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Household heterogeneity in macroeconomic models: a historical perspective

Beatrice Cherrier, Pedro Duarte, Aurélien Saïdi¹

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Abstract: In this paper, we trace the rise of heterogeneous household models in mainstream macroeconomics from the turn of the 1980s to the early 2000s, when these models evolved into an identifiable and consistent literature. We show that different communities across the US and Europe considered heterogeneous agents for various reasons and developed models that differed in their theoretical and empirical strategies. Minnesota economists primarily focused on incorporating stochastic heterogeneity into general equilibrium models. Other researchers refined growth models or tried to find alternatives to the permanent income hypothesis, leading them to explore more structural heterogeneity. We also document the computational challenges that some of these communities faced, how they gradually became aware of each other's work, and how they faced criticisms from macro- and microeconomists, many of them trained in European countries and dissatisfied with the theoretical and empirical aggregation strategies underlying these models.

Keywords: history of macroeconomics; heterogeneous agents; Bewley models; permanent income hypothesis, aggregation; equity premium puzzle, precautionary savings.

JEL Codes: B21, B22, B23, E13, E65.

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1. Introduction: overlapping narratives

As mainstream macroeconomists continue to refine their models, they often reflect on the evolution of their field.² However, in constructing an overarching history of macroeconomics, they tend to make problematic claims. First, they usually emphasize that the Keynesian revolution in the 1930s engendered a new breed of economists solely focused on aggregates (Sargent 2015, Kaplan and Violante 2018). Thomas Sargent, for instance, referred to Tobin who allegedly “said that macroeconomics is a subject that attains workable approximations by ignoring effects on aggregates of distributions of wealth and income” (Sargent 2015, 47; echoed by Kaplan and Violante 2018, 169).³ Second, macroeconomists view the turn to microfoundations engineered by Robert Lucas and others in the 1970s as the end of this exclusive focus on aggregative models. Third, the narrative goes, the microfoundations challenge resulted in a widespread use of representative agent models, with the consequence that, up to the Great Financial crisis of 2008, heterogeneity in agents’ optimizing behavior was mostly neglected.⁴

In this paper, we aim to challenge some of these oversimplified histories by qualifying the idea of a hegemony of representative-agent modeling in macroeconomics since the 1980s, one that resulted in the stabilization of so-called DSGE models in the late 1990s.⁵ In doing so, we unpack a long, multifarious but uninterrupted tradition of reflecting about the sources of agent heterogeneity and its consequences for business cycles, monetary phenomena, savings and growth patterns. While some contemporary macroeconomists acknowledge the origins of their program to Minnesota in the 1980s, our paper is the first to propose a history of household

² In this paper, we only focus on those works that are published in top economic journals. See Davis (2008) and Dequech (2007) for a discussion of the term “mainstream.”

³ We were unable to locate this Tobin quote. The hint suggested by Tobin’s biographer, Robert Dimand, was the introduction of Tobin’s 1980 book, where we find him being very conscious about the micro-macro relationships: “Candid macro economists have never deceived themselves that the equations of their models were more than simple and approximate descriptions of the diverse responses of individual agents of ever-changing relative weights in the aggregates” (Tobin 1980, x).

⁴ For instance, British consumption specialist John Muellbauer (2016) wrote: “The New Keynesian DSGE models that dominated the macroeconomic profession and central bank thinking for the last two decades were based on several principles. The first was formal derivation from micro-foundations, assuming optimizing behavior of consumers and firms with rational or ‘model-consistent’ expectations of future conditions. For such derivation to result in a tractable model, it was assumed that the behavior of firms and of consumers corresponded to that of a ‘representative’ firm and a ‘representative’ consumer. In turn, this entailed the absence of necessarily heterogeneous credit or liquidity constraints.” (see <https://voxeu.org/article/why-central-bank-models-failed-and-how-repair-them>).

⁵ For the history of the DSGE models see Duarte (2012) and De Vroey (2016, chs. 9-18).

heterogeneity in macroeconomics since the 1980s to the early 2000s, before “heterogeneous-agent macroeconomics” became perceived as a consistent literature worthy of being surveyed.⁶

Yet, this paper is not such a survey. Thus, it is not exhaustive and is not intended to evaluate the different approaches. Missing names should not be construed as a judgment that the associated contributions are minor. In line with recent historiographical practices (Forget and Goodwin 2011), we rather focus on several representative communities across the United States and Europe who have endeavored to bring different types of heterogeneity into their models. Modeling household heterogeneity was never a goal in itself. Also, only a minority of contributors to this literature were interested in studying how business cycles and macroeconomic policy responses affect inequalities and how inequalities reciprocally affect the business cycle. Comparing those communities and examining their relationships can help us understand their various motives, choices, successes, frustrations and criticisms of one another. To study these communities, we rely on published works, institutional records (such as the NBER reports), cursory archival material, and a round of more than ten extensive semi-directed interviews with the key players in this story.

The University of Minnesota was an important hub for the development of heterogeneous macroeconomics. While the rise of self-christened “new classical” macroeconomics is often associated with Chicago, where Robert Lucas spent most of his career, Minnesota became the top US department of economics with respect to macroeconomics and econometrics in 1980s according to most rankings. It boasted Neil Wallace, Thomas Sargent, Christopher Sims and Edward Prescott. They were dubbed “the four horsemen” of macroeconomics in the press, and the last three went on to become Nobel laureates. In section 2, we document why and how many of their colleagues and students developed heterogeneous-agent general equilibrium models. While Richard Rogerson, Gary Hansen, Ayse Imrohoroglu, José-Víctor Ríos-Rull, Rao Aiyagari, Mark Huggett or Per Krusell, among others, were sometimes disappointed by how little heterogeneity seemed to affect the behavior of aggregates, they nonetheless slowly developed both new theoretical structures and new computational strategies to solve these models.⁷

⁶ Among the several surveys of heterogeneous agent macroeconomics see eathcote, Storesletten and Violante (2009), Attanasio and Weber (2010), Guvenen (2011), Kaplan and Violante (2018), Ragot (2018).

⁷ Other students, such as Hugo Hopenhayn, sought to model firm heterogeneity into either overlapping-generations or general-equilibrium models. The literature on firm heterogeneity is not covered in this paper.

The kind of heterogeneity Minnesota macroeconomists worked on was mostly “stochastic,” in that they modeled *ex ante* identical agents who differed *ex post* due to the consequences of idiosyncratic shocks. In contrast, macroeconomists interested in growth theory explored “structural” heterogeneity in which agents’ behavior differs *ex ante*. From Robert Becker (1980) to Truman Bewley (1982), they built on a tradition originated by the mathematician Frank Ramsey in the 1920s.⁸ This line of inquiry was concerned with the mathematical and economic consequences of heterogeneity in agents’ time preferences, formalized by different discount rates. In section 3, we recount these efforts, as well as those of other distinct communities who coincidentally sought to introduce structural heterogeneity in life-cycle or permanent-income models. These contributors included macroeconomists trained or working at MIT or Harvard (such as Christopher Carroll and Stephen Zeldes), as well as a host of UK-trained microeconomists, among whom Angus Deaton, Orazio Attanasio and Martin Browning. Unlike their Minnesota colleagues, these economists were more focused on incorporating the many new stylized facts on marginal propensities to consume, liquidity constraints, precautionary motives or the distribution of income and wealth that emerged with the growing availability of national panel surveys. To that end, they were willing to use all sorts of models, whether general or partial equilibrium, with two or more types of agents exhibiting a wide range of structural heterogeneity. A key challenge was building convincing empirical strategies to estimate these models.

In section 4, we discuss the importance of UK-trained economists in our narrative. We show that Kydland and Prescott’s calibration method blurred the distinction between microfoundations and aggregation concerns, allowing applied microeconomists such as James Heckman to enter macroeconomic debates in this period. Applied microeconomists challenged the aggregation theories and empirical practices underlying Kydland and Prescott’s use of the representative agent and their associated calibration methodology, as well as Minnesota heterogeneity models. British economists such as William Gorman to Richard Stone, John Muellbauer and Angus Deaton had long been interested in deriving aggregate (demand) functions from individual functions of agents who differed in taste, endowments, or time preferences. Like their US-based colleagues, in the 1980s, they became more concerned with how to consistently use empirical micro estimates for macro models and vice versa. They were

⁸ The labels “stochastic” and “structural” heterogeneity came later, in the early 1990s, with economists such as Ricardo Caballero and Eduardo Engel (1991, 1659).

also aware that most aggregation results of the previous decades had been achieved by neutralizing in a way or another distributional effects, which was now becoming a key concern. Simultaneously, the behavioral heterogeneity theory proposed in a general equilibrium framework by Jean-Michel Grandmont in France or Werner Hildenbrand in Germany shows how diverse the theoretical and empirical concerns with aggregation have been.

