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► **To cite this version:**

Olivier Besson. Adaptive Detection of Gaussian Rank-One Signals Using Adaptively Whitened Data and Rao, Gradient and Durbin Tests. IEEE Signal Processing Letters, 2023, 30, pp.399-402. <10.1109/LSP.2023.3265802>. <hal-04107668>

HAL Id: hal-04107668

<https://hal.science/hal-04107668v1>

Submitted on 26 May 2023

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Adaptive Detection of Gaussian Rank-One Signals Using Adaptively Whitened Data and Rao, Gradient and Durbin Tests

Olivier Besson 

Abstract—We address the problem of detecting a Gaussian rank-one signal using training samples to learn the covariance matrix of the noise present in the samples under test. Towards this end, we propose to use the latter after they have been whitened by the sample covariance matrix of the training samples. As an alternative to the generalized likelihood ratio test, we investigate three simpler alternatives, namely the Rao, gradient and Durbin tests. Closed-form expressions of the corresponding test statistics are derived and the detectors are shown to have a constant false alarm rate. Their performance is assessed via numerical simulations.

Index Terms—Adaptive detection, Durbin’s test, gradient test, Rao’s test, Swerling model, whitened data.

I. INTRODUCTION

SINCE the fundamental works carried out by Kelly in a series of papers and technical reports [1], [2], [3], [4], the generalized likelihood ratio test (GLRT) has imposed itself as the main, not to say systematic, approach to solve composite hypotheses testing problems in adaptive radar detection [5], [6], [7]. Kelly’s framework consists of a set of test samples \mathbf{X} , which possibly contain a target-like signal in addition to Gaussian noise, along with target-free training samples \mathbf{Z} which are used to learn the unknown covariance matrix Σ of the noise present in the test samples. The GLR approach is favored for its numerous advantages, including the facts that it usually satisfies the natural invariances of the problem and that it results in a constant false alarm rate (CFAR) detector, a property of utmost importance in radar detection. It has been declined under multiple variations along the last forty years. A classical variation is to resort to a two-steps GLRT, which consists first in deriving the GLRT assuming that Σ is known, then replacing the latter by the sample covariance matrix of the training samples. The adaptive matched filter [8] is certainly the most famous example of such approach. Discrepancies between the noise in the test and training samples can also be handled, e.g., the adaptive coherence estimator (ACE) which coincides with the GLRT for a partially homogeneous environment for which the noise covariance matrices of the test and training samples differ by

a scaling factor [9]. The GLRT has also been derived based on different models of the target signal. In [10] uncertain rank-one waveforms are considered. Adaptive detection of subspace target signals is addressed in [11], [12]. The case where the assumed target signature differs from the true one has attracted much attention and Kelly analyzed his GLRT under this mismatch in [4]. In this case, one is led to address the trade-off between robustness and selectivity. This can be done by introducing a tuning parameter in Kelly’s GLRT [13], by modifying the null hypothesis and introducing a fictitious signal [14], [15] or by using two-stage detectors [16], [17], [18].

For most adaptive detectors so far, two patterns can be recognized. The first is that most of the time a deterministic model is adopted for the target signal. More precisely the latter is a rank-one vector or matrix with a known signature and an unknown deterministic amplitude. However a stochastic model also makes sense and it is actually what is proposed in Swerling models [19], [20] where the target amplitude is assumed to be random and modeled as the square root of a chi-square distributed random variable while its phase is uniformly distributed. As a result the mean of the test samples distribution is not modified (as is the case for a deterministic model) but its covariance matrix undergoes a rank-one modification under the alternative hypothesis. The GLRT for Swerling I targets was derived in [21] and a high signal to noise ratio approximation was proposed in [22]. A second observation is that, most often, the test statistic depends on the adaptively whitened data $\mathbf{S}^{-1/2}\mathbf{X}$ where \mathbf{S} stands for the sample covariance matrix of the training samples. In [23] we derived the GLRT for Swerling I targets using these adaptively whitened samples. The new detector was shown to perform as well and even better than the GLRT based on the initial data (\mathbf{X}, \mathbf{Z}) .

In this letter, we adopt the same approach as in [23] but look at alternatives to the GLRT. In particular we investigate Rao’s test [24], the gradient test [25], [26] and Durbin’s test [27], see [28], [29] for an overview of these detectors. Rao’s test has been extensively used for adaptive radar detection, see e.g., [30], [31], [32] while the gradient test has been used less often [33]. The advantage of Rao’s test is that it does not require any estimation under the alternative hypothesis, only the Fisher information matrix (FIM) or an estimate of it [34], and is usually rather simple. The gradient test requires the maximum likelihood estimates (MLE) of the unknown parameters under the alternative hypothesis as well as the the score function but not the Fisher information matrix. Durbin’s test is based on the

FIM and the MLE under the alternative hypothesis. Herein we derive these three tests and examine their performance compared to the GLRT.

