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# **A multi-model structural analysis of the vaults of Notre-Dame de Paris Cathedral after the 2019 fire and a proposal for a hybrid model merging continuum and discrete approaches**

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## *Abstract*

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After the Notre-Dame de Paris (NDP) Cathedral fire, a structural analysis was undertaken to provide decision support for architects in charge of diagnosis and repair operations. Due to the potential impact of the results on the renovation project, calculations on the vaults and walls of the monument were compared to increase their reliability. This article presents the 3 modelling strategies implemented: 2 discrete block-to-block 3D approaches (DEM and FEM) and 1 continuum 3D approach (FEM). The assumptions common to the 3 approaches are presented. They mainly concern the geometrical model, the thermal actions and a diagnosis methodology called the working point method, previously used by the authors on other Gothic vaults. Comparisons of the results with each other and with on-site deflection measurements should lead to validation of the calculation strategies. The analysis will highlight the strengths and weaknesses of the computational approaches and propose research perspectives. Future developments concern the determination of models of homogenized thermomechanical behaviour of masonry and the development of a new hybrid calculation tool taking advantage of the continuum and discontinuum approaches detailed in this article.

KEY WORDS: Masonry, DEM, FEM, Failure analysis, Structural assessment, Damage model, FCZM model, Notre-Dame de Paris

# 1. Introduction

Masonry is one of the oldest materials used worldwide. However, the understanding and modelling of its mechanical behaviour still faces real scientific obstacles. The reasons for these difficulties lie mainly in the fact that masonry is a multi-scale, anisotropic, heterogeneous composite composed of materials exhibiting non-linear mechanical behaviour [1]. Today, this composite material is the subject of in-depth research for its use in new construction and for the preservation of built heritage.

Masonry constructions, such as buildings classified as Historical Monuments, must be preserved, restored and, as a last resort, rebuilt identically by following the principles of the International Council on Monuments and Sites (ICOMOS - Venice Charter) [2][3]. The question of stability of an old masonry structure having suffered a fire arises in this context [4]. The event of April 15, 2019 at Notre-Dame de Paris (NDP) Cathedral is there to remind us that the masonry heritage is fragile with regard to fire hazards. Thus, the “Structure” working group (WG) of the “CNRS / MC” scientific project was formed around the question of post-fire stability of the vaults of the NDP Cathedral. The aim of the study is to assess the post-fire stability of the vaults and thus to help the architects of NDP with diagnosis and repair operations.

In order to make the results more reliable, this WG launched a comparison of calculation methods in 2020 on the before/after fire structural analysis of NDP masonry elements. Three teams were formed, each using a different approach to the calculation of masonry structures. Therefore, the article focuses, first, on the structures studied, then on the basic assumptions and principles common to the 3 teams. At the time of writing, the results cannot be revealed for confidentiality reasons. However, the 3 kinds of calculation approaches are detailed and compared. Finally, some scientific perspectives resulting from this comparison are discussed.

The structural analysis of masonry structures such as Gothic cathedrals still raises modelling and calculation issues due to the complexity of their 3D geometry and to the materials involved, which are anisotropic, non-linear, highly heterogeneous, and possibly deteriorated. Many tools have been developed to tackle these difficulties specific to old masonry buildings [5]. D’Altri [6] recently proposed a classification of modelling strategies into 4 families:

- (i) Block-Based Models (BBM) with a block-to-block description of the masonry considering the mechanical interactions between blocks.
- (ii) Continuum Homogeneous Models (CHM) in which a homogenized masonry material is considered.
- (iii) Geometry-Based Models (GBM) in which the structure is modelled as a rigid body.
- (iv) Equivalent Frame Models (EFM) in which the structure is idealized into panel-scale structural components.

The two first families, BBM and CHM, are applied in this work. In the continuum model (CHM), the finite element method (FEM) associated with non-linear models coupling plasticity and orthotropic damage is used [7][8][9]. This kind of model has proved its worth for the study of the quasi-static stability of Gothic cross-ribbed vaults [10] and in the assessment of the stability of historical masonry structures under seismic actions [11][12].

Unlike the continuum models, the block-based models (BBM) consider the actual assembly of stones as well as the distinct mechanical properties of each of the constituents, namely blocks, mortar and mortar-block interface. Thus, the intrinsic heterogenic and anisotropic character of the masonry is taken into account. The failure and cracking pattern resulting from this type of model are described in detail in the block-mortar interfaces. The non-linear behaviour of block-mortar interfaces can be considered through joint-elements implemented in FEM codes (Interface-element-based approach according to D'Altri) [13][14] or by contact laws implemented in DEM codes (contact-based approaches according to D'Altri) [15][16][17].

In this comparison, 2 BBM strategies are employed: a “*contact-based approach*” [17] (section 4.1) and an “*Interface-element-based approach*” [18][19][20] (section 4.2). A CHM approach is also used (section 4.3) [7][10]. These 3 calculation strategies need different 3D geometrical models (section 3.3). All of these approaches had previously been used by the authors to simulate old masonry structures and for the design and validation of new structures, but never in the context of thermal action due to fire. It should be mentioned that an analytical approach based on the “*Yield design*” theory [21][22] was used on a simplified 2D geometry to provide an upper bound for numerical simulations; these calculations will not be detailed here.

After a brief presentation of the NDP cathedral and the damage caused by the fire (section 3.1), the first part of this article is devoted to the common calculation hypotheses, and also to an original computation methodology, the so-called “*working point analysis*” described in section 0. Then, the 3 strategies used are detailed, with their specific material models. Finally, some research perspectives are announced, namely, on the one hand, an experimental campaign for the identification of thermomechanical properties and, on the other hand, the implementation of a hybrid calculation strategy.

## 2. Research aims

While the research work presented here obviously has an academic aspect, it also aims to provide decision support for NDP architects in charge of diagnosis and repair operations. The goals are:

- (i) to conduct a structural assessment of the vaults of the NDP cathedral before and after the fire, by using the 3 modelling strategies for masonry developed by 3 teams: 2 BBM and 1 CHM approach;
- (ii) to benchmark the effectiveness of these strategies by confronting their results in order to highlight their strengths and weaknesses in the perspective of further in-depth research on the mechanical behaviour of NDP masonry, and development of a new calculation tool dedicated to historic masonry monuments (DEMMEFI project funded by the French National Research Agency (ANR) [23]).