In conclusion, we tie all these works to the epistemic tradition of their respective communities, e.g. what modeling choices they were unwilling to relinquish. We provide a broader perspective and challenge the idea that prior to the Lucasian revolution macroeconomists were exclusively concerned with aggregates. Historians of macroeconomics have shown that nearly all major contributors to the history of macroeconomics were concerned with the relationship between aggregates and those individual behavior which they knew were extremely heterogeneous. It is only with the neutralization of heterogeneity concerns resulting from the 1970s microfoundational program that “heterogeneity” later became a “modeling choice.”

2. The puzzles Minnesota macroeconomists faced

The development of infinitely-lived representative-agent models in the real business cycle literature, fostered at Minnesota by Prescott, came, from the start, with limitations that pointed to the need for more heterogeneity in their models. This is seen in the concluding paragraphs of Finn Kydland and Prescott’s 1982 “Time to Build and Aggregate Fluctuations.” In this paper, they presented a growth model without monetary shocks which succeeded in replicating selected features of US postwar business cycles. The model exhibited an infinitely-lived representative agent, which they explained was “standard in growth theory” at the time (p. 1345). However, they acknowledged the challenge of matching key stylized facts, such as the greater variability of hours worked compared to labor productivity. Matching empirical data required that the calibrated value of the elasticity of labor supply to be much higher than the values estimated by labor economists. Kydland and Prescott pointed to measurement errors, specific forms of non-separable preferences, or the lack of heterogeneity: “all members of the household may not be equally productive, say due to differing stocks of human capital,” they wrote (p. 1365), leaving the issue unexplored.

Kydland (1994) addressed this challenge in a paper prepared for a 1993 NBER Conference on Macroeconomics. He presented a model with two types of agents, which differed according to their human capital level. This approach increased labor volatility relative to productivity volatility and drew on microeconomic studies of labor force heterogeneity by researchers such as Jacob Mincer (1966) or Clark and Summers (1981). Two of Prescott's graduate students, Richard Rogerson and Gary Hansen, came up with further ways to reconcile the large calibrated macroeconomic and the small estimated microeconomic elasticities of labor supply.⁹ They relied on the introduction of stochastic heterogeneity in Kydland-Prescott types of models. Hansen (1985) adopted in a Kydland-Prescott style model a device already developed by Rogerson in his dissertation and later published in 1988. He conceived the labor supply of agents as indivisible, so that they may be hired just for a previously negotiated number of hours. Identical workers optimized their working hours and sold a contract where they committed to work for this amount of time with some probability. A lottery then determined which agent worked and which did not (Hansen 1985, 316). This was meant to guarantee that the consumption possibility set remained convex. As explained by Hansen (1985, 316), "although households are ex ante identical, they will differ ex post depending on the outcome of the lottery." In this case, he concluded, a rather inelastic individual labor supply could be consistent with an infinitely elastic aggregate labor supply.

It so happened that Prescott was also exploring heterogeneity to improve the matching of stylized facts in another research project he was pursuing with his former Carnegie Mellon graduate student Rajnish Mehra. Working on the kind of asset pricing model in a pure exchange economy that Lucas had developed in 1978, they found that a calibrated representative-agent model with perfect competition and reasonable risk aversion levels generated an equity premium of less than 1% compared with returns on riskless assets (Mehra and Prescott 1982, 1985). Yet the average premium observed in the data was around 6%, a discrepancy that became known as the equity premium puzzle. Their prospective solution was again to introduce heterogeneity into the model, something that Mehra's other thesis supervisor, finance specialist Costas Constantinides (1982) was working on at that time. The heterogeneity that they

⁹ See Clerc and Le Fur (2022) for a comprehensive overview of Rogerson and Hansen's work in the context of debates on the microfoundations of the aggregate labor supply. They also discuss the early attempts by Sergio Rebelo, then a graduate student at the University of Rochester under the supervision of Robert King and Paul Romer, to incorporate structural heterogeneity in preferences and wealth into the Real Business Cycle model following Kydland's early attempts. Rebelo's resulting works remained unpublished. Clerc and Le Fur (2022) analyze as well Rogerson's later attempts to study heterogeneity in productivity with his Rochester student Jang-Ok Cho.

envisioned was not in the structural characteristics of agents, but relied on restrictions to market participation for otherwise identical agents:

Perhaps introducing some features that make certain types of intertemporal trades among agents infeasible will resolve the puzzle. In the absence of such markets, there can be variability in individual consumptions, yet little variability in aggregate consumption. The fact that certain types of contracts may be non-enforceable is one reason for the non-existence of markets that would otherwise arise to share risk. Similarly, entering into contracts with as yet unborn generations is not feasible. (Mehra and Prescott 1985, p. 159)

Though seldom used by Minnesota faculty like Prescott or Wallace, a handful of their graduate students began exploring the idea that agent heterogeneity could help improve the performance of macroeconomic models in terms of matching empirical stylized facts or offering fine-grained analysis of the consequences of the business cycle or macro policies. Around 1986, Prescott's graduate student Ayse Imrohoroglu sought to build a model with poor and rich agents to re-examine Lucas's (1987) statement that the welfare costs of business cycles were small. She resorted to a modeling trick introduced by Cowles theorist Bewley (1977) to provide "rigorous" microfoundations to Milton Friedman's permanent income hypothesis (reinterpreted as a constancy of the marginal utility of money over time). Drawing on Friedman's remark that "consumers are subject to random shocks," Bewley modeled a single agent who faced heterogeneous income shocks and a borrowing constraint. In this exchange economy with no markets for contingent claim contracts, money is the only store of value. He tackled the problem of precautionary saving and demonstrated the conditions for a monetary equilibrium to exist.

Imrohoroglu expanded on Bewley's idea by proposing a model where exogenous probabilities to find a job generated uninsurable idiosyncratic income uncertainty. Because agents faced borrowing constraints, they couldn't fully offset this uncertainty and were forced to hold precautionary assets to self-insure. Computing equilibria for such models was challenging since equilibrium prices were not just functions of aggregate variables, but also of the entire wealth and income distribution of agents that changes endogenously over time. The usual linear quadratic approximation approach could not be implemented, so that a major

contribution of Imrohoroglu's paper was to propose a new computational strategy.¹⁰ Her numerical method relied on the discretization of the state space through the construction of a grid for asset values, and the iterated computation of a numerical approximation of the value function associated with agents' decision until it converged. For the sake of simplicity, the interest rate was pinned down by the storage technology and remained constant over time.¹¹ She coded this loop in FORTRAN, inputted punch cards on the Minnesota Supercomputer Institute's Cray, and waited a full night to get her results (interview). Her conclusion was that in specific settings and for some parameters, the welfare cost of business cycle was five times larger than the one in a social planner economy with perfect insurance. She found her result disappointing because it was still very small (0.5 percent of total U.S. consumption). If Kydland and Prescott had provided the blueprint for computational experiments in macroeconomics, then, Imrohoroglu was the one who provided the first operationalization for a model with heterogeneity.¹²

Another Prescott student, Mark Huggett (PhD 1991) saw her presenting her research at a job market seminar (interview). He had been searching for a dissertation topic, was familiar with Bewley's early theoretical work but became intrigued by Imrohoroglu's approach to modeling. Taking stock of the equity premium puzzle highlighted by his supervisor, he wanted to investigate the effect of incomplete markets and precautionary savings in pushing down the riskless interest rate (Huggett 1993). Initially, he attempted to write a model that exhibited both aggregate and idiosyncratic shocks. He could not solve it, so he settled on a model where restricted participation resulted in uninsurable idiosyncratic endowment shocks, thus higher precautionary savings (interview). He solved the model for each agent's program for bond holding, finding the unique stationary distribution at a given interest rate. This first loop he nested in a second loop designed to repeat the process until the interest rate that cleared the bond market was found, which allowed him to relax Imrohoroglu's simplifying hypothesis about the storage technology.¹³

¹⁰ Approximating economies around their steady state meant assuming that agents receive their average income at each period, which shuts down motives for holding assets (no precautionary savings) and is likely to violate the non-negativity constraint on the asset.

¹¹ Similar simplifying assumptions were also postulated by Díaz-Giménez and Prescott (1992), who refined the original discretization method of Imrohoroglu.

¹² Her code was circulated among graduate students. At that time, there existed neither training nor textbook to handle these models computationally.

¹³ The simplifying hypothesis used by Imrohoroglu often leads contemporary macroeconomists to consider her model as partial equilibrium and Huggett as extending it to general equilibrium. This interpretation is questionable. Imrohoroglu (1989, 1364) herself talked about examining "simple general equilibrium models."

