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Wilkinson Microwave Anisotropy Probe data and the curvature of space

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ABSTRACT

Inter alia, the high-precision *Wilkinson Microwave Anisotropy Probe* (*WMAP*) data on cosmic background radiation marginally indicate that the Universe has positively curved (and hence spherical) spatial sections. In this Letter, we take this data seriously and consider some of the consequences for the background dynamics. In particular, we show that this implies a limit to the number of e-foldings that could have taken place in the inflationary epoch; however, this limit is consistent with some inflationary models that solve all the usual cosmological problems and that are consistent with standard structure formation theory.

Key words: cosmic microwave background – cosmological parameters – cosmology: miscellaneous – early Universe.

1 INTRODUCTION

The *Wilkinson Microwave Anisotropy Probe* (*WMAP*) has recently provided high-resolution cosmic microwave background data (Bennett et al. 2003; Spergel et al. 2003) of importance for cosmology – indeed, they constitute a significant contribution towards the goal of developing precision cosmology. Among the interesting conclusions that have been reached from this data are constraints on the present value Ω_0 of the total density parameter of the Universe. Interestingly, the *WMAP* results indicate that although the Universe is close to being flat ($\Omega_0 \simeq 1$), a closed universe is marginally preferred: $\Omega_0 > 1$ (Bennett et al. 2003). In particular, with a prior on the Hubble constant, one gets that $\Omega_0 = 1.03 \pm 0.05$ at 95 per cent confidence level, whereas combining *WMAP* data with type Ia supernovae leads to $\Omega_0 = 1.02 \pm 0.04$ or to $\Omega_0 = 1.02 \pm 0.02$, without and with a prior on the Hubble parameter, respectively. The latter may be regarded as the present best estimate of this parameter. Note also that this tendency to point toward a closed universe is, strictly speaking, neither new nor restricted to the *WMAP* data. For instance, with a prior on the nature of the initial conditions, the Hubble parameter and the age of the Universe, analysis of the *DASI*, *BOOMERanG*, *MAXIMA* and *DMR* data (Sievers et al. 2003; Netterfield et al. 2002) lead to $\Omega_0 = 0.99 \pm 0.12$, but to $\Omega_0 = 1.04 \pm 0.05$ (both at 1σ level) if one takes into account only the *DASI*, *BOOMERanG* and *CBI* data. The *Archeops* balloon experiment (Benoît et al. 2003a,b) points towards $\Omega_0 = 1.00^{+0.03}_{-0.02}$ with data from the *Hubble Space Telescope* (*HST*) and a prior on the Hubble constant, but to values in the range $\Omega_0 = 1.16^{+0.24}_{-0.20}$ from combined cosmic background radiation (CBR) data alone and to $\Omega_0 = 1.04^{+0.02}_{-0.04}$ from combined CBR and supernova data, all at a 68 per cent confidence level. The improved precision from *WMAP* is clear. In this Letter, we draw the implication of the existence of such a small

curvature on the dynamics of inflation. Besides, the low quadrupole of the CMB power spectrum may be related to the dynamics of inflationary models with positive spatial curvature (see Efstathiou 2003, for example, and the note added at the end of this Letter).

The confirmation of the existence of Doppler peaks and their respective locations tends to confirm the inflationary paradigm (Barger, Lee & Marfatia; Peiris et al. 2003), as does the existence of an almost scale-invariant power spectrum. Furthermore, a nearly flat universe, argued to be predicted generically by most inflationary models (Guth 1981; Albrecht & Steinhardt 1982; Hawking 1982; Linde 1982; Starobinsky 1982), is usually seen as a further evidence. Because

$$\delta(t) \equiv \Omega(t) - 1 = \frac{K}{a^2 H^2}, \quad (1)$$

where $a(t)$ is the scale function and $H(t) = \dot{a}/a$ is the Hubble parameter, the *WMAP* constraints on the curvature of space imply that the curvature density parameter, $\Omega_K(t) \equiv K/a^2 H^2$, has been smaller than a few per cent from the time the largest observable scale, $k_{\min} \sim a_0 H_0$, crossed the horizon during inflation up to today. It is then a good approximation to neglect the curvature from the time k_{\min} crossed the horizon to today when performing structure formation calculations, mainly because $\Omega_{k_{\min}} = \Omega_{\text{today}}$ (see Fig. 1). Thus the constraints on the curvature of the Universe back up the validity of calculations underlying the tests of inflation performed up to now that assume a flat universe and focus on the properties of the perturbations (spectral indices, tensor modes, statistics, etc.).