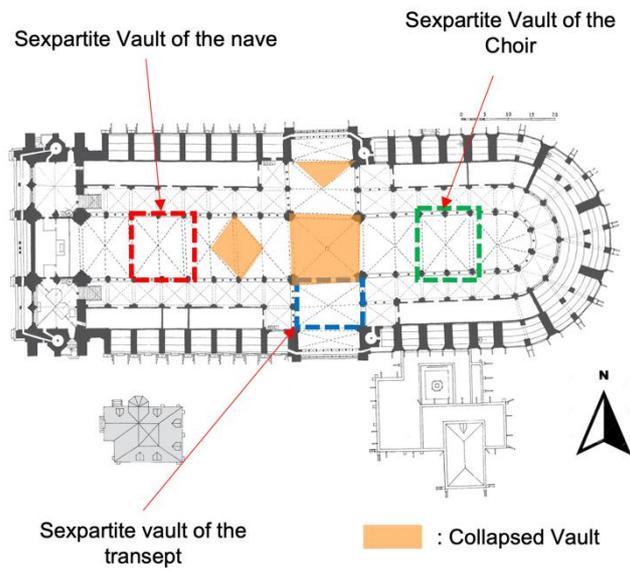
## 3. Common assumptions of the test case

The calculation strategies are compared on the basis of common assumptions related to the geometry (although adapted to each model): the thermomechanical actions, the boundary conditions and above all, the “*working point analysis*”. Before specifying each of these points, some fundamental data on the NDP cathedral should be recalled.

### 3.1. Case study: *Notre-Dame de Paris Cathedral*

The NDP Cathedral is a typical monument of early Gothic architecture. Construction began in 1163. The central vessel is composed by the nave to the West and the choir to the East. The

transept crossing provides the junction between the central vessel and the North and South transepts (Figure 1(a)).



(a)

(b)

Figure 1 : (a) Plan view of NDP Cathedral; in orange vaults that collapsed after the fire and (b)

*Collapse of the transept crossing*

The central vessel is covered by sexpartite ribbed vaults alternately supported by “strong” piers (receiving the diagonal and lateral transverse rib arches) and “weak” piers (receiving the central transverse rib arches) (Figure 2). NDP Cathedral has a double-level interior gallery (first aisle) and a single-level exterior gallery (second aisle). These aisles are covered by quadripartite ribbed vaults. The upper flying buttresses, the slenderest, are supported by the longitudinal side walls of the central vessel, called “gouttereau walls”. The “tas de charge” is the junction between the vaults and the gouttereau walls. The massive buttresses are abutments rising outside the cathedral. They receive the upper and lower flying buttresses.

Crucial parts of the masonry structure were impacted by the fire [4]. In particular, the vault of the transept crossing totally collapsed as a result of the impact of the spire falling while it was aflame (Figure 1(b)). This vault needs to be rebuilt and, therefore, will not be discussed in this work. However, the bearing capacity of most of the vaults, which are still fully in place, must be assessed. They are the subject of the research presented here.

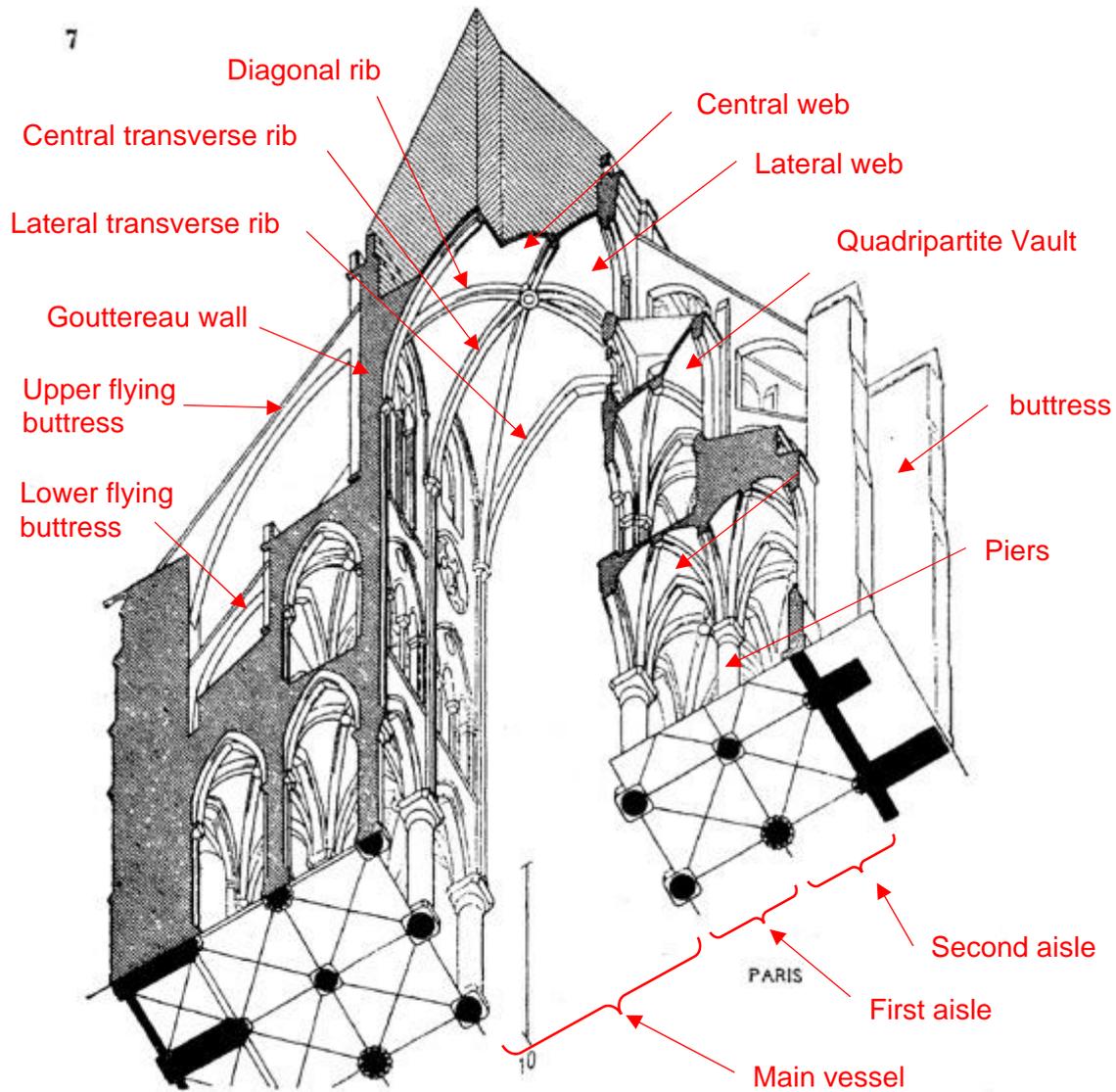


Figure 2 : 3D view of a typical span of Notre-Dame de Paris Cathedral according to [24]

### 3.2. Working point analysis

Structural assessment requires identification of the nature of the external actions (loads and/or displacements) applied to the structure, which could, by their increase, lead to a lack of stability of the structure. For a vault-type structure, the question is usually posed in terms of vertical overloads applied on its extrados. However, this kind of loading is very rare in the central vessel vaults of gothic cathedrals insofar as the extrados of these vaults are generally free from any mechanical actions. The vaults of the central vessel of NDP are part of this framework and, consequently, must be studied with respect to potential actions acting on the vessel structure and possibly leading, for a vault, to displacements of its piers.