Prescott may have shared Huggett's paper with Rao Aiyagari, but became intrigued by Imrohoroglu's approach to modeling (Huggett interview). In his dissertation, Aiyagari compared infinitely-lived agent growth models with a class of models which, by construction, introduced demographic heterogeneity and restrictions to market participation. Those overlapping generation models (hereafter OLGs) where agents differed by their birth and death periods had become wedded to the examination of the role of money in the economy. He showed that under certain conditions, the two classes of model generated the same dynamics. In the early 1990s, Aiyagari sought to understand the role of precautionary motives were in determining aggregate wealth, and to compare the dynamics of representative agent models with those of models with heterogeneity resulting from limited access to markets. He wrote a general equilibrium Bewley model with production, where agents differ because they face uninsurable idiosyncratic labor endowment shocks and thus trade assets among themselves. Like Imrohoroglu and Huggett, he calibrated the model using microeconomic data, and he relied on numerical methods to find the equilibrium interest rate and aggregate wealth. Although his stationary equilibrium showed a higher *per capita* capital stock than a full insurance market economy (because agents save more), this stock was still much lower than that suggested by other studies on the importance of the precautionary motives (Aiyagari 1994).¹⁴

In 1992, as Aiyagari completed the first version of his paper, he organized a conference on “macroeconomics with heterogeneous agents, incomplete markets, liquidity constraints and transaction costs” at the Federal Bank of Minneapolis, where he was employed. Held in September 1993, the small conference included his paper, an early attempt to propose a new concept of solution of models of asset pricing with incomplete markets by young USCD professor Wouter den Haan, and an article in which former Minnesota PhD student Per Krusell and his colleague from Queens and Carnegie, Tony Smith, outlined a general equilibrium model with heterogeneous agents who faced both idiosyncratic and aggregate shocks. They wanted to compare how the distribution of income and wealth affect the business cycle behavior of macroeconomic aggregates in the case of a representative agent vs heterogeneous agents. The task was ambitious, as it required rational agents to know future prices, determined by aggregate

¹⁴ Aiyagari (1995) used his heterogeneous-agent framework for policy evaluation purpose. He challenged Christophe Chamley's (1986) result that positive capital taxation may be suboptimal in a finitely-lived agent framework.

savings, to make their consumption and savings decisions. Savings in turn depended on how aggregate and idiosyncratic shocks affected the whole distribution of wealth and income. The computational challenge was immense, given that the state variable was an infinite dimensional object.

To tackle this challenge, Krusell and Smith turned to the concept of bounded rationality, an idea they had discussed since meeting at Minnesota in 1989. They sought to provide more “realistic” and “quantitatively reasonable” microfoundations to general equilibrium model (Krusell and Smith 1996, 522) by assuming that agents follow unsophisticated decision rules (i.e., they are rationally bounded). In an unpublished working paper, Smith (1992) had previously shown that the gains from being sophisticated in saving decision were small, so that agents could save a constant fraction of the income. In response to Prescott’s comment that a sophisticated agent could take advantage of this, Krusell and Smith wrote a model that assumed explicit costs of following sophisticated savings rules in an otherwise standard stochastic growth model.¹⁵ At the 1993 conference, they presented a paper that examined the consequences of letting agents choose between unsophisticated and costly sophisticated saving decision rules in a general equilibrium model with both aggregate and uninsurable idiosyncratic shocks on employment. They found that if the costs of following a sophisticated decision were small (less than 0.1% of consumption), the behavior of the aggregate was not different from a model with a representative agent. In line with their focus on unsophisticated decision rules, Smith coded an algorithm in which he tried to reduce the dimension of the state space by assuming that agents make decision solely based the mean of the wealth distribution (interview).

To Krusell and Smith’s surprise, agents could accurately forecast all relevant prices based on this simplifying assumption. They called the irrelevance of other moments of the wealth distribution for computing aggregate saving decisions “approximate aggregation.” As they revised the paper, they played down references to their sophisticated vs unsophisticated decision rule setting (interview).¹⁶ They also articulated more clearly that if aggregates did not

¹⁵ The model thus exhibited a kind of structural heterogeneity. The paper was written in 1991-1992, and eventually published as Krusell and Smith (1996).

¹⁶ Hints of their previous framework survived in their explanation for singling out the mean of the wealth distribution: “our approach is to calculate equilibria in which, by assumption, agents have a limited ability to predict the evolution of this distribution. We then show that this bound on ability almost does not constrain the agents at all,” Krusell and Smith (1998, 870) explained in the published version.

behave differently in their heterogeneous-agent setting from those in a representative agent model, it was because the shocks they faced were small enough so that most agents can insure themselves. Their current income and wealth thus became irrelevant. Borrowing constraints mattered for the poorest agents only, yet those did not significantly affect aggregate wealth. To better match the empirical distribution of wealth, they drew inspiration from Bewley's work on the permanent income hypothesis (1980, 1983; see section 3.) and introduced heterogeneous discount factors. The equilibrium resulting from their version of the model with both stochastic and structural heterogeneity was well defined, with the interest rate being driven by the patient agents who save a lot and end up possessing most of the wealth. In this case, the behavior of aggregates was significantly different from the representative agent setting: "poor agents have a large influence on aggregate consumption . . . they can be characterized as 'hand-to-mouth' consumers," they noted (p. 870).

As early as 1993, the paper was hailed as a methodological success. In his Carnegie-Mellon course notes (published in 1995), José-Víctor Ríos-Rull attributed the development of heterogeneous agent models to "cheaper and more powerful machines" and to economists' "computation literacy."¹⁷ Despite having different purposes in mind (policy evaluation, matching stylized facts, comparing the dynamic of aggregates across classes of models, exploring precautionary savings), the various models proposed by Minnesota students were increasingly viewed as part of a common literature. By the early 1990s, models drawing on Bewley's works were already associated with his name, and referred to as a "class" of models (see, for instance, Aiyagari 1994).¹⁸

But this community was also pervaded by a sense of disappointment. Though Krusell and Smith had eventually shown that some types of heterogeneity affected the behavior of aggregates, this late addition was often overlooked, and their paper was often lumped together with Aiyagari's as evidence that the addition of agent heterogeneity to macroeconomic models

¹⁷ Greater computation literacy also meant that graduate students in the mid-1990s would now have access to lecture notes, textbooks and even courses on computational economics. For instance, Jesús Fernández-Villaverde (interview), who started his PhD at Minnesota in 1996, told us that Minnesota students interested in heterogeneous-agents models then had available: Ríos-Hull's lecture notes on how to solve models with stochastic heterogeneity in practice, as well as his chapter in Cooley (1995) and photocopies of Kenneth Judd's lecture notes from Stanford. Towards the end of that decade the influential books by Judd (1998) and by Ramon Marimon and Andrew Scott (1999) were published and Minnesota students could take Ellen McGrattan's course in computational methods.

¹⁸ While Lucas (1980) and former Chicago PhD student Bert Taub used the same modeling strategy, most early adopters were Minnesota graduate students of Prescott: these included Rodolfo Manuelli, and Javier Díaz-Giménez.

was not worth the effort. Ríos-Rull, another student of Prescott's, had already reached similar conclusions in an OLG framework several years earlier. Trained as a quantitative sociologist and demographer before entering the Minnesota graduate program in economics, Ríos-Rull had retained a specific attention to the demographic structure of macroeconomic models, as well as an early interest in how these models could explain inequalities (interview). Both his course notes and his early articles show that he understood that models with “*ex ante*” and “*ex post*” heterogeneity may help understand the behavior of aggregates better. In 1993, he studied the consequences of allowing agents to acquire skills and thus differ *ex post*. In the following year, he published a paper that concluded that those calibrated large OLG models imposing restricted access to contingent markets and the complete-market representative agent models behave similarly (Ríos-Rull 1994).

The negative interpretation of this first decade of work on heterogeneity was in Kydland (1994, 860)’s address to the fifteenth Annual conference for the Society for Economic Dynamics and Control in June 1993: “the prototype real business-cycle economy is inhabited by immortal consumers, that is, no life-cycle behavior is assumed. A question is: If we addressed the same sort of question, say, about the role of technology shocks in an [OLG] economy with mortal consumers, would the findings be different? Victor [Ríos-Rull]’s answer is no. Sure, there are some minor differences in the properties of some of the model statistics, but none that would lead one to question the basic findings from the immortal-consumer framework,” he explained (see also Kydland and Prescott 1996, 73).

While most of the models proposed by the Minnesota community in the 1980s and 1990s introduced heterogeneity with agents that are “*ex ante* identical” (Ríos-Rull 1993, 905), there had been exploration of the differences in agents’ preferences, attitudes to risk, impatience, and their aggregate implications. Two literatures that were active in the 1980s and 1990s dealt with growth, as well as saving and consumption behavior. Rather than reading these other strands of the literature, Minnesota students were encouraged to improve the kind of quantitative general equilibrium models that they have been trained to use. It thus took years for Minnesota economists to get acquainted with the models proposed by other macroeconomists working on heterogeneity.

3. Structural heterogeneity in growth model: the Ramsey tradition?

The coexistence of structural and stochastic, heterogeneities in macroeconomic models is evident in the agenda of a key protagonist of our story: Bewley himself. In trying to build general equilibrium models that would conform to Friedman's ideas on permanent income or money, he included both idiosyncratic shocks on endowments and preferences (to generate stochastic heterogeneity), and heterogeneous individual discount rates (structural heterogeneity). In his 1977 paper, he modeled only one consumer, but in 1980 he expanded to a finite number of consumers with possibly different discount rates, initial stochastic endowments and a borrowing constraint. Bewley (1980) then studied the conditions under which a rational expectations monetary equilibrium exists. In his general equilibrium model the value of money resulted from both the agents' structural heterogeneity and the stochastic environment (see also Bewley 1983, 1986). His interests were influenced by the new literature on general equilibrium theory that was expanding the Arrow-Debreu model in different dimensions: monopolistic competition (Negishi 1961), uncertainty (Radner 1968), incomplete markets (Diamond 1967), intertemporal models, transactions costs, money, among others (see Weintraub's 1977 survey, pp. 8-18). This literature also influenced his and others' efforts to bring structural heterogeneity to growth models by introducing agents with different discount rates.