2 SPACE CURVATURE AND INFLATION

Nevertheless, when $K \neq 0$ (which is highly favoured over the case $K = 0$ by probability considerations: the latter are of measure zero in the space of Friedman–Lemaître–Robertson–Walker universes in most measures, where we note that $K = 0$ corresponds not to an extremely large but nevertheless finite radius curvature, but to

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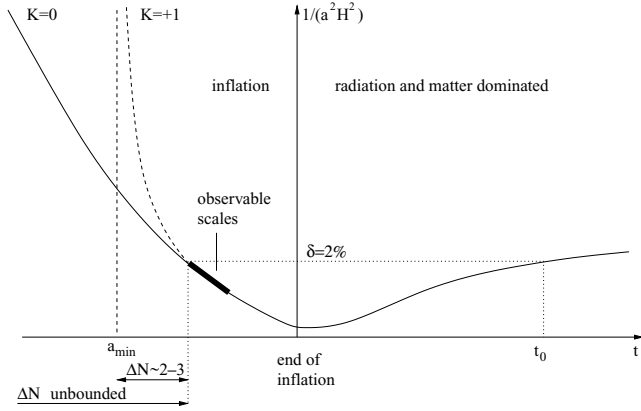


Figure 1. The curvature ($1/a^2 H^2$) as a function of time (schematically). The solid line represents the case of a flat universe ($K = 0$) with exponential inflation, whereas the dashed line depicts the case of a closed universe ($K = +1$). Before the end of inflation, aH scales either as $e^{\lambda t}$ ($K = 0$) or $1/\sinh(\lambda t)$ ($K = +1$), whereas it scales as $\alpha t^{\alpha-1}$ during matter- and radiation-dominated eras. The largest observed scale is depicted as well, and one can easily see that Ω_K will be smaller than a few per-cent between the time it crosses the horizon during inflation to today when it re-enters the horizon.

the actually infinite limit of this radius), Ω_K is driven toward 0 exponentially during inflation. Therefore the curvature term was necessarily dominant at early enough times, if inflation undergoes a sufficient number of e-foldings, mainly because the curvature term behaves as a^{-2} and will tend to dominate over any kind of matter having an equation of state less stiff than $-1/3$, and thus over a slow-rolling scalar field or a cosmological constant, in the early inflationary era. Note that we are not attempting to resolve issues of probability here; simply, we proceed on the basis that if the data indicate a non-zero value of k , however that came about, we need to take this seriously and understand how this relates to limits on possible evolutionary histories imposed by the standard dynamical equations.

We argue that the study of dynamics of the background for such models can give interesting insight into the inflationary era and its dynamics prior to k_{\min} horizon exit. Working backwards in time from the present, we aim to emphasize the constraints that arise on the inflationary models from the *WMAP* data. In doing so, we take seriously the indication that a value of Ω_0 slightly larger than 1 is favoured. During a slow-roll inflationary phase, the Universe is described by an almost de Sitter space-time with the $K = +1$ scale factor

$$a(t) \propto \cosh(Ht) \quad (2)$$

instead of the exponential growth obtained in the flat case. It follows that the number of e-foldings between the onset and the end of inflation cannot be arbitrarily large, when we take present day cosmological parameters into account. The curvature enhances the curvature, \ddot{a} , of the scale factor in the early stage of inflation so that there exists a turn-around point in the past. It follows (Ellis et al. 2002a,b) that the number of e-foldings during inflation is bounded by a maximum possible number of e-foldings, N_{\max} , related to the present day density parameter Ω_0 .