II. ADAPTIVE DETECTION

In this section we first define the problem at hand and briefly review the results of [23] which will be necessary. Then we derive the Rao, gradient and Durbin tests.

A. The Composite Hypotheses Testing Problem

We let N denote the size of the observation space and we let $\mathbf{X} \in \mathbb{C}^{N \times K_p}$ be the matrix of the test samples and $\mathbf{Z} \in \mathbb{C}^{N \times K_s}$ be the training samples with $K_s \geq N$. Under the null hypothesis \mathbb{H}_0 the columns of \mathbf{X} and \mathbf{Z} are assumed to be independent and distributed according to a complex multivariate Gaussian distribution with covariance matrix Σ . Under the alternative hypothesis \mathbb{H}_1 the covariance matrix of \mathbf{X} becomes $\Sigma + P\mathbf{v}\mathbf{v}^H$ where P stands for the target power and \mathbf{v} for its (unit-norm) signature. We let $\mathbf{Q} = [\mathbf{V}_\perp \ \mathbf{v}]$ be a unitary matrix with $\mathbf{V}_\perp^H \mathbf{v} = \mathbf{0}$ so that $\mathbf{Q}^H \mathbf{v} = \mathbf{e}_N = [0 \ \dots \ 0 \ 1]^T$. The data is transformed to $\tilde{\mathbf{X}} = \mathbf{Q}^H \mathbf{X}$, $\tilde{\mathbf{Z}} = \mathbf{Q}^H \mathbf{Z}$ and we let $\tilde{\mathbf{L}}$ be the lower triangular Cholesky factor of $\tilde{\mathbf{Z}}\tilde{\mathbf{Z}}^H$. Finally we define the adaptively whitened data as $\mathbf{Y} = \tilde{\mathbf{L}}^{-1} \tilde{\mathbf{X}}$ and we partition it as

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} \begin{matrix} N-1|K_p \\ 1|K_p \end{matrix} \quad (1)$$

Then the probability density function (p.d.f.) of $(\mathbf{Y}_1, \mathbf{Y}_2)$ is given by $p(\mathbf{Y}_1, \mathbf{Y}_2) = p(\mathbf{Y}_1)p(\mathbf{Y}_2|\mathbf{Y}_1)$ where $p(\mathbf{Y}_1)$ does not depend on any parameter and

$$p(\mathbf{Y}_2|\mathbf{Y}_1) = \frac{\Gamma(K_p + L)}{\pi^{K_p} \Gamma(L)} |\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1|^{-1} \\ \times [1 + \mathbf{Y}_2 (\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1)^{-1} \mathbf{Y}_2^H]^{-(K_p + L)} \quad (2)$$

with $L = K_s - N + 1$ and $\lambda = 1 + P\mathbf{v}^H \Sigma^{-1} \mathbf{v}$. In other words $\mathbf{Y}_2|\mathbf{Y}_1$ follows a complex multivariate Student distribution with L degrees of freedom and parameter matrix $\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1$. It follows that, considering \mathbf{Y} as our data, we have a simple binary hypotheses testing problem of

$$\mathbb{H}_0 : \lambda = 1 \text{ versus } \mathbb{H}_1 : \lambda > 1 \quad (3)$$

We would like to emphasize that the initial composite hypotheses testing problem has been considerably simplified. Under \mathbb{H}_0 the distribution of \mathbf{Y} is completely known, while under \mathbb{H}_1 it depends on a scalar λ related to the signal to noise ratio. Despite this simplification the GLRT using \mathbf{Y} performs as well and even better than the GLRT based on (\mathbf{X}, \mathbf{Z}) . Moreover, since the distribution of \mathbf{Y} does not depend on any parameter under \mathbb{H}_0 all tests based on \mathbf{Y} will be CFAR.