Working point analysis (also called equilibrium point methodology), developed in 2014 [10], is used for the structural assessment of three sexpartite vaults of NDP still in place and corresponding to the vaults of the choir (in green), the nave (in red) and the transept (in blue) identified in Figure 1(a). For a given vault, the working point methodology consists of separately estimating the mechanical behaviour of the vault and the supports and establishing the equilibrium point (or working point) of the two systems combined: vault and supports. Note that the structural assessment is reduced to the study of horizontal loading here, i.e. loading perpendicular to the gouttereau walls (induced by wind, for example), leading to symmetrical horizontal loading of the opposite vault-support interfaces.

The first step is to separate the sexpartite vault (part in blue in Figure 3) from its supports (structural set in green in Figure 3). The interface between these two sets corresponds to (i) the vertical planes of the internal faces of the gouttereau walls (the so-called “bahut” walls) and (ii) the horizontal planes located halfway up the horizontal cross sections between the upper flying-buttress and the gouttereau walls. Note that these horizontal planes pass approximately through the upper level of the fillings of the haunches of the vaults as shown in Figure 3.

At this interface between the two substructures (vault and support), the resulting horizontal force acting on the vault-support interface is noted  $\vec{F}$  and its associated displacement is noted  $\vec{\delta}$ . The numerical simulations consist of estimating the variation of the horizontal force  $\vec{F}$  as a function of its associated displacement  $\vec{\delta}$  (mechanical behaviour law) for the two substructures independently, for each vault-support interface (Figure 3 (c)). For a given interface, the working point is the intersection point between the two curves and hence corresponds to the equilibrium point (couple pushing force – associated displacement) at the vault-support interface. Furthermore, plotting the mechanical behaviour laws of the vault and of its support on the same graph allows the load-bearing capacity of the vault and its support to be estimated directly.

The working point and the horizontal load-bearing capacity are expected to be different for “strong” and “weak” piers, due to the specific typology of sexpartite vaults. These aspects will be particularly analysed from the structural assessment.

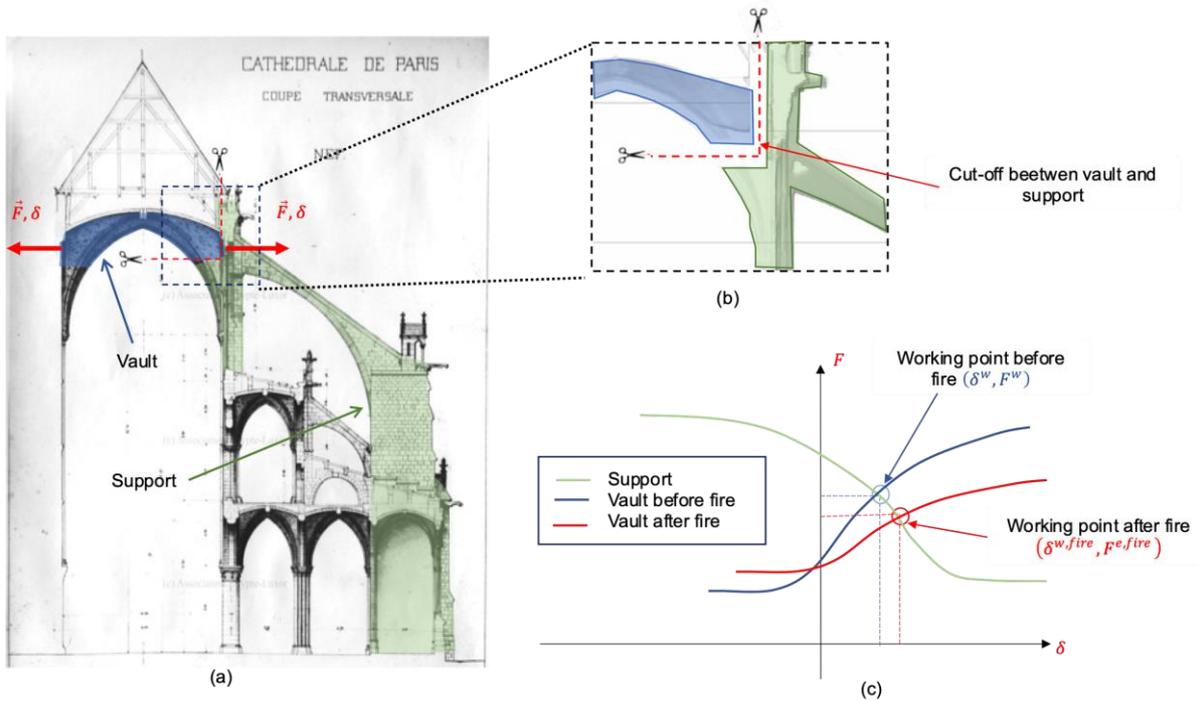


Figure 3 : (a) General view of the cut-off needed for the working point analysis (b) Zoom on the zone of interest (c) Schematic of the Force-Displacement law expected for the support and the vault before and after fire.

For each of the three vaults studied (choir, nave, transept), the working point methodology is implemented before and after fire (Figure 3(c)) in order to estimate the evolutions of the equilibrium displacement and of the horizontal load-bearing. The red curve in Figure 3(c) represents the residual mechanical behaviour law of the vault after fire.

The thermal action is further considered in section 3.5 while the mechanical characteristics of materials are estimated from literature values [10][16][28] previously used by the authors, in agreement with the NDP architects (section 3.3).

The advantages of the working point methodology are worth highlighting. First, the fact of considering the vault and its supports separately makes it possible to limit the computation times. Second, the disjunct study of the vault and its supports allows the mechanical behaviour of each component of the structure to be analysed independently, from the equilibrium state to the assessment of the residual capacity. Finally, this type of representation of the structure equilibrium is intuitive and educational, which is of great help when the research is used by those involved in heritage restoration during the decision phase.

### 3.3. Mechanical properties of materials

The mechanical properties of the limestone, mortar joints and stone/mortar interfaces are determined from:

- (i) Studies carried out by the LRMH laboratory on limestone cores drilled from NDP and tested in laboratory [29]. Cores were extracted from webs, ribs and lateral walls.
- (ii) Non-destructive campaign based on sound velocity measurements to complete the characterization of the limestone of different structural elements of the Cathedral using correlation between sound velocity and mechanical properties [36].
- (iii) literature values previously used by the authors, in agreement with the NDP architects [10][16][28].

In summary, the main mechanical properties of the masonry homogenized material used in the continuum model (CHM) are reported in Table 1 while the main properties of the block mortar interface used in the 2 block-based model (BBM) are reported in Table 2. It can be noted that the mechanical properties of each of the 3 models were chosen in order to obtain equivalent mechanical behaviors of the masonry, and that a quasi no-tension approach ( $R_t = \sigma_I^e = 0.1 \text{ MPa}$ ) were implemented.