Bewley, who obtained two PhDs from the University of California, Berkeley, one in economics and the other in mathematics,¹⁹ aimed to bridge the literatures on general equilibrium theory and on growth, often referred to as capital theory. He developed a general equilibrium model with a finite number of infinitely lived consumers and made no connections with his earlier works on permanent income and money (Bewley 1982).²⁰ Among other things he proved an analog of a turnpike theorem by José Alexandre Scheinkman (1976), a student of Lionel McKenzie, when consumers have the same time discount rate. Finally, he concluded that if agents have different discount rates, "then the less patient consumers eventually consume nothing in equilibrium" (233). Bewley thus proved a conjecture that the general equilibrium literature had connected to Frank Ramsey's 1928 article that discussed how much a nation

¹⁹ The economics dissertation was on "Equilibrium theory with an infinite dimensional commodity space," (1970) and the mathematics one "Non-genericity of first integrals," 1971, advised by Calvin Cooper Moore.

²⁰ Because the horizon is infinite, we have a general equilibrium model with infinitely many commodities, whose existence of equilibria he had already proven a decade earlier (Bewley 1972).

should save.²¹ He opened his famous paper by rejecting time-discounting to study the intertemporal decisions of a community because he found it inconsistent with intergenerational justice. Nevertheless, he introduced discount rates for future utility in the analysis of intertemporal decisions by an individual (Duarte 2016). After deriving the now famous “Keynes-Ramsey rule” from intertemporal utility maximization with no discounting, Ramsey (1928) explored the determination of the interest rate in several special cases, including that of individuals with different discount rates. He closed his article in what later generations of economists have dubbed the “Ramsey conjecture”: “In such a case [heterogeneous time discounting], therefore, equilibrium would be attained by a division of society into two classes, the thrifty [with discount rates lower than the interest rate] enjoying bliss and the improvident [those with discount rates greater than the interest rate] at the subsistence level” (Ramsey 1928, 559).

Such conjecture on the properties of a growth equilibrium with discount rate heterogeneity was first proven by Robert Becker (1980), then a PhD student at Rochester under McKenzie.²² Becker’s demonstration fueled a stream of papers that reexamined the optimal growth paths in the presence of heterogeneity, in which Bewley participated.²³ The primary concern of contributors was to prove the existence of an equilibrium and to study its properties (uniqueness and stability), with the goal of deriving macroeconomic implications from their models. Later in the 1980s, a news wave of interest on the heterogeneity in time discount rates emerged. Bewley (1986) and Scheinkman and Weiss (1986) demonstrated that market incompleteness could be a source of equilibrium fluctuation. Becker and Foias (1987) introduced market

²¹ The idea of such “Ramsey conjecture,” according to Bewley (1982), had already appeared in the works of Trout Rader (1971, ch. 1; 1981). Bewley (1982) cited a working paper version of Rader (1981). In the published paper, Rader (1981, 232, fn. 7) does connect his results to Ramsey (1928).

²² Becker may have been introduced to the conjecture by William Brock, who had stated a related conjecture in some unpublished lecture notes of 1973 (Becker 1980, 375, fn. 2). Brock was then at the University of Chicago, but also visiting Rochester in the Fall of 1973, where Becker was studying since the previous year. His 1980 article is a modified version of one chapter of his thesis, completed in 1978, as he acknowledged (Becker 1980, 375). In the late 1970s Bewley stumbled on a version of the existence chapter in Becker’s thesis, phoned him and got to know all about his PhD thesis (Becker interview). As an undergraduate student at Washington University, Becker had been taught by Trout Rader, who exposed his students to an exchange economy with heterogeneous agents (Becker interview). Becker’s research agenda on general equilibrium didn’t preclude him from working also with a one-sectoral growth model with an infinitely-lived representative agent (e.g. Becker 1983 and the other papers he published in the same issue).

²³ Becker (1980, 381) understood well that they were moving much beyond Ramsey (1928): “In contrast to his detailed and explicit model of optimal growth, Ramsey left a model of equilibrium undeveloped.” Strictly speaking, Ramsey did not have a model with economic growth (there is no population growth and no technological innovations): it is a model of intertemporal decisions. By the 1960s, however, Ramsey had already been paired with Tjalling Koopmans and David Cass to name the optimal growth model known as the “Ramsey-Cass-Koopmans model” (Duarte 2009).

incompleteness and demonstrated under which conditions the Ramsey conjecture would hold (now with the conjecture renamed to “Rae-Fisher-Ramsey conjecture”).²⁴ In such models, multiplicity of equilibria was pervasive, with one one stable and one with periodic cycles (Becker and Foias 1987). Their analysis presupposed the existence of a Ramsey equilibrium, an issue that they proved in a 1985 working paper, eventually published years later with an additional co-author and Becker PhD student, John Boyd III (Becker, Boyd III and Foias 1991, see also Becker and Foias 2000 for later work on incomplete markets and aggregate fluctuations).²⁵ Other economists also studied heterogeneity with incomplete markets in these years, such as Michael Woodford (1989).²⁶

Agents with high vs low discount rates was in fact the modeling strategy Krusell and Smith (1998) later adopted to match the wealth distribution in their model. At that time, however, they did not know about the Ramsey conjecture nor about the more recent literature striving to demonstrate it. Nor was Becker aware of their work. When he encountered it during the 1994 Summer Workshop on Applied General Equilibrium Analysis at the University of Northwestern, he was puzzled that they would simply assume the existence of an equilibrium under incomplete markets which he had such a hard time demonstrating (Becker, interview).²⁷

²⁴ Ciprian Foias was in the mathematics department of Indiana (while Becker was in the economics department), with a Ph.D. from the Institute of Mathematics, Bucharest, in 1962, and became a close collaborator of Becker over the years. Becker (interview) told that at Rochester he read a lot of Irving Fisher (as well as Walras and Marshall), in particular his book *The Theory of Interest*, and that Fisher also got him to think about heterogeneity (after the earlier exposure to it in Rader’s undergraduate course). He also mentioned the importance of the micro course with James W. Friedman (PhD Yale 1963) when he learned about supergames where all agents had the same discount rate.

²⁵ Boyd III got his PhD from Indiana in 1986 with the thesis “Preferences, Technology and Dynamic Equilibria.” He got his first job at Rochester’s economics department, where Lionel McKenzie was, and stayed there until 1998.

²⁶ Though the Ramsey conjecture was at the heart of optimizing general equilibrium models with growth, another strand of literature brought structural heterogeneity into non-optimizing growth models. In the 1970s distribution theory was a hot topic due to the works of Nicholas Kaldor, Luigi Pasinetti and others. Joseph Stiglitz (PhD from MIT in 1966 under Robert Solow) was interested on the distribution of wealth and income among individuals, instead of the existing focus on functional income distribution. In his PhD thesis he linked functional income distribution with that among individuals by considered two classes, workers and capitalists, with different savings rates. This led to his 1969 paper in which he considered different “‘forces for inequality’ [in a Solow model]: heterogeneity of labor force, class saving behavior, and alternative inheritance policies” (Stiglitz 1969, 382). He came close to the spirit of the Ramsey conjecture when he added a borrowing constraint and found out that the least productive and poorer ate up all their income, while the most productive and richest eventually accumulate all the wealth. But this developed as a separate literature later on as indicated in the survey by Sahota (1978).

²⁷ Becker and Itzhak Zilcha (1997) showed that stationary solutions in stationary stochastic environments where agents have precautionary motives may not exist. They provided an existence proof of a stationary Nash equilibrium for some specific setups. Their paper, completed in 1995, is remarkable in that, in the wake of their encounter with Krusell and Smith’s research, they referenced the latter’s research in addition to works by Huggett and Rios-Rull which they knew from the Northwestern meetings. When doing so, they emphasized that their model “lead to results that differ from the representative agent case,” in particular with respect to the wealth distribution

This shows how different traditions relying on modeling heterogeneity were at that time. Growth theorists working on the Ramsey conjecture focused on demonstrating theoretical properties of these models. Krusell and Smith's approach was different, simply assuming an equilibrium and approximating it numerically.²⁸ Other communities of economists associated with Harvard, MIT or major British research centers, adopted other empirical approaches, eschewing general equilibrium models and writing simple optimization problems designed to fit the growing body of micro data that pointed to heterogeneous behavior through simulation exercises.

4 - Structural heterogeneity in saving and consumption research: going from micro to macro?