Assuming that the inflation is driven simply by a cosmological constant, the scale factor of a closed universe in this epoch is given by

$$a(t) = a_{\min} \cosh(\lambda t) \quad (3)$$

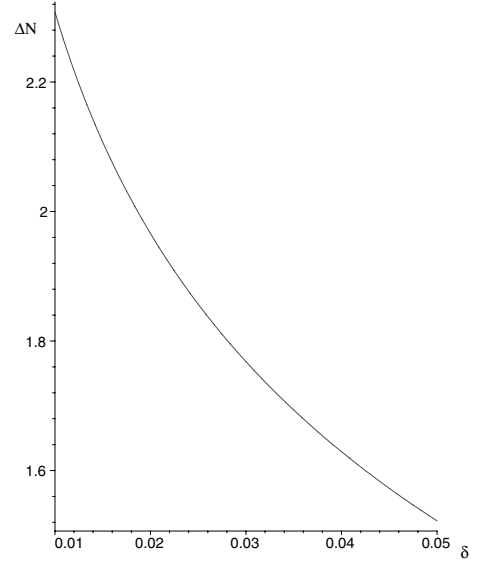


Figure 2. The maximum number of e-foldings allowed before the largest observable scale, k_{\min} , exits the horizon during inflation, as a function of δ_0 .

with $\lambda \equiv \sqrt{\Lambda/3}$ and $t = 0$ corresponding to the minimum expansion of the Universe. It follows that $\delta = \Omega - 1 = 1/\sinh^2(\lambda t)$. Thus the time t_* at which $k_{\min} = a_0 H_0$ crosses the horizon is obtained to be

$$\lambda t_* = \operatorname{arcsinh}(1/\sqrt{\delta_0}). \quad (4)$$

The number of e-foldings between $t = 0$ and t_* is simply

$$\Delta N_{\max} = \ln \frac{a_{k_{\min}}}{a_{\min}}, \quad (5)$$

i.e.

$$\Delta N_{\max} = \frac{1}{2} \ln \left(1 + \frac{1}{\delta_0} \right). \quad (6)$$

Typically, it ranges from 2.3 to 1.5 when δ_0 varies from 0.01 to 0.05 (see Fig. 2).

This argument also holds for negatively curved universes, in which case the scale factor behaves as $a \propto \sinh(\lambda t)$. For this scenario, there will not be any maximal number of e-foldings, but the Universe starts off from a singularity. This will enhance the trans-Planckian problem (Brandenberger & Martin 2001; Martin & Brandenberger 2001; Lemoine et al. 2002).

Another way to look at the existence of a maximum number of e-foldings in the $K = +1$ case is to relate the number of e-foldings during inflation to the later history of the Universe (Ellis et al. 2002a,b). During inflation, the comoving Hubble length aH is decreasing, whereas it is increasing at all further times. A given comoving scale, k , crosses the horizon when $k = a(t_k)H(t_k) \equiv a_k H_k$. The number of e-foldings, $N(k)$, between when this scale crosses the horizon and the end of inflation is obtained to be

$$N(k) = \ln \left(\frac{a_{\text{end}} H_{\text{end}}}{a_k H_k} \right). \quad (7)$$

The determination of $N(k)$ requires that we compute

$$\mathcal{R} \equiv \frac{a_0}{a_{\text{end}}}, \quad (8)$$

which requires a complete history of the evolution of the Universe from the end of inflation to today. If one assumes that the Universe is matter-dominated from the end of (slow-roll) inflation to reheating,

that radiation dominated from reheating to equality and that matter dominated up to now,[†] then (Lyth & Liddle 1993; Lidsey et al. 1997)

$$N(k) \simeq 62 - \ln \left(\frac{k}{a_0 H_0} \right) - \ln \left(\frac{10^{16} \text{ GeV}}{V_k^{1/4}} \right) + \frac{1}{4} \ln \frac{V_k}{V_{\text{end}}} - \frac{1}{3} \ln \left(\frac{V_{\text{end}}^{1/4}}{\rho_{\text{reh}}^{1/4}} \right) - \ln h, \quad (9)$$

with h being the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. For our purpose, it will be more convenient to relate $N(k)$ to \mathcal{R} , which can easily be found to be given by

$$-\ln \mathcal{R} \simeq -66 - \frac{1}{3} \ln \left(\frac{V_{\text{end}}^{1/4}}{\rho_{\text{reh}}^{1/4}} \right) + \ln \left(\frac{10^{16} \text{ GeV}}{V_{\text{end}}^{1/4}} \right). \quad (10)$$

It follows that

$$N(k) \simeq 128 - \ln \mathcal{R} - \ln \left(\frac{k}{a_0 H_0} \right) - 2 \ln \left(\frac{10^{16} \text{ GeV}}{V_k^{1/4}} \right) - \ln h. \quad (11)$$

The cosmic microwave background roughly probes scales from 10 Mpc (i.e. of the order of the thickness of the last scattering surface) to 10^4 Mpc (i.e. of the order of the size of the observable Universe), whereas galaxy surveys can probe the range 1 to 100 Mpc.