B. Rao, Gradient and Durbin Test Statistics

We begin with Rao's test based on \mathbf{Y} , which is given by

$$t_{\text{Rao}} = \left[\left(\frac{\partial \log p(\mathbf{Y}_1, \mathbf{Y}_2)}{\partial \lambda} \right) \right]_{\lambda=1} I^{-1}(\lambda = 1) \quad (4)$$

where $I(\lambda)$ is the Fisher information and the term between parentheses is the score function which we will note $s(\lambda)$ for the ease of notation. The latter is given by

$$s(\lambda) = \frac{\partial \log p(\mathbf{Y}_1, \mathbf{Y}_2)}{\partial \lambda} = \frac{\partial \log p(\mathbf{Y}_2|\mathbf{Y}_1)}{\partial \lambda} \\ = -\text{Tr} \{ (\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1)^{-1} \} \\ + (K_p + L) \frac{\mathbf{Y}_2 (\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1)^{-2} \mathbf{Y}_2^H}{1 + \mathbf{Y}_2 (\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1)^{-1} \mathbf{Y}_2^H} \quad (5)$$

We now look at the Fisher information which writes

$$I(\lambda) = \mathbb{E} \left\{ -\frac{\partial^2 \log p(\mathbf{Y}_1, \mathbf{Y}_2)}{\partial \lambda^2} \right\} \\ = \mathbb{E}_{\mathbf{Y}_1} \left\{ \mathbb{E}_{\mathbf{Y}_2|\mathbf{Y}_1} \left\{ -\frac{\partial^2 \log p(\mathbf{Y}_2|\mathbf{Y}_1)}{\partial \lambda^2} \right\} \right\} \\ = \mathbb{E}_{\mathbf{Y}_1} \{ I_{2|1}(\lambda) \} \quad (6)$$

where $I_{2|1}(\lambda)$ is the Fisher information corresponding to the conditional distribution of $\mathbf{Y}_2|\mathbf{Y}_1$. Let us temporarily define $\mathbf{B}_\lambda = (\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1)^{-1}$. Since $\mathbf{Y}_2|\mathbf{Y}_1$ follows a complex multivariate Student distribution with L degrees of freedom and parameter matrix \mathbf{B}_λ^{-1} it follows that [35]

$$I_{2|1}(\lambda) = \frac{-1}{K_p + L + 1} \text{Tr} \left\{ \mathbf{B}_\lambda \frac{\partial \mathbf{B}_\lambda^{-1}}{\partial \lambda} \right\}^2 \\ + \frac{K_p + L}{K_p + L + 1} \text{Tr} \left\{ \mathbf{B}_\lambda \frac{\partial \mathbf{B}_\lambda^{-1}}{\partial \lambda} \mathbf{B}_\lambda \frac{\partial \mathbf{B}_\lambda^{-1}}{\partial \lambda} \right\} \\ = \frac{-\text{Tr} \{ \mathbf{B}_\lambda \}^2 + (K_p + L) \text{Tr} \{ \mathbf{B}_\lambda^2 \}}{K_p + L + 1} \quad (7)$$

and therefore

$$I(\lambda) = \frac{-\mathbb{E} \left\{ \text{Tr} \{ \mathbf{B}_\lambda \}^2 \right\} + (K_p + L) \mathbb{E} \left\{ \text{Tr} \{ \mathbf{B}_\lambda^2 \} \right\}}{K_p + L + 1} \quad (8)$$

As shown in [23] $\mathbf{Y}_1^H \mathbf{Y}_1 \stackrel{d}{=} \mathbf{N}_1^H \mathbf{W}_1^{-1} \mathbf{N}_1$ where \mathbf{N}_1 follows a Gaussian matrix variate distribution with covariance matrix $\mathbf{I}_{N-1} \otimes \mathbf{I}_{K_p}$ and \mathbf{W}_1 follows a complex Wishart distribution with K_s degrees of freedom and parameter matrix \mathbf{I}_{N-1} . Therefore $\mathbf{F}_1 = \mathbf{Y}_1^H \mathbf{Y}_1 \stackrel{d}{=} \mathcal{CF}_{K_p}(N-1, K_p + L)$ where $\mathcal{CF}_p(a, b)$ stands for the complex matrix-variate F distribution with p.d.f. $p(\mathbf{F}) \propto |\mathbf{F}|^{a-p} |\mathbf{I}_p - \mathbf{F}|^{-(a+b)}$. While obtaining the distribution of $\mathbf{B}_\lambda = (\lambda \mathbf{I}_{K_p} + \mathbf{F}_1)^{-1}$ is not difficult, calculating its moments for arbitrary λ seems to be very complicated. At least we did not manage to obtain expressions of $\mathbb{E} \{ \text{Tr} \{ \mathbf{B}_\lambda \}^2 \}$ and $\mathbb{E} \{ \text{Tr} \{ \mathbf{B}_\lambda^2 \} \}$. However, for the purpose of deriving Rao's test, we only need the moments of \mathbf{B}_1 . Now, $\mathbf{B}_1 \stackrel{d}{=} \mathcal{CB}_{K_p}(K_p + L, N-1)$ follows a matrix-variate complex Beta distribution with p.d.f.