Those economists scattered across MIT, Harvard, as well as at the Institute for Fiscal Studies (IFS), LSE or Cambridge in the UK ventured to tackle the consequences of heterogeneity on macroeconomic aggregates because they found it in the growing body of consumers' data. This data became available due to the now-mature consumer surveys launched in the late 1960s, such as the Michigan's Panel Survey on Income Dynamics (PSID) which launched in 1968 and asked households questions about food consumption, the Consumer Expenditure Survey of the Bureau of Labor Statistics (which started in 1888 but became a continuous survey only in 1980), or the British Family Expenditure Survey (in existence since 1957). These East coast economists typically encountered those body of data through their doctoral studies or internships institutions like the NBER or the Council of Economic Advisors.²⁹

As they delved deeper, these economists grew increasingly concerned about the same set of aggregate "puzzles," such as the equity premium puzzle, the high level of aggregate wealth held by US households, or the late-1980s consumption boom in the UK that existing macroeconomic models were unable to account for. Equally concerning was the lack of fit

(p.124). They had also already referenced, in earlier versions, Aiyagari's work, Deaton's line of research precautionary savings, and Kirman and Harley's challenges to representative agent models.

²⁸ While Krusell and Smith didn't connect their work to the growth literature of Becker and others, Aiyagari was different in this respect. According to Becker (interview), Aiyagari knew about the growth literature and Becker and others knew about his work. Huggett (1997) likewise explained that part of his work applied to models with heterogeneity in discount factors, and mentioned Becker and Foias (1987).

²⁹ For instance, Greg Mankiw remembers that in the early 1980s, his interest in applied macroeconomics was aroused when working as a junior assistant to Lawrence Summers at the Council of Economic Advisors (interview).

between microeconomic data and the then-prevalent theoretical models of consumption such as Friedman's permanent income hypothesis, or Franco Modigliani and Richard Brumberg's life cycle hypothesis, which MIT macroeconomist Robert Hall reframed in a rational expectations setting in 1978.

These models predicted a very small marginal propensity to consume out of transitory income (less than 5%), that is, consumption should not respond much to current income. Under the rational expectations hypothesis, it should even follow a random walk, and only unexpected income changes should affect consumption (Hall 1978). A host of studies, including Marjorie Flavin's MIT dissertation (written under the supervision of Hall), showed otherwise: that aggregate consumption responds to both unanticipated and anticipated changes in income.³⁰ Hall and Frederic Mishkin (1982) found evidence of a 20% aggregate marginal propensity to consume. Another MIT student, Stephen Zeldes (1989) wrote a model deriving testable implications from the existence of borrowing constraints. These constraints were often given as an explanation for the empirical rejection of the restrictions on the data implied by the stochastic version of the permanent income hypothesis. Based on the PSID micro panel data, Zeldes concluded that the results were (partially) supportive of the influence of liquidity constraints on consumption.

At the same time, Zeldes's coauthor and former fellow student Gregory Mankiw (PhD MIT 1984) pooled with Yale economist John Campbell to provide evidence that 50% American household were either hand-to-mouth (e.g. consuming all their current income) or following a rule of thumb rather than relying permanent income to make their consumption (Campbell and Mankiw 1989). Mankiw and Zeldes (1991) further documented that two thirds of food in the U.S. was consumed by agents who held no stocks. With Glenn Hubbard from Columbia and Jonathan Skinner from the University of Virginia, Zeldes showed that a significant fraction of household accumulate no wealth at all, even by the time they retire. Existing models could not explain those facts. In that same year, Harvard professor Lawrence Summers and his former assistant at the Council of Economic Advisors, Christopher Carroll (PhD MIT 1990), presented the results of a survey they had done and called for "increased emphasis on liquidity constraints and short run precautionary saving as determinants of consumption behavior" (Carroll and Summers 1991, 306). Drawing on data from the US Consumer Expenditure Survey and cross-

³⁰ These studies are too numerous to be exhaustively referenced here, but Deaton (1991) can be read as an extensive account of the research done in the 1980s and 1990s.

country data, they showed that patterns of consumption differed among age, education and occupational groups, and that some consumption and income profiles were synchronized over the life cycle. Similar conclusions were reached by British economists Martin Browning, Angus Deaton and Margaret Irish (1985).

All these studies pointed to the heterogeneity of individual marginal propensities to consume, which suggested that *microeconomic* heterogeneity might help solve *macroeconomic* puzzles. Indeed the problem was not that agents do not hold stock, face borrowing constraints or are hand-to-mouth, but that the micro evidence showed that *some of them* do so. To address this issue, these economists developed models that included structural parameters that allowed for heterogeneity among rich vs. poor, young vs. old, or eligible vs. non-eligible agents, consistent with the patterns emerging from micro data. These models were then simulated using microeconomic data to determine whether they could generate the kind of aggregate dynamics observed in the real world. Mankiw (1986) for instance, had long been concerned with the equity premium puzzle, which he aimed to explain by the heterogenous effect that aggregate shocks on consumption had on different types of consumers. With Zeldes, he therefore wrote a partial-equilibrium CAPM model with two kinds of agents, stockholders and non-stockholders. They found out that the equity premium could be explained by the stockholders' consumption varying more strongly with the stock-market. Mankiw and Campbell also studied fiscal policy with a model that assumed that half of agents behave according to the permanent income hypothesis while the other half was hand-to-mouth.

The existence of borrowing constraints also caught the attention of British consumption microeconomist Deaton, who was then working at Princeton.³¹ He questioned whether such constraints could lead agents to develop a precautionary saving motive. At the 1989 meeting of the Econometric Society, he presented a paper that demonstrated “the failure of the representative agent model” to match U.S. aggregate saving behavior under liquidity constraints (Deaton 1991, 1242). He proposed to “work from the bottom up, starting not from the aggregate time-series process, but from those observed in the micro data” (p. 1243). His alternative model combined patient consumers who accumulate assets and impatient hand-to-mouth consumers with liquidity constraints. When faced with uncertainty, Deaton concluded, rational agents cannot always balance their Euler equations. At the same time, Carroll was completing related

³¹ Deaton credited the aforementioned research by Carroll and Summers (1991) with changing his views of the relevance of Modigliani's theory (personal communication).

investigation, yet, without relying on borrowing constraints. He proposed that agents' rational way to respond to high income uncertainty was to accumulate assets to meet a "buffer stock," that is a target wealth-to-permanent income ratio. He took the term from Deaton's 1991 paper. Such rational response to a stochastic environment, he claimed, was consistent with the stylized facts he and Summers documented in their 1989 survey.³² Even those agents who did not face liquidity constraints could therefore save more than the certainty equivalent life-cycle workhorse model predicted, he concluded.

In his model, Carroll described agents' behavior as a mix of "impatience" and "prudence".³³ He introduced stochastic heterogeneity into an infinite-horizon version of the model by incorporating idiosyncratic shocks on labor income. They aimed to capture income uncertainty and mimic the actual fluctuations of labor income as observed in PSID data. One of the key predictions of Carroll's model was that marginal propensity to consume is a strictly decreasing function of the level of wealth, which implies that distributional effects should be taken into account when studying aggregate variables. This conclusion was strengthened when switching to a life-cycle version of the model. Carroll considered several age/income profiles (as well as several occupations), calibrated to match the structural (wealth and demographic) heterogeneity in US micro data. Building on previous work by Deaton and Carroll, Hubbard, Skinner and Zeldes (1994) offered a revised version of the life-cycle model of consumption with uncertainty about earnings, medical expenses and lifespan. This model generated large precautionary savings that played a role in determining aggregate savings. Relying on numerical solutions, the authors emphasized "the importance of the *variance* in earnings and health expenses in determining aggregate savings and wealth," which suggested that the design of tax policies and spending programs could affect aggregate saving behavior. Therefore, the introduction of structural heterogeneity in economic models helped provide new explanations for macro puzzles.

³² According to Carroll, these stylized facts consist of a "consumption/income parallel" in low frequency data (that is an almost constant consumption to income ratio in the long run), a "consumption/income divergence" in high frequency data (with a decrease of both average and marginal propensity to consume in cross-sectional data or in the short and medium runs), and an average financial wealth small, but still positive, over the entire working lifetime.

³³ Impatience needed to be sufficiently high, but all agents had the same discount rate. Prudence is a term coined by Miles Kimball (1990) to characterize a positive third derivative of the utility function, that is the existence of a precautionary saving.