Concerning the characteristic scales involved in the problem, we can assume that $V_k \sim V_{\text{end}}$, as long as slow rolling holds. The reheating temperature can be argued to be larger than $\rho_{\text{reh}}^{1/4} > 10^{10} \text{ GeV}$ to avoid the gravitino problem (Sarkar 1996) and may be pushed in the extreme case to 10^3 GeV , i.e. just before the electro-weak transition, so that baryogenesis can take place. The amplitude of the cosmological fluctuations (typically of the order of 2×10^{-5} on Hubble scales) roughly implies that $V_{\text{end}}^{1/4}$ is smaller than a few times 10^{16} GeV and, for the same reason as above, has to be larger than 10^{10} GeV in the extreme case.

This implies that the number of e-foldings has to lie approximately between 50 and 70, which is the order of magnitude also required to solve the horizon and flatness problem when $K = 0$ (Guth 1981; Albrecht & Steinhardt 1982; Linde 1982; Hawking 1982; Starobinsky 1982), but it can be lowered to 25 in the extreme case of thermal inflation (Lyth & Stewart 1995), which we are not considering here.

In the following, we restrict our analysis to the case where reheating takes place just at the end of inflation, i.e. $\rho_{\text{reh}} = V_{\text{end}}$. It has been shown (Ellis et al. 2002a) that, in that case, the existence of a positive curvature limits the number of e-foldings to

$$N_{\text{max}}(\mathcal{R}, \delta_0) = \ln \sqrt{\frac{\alpha}{\delta_0}}, \quad (12)$$

with

$$\alpha = (1 + \delta_0 - \Omega_{\text{rad}0})\mathcal{R} + \Omega_{\text{rad}0}\mathcal{R}^2, \quad (13)$$

where $\delta_0 \equiv \Omega_0 - 1$ and $\Omega_{\text{rad}0} \simeq 4.17 \times 10^{-5} h^{-2}$ is the radiation density parameter today. Let us emphasize that the derivation of equation (12), as well as equation (6), does not make any assumption concerning what happened before the start of inflation.

This allows us to estimate the number of e-foldings allowed prior to the time at which k_{min} crossed the horizon,

$$\Delta N = N_{\text{max}}[\mathcal{R}(V_{\text{end}}^{1/4}), \delta_0] - N(k_{\text{min}}, V_{\text{end}}^{1/4}), \quad (14)$$

[†]The results will be only marginally affected by a late-time cosmological constant, as indicated by the supernova data.

where we will let δ_0 vary in the range 0.01–0.05 (see Fig. 2). It can be checked, as expected from equation (12), that ΔN depends slightly on $V_{\text{end}}^{1/4}$.

We see that the allowed number of e-foldings is compatible with the requirements of structure formation. ‘Flatness’ is, of course, solved to the accuracy represented by δ_0 . The scenarios sketched here do not lead to as small a value of δ_0 as is often supposed – and thereby is compatible with the best-fitting *WMAP* data. However, for the mean value of the curvature, one obtains that

$$\delta_0 = 0.02 \iff \Delta N_{\text{max}} = 1.97. \quad (15)$$

With standard parameter values, this gives the maximum possible number of e-foldings as about $N_{\text{max}} = 62 + 1.97 \simeq 64$ – compatible with the estimates given above, but not nearly as large as suggested in some models of inflation.

In view of the bounds on N_{max} , there are also bounds on the ability of inflation to solve the horizon problem when one considers inflationary dynamics with $K = +1$ (Ellis et al. 2002b). Roughly speaking, the horizon problem is only solved during the inflationary era itself if the start of the inflationary era is close enough to the throat at $t = 0$ in the expansion (see equation 3); that is, if the total number N of e-foldings during inflation is close to the total N_{max} allowed, i.e. $N \simeq N_{\text{max}}$. If inflation starts sufficiently far from that throat, then it does not matter how many e-foldings take place – they cannot provide causal connectivity on large enough scale to solve the horizon problem. Rather, that connectivity has to be set up in the pre-inflationary era (and is evidenced by a smooth enough small patch, which is then expanded to large scales by inflation). However, inflation certainly can start near enough the throat to solve the horizon problem during the inflationary era and still lead to $\delta_0 = 0.02$. Examples of non-slow-rolling cases are the emergent and awakening universes (Ellis & Maartens 2003) that start form an Einstein static state and are singularity free. The above formula (15) limiting the number of e-foldings will not apply in these cases.