Table 1 : Main mechanicals properties of the homogenized material (CHM model)

Mechanical properties	Notation	Value	Unit
Bulk Density	$\rho$	1950	$kg.m^{-3}$
Homogenized Young modulus	$E$	1.3	$GPa$
Poisson ratio	$\nu$	0.2	-
Masonry compressive strength	$R_c$	10.0	$MPa$
Masonry tensile strength	$R_t$	0.1 (interface)	$MPa$
Tensile Fracture energy	$G_t$	4.5	$J.m^{-2}$

Table 2 : Main mechanicals properties of the block/mortar interface (BBM models)

Mechanical properties	Notation	Value	Unit	
Mode I (Normal direction)	Maximum stress	$\sigma_I^e$	0.1	$MPa$
	Cohesive energy	$Gf_I$	4.5	$J.m^{-2}$
Mode II (Tangential direction)	Maximum stress	$\sigma_{II}^e$	0.5	$MPa$
	Cohesive energy	$Gf_{II}$	300	$J.m^{-2}$
Frictional parameter	Frictional coefficient	$\mu$	0.80	-

### 3.4. Geometry

As mentioned above, it was decided to restrict the study area to three typical spans of the central vessels of the NDP Cathedral, chosen among the vaults that were still in place. The structural elements composing these modules are: (i) sexpartite vault, (ii) main flying buttress, (iii) abutment and (iv) upper part of the gouttereau wall. In order to limit the mesh preparation time and the calculation times, the (approximate) double symmetry of the sexpartite vaults and supporting systems was taken into account. Thus, the calculations were carried out on a quarter of the module.

The geometric envelope of the structure has been defined by close fitting to the point-cloud data collected before the fire in a laser-scanner survey carried out by Andrew Tallon [30]. This envelope defines the geometry of the model for both CHM and BBM approaches. The fitting of the envelope to the point-cloud is performed manually for simple geometries (buttress, flying buttress, gouttereau walls) and parametrically for the vault ribs and webs, using non-linear regression fitting of b-splines on the point-cloud and mean surface parameterization.

In the case of CHM (Figure 4(a)), the volume inside the envelope is occupied by a homogeneous, isotropic material, the behaviour of which is described later. In the case of BBM, information on the geometry needs to be enriched by a segmentation of the volume into blocks (Figure 4(b)). For the flying buttress and for the ribs of the vault, this segmentation conforms to the stereotomy as it actually appears on the surface of these elements. On the other hand, the pattern of stone masonry of the vaults was inferred from a specific analysis. Based on observations (only a limited number of beds being observable), a plausible discretization strategy for the vaults was adopted. A parametric set of blocks was generated using the “fishnet” algorithm: their vertices were arranged on the double-curved surface like those of a Chebyshev network [25][26][27], with one of the generating directions being parallel to the beds, as is observed on the real structure.

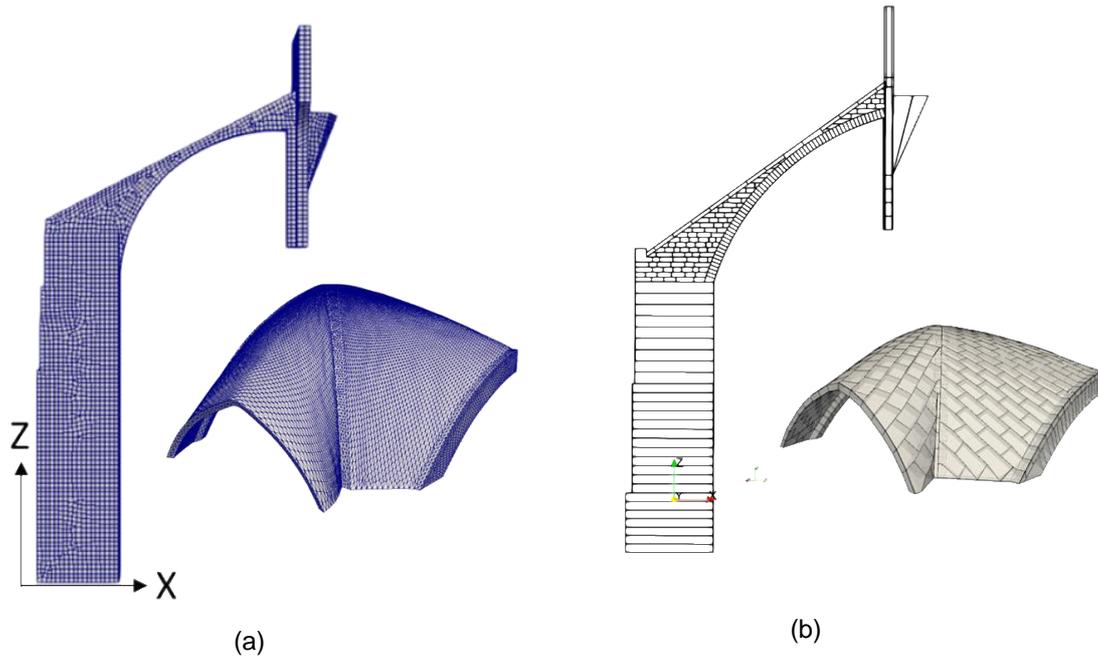


Figure 4 : Mesh of the strong pier and quarter of the sexpartite vault of the choir: (a) continuum and (b) block-based approaches

### 3.5. Actions: mechanical and thermal loads

The sexpartite vaults have been subjected to several actions: (i) during the fire, thermal flux action on the limestone blocks and lime mortar of the vaults of the central vessel, (ii) water action during the extinguishing of the fire by the firefighters on these same materials, (iii) mechanical shocks generated by the fall of structural elements and (iv) overloading of the vaults due to burnt wooden beams, the supply of extinguishing water and the weight of the molten lead of the roof.

In this work, the behaviour of the structural elements was calculated by considering only their self-weight and the residual thermal damage of the vault materials caused by the fire. It was assumed that the buttresses and the gouttereau walls were not impacted by the fire (Figure 3(c)). The before/after comparison then relied on two types of calculation: ambient calculation before the fire and ambient calculation once the fire had been extinguished. The difference between the two calculations allowed the mechanical impact of the fire to be estimated, in terms of a degradation gradient of the material mechanical properties (Young's modulus and strengths).

It was therefore necessary to first estimate the maximum temperature gradient reached in the stones during the fire.