The 1990s thus saw the development of two macroeconomic literatures that attempted to explain aggregate behaviors by relying on both stochastic and structural heterogeneity. These literatures employed different methods and reached diverging conclusions on the impact of heterogeneity on aggregates. The two sets of contributors acknowledged each other. Most Minnesota macroeconomists mentioned Zeldes's 1989 empirical study to support the importance of borrowing constraints.³⁴ In their final footnote, Hubbard, Skinner and Zeldes (1994, 94 ft 39). noted the discrepancy between their findings and those of Aiyagari. Summarizing these two strands in an AEA session on "the role of microeconomic heterogeneity in macroeconomics," Carroll contrasted the Deaton-Zeldes "bottom-up" approach "to model microeconomic consumption behavior carefully and then to aggregate," which concluded that "precautionary saving and microeconomic heterogeneity can profoundly change behavior," with the Aiyagari-Krusell-Smith "top-down" approach in which "precautionary saving is of little importance in determining the aggregate capital stock" (Carroll 2000, 110).³⁵

Most importantly, Carroll's comment, titled "Requiem for the Representative Consumer?" highlights what many economists perceived to be the underlying fault lines in the variegated approaches to using microeconomic heterogeneity to understand macroeconomic outcomes: underlying microfoundations, in particular theoretical and empirical rationales for aggregation. "This paper argues that the models that produce this 'approximate aggregation' result do not really have solid microfoundations, in the sense that they do not match the key microeconomic facts of a skewed wealth distribution and a high marginal propensity to consume. When the model is modified in ways that help it to capture these microeconomic facts, the behavior of the resulting aggregate economy differs from the behavior of the representative-agent economy in ways that may be very important for understanding aggregate fluctuations and analyzing the effects of economic policies," he concluded (Carroll 2000, 114). His criticisms echoed a long line of objections to Kydland and Prescott's theoretical model of a representative-agent and calibration based empirical strategy. While heterogeneity was often introduced into models in response to concerns with the underlying aggregation strategies in various macro models, those criticisms remained at the core of debates on which types of heterogeneous-agent models were more promising lines of research throughout the 1990s and early 2000s, as we discuss below.

³⁴ One of the referees of Huggett's 1993 paper also asked him how his treatment of precautionary savings compared to that proposed by the Carroll-Deaton-Zeldes community (interview).

³⁵ In footnote 9, Carroll conceded that "Krusell and Smith (1998) also show that adding heterogenous preferences results in a much more realistic distribution of wealth, and a higher correlation between aggregate consumption and income."

5. Microfoundations, heterogeneity and aggregation

When it was published in 1982, Kydland and Prescott's calibration strategy immediately drew criticisms from Heckman. At the November 1983 Carnegie-Rochester conference, he commented on Kydland (1984)'s paper on labor-force heterogeneity, stating that his choice of a "representative consumer model" forced him to concentrate solely on man hours and to overlook employment entry decisions (working vs not working). Heckman believed that such a model prevented the use of micro evidence to calibrate macro models. "By introducing heterogeneous agents into a dynamic equilibrium model, Kydland takes an important first step toward accommodating the wealth of microeconomic findings that indicate considerable microeconomic diversity in preferences and endowments. There are numerous puzzles in macroeconomics that may simply be aggregation phenomena" Heckman (1984, 213-4) wrote.³⁶ He acknowledged the computational costs of introducing heterogeneity, but challenged Kydland's solution, which was to introduce a social welfare function. "It is not obvious to me that any such function describes aggregate behavior or that the costless lump-sum redistributions among agents that are implicit in Kydland's model are possible," he explained. He warned that "the micro data contradict representative worker, firm, or consumer models. A macro fiction can be constructed that 'explains' the data, but no micro counterpart of these fictional behavioral functions can be found. Accordingly, micro findings do not and could not be expected to produce macroparameters unless an appropriate aggregation procedure is developed." Heckman's criticisms were in line with his interest on estimating labor supply that emerged from his PhD thesis at Princeton (1971) and with his quest for models that fit the distribution of wages for the US labor market. At that time, Heckman and the Chicago PhD Guilherme Sedlacek (1985) were developing a model of self-selection for workers with heterogeneous skills and two market sectors. They then used this model to estimate the aggregation bias in measured aggregate real wages.³⁷

³⁶ Kydland and Prescott, like Hansen and Rogerson, looked for interior solutions to the representative agent's program. Building on a large microeconomic literature, Heckman pointed out that the consumer program regularly leads to corner solutions at the micro level (e.g. when it is optimal not to work). Muellbauer (1981, 33), among others, discussed how "corner solution and other nonlinearities" prevent consistent aggregation in the aggregate labor supply function.

³⁷ Clerc and Le Fur (2019) discussed in further details Heckman's criticisms to calibration and the aggregation problems in labor supply.

Hansen (interview) and Rogerson were aware of these criticisms and had framed their contribution with an explicit acknowledgment of aggregation problems: “since we are allowing households to choose any level of unemployment insurance they wish, we have to allow for the heterogeneity that may come about because different households will have different income streams . . . however, this heterogeneity will disappear in equilibrium since all households will choose full insurance” (Hansen 1985, 325). In 1987, Rogerson and his PhD student Jang-Ok Cho (1988, 233) further this idea by producing a model that “considers an economy which is populated by a continuum of identical families consisting of two members . . . Although both family members have the same preferences, they display very different elasticities of labor supply [...] it is seen that this cross-sectional heterogeneity results in interesting forms of aggregation bias.” In his Nobel lecture, Prescott later emphasized that his graduate students’ work provided him with adequate aggregation procedures, in which “no conflict arises between micro and macro observations.”³⁸

Kydland and Prescott’s attempt to base an aggregation strategy on Rogerson’s and Hansen’s works left Heckman unimpressed. His further criticisms were not limited to whether their theoretical aggregation allowed borrow a labor parameter from the microeconomic literature, but targeted their empirical strategy at large. In a 1996 issue of the *Journal of Economic Perspective* on calibration, he teamed up with Lars Hansen from the university of Chicago to explain why, in spite of being elegant, stimulating and provocative, real business cycle models rested on weak empirical foundations: “The deliberately limited use of available information in such computational experiments runs the danger of making many economic models with very different welfare implications compatible with the evidence. We suggest that Kydland and Prescott’s account of the availability and value of micro estimates for macro models is dramatically overstated” (Hansen and Heckman 1996, 90). This time, they also addressed the rise of heterogenous agent models at Minnesota, one that they interpreted as an

³⁸ Prescott (2006, 216-227) explained that “the aggregation theory underlying this aggregate household is based in part on the first welfare theory, namely, that a competitive equilibrium maximized some weighted average of individual utilities...But in order for the model to be used to study business cycle fluctuations, the labor supply decision must be endogenized as well . . . [Rogerson and Hansen’s] aggregation theory implies that whenever the principal margin of adjustment is the fraction employed and not hours per person employed, the aggregate labor supply elasticity [unlike individual elasticities] is large. This finding is consistent with all the micro-observations, so no conflict arises between micro and macro observations.” Clerc and Le Fur (2022) however point that Hansen and Rogerson’s work broke the correspondence between individual behavior and that of the representative agent. In their framework, the preferences and elasticities of the representative agent could not be interpreted as mimicking that of individual identical agents, but at best as that of a social planner, an aggregate construct that can be interpreted as a stand-in for a labor market phenomenon.

attempt to narrow the range of specification errors in calibrating with microeconomic data. They however remarked that little emphasis has been given to the transition from micro to macro in the RBC literature, which they found surprising “given that understanding the distribution of heterogeneity is central to making this transition” (p. 101).

A further paper published in the *Handbook of Macroeconomics* in 1999 allowed Hansen and Heckman, now joined by Browning, to expand their criticism. They insisted that closing the micro-macro gap required accounting for the heterogeneity in preferences, discount rates, risk aversion, constraints, labor supply and skills and human capital highlighted by microeconomic studies. They showed that using a one-sector growth model was problematic for assessing the welfare consequences of macroeconomic policies on individuals because the diversity of subjective account factors in the population needed to be taken into account. They also addressed the perceived failure of models of the Imrohoroglu-Huggett-Aiyagari and Krusell-Smith type — the finding that microeconomic uncertainty had not much impact on capital accumulation and wealth distribution. They explained that aggregating income from agents in different risk classes was “a potentially dangerous practice” (p. 567).

Heckman, Hansen and Browning’s criticisms of the representative agent framework, calibration and of efforts to introduce stochastic heterogeneity were echoed by a coauthor of the latter, Deaton. In his 1992 book on *Understanding Consumption*, Deaton explained that his attempts to provide new theoretical foundations for consumption behavior were grounded on his dissatisfaction with representative agent models: “Representative agents have two great failings; they know too much, and they live too long. An aggregate of individuals with finite lives, and with limited and heterogeneous information is not likely to behave like the single individual of the textbook. We are likely to learn more about aggregate consumption by looking at microeconomic behavior, and by thinking seriously about aggregation from the bottom up” (Deaton 1992, ix). Indeed, how to aggregate heterogeneous individual demands had also laid at the core of Deaton’s research agenda since the 1960s. In this, he furthered a century-long British tradition of thinking about theoretical and empirical aggregation.

