_1 + \mu I_2 + \sum_1^{n_\alpha} \alpha_i I_4^i + \sum_1^{n_\beta} \beta_i I_5^i + \sum_1^{n_\gamma} \gamma_i I_6^i \quad (2)$$

where:

$$\begin{aligned} I_1 &= \text{tr}(\mathbf{S}), \quad I_2 = \text{tr}(\mathbf{S}^2), \quad I_4 = \mathbf{B} \cdot \mathbf{B} \\ I_5 &= \mathbf{B} \cdot \tilde{\mathbf{S}} \mathbf{B}, \quad I_6 = \mathbf{B} \cdot \tilde{\mathbf{S}}^2 \mathbf{B}, \end{aligned} \quad (3)$$

with \mathbf{B} the magnetic induction, \mathbf{H} the magnetic field, $\tilde{\mathbf{S}}$ the deviatoric part of the strain, λ and μ the Lamé coefficients of the material. The magnetic and mechanical behavioral laws can then be obtained by the analytical differentiation of the energy ψ :

$$\mathbf{T}(\mathbf{B}, \mathbf{S}) = \frac{\partial \psi}{\partial \mathbf{S}}, \quad \mathbf{H}(\mathbf{B}, \mathbf{S}) = \frac{\partial \psi}{\partial \mathbf{B}}. \quad (4)$$

with \mathbf{H} the magnetic field. The behavior of magnetolectric composite structures can be obtained through the combination of (1) and (4).

III. FORMULATIONS AND RESOLUTION METHOD

The electrical and mechanical problems are limited to the active material's domain Ω_m . Considering static behaviour, relation (1) and appropriate boundary conditions, the electric weak form in terms of electric potential reads, find φ such that:

$$\int_{\Omega_m} \nabla \delta \varphi \cdot \mathbf{D}(\mathbf{S}, \mathbf{E}) \, d\Omega = 0 \quad \forall \delta \varphi. \quad (5)$$

From (1), (4) and the balance of linear momentum, the mechanical displacement weak form reads, find \mathbf{u} such that:

$$\int_{\Omega_m} \nabla^S \delta \mathbf{u} : \mathbf{T}(\mathbf{S}, \mathbf{E}, \mathbf{B}) \, d\Omega = 0 \quad \forall \delta \mathbf{u}, \quad (6)$$

where $\nabla^S \mathbf{u} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^t)$. Both (5) and (6) are discretized using first order FEM and nodal shape functions. Similarly to [3], the weak form of the magnetic problem is obtained by testing Maxwell-Ampere's equation keeping the boundary term. It reads, find (\mathbf{a}, ϕ_{red}) such that:

$$\int_{\Omega_m} \nabla \times \delta \mathbf{a} \cdot \mathbf{H}(\mathbf{B}, \mathbf{S}) d\Omega + \int_{\partial\Omega_m} \nabla \times \delta \mathbf{a} \cdot \mathbf{n} \phi_{red} d\partial\Omega \quad (7)$$

$$= \int_{\partial\Omega_m} (\delta \mathbf{a} \times \mathbf{n}) \cdot \mathbf{H}_0 d\partial\Omega \quad \forall \delta \mathbf{a},$$

with \mathbf{a} the magnetic vector potential, ϕ_{red} the magnetic reduced scalar potential, \mathbf{H}_0 the field created by the currents external to the domain calculated by the Biot-Savart law and \mathbf{n} the outwards normal vector to $\partial\Omega_m$. The previous equation is discretized using edge elements for \mathbf{a} and 0-order surface elements for ϕ_{red} . To describe the behavior of the magnetic field at the boundary, we add to our system of equations a discretized form of Green's third identity applied to the reduced magnetic scalar potential ϕ_{red} , with G the kernel of the Laplacian operator.

$$\frac{1}{2} \phi_{red} + \int_{\partial\Omega_m} G \frac{\mathbf{B} \cdot \mathbf{n}}{\mu_0} d\partial\Omega - \int_{\partial\Omega_m} \phi_{red} \frac{\partial G}{\partial n} d\partial\Omega \quad (8)$$

$$= \int_{\partial\Omega_m} G \mathbf{H}_0 \cdot \mathbf{n} d\partial\Omega.$$

The previous equation is then projected onto surface test functions ω_0 constant by surface cell element, ϕ_{red} is discretized with the same constant by facet element shape functions. The magnetic induction leaks, B_n , are linked to the magnetic vector potential in edge elements via an incidence matrix through a local Stokes theorem applied to the facet element, and its 1-form boundary. The BEM matrices are full and are compressed using the Fast Multipole Method (FMM) to reduce storage space. The magnetostatic and magnetomechanical formulations were validated against an analytical formula and the experimental $\mathbf{H}(\mathbf{B})$ curves.