In the absence of fire models sufficiently advanced to describe the cathedral fire, two thermal scenarios have been considered *a priori* in this study. The first scenario predicts a rapid gas temperature rise (equivalent to the temperature rise predicted by the conventional "ISO 834-1" fire curve [32] followed by a linear cooling phase of one hour. The second thermal scenario shows a less severe gas temperature rise but with a longer temperature stabilization (500°C for 6 hours). The first scenario could be applied to the interior cladding of the gouttereau walls, which were mainly stressed by radiation and convection from the burning timber roof. The second scenario could be applied to the extrados of the vaults, which could have been solicited by burning wood and then by piles of embers. The heat diffusion is then simulated by the finite element code Cast3M [33] on a stone thickness of 20 cm (representative of the vaults of the Cathedral). The heat exchange coefficients (convection and radiation) are taken according to the requirements of Eurocode 1, fire part [34] for a conventional fire curve. In the absence of precise data on the cathedral limestone, two sets of literature data are used [35], allowing to simulate on the one hand an insulating limestone (thermal conductivity  $\lambda = 1 \text{ W/mK}$ ) and on the other hand a rather diffusive limestone (thermal conductivity  $\lambda = 2 \text{ W/mK}$ ). Figure 5 shows the maximum temperature profile reached for each simulation case (thermal scenario and thermal properties of the stone). In this figure, the shaded area therefore corresponds to the range of temperature potentially reached by the material during the fire, depending on the depth in the stone. To perform a safe mechanical calculation, the upper envelope curve of the simulated thermal profile is finally retained.

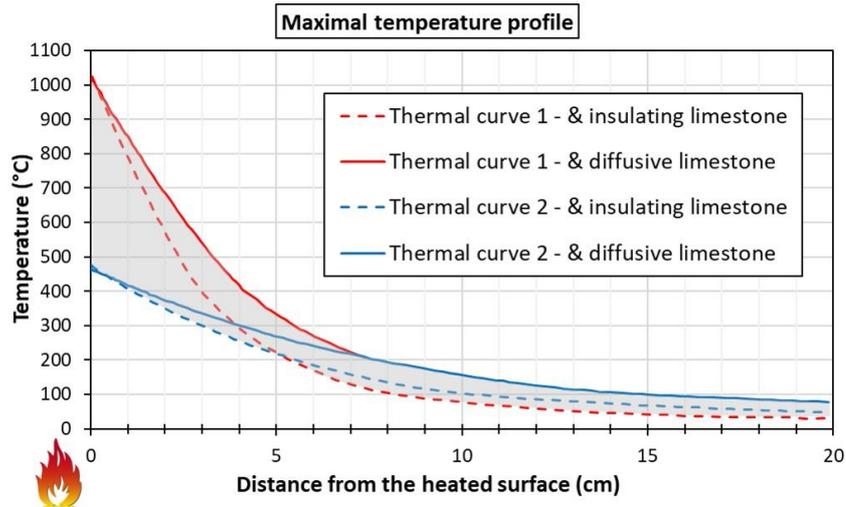


Figure 5 : simulated maximal temperature profiles in the stone (20 cm thick) after the fire, depending on the thermal scenario and the stone thermal property.

Based on this maximum estimated temperature gradient reached in the stone, information available in the scientific literature [37] was used to estimate the residual mechanical characteristics of the limestone. A gradient of mechanical properties in the direction normal to the vaults was then imposed from the extrados to the intrados. Figure 6(a) schematically describes the progressive decrease in Young's modulus from the fire-exposed surface (extrados) to a depth of 20 cm, at which the initial modulus is conserved.

This simplified approach will be further developed in the framework of the research projects following this work. In particular, the thermal gradients reached in the stones will be refined on supplementary cores and thermal analyses (ATG, DSC, SEM...). The temperature evolution of mechanical properties in the different constituent materials of the vaults (limestone, lime mortar and interface) will be studied in depth.

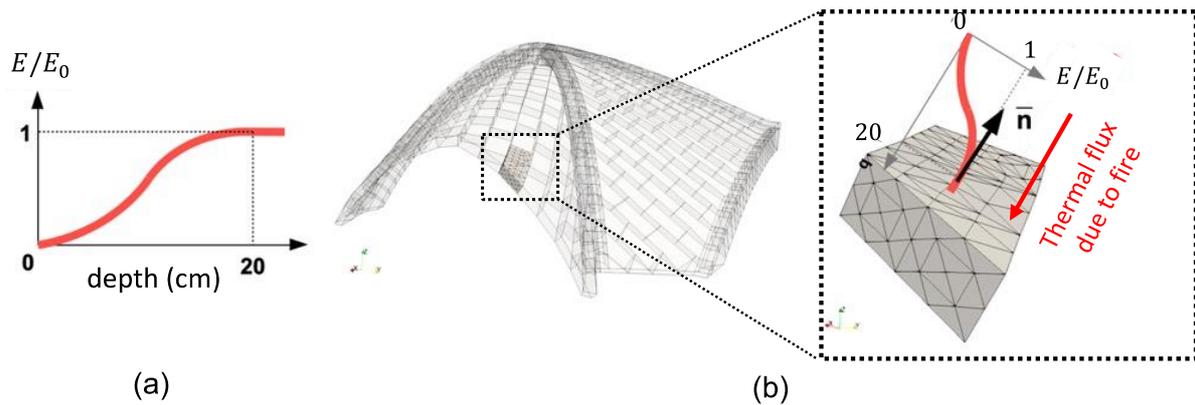


Figure 6 : Progressive degradation of the mechanical properties through the thickness of the stones of the vaults due to the fire (a thickness of 0 corresponds to the extrados of the vaults): (a) damage law implemented in the simulations for all the vault stones (b) for each individual stone of the vault, the degradation profile is steered along the normal vector to the exposed surface (from extrados to intrados).

#### 4. Multi-model mechanical analysis of a span of Notre-Dame Cathedral

In this section, the three calculation models presented in the introduction are detailed successively: (i) the “*DEM Block-Based Model*”, (ii) the “*FEM Block-Based Model*” and (iii) the “*FEM Continuum Model*”.

##### 4.1. *DEM Block-Based Model*

First of all, a distinct element approach is used with LMGC90 software [17]. A 3D model is designed based on (i) a block-by-block discretization of the masonry and (ii) an interaction law which describes contact between blocks by modelling the behaviour of the mortar-block interface (Figure 7).

Blocks are considered deformable and modelled by finite elements here. An homogenized isotropic elastic behaviour is used to take into account both the elasticity of the joint and the stone [1]. Contact laws between blocks describe the linear, damageable and post-rupture frictional behaviour of masonry joints [15]. The approach is based on the notion of hard contact – no interpenetration of blocks– and is solved using the NSCD algorithm [31] (implicit method), which is the very basis of the LMGC90 software [17].