3 CONCLUSION

In this Letter, we have argued that the curvature of space may play an important role in the dynamics of inflation in its early stage. We have argued that a maximum number of 1.5–2.5 e-foldings can take place before the largest scale observed in the Universe crossed the horizon during slow-roll inflation. However, despite the fact that the curvature term will be smaller than a few per cent during all times, ranging from the horizon exit of the largest observable mode to its re-entry today, a positive curvature term as small as the one indicated by current data will change the early phase of inflation dramatically and hence lead to the limits on the number of e-foldings allowed as described above. These restrict the families of inflationary universes compatible with the current data but do not exclude inflationary models, even though the value of δ_0 indicated by *WMAP* is larger than supposed in most inflationary scenarios.

Note the following possible way around the argument. In the case that inflation is driven by a scalar field, the existence of a maximal number of e-foldings may be avoided if it exits the slow-roll regime. However, it was shown that the dynamics of closed universes filled with a scalar field is chaotic (Hawking 1984; Page 1984; Cornish & Shellard 1998; Toporensky 2001) and can lead to the existence of singularity free universes (Parker & Fulling 1973; Starobinsky 1978) with periodic (Hawking 1984) or aperiodic (Page 1984) trajectories. In that case, again, a positive curvature affects the dynamics of the early phase of inflation and the slow-roll regime has to be left

before N_{\max} is reached, which is also at odds with the standard picture.

Finally, let us consider further features of spherical universes that may turn out to be useful in understanding the *WMAP* data. Surprisingly, the *WMAP* angular correlation function seems to lack signal strength on angular scales larger than 60° (Bennett et al. 2003; Spergel et al. 2003). This may indicate a possible discreteness of the initial power spectrum, as expected, for example, from non-standard topology. The case for topology becomes most interesting in the case of spherical spaces (Gausmann et al. 2001). First, the spectrum of the Laplacian in spherical spaces is always discrete (Lehoucq, Uzan & Weeks 2002; Lehoucq et al. 2002). Secondly, and as emphasized by Weeks, Lehoucq & Uzan (2003), a non-trivial topology is most likely to be detectable in the case of spherical spaces and examples of the resulting observational effects are discussed by Riazuelo et al. (2003) and Uzan et al. (2003).

The same conclusion cannot be reached for negatively curved universes. It is unlikely that we could detect a compact hyperbolic space because, for an observer at a generic point, the topology scale is comparable to the curvature radius or longer. The possibility of a detection would require the observer to sit near a short closed geodesic (Weeks 2002; Gomero, Rebouças & Tavakol 2001, 2002).

In conclusion, if the indication that Ω_0 differs from unity at a level of a few per cent is confirmed, then it may imply important results concerning the dynamics of the early stage of our Universe. The $K = +1$ background de Sitter model (3) differs fundamentally from the scale-free de Sitter model in the $K = 0$ frame – not least in terms of being geodesically complete. The *WMAP* observations suggest that the former may be the appropriate model to use in investigating early inflationary dynamics.

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NOTE ADDED IN PRESS

After our work was completed, Efstathiou 2003 attempted to link the lack of power on large scales to the possibility of positive curvature. His argument relies on the fact that if Ω is slightly larger than 1, then why should we have a scale-invariant power spectrum up to the largest scales? He proposed to change the initial power spectrum but did not exhibit any specific model to make this happen. Linde (2003) then emphasized that it is possible to get $\Omega > 1$ from inflation. Then a series of papers (e.g. Cline et al. 2003; Contaldi et al. 2003; Lasenby & Doran 2003; Luminet et al. 2003) attempted such a link in various ways. Lasenby & Doran showed that the model proposed there is consistent with the bounds put forward in this Letter.