Similarly to [3], the different physics are solved iteratively using partial solutions until all single-physics solutions converge to the desired relative tolerance (10^{-5}). MUMPS direct solver is used to solve for the discretized electric equation, Newton-Raphson schemes are used to solve for the nonlinear mechanical and magnetic problems, they are based on a preconditioned BiCGSTAB for the discretized mechanical equation (no Dirichlet boundary conditions are imposed) and a preconditioned GMRES for the discretized magnetic problem.

IV. TEST CASE

As a test case we selected a low torque motor made of a magnetoelectric composite and a rotating magnet. It works as follows: a magnetostrictive disk (Galfenol) is positioned on top of, and perfectly bonded to, a piezoelectric phase (PZT-5A). As the piezoelectric phase is excited by a voltage (1 kV) applied to the electrodes, the piezoelectric phase drives the deformation of the magnetostrictive phase. The piezoelectric layer is considered poled by the previous electrodes, so, it creates traction and the deformation of the magnetostrictive phase in the direction of the applied electric field, seen in Figure (2a). This deformation causes the magnetic properties of the magnetostrictive phase to change, in particular, as shown in Figure (2b), a preferred magnetization direction appears.

The piezoelectric material tensors \mathbf{C} , \mathbf{e} and $\boldsymbol{\varepsilon}$ are usually available with materials poled along the (z) direction. If the configuration of the poling electrodes and the tensors are

known in their poled state, then, to take account of the poling pattern, an electrostatic resolution can be performed and then the coupling tensors rotated in the direction of the electric field.

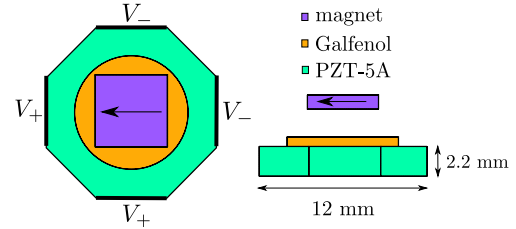


Fig. 1. Test device, the arrow corresponds to the direction of the remnant field in the magnet

A magnet is placed above the composite structure. By the shifting of the magnetization direction of the magnetostrictive layer, a mechanical torque appears in the magnet and the magnetostrictive layer due to their magnetic interaction, it is calculated by the equivalent charge method. The coefficients for the invariant approach are obtained by the fitting of the $\mathbf{H}(\mathbf{B})$ curves of Galfenol at 0 MPa presented in [2].

V. CONCLUSION

In this paper we used a FEM-BEM coupling strategy to the modeling of magnetoelectric effects in composite structures. This allowed us not to mesh the free space region. We used an invariant description of the Helmholtz free energy to derive the nonlinear expressions of the Cauchy stress tensor and the magnetic field in order to model the magneto-mechanical behavior of Galfenol. We solved for the complete multi-physics problem by solving iteratively the three derived single-physics problems, and the full algorithm was successfully tested on the modeling of a low torque magnetic motor.

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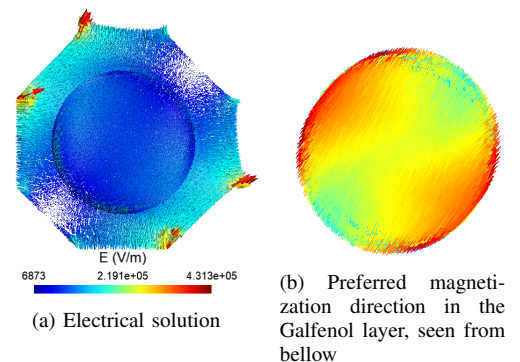


Fig. 2. Coupled solution to the multiphysics problem. Figure (2b) is given by the direction of the eigenvector corresponding to the spectral radius of the susceptibility matrix χ by element, the color corresponds to the value of the spectral radius of χ .