The interaction law is solved using a modified friction contact model. The constitutive equations of the model are described below (eq. 1 to eq. 5). Where  $\sigma \begin{pmatrix} \sigma_I \\ \sigma_{II} \end{pmatrix}$  is contact force vector,  $\sigma_I^e$  and  $\sigma_{II}^e$  are respectively the normal and tangential cohesive strength,  $\delta \begin{pmatrix} \delta_I \\ \delta_{II} \end{pmatrix}$  the displacement jump of the interface,  $d$  the damage parameter,  $\mu_c$  the frictional coefficient and  $n$  a parameter describing the coupling between cohesive and frictional behaviours. eq. 1 describes the normal part of the cohesive interaction law and eq. 2 describes the tangential part of the cohesive interaction law (if the equality is verified, there is sliding and if it is strictly inferior, there is sticking). eq. 3 gives the evolution of friction coefficient with respect to the damage, eq. 4 gives the evolution of cohesive strength and eq. 5 is a damage evolution function.

$$\sigma_I - \sigma_I^e \geq 0 \perp \delta_I \geq 0 \quad \text{eq. 1}$$

$$|\sigma_{II} - \sigma_{II}^e| \leq \mu(d)(\sigma_I - \sigma_I^e) \quad \text{eq. 2}$$

$$\mu(d) = d^n \mu_c \quad \text{eq. 3}$$

$$\begin{pmatrix} \sigma_I \\ \sigma_{II} \end{pmatrix} = (1-d) \begin{bmatrix} C_N & 0 \\ 0 & C_T \end{bmatrix} \begin{pmatrix} \delta_I \\ \delta_{II} \end{pmatrix} \quad \text{eq. 4}$$

$$g(d, \delta) = 0 \quad \text{eq. 5}$$

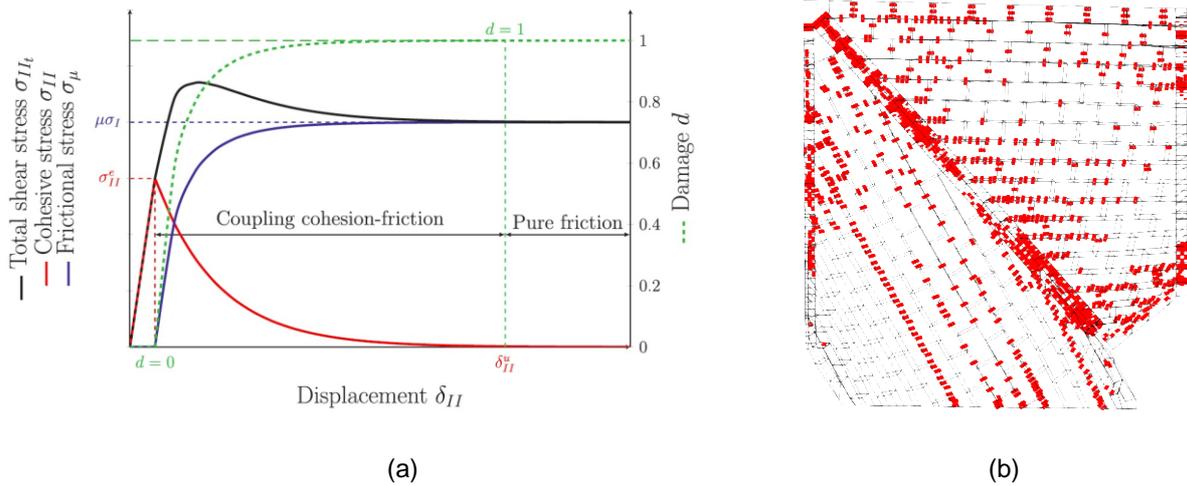


Figure 7 : DEM Block-Based Model (a) Mode II FCZM law and (b) Mapping of interfaces damaged during displacement of the arch supports towards the interior of the nave, viewed from above.

The cohesive zone law has the following specificities. Under combined traction and shear loadings, the interface is subjected to mixed-mode I + II. The parameters of the mixed-mode law are computed thanks to a mixing ratio, and from two criteria: a so-called damage initiation criterion and a failure criterion making it possible to estimate the failure energy in mixed mode. Under combined compression and shear loadings, the stress-displacement law of the interface takes account of the coupling between the cohesive behaviour of mode II of the interface and the frictional behaviour (Coulomb's friction) with respect to the damage level of the interface (Figure 7(a)).

#### *4.2. FEM Block-Based Model*

The second model is also based on a block-to-block discretization of the masonry but only uses the finite element method (Cast3M code) [19][20]. The stone blocks are modelled with homogeneous, isotropic, elastic three-dimensional elements and the mortar joints with interface elements showing unilateral, non-linear behaviour with (i) a forbidden penetration (by penalization), (ii) decohesion (rupture in mode I) and slipping (rupture in mode II) authorized according to a Mohr-Coulomb criterion with an associated flow rule. The model response in mode I of the joints presents firstly an initial stiffness and then a perfect plastic flow in tension (Figure 8(a)). The block Young's modulus and the stiffness of joint elements were both readjusted in order to match the homogenized modulus value used in the block-based DEM approach.

In order to better compare the results of this elastoplastic approach with the results obtained by the other two approaches, which both involve progressive damage, each calculation was systematically carried out in two ways: (i) with the tensile strength and the cohesion of the joints considered equal to zero and (ii) with these two parameters considered equal to a fraction of the values of elastic limit stress in mode I and in mode II used in this work by the other two models. It was thus possible to surround the results of the other two approaches by a lower bound, similar to an assembly in dry frictional joints, and an upper bound, for which the results must be analysed critically in cases of large deformations. The dry frictional joints simulation, which uses the simple Coulomb cone criteria (Figure 8(c)), represents a conservative approach. On the other hand, the cohesive joints simulation, which uses the truncated Mohr-Coulomb cone criteria (Figure 8(d)), allows a cohesion and resistance in traction but overestimate the dissipated plastic energy after important relative displacement between two blocks.