This tradition was rooted in trade theory and welfare economics, centered around the concept of “community indifference curves.” The notion of community indifference was taken up by Nicholas Kaldor and was central to the 1953 paper in which Birmingham theorist Terence (W. M.) Gorman established the conditions under which individual preferences can be summed

or averaged into a collective preference function (a result known as exact aggregation).³⁹ Gorman's results and subsequent aggregation theorems imposed stringent and unrealistic linearity restrictions on individual Engel curves, which *de facto* pushed to the side those distributional concerns that became important in the 1990s (Deaton and Muellbauer 1980, 120). Nonetheless, the British tradition sought to find non-linear functional forms for Engle curves that would allow exact aggregation and be consistent with empirical data, such as the Translog model (Chai and Moneta 2010). The Cambridge Department for Applied Economics (DAE) played a critical role in this quest, with contributions by DAE theorists such as Michael Farrell, Alan Prest, Derek Rowe, Andrew Roy, as well as Houthakker and Sigbert Prais, and DAE director Richard Stone's empirical aggregation work which led to the Linear Expenditure model of consumers' expenditures.⁴⁰ The founder of microsimulation, Guy Orcutt (1990, 15), later reflected on the influence of this unique Cambridge intellectual environment: "around 1955 [while visiting the DAE], I became increasingly concerned about the large gap between microanalytic research and the application of the results of research to policy problems at the national level . . . There simply was no known way of satisfactorily aggregating relations about microcomponents into macroeconomic relations." This focus on sound empirical aggregation also highlighted that aggregate data only partially reflected individual behaviors.

Both Deaton, who worked under Stone on consumer demand models, and his coauthor John Muellbauer, a Cambridge undergraduate who completed a dissertation on the measurement of quality and price indices for producer and consumer durables under Hall at Berkeley, followed the British aggregation tradition. They worked on the aggregation of both individuals and of goods. In 1976, Muellbauer proposed further analysis of the conditions for the existence of a "representative consumer." He extended Gorman's results to a class of PIGL (price independent generalized linearity) preferences which allowed for heterogeneity in tastes. Together, Deaton and Muellbauer (1980) developed a flexible functional form in which average demand depended on prices and a representative level of total expenditures (itself a function of expenditures distribution), rather than on the average total expenditure. This resulted in the Almost Ideal Demand System (AIDS). In their book, Deaton and Muellbauer (1980, 148) again

³⁹ These results had in fact been discovered by Giovanni Battista Antonelli in 1886, coincidentally rediscovered by Gorman in 1953, and André Nataf (1953) and Henry Theil (1954) in an econometric perspective.

⁴⁰ Houthakker and Prais (1955) showed that a semi-log form was more appropriate for Engle curves for necessity goods while a double log was better for luxury goods. Richard Stone's Linear Expenditure model aimed to make empirical estimates compatible with the theoretical approach, by integrating to the system the salient assumptions of demand theory, especially the homogeneity and symmetry conditions of the Slutsky equations, assumptions that would eventually be tested and rejected by subsequent empirical studies.

pointed to the problematic relationship of aggregates to micro data: “If [...] as is frequently the case, the data are available only for aggregates of households, there are no obvious grounds why the theory, formulated for individual households, should be directly applicable.” They clarified that understanding aggregate behavior went, in their view, beyond what U.S. macroeconomists would call microfoundations, that is, looking for an individual behavior to ground aggregates: “in general, it is neither necessary, nor necessarily desirable, that macroeconomic relations should replicate their microeconomic foundations so that exact aggregation is possible . . . A correct treatment requires that both these effects be adequately modeled, but this is impossible if aggregate demand is treated as coming from a ‘representative’ consumer who buys some of all of the goods” (*ibid.*, 148-149).⁴¹ This was a charge that Deaton (1991, 38) reprised when he noted that the representative agent was “doing away with aggregation.”⁴²

The U.S. macroeconomists’ appeal to “microfoundations,” both in the theoretical write-up of behavioral equations and the empirical procedures used to borrow parameters from the microeconomic literature thus immediately raised the problem of heterogeneity in these individual characteristics, circumstances, and behaviors, how to move from microeconomic to macroeconomic evidence, and which type of heterogeneity affected the aggregates. These were issues that microeconomists, both in the US and in the UK, were accustomed to. As Hansen and Heckman (1996) remarked, Sonnenschein and Mantel’s conclusion on the arbitrary shape of excess aggregate demand functions left economists who wanted to carry their model to the data to impose arbitrary restriction of their assumed degree of heterogeneity, or to rely on Gorman’s 1953 result to use a representative consumer. This was a huge problem, as Gorman and others’ results implied neutralizing heterogeneity and distributional effects that economists were now interested in.

⁴¹A more radical criticism of how mainstream macroeconomists were handling heterogeneity was proposed by Alan Kirman. After summarizing most criticisms against the representative agent, Kirman (1992) advertised complexity economics, where modeling the interactions of a large set of heterogeneous agents following simple rules yield interesting emergent properties.

⁴²Deaton later reminisced about the influence of the two Cambridge controversy that had also cradled Stiglitz’s attempts to bring structural heterogeneity in growth model: “in Cambridge in the 1960s and 1970s, we all listened to the Cambridge Keynesians, their denunciations of the validity of the non-substitution theorem, and their insistence on the interdependence of equilibrium prices and income distribution; could it really be true that a sweeping increase in income inequality would leave consumer demands unchanged?,” Deaton (2018, 380) explained in his Nobel address.

Empirical economists also pointed to inconsistencies both in macroeconomists borrowing microeconomic parameters, as Kydland and Prescott proposed, and conversely in microeconomists using aggregate data to discipline micro models, like the life-cycle hypothesis. In 1993, Browning and Orazio Attanasio (PhD LSE 1988) contended that the life-cycle theory could only be studied with aggregate data if very stringent aggregation conditions were satisfied because it is based on individual optimization. “Under what conditions will aggregate time series behave as if they were generated by representative individual? Under what conditions is it possible to identify the parameters of individual behavior from aggregate data? The most common reaction of macroeconomists to these problems has been to sweep them under the carpet. Most of the empirical work on testing the life cycle models *assumes* the existence of a representative consumer,” they explained (Attanasio and Browning 1993, 3).

The pair met in Stanford at the turn of the 1990s, but Attanasio was introduced to aggregation in the first microeconomic courses that he took as an LSE student back in the 1980s.⁴³ One was taught by Steve Nickell, who discussed those aggregation issues that arise when economic relationships are non-linear (interview). Together with former fellow graduate student Guglielmo Weber (PhD LSE 1988 under Christopher Pissarides), Attanasio also investigated the gap between macro and micro estimations of the intertemporal substitution in consumption based on the British Family Expenditure Survey for 1970-1986. They reconstructed consumption means for cohort aged from 30 to 40 at the beginning of the sample. This pointed to aggregation biases arising from disregarding demographic effects (like the age distribution), non-linearities at the individual level and time varying preferences.⁴⁴

Browning, Attanasio and Weber were all associated with the Institute for Fiscal Studies, which had become the premier institute in microeconomic research in the UK. Founded in 1969, its many members generally self-identified as microeconomists who pursued a longstanding tradition of investigating the macro consequences of micro research through the development theoretical and empirical designs for aggregation. It was eventually IFS director Richard Blundell who, in a 2005 survey with MIT’s Tom Stoker, clearly articulated how the introduction of heterogeneity in economic models created aggregation challenges: “to describe

⁴³ Attanasio was hired as a macroeconomist at Stanford in 1988, where he shared undergraduate and graduate macroeconomics courses with Robert Hall.

⁴⁴ More specifically, Attanasio and Weber (1995) documented large differences between computing the log of mean consumption (as is done when working with aggregate data) and the mean of log consumption (as is done when working with household-level data).

the behavior of a group, one must come to grips with this heterogeneity. In terms of empirical research in economics, this means facing and resolving aggregation problems,” they explained (p. 347). They warned that Krusell and Smith’s approximate aggregation result was due to their specific setup, in which savings become concentrated in the hands of a group that exhibits an almost constant marginal propensity to save. It could probably not be generalized. In their conclusion, the authors praised the “embedding” by Lucas, Kydland and Prescott of “the specification of macroeconomic models and the understanding of the impact of economic policies . . . in the rule of optimal individual behavior.” But they added that such program “cannot be right without taking account of aggregation . . . there must be an explicit bridge to economic aggregates because real people and their situations are so very heterogeneous. Aggregation is essential, because heterogeneity is a pervasive and indisputable fact of life” (p. 384). They argued that data availability, better computers and new techniques to incorporate distributional information into aggregate relationships were reasons to stop “closeting aggregation problems as ‘just too hard’” (p. 385).⁴⁵

At a time when UK economists used rich micro data to explore empirically the influence of microeconomic heterogeneity on aggregates, other researchers in Europe pursued more theoretical investigations on aggregation issues. One such question was whether introducing some degree of heterogeneity on consumers’ incomes and preferences in an otherwise standard Arrow-Debreu general equilibrium framework could overcome the negative conclusions of the Sonnenschein-Mantel-Debreu theorem.⁴⁶ This was the research program developed by German economist Werner Hildenbrand. He was looking for restrictions on the income distribution and preferences that would insure remarkable properties for the aggregate demand functions (especially weak axiom of revealed preferences and gross substitutability). In 1983, Hildenbrand proposed a model in which individuals are grouped in pools where preferences are identical, but with a constraint on the distribution of individual expenditures. The resulting aggregate demand function was monotonic and diminishing with price. However, he tested his hypothesis on the distribution of expenditures on the British data that Anthony Atkinson had given him and concluded that it was not supported empirically.