REFERENCES

- Albrecht A., Steinhardt P. J., 1982, *Phys. Rev. Lett.*, 48, 1220
 Barger V., Lee H. S., Marfatia D., 2003, *Phys. Lett. B*, 565, 33
 Bennett C. L. et al., 2003, *ApJ*, in press (astro-ph/0302207)
 Benoît A. et al., 2003a, *A&A*, 399, L19
 Benoît A. et al., 2003b, *A&A*, 399, L25
 Brandenberger R., Martin J., 2001, *Mod. Phys. Lett. A*, 16, 999
 Cline J. M., Crotty P., Lesgourgues J., 2003, preprint (astro-ph/0304558)
 Contaldi C. R., Peloso M., Kofman L., Linde A., 2003, *JCAP*, 07, 002
 Cornish N., Shellard E. P. S., 1998, *Phys. Rev. Lett.*, 81, 3571
 Efstathiou G., 2003, *MNRAS*, 343, L95
 Ellis G. F. R., Maartens R., 2003, preprint (gr-qc/0211082)
 Ellis G. F. R., Stoeger W., McEwan P., Dunsby P., 2002a, *Gen. Rel. Grav.*, 34, 1445
 Ellis G. F. R., McEwan P., Stoeger W., Dunsby P., 2002b, *Gen. Rel. Grav.*, 34, 1461
 Gausmann E., Lehoucq R., Luminet J.-P., Uzan J.-P., Weeks J., 2001, *Class. Quant. Grav.*, 18, 5155
 Gomero G. I., Rebouças M. J., Tavakol R., 2001, *Class. Quant. Grav.*, 18, 4461
 Gomero G. I., Rebouças M. J., Tavakol R., 2002, *Int. J. Mod. Phys. A*, 17, 4261
 Guth A. H., 1981, *Phys. Rev. D*, 23, 347
 Hawking S. W., 1982, *Phys. Lett. B*, 115, 295
 Hawking S. W., 1984, in de Witt B. S., Stora R., eds, *Relativity, Group and Topology II*. North Holland, Amsterdam, p. 333
 Lasenby A., Doran C., 2003, preprint (astro-ph/0307311)
 Lehoucq R., Weeks J., Uzan J.-P., Gausmann E., Luminet J.-P., 2002, *Class. Quant. Grav.*, 19, 4683
 Lehoucq R., Uzan J.-P., Weeks J., 2003, *Kodai Math. J.*, 26, 119
 Lemoine M., Lubo M., Martin J., Uzan J.-P., 2002, *Phys. Rev. D*, 65, 023510
 Lidsey J. E. et al., 1997, *Rev. Mod. Phys.*, 69, 373
 Linde A. D., 1982, *Phys. Lett. B*, 108, 389
 Linde A., 2003, *JCAP*, 05, 002
 Luminet J.-P., Weeks J., Riazuelo A., Lehoucq R., Uzan J.-P., 2003, *Nat.*, submitted
 Lyth D. H., Liddle A., 1993, *Phys. Rept.*, 231, 1
 Lyth D. H., Stewart E. D., 1995, *Phys. Rev. Lett.*, 75, 201
 Martin J., Brandenberger R., 2001, *Phys. Rev. D*, 65, 123501
 Netterfield C. B. et al., 2002, *ApJ*, 571, 604
 Page D. N., 1984, *Class. Quant. Grav.*, 1, 417
 Parker L., Fulling S. A., 1973, *Phys. Rev. D*, 7, 2357
 Peiris H. V. et al., 2003, *ApJ*, in press (astro-ph/0302225)
 Riazuelo A., Uzan J.-P., Lehoucq R., Weeks J., 2003, *Phys. Rev. D*, submitted
 Sarkar S., 1996, *Rep. Prog. Phys.*, 59, 1493
 Sievers J. L. et al., 2003, *ApJ*, 591, 599
 Spergel D. N. et al., 2003, *ApJ*, in press (astro-ph/0302209)
 Starobinsky A. A., 1978, *Soviet Astron. Lett.*, 4, 82
 Starobinsky A. A., 1982, *Phys. Lett. B*, 117, 175
 Toporensky A. V., 2001, (gr-qc/0011113)
 Uzan J.-P., Riazuelo A., Lehoucq R., Weeks J., 2003, *Phys. Rev. Lett.*, submitted (astro-ph/0303580)
 Weeks J., 2002, *Int. J. Mod. Phys. A*, submitted
 Weeks J., Lehoucq R., Uzan J.-P., 2003, *Class. Quant. Grav.*, 20, 1529

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