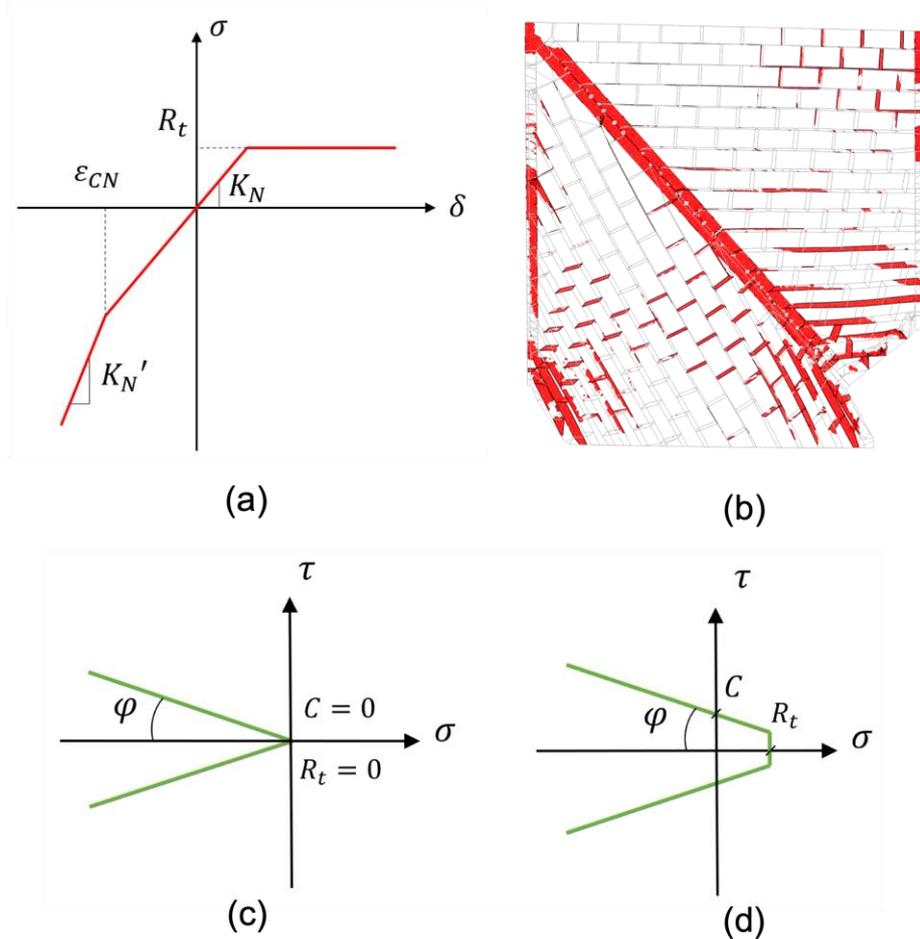


Figure 8 : FEM Block-Based Model (a) Mode I interface law with non-zero interface tensile strength (b) Mapping of joints experiencing a positive opening crack during displacement of the arch supports towards the exterior of the nave, viewed from above, (c) Mohr Coulomb criteria for the dry frictional joints simulation and (d) Mohr Coulomb criteria for the cohesive joint simulation

### 4.3. FEM Continuum Model

The third model is based on the finite element method (Code\_Aster) associated with a non-linear model coupling plasticity and orthotropic damage due to induced cracking using the rotating crack model [7]. The linear homogenized characteristics are isotropic. This constitutive law has already proved its worth in the study of the quasi-static stability of Gothic cross-ribbed vaults [10]. It takes account of (Figure 9(a and b)):

- The asymmetry of tensile/compressive behaviour of the material, indeed, the tensile strength of the masonry is usually really small compare to the compressive strength

- The anisotropic post-peak tensile damage (through cracking) due to a softening behaviour that causes the apparent modulus and the residual tensile strength decreases,
- The calculation of the crack openings resulting from the plastic strains and the Hillerborg method to regularize the dissipated energy (Figure 9(c)),
- The compressive-shear damage with a Drucker-Prager criterion which considers the increase of the shear strength with the normal stress;
- The possibility of reclosing tensile cracks in case of unloading (reduction of the applied tensile force) or of compressive reloading of an element previously damaged in tension,
- Compression dilatancy (increase in volume as the peak approaches),
- Thermal damage that influences non-linearly strengths and modulus with the increase of temperature,
- Thermal expansion is taken into account with a non-linear coefficient of thermal expansion varying with the temperature.

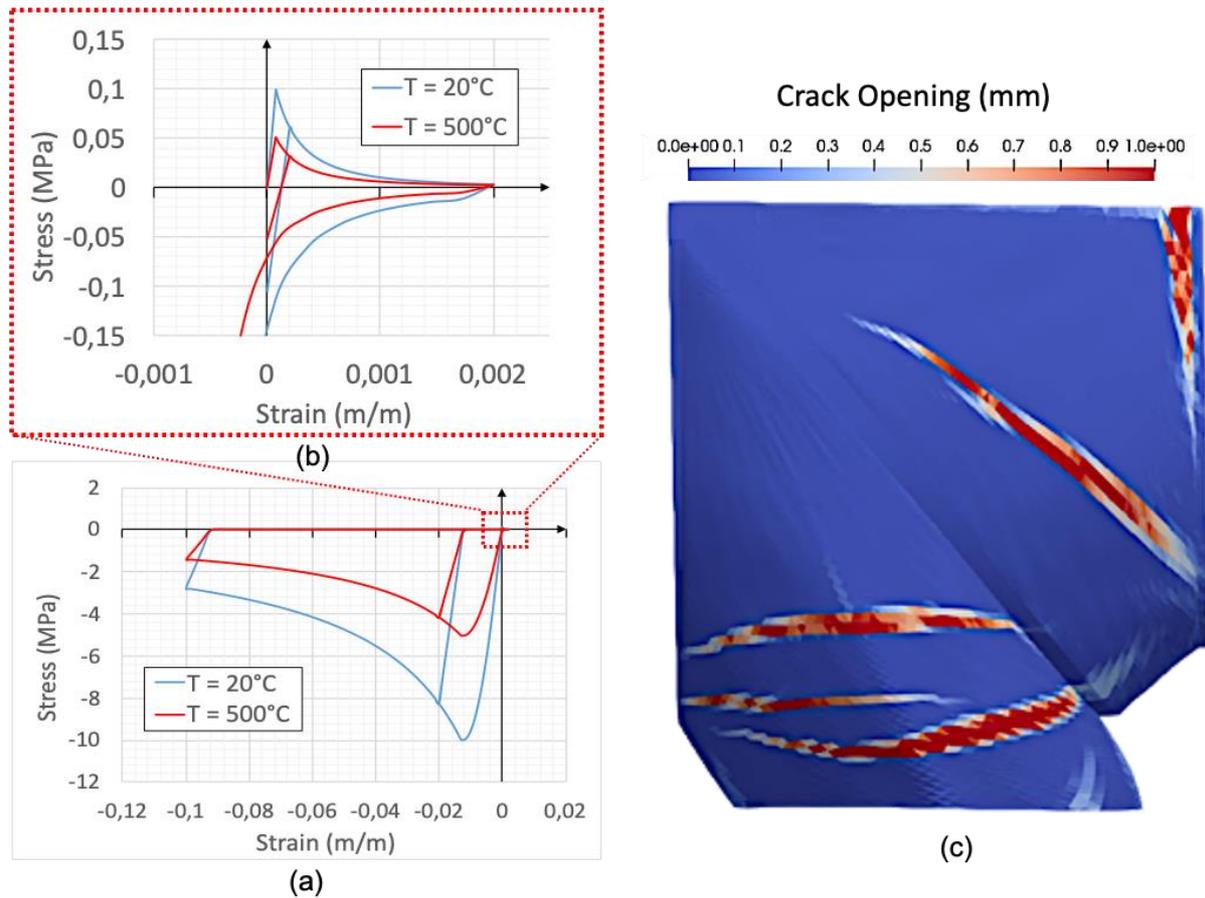


Figure 9: Continuum model based on FEM. (a) Response of the orthotropic damage law in cyclic tension and compression at two temperatures (20°C and 500°C), (b) Zoom on the tensile part and (c) Mapping of crack openings during displacement of the arch supports towards the interior of the nave, viewed from above.