⁴⁵ By that time, Minnesota students were following the microeconomic work of Blundell closely, and tightening their relations with UCL through exchanges and recruitments (Fernandez-Villaverde, Interview)

⁴⁶ Hugo Sonnenschein (1972), Rolf Mantel (1974) and Gerard Debreu (1974) showed that central properties of individual demand functions (gross substitutability, weak axiom of revealed preferences, etc.) are generally not preserved at the aggregate level.

In a 1990, French theorist Jean-Michel Grandmont, who completed his PhD at Berkeley with Debreu in 1971, took up the problem of finding realistic constraints on the income distribution that were consistent with the existence of a well-behaved aggregate demand function. He conducted this research at Yale, while interacting with Bewley, and acknowledged his intellectual debt to Hildenbrand in the resulting article (Grandmont 1992, 1).⁴⁷ Grandmont generated classes of consumers based on different income profiles. Within the same income class, multiple consumer profiles are created from a referent utility function that is sometimes compressed or sometimes stretched in order to produce what he called “behavioral heterogeneity.” This procedure, he demonstrated, guaranteed the emergence of key properties like monotonicity of the aggregate demand function, as well as unicity and stability of the competitive equilibrium, without any particular assumption about rationality and individual demand functions (other than homogeneity and Walras’s law).⁴⁸ In contrast to the introduction of stochastic and structural heterogeneity in the Minnesota and growth literatures, which made computing solutions more difficult, Grandmont’s behavioral heterogeneity conferred simple and remarkable characteristics to aggregate demand functions which did not exist at the individual level. Heterogeneity was thus not treated as a kind of necessary evil to make models more realistic or consistent with microeconomic data. It became a desirable theoretical characteristic of models: “in contrast to the naive representative agent viewpoint . . . this distributional approach might . . . generate specific hypotheses not only about aggregate demand but also about the *distribution* of the agents’ actual observable choices, that could then be tested on cross-sectional or panel data,” Grandmont (1992, 3) explained. He viewed his contribution as an improvement over the “recent efforts to provide systematic theoretical ‘microfoundations’ to quantitative dynamic macroeconomics through models involving a single optimizing representative agent” (p. 3). Further work examined the matter in a dynamic context. Hildenbrand and Kneip (1999) argued that in this case some consumption profiles become dominant in a class of preferences, generating those simple characteristics of aggregate demand functions.

The methodological divides between the theoretical and empirical literatures in which economists attempted to model the impact of heterogeneous agents on macroeconomic

⁴⁷ Hildenbrand was a visiting professor at Berkeley on several occasions during Bewley and Grandmont’s time as Debreu PhD students.

⁴⁸ For those remarkable properties result from the neutralization of opposite decisions, Berkeley-trained economist and Oxford professor John Quah (2001) refers to “sign-balanced heterogeneity.”

aggregates and the consequences for policy evaluations were significant. This caught the attention of Hall, who was then leading the *Economic Fluctuations and Growth* program of the NBER. Around 1992, he thus asked Attanasio to set up a working group on the “Aggregate implications of microeconomic consumption behavior” to discuss these issues, which eventually became known as the “consumption: micro to macro” group. A couple of years later, Attanasio in turn asked Ríos-Rull – who, he believed, represented the “Minnesota approach” – and Carroll – from the “Cambridge approach” – to co-chair the group with him, so as to “maximize the diversity of approaches represented and ensuing conversations” (interviews with Attanasio, Carroll, and Rios-Rull). He would represent the “London crowd,” who thought about the evidence from micro data and worried about aggregation. He understood that, at the turn of the 1990s, these economists had diverging understanding of what was a good strategy to model the relationships between microeconomic heterogeneity and aggregates, and conceived the group as a forum so that advocates of these approach could “better understand each other” (interview with Attanasio).⁴⁹ The generation that was nurtured in part within this forum, which gradually came to focus on the macroeconomic causes and consequences of inequality, is now the present of macroeconomics.

6. Concluding thoughts

The story we have tried to tell here is far from exhaustive, but we hope that it is representative of interesting features in the development of macroeconomics. We have attempted to present an overview of the development of heterogeneous agents models in macroeconomics, although it is by no means comprehensive. We have highlighted, first, that if macroeconomics encompasses all work related to the study, explanation, forecasting, and policy implications of aggregate behavior, then a historical narrative that emphasizes the exclusive focus on aggregates before the 1970s, followed by a turn toward microfoundations resulting in a reliance on representative agents, and a post-2008 crisis rediscovery of heterogeneity, is a flawed one. We have demonstrated that the development of representative agent general equilibrium models was accompanied by efforts to incorporate agent heterogeneity to improve their ability to match the growing body of stylized facts, at least in the U.S. We also found that the use of stochastic and structural heterogeneity in aggregate models of the business cycle and growth patterns

⁴⁹ See *NBER Reporter*, Fall 1996, pp. 5 and 8. The *Economic Fluctuations* program also hosted an “Income Distribution and Macroeconomics” group (led by Robert Benabou and Steven Durlauf) and a “Micro and Macro Perspectives on the Aggregate Labor Market” group (led by Richard Rogerson and Randall Wright).

respectively was complemented by other traditions where microeconomists studied the effects of structural heterogeneity in tastes, skills, time preferences, incomes, ages, and other factors on the behavior of aggregates. In some cases, these developments resulted from a desire to match new empirical evidence, study reactions to shocks, improve policy evaluation as well as from an acknowledgment that heterogeneity had been a feature of economic behavior and that (dis)aggregation, that is how to move from heterogeneous micro behavior to macro aggregates (and vice-versa), was an essential component of economists' work.

The history of bringing heterogeneity in macroeconomics models shows that different communities of economists made distinct epistemic choices. Minnesota researchers considered it essential to operate in a quantitative general equilibrium framework with microfoundations, which partly determined their focus on stochastic heterogeneity. This type of heterogeneity initially failed to make much of a difference compared to representative-agent models. In contrast, growth theorists working with structural heterogeneity attempted to expand the theoretical results from the general equilibrium literature that arose out of the works of Arrow, Debreu and McKenzie. Other US- and UK-based macro- and microeconomists proved more willing to abandon general equilibrium settings yet retain theoretical microfoundations or structural estimation. This allowed them to simulate a larger range of models in which various types of structural heterogeneity affected aggregate consumption, wealth, the elasticity of labor supply or interest rates. A more radical break from microfoundations, not covered in this article, resulted in the exploration of agent-based models with a collection of heterogeneous agents following simple behavioral rules.⁵⁰

As documented by Kevin Hoover (2012), Pedro Duarte and Gilberto Lima (2012), or James Hartley (1997), discussing the relationships between individual behavior and aggregates has always been pervasive in the history of macroeconomics. "The relations between aggregates have to be consistent, to be sure, with our knowledge of the behavior of single firms and households with regard to single goods," Cowles Director Jacob Marschak said in the introduction of his 1948-1949 Chicago course, not yet on "macroeconomics", but on "income, employment and the price level" (Marschak 1951, 1.1). Marschak proceeded to study consumption and investment expenditure and to spend one fourth of the course on aggregation.

⁵⁰ See Truc and Dal Pont 2022 for a history of the development of agent-based models, in particular in macroeconomics. For an overview of those macroeconomic traditions which rejected both general equilibrium and microfoundations, including post-keynesians, Austrians and institutionalists, see King (2012).

Similarly, Klein (1947, 56) pointed to the “derivation of a theory in terms of communities of individuals and groups of communities from a basic theory in terms of individuals and single commodities . . . passing from micro to macro economics, i.e., aggregation.” He found that the problem had not been adequately considered by Keynesians. Solow (1986, 196) later reflected that “in a sense, macroeconomics always had micro-foundations. Whether Keynes or Pigou or anyone proposed an aggregative relationship, its particular form was always defended by a microeconomic story.” Lucas traced the quest for microfoundations back to Patinkin, while Sargent (2022, 7) acknowledged that Tobin, Modigliani, Solow and Jorgenson pursued “a longstanding project of providing microfoundations of Keynesian economics.”

An interesting consequence of this alternative narrative, in which “aggregation theory and empirical work” are restored as a key part of economists’ activity across the 20th century, is that it blurs the boundaries between micro and macro. At least, it creates a difference between studying economists who self-identify as macroeconomists and studying economists who actually contribute to macroeconomics. In the 1980s and 1990s, many of them, across Europe and the US, either identified as microeconomists, or econometricians, or aggregation theorists. How much history of micro is necessary to write the history of macro is thus an open question.

Online Interviews

Robert Becker (Feb. 23 2022)
Truman Bewley (Feb. 23 2022)
Mark Huggett (Feb. 24 2022)
Ayse Imrohoroglu (Feb. 28 2022)s
Per Krusell (March 2 2022)
Anthony Smith (March 14 2022)
Gary Hansen (March 15 2022)
Christopher Carroll (May 17 2022)
José-Víctor Ríos-Rull (May 27 2022)
N. Gregory Mankiw (May 27 2022)
Orazio Attanasio (Sept. 23 2022)
Jesús Fernández-Villaverde (Jan 23 2023)

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