#### 4.4. Review and comparison of models

At the time of writing, the studies are being finalized. The results of this comparison of calculation methods will be published later. However, it can be emphasized that the strategies implemented showed their efficiency in simulating the mechanical behaviour of NDP vaults and gave comparable results. In addition, *in situ* measurements to quantify the impact of fire were used to validate the numerical simulation. These experimental measures consisted of a comparative study of the point-clouds before and after the fire, which was carried out by the BESTREMA engineering office. It provided fields of vertical displacements before and after the fire, at the intrados of the vaults studied.

The second objective of this work was to compare the results of the 3 teams in order to highlight the advantages and limitations of the discontinuum (BBM) and continuum (CHM) approaches in the evaluation of monumental masonry structures. It appears that the CHM approach (FEM continuum model), which is less costly in terms of computational resources, is faster to implement than the discontinuum approaches. This model has the advantage of precisely simulating the non-linear behaviour of the homogenized masonry material [6]. However, this kind of model cannot take the influence of stereotomy on the cracking pattern into account and cannot consider post-peak frictional behaviour of broken masonry joints, since the nodes of the finite elements remain linked with each other. As a result, the kinematics of certain failures and instability phenomena cannot be modelled correctly.

On the other hand, the discontinuum approaches (FEM and DEM Block Based model) have shown their ability to take the non-linearities observed at the stone-mortar interfaces into account, and to handle the complex failure kinematics. Nevertheless, the approach used has the disadvantage of not being able to correctly simulate the damage of continuum block elements. Another limitation of this kind of approach lies in the preparation time of the block-to-block geometric models and also in the higher cost of computational resources.

Finally, the first results were obtained in a very short time (about 6 months) considering the complexity of the task and are now helping those involved the reconstruction, project managers and owners, to refine their diagnosis of the vaults and to decide on possible reinforcement solutions.

## 5. Research actions following this multi-model mechanical analysis

Following this work, long-term research projects have been started with the aim of refining post-fire diagnoses and developing a more efficient masonry calculation tool. These research projects are both experimental and numerical.

### *5.1. Thermomechanical properties of the materials*

The mechanical and thermal characteristics of stones, mortar joints and stone/mortar interfaces will be obtained on the basis of experimental tests. Several complementary approaches will be used. For limestone, mechanical tests will be performed on cores drilled *in situ* to evaluate the elastic properties (Young's modulus and Poisson coefficient) and the fracture properties (compressive and tensile strength, post peak behaviour). As the number of cores must be limited on this type of structure with such high architectural value, a non-destructive campaign based on sound velocity measurements will complete the characterization of the limestone. The speed and ease of this type of non-destructive measurement implies that all of the cathedral's structural elements can be covered. The heterogeneity of the mechanical properties will thus be estimated at the scale of the block and at the scale of the structure. Correlation laws between sound velocity and mechanical properties of limestones will be used to estimate the mechanical properties and their dispersion [36]. The knowledge of the dispersion of mechanical properties will allow for sensitivity analyses to be carried out in order to identify influential and non-influential mechanical parameters on the behaviour of the vaults.

In parallel, an *in-situ* identification of the materials will be carried out. Equivalent materials, with characteristics close to those of the on-site materials, will then be defined for the limestone, the lime mortar and the interface. On this basis, an experimental campaign will be undertaken with the aim of identifying the thermomechanical behaviour of the limestone, the mortar and the stone-mortar interface. The objective is to determine the thermomechanical behaviour of equivalent materials in linear and non-linear phases up to failure, in both cold and hot conditions [37].

### *5.2. Toward a hybrid model merging continuum and discrete approaches*

The model comparison carried out for the structural evaluation of NDP allows the advantages and limitations of the block-to-block (BBM) and continuum (CHM) models to be

benchmarked for the evaluation of monumental masonry structures. From this comparison, the objective is to build a hybrid computational method that combines the advantages of the block-to-block approach and the continuum approach. The hybrid tool is being developed in the framework of the “DEMMEFI” project funded by the French National Agency for Research (ANR) [23]. The “hybrid” model will combine (i) a continuum approach using a finite element method with damage to model the blocks and (ii) a discrete approach implemented in a discrete element method with a frictional cohesive zone model.

At the mesoscopic scale, the hybrid model will simultaneously describe damage in continuum blocks and joints, and the frictional cohesive behaviour of the block/mortar interfaces, so as to finally evaluate the non-linear homogenization of the masonry material. Moreover, at the structural scale, the hybrid approach will allow sub-structures requiring a fine block-to-block description and others using a continuum form to be modelled simultaneously in a single numerical simulation, their properties having been previously homogenized. This flexibility offered in the modelling of the structure will reduce the computation time with respect to a full block-to-block approach.

The implementation will consist of adding the existing sources of the orthotropic damage model [7] to the finite element module of the LMGC90 code [17][38]. In particular, the work will establish the relationship between the strains, the stresses, the internal variable management system of the ENDO3D law [7] and the global variables of LMGC90. Test cases addressing the maximum number of multidirectional mechanical and thermal loading cases will be run on finite elements to validate the implementation. The first hybrid calculations will then be considered.

Once the damage model has been implemented and tested on simple cases, the different hybrid approaches will be tested, and probably optimized, on real representative structures such as walls and fitted vaults. The role of the thermomechanical loads, and the strongly non-linear behaviours of the interfaces and the limestone blocks model will be analysed. The numerical cohabitation of the continuum damage model [7] and the cohesive zone interface model FCZM [15] will be tested and validated.

## 6. Conclusion

The present article exposes the principles of a post-fire structural analysis undertaken in the few months following the fire of the NDP Cathedral. Model comparisons have been carried out by several research teams, using distinct calculation strategies representative of the diversity of the currently available advanced approaches to model masonry structures. The results obtained have proved the complementarity of the teams and the efficiency and robustness of the tools for numerically simulating the mechanical behaviour of the main structural elements of NDP Cathedral and for establishing a comparative diagnosis of the stability of the structure before/after the fire, despite a simplification of hypotheses on the thermomechanical behaviour of materials under the action of fire and its extinction. Further research actions of the members of the “Structure” WG of the CNRS/MC Notre-Dame scientific project will be precisely aimed at increasing knowledge on the thermomechanical behaviour of limestone masonry in order to better take account of the thermal load.

This comparison has highlighted the strengths and weaknesses of the different calculation strategies used at present to assess the structural behaviour of masonry structures. The expertise acquired by the research teams while performing and comparing discrete and continuum simulations on a highly intricate and emblematic medieval construction has led to the development of a hybrid model, coupling discrete and continuum approaches, in order to benefit from the advantages of both approaches.

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