$$p(\mathbf{B}_1) \propto |\mathbf{B}_1|^L |\mathbf{I}_{K_p} - \mathbf{B}_1|^{N-1-K_p} \quad (9)$$

Moments of the matrix-variate complex Beta distribution have been derived in [36] where it is shown that

$$\mathbb{E} \left\{ \text{Tr} \{ \mathbf{B}_1 \}^2 \right\} = \frac{K_p(K_p + L)[K_p(K_p + L)K_t + N - 1 - K_p]}{(K_t - 1)K_t(K_t + 1)} \quad (10a)$$

TABLE I
TEST STATISTICS OF RAO, GRADIENT AND DURBIN TESTS

Rao	Gradient	Durbin
$[s(1)]^2$	$s(1)\hat{\lambda}_1$	$\hat{\lambda}_1$

$$\mathbb{E} \{ \text{Tr} \{ \mathbf{B}_1^2 \} \} = \frac{K_p(K_p+L)[(K_p+L)K_t+K_p(N-1)-1]}{(K_t-1)K_t(K_t+1)} \quad (10b)$$

where $K_t = K_p + K_s$ is the total number of samples. Plugging (10) in (8) yields $I(\lambda = 1)$. We note that the latter depends only on (N, K_p, K_s) and is therefore a constant. It ensues that

$$t_{\text{Rao}} \equiv [s(1)]^2 \quad (11)$$

Let us now turn to the gradient test [25], [26] which is given by

$$t_{\text{gradient}} = s(1)\hat{\lambda}_1 \quad (12)$$

where $s(1)$ can be computed from (5) and $\hat{\lambda}_1$ is the maximum likelihood estimate of λ under \mathbb{H}_1 and is given by

$$\hat{\lambda}_1 = \arg \max_{\lambda \geq 1} |\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1|^{-1} \times [1 + \mathbf{Y}_2(\lambda \mathbf{I}_{K_p} + \mathbf{Y}_1^H \mathbf{Y}_1)^{-1} \mathbf{Y}_2^H]^{-(K_p+L)} \quad (13)$$

Finally, we investigate Durbin's test [27] whose expression is

$$t_{\text{Durbin}} = (\hat{\lambda}_1 - 1)^2 I(\lambda = 1) \equiv \hat{\lambda}_1 \quad (14)$$

C. Summary

In Table I we summarize the three new detectors. It is interesting to note that the gradient test sort of mix Rao's test and Durbin's test. Rao's test depends only on the score function evaluated under \mathbb{H}_0 , Durbin's test only on the estimate of λ under \mathbb{H}_1 and the gradient test depends on both. Observe also, as already mentioned, that all detectors have a constant false alarm rate since the distribution of \mathbf{Y} does not depend on any parameter under \mathbb{H}_0 .

III. NUMERICAL EVALUATION

In this section we conduct a numerical study of the probability of detection P_d of the detectors introduced above. We consider a scenario where $N = 16$. The noise is the sum of a correlated noise and white noise, i.e., its covariance matrix is given by $\Sigma = \Sigma_c + \sigma_n^2 \mathbf{I}_N$ where $[\Sigma_c]_{m_1, m_2} = \exp\{-2\pi^2 \sigma_f^2 (m_1 - m_2)^2\}$ with $\sigma_f = 0.01$. We set σ_n^2 so that $\text{Tr}\{\Sigma_c\}/\text{Tr}\{\sigma_n^2 \mathbf{I}_N\} = 20\text{dB}$. The actual signal of interest is given by $\bar{\mathbf{v}} = \mathbf{e}(f_t)$ where $\mathbf{e}(f) = [1 \ e^{i2\pi f} \ \dots \ e^{i2\pi(N-1)f}]^T$. It may differ from the test signal $\mathbf{v} = \mathbf{e}(f_t)$ where f_t is possibly different from \bar{f}_t . The signal-to-noise ratio is defined as $\text{SNR} = P\bar{\mathbf{v}}^H \Sigma^{-1} \bar{\mathbf{v}}$.

First we consider the matched case where $\mathbf{v} = \bar{\mathbf{v}}$. Fig. 1 displays P_d versus SNR for different values of K_p and K_s . From these figures we can observe that the gradient test is equivalent to the GLRT for $K_s = 32$ and incurs a marginal loss when $K_s = 20$, whatever K_p . Durbin's test is very close to the gradient test for $K_s = 32$ but departs from it when $K_p = 20$, which shows the interest of weighting the MLE $\hat{\lambda}_1$ by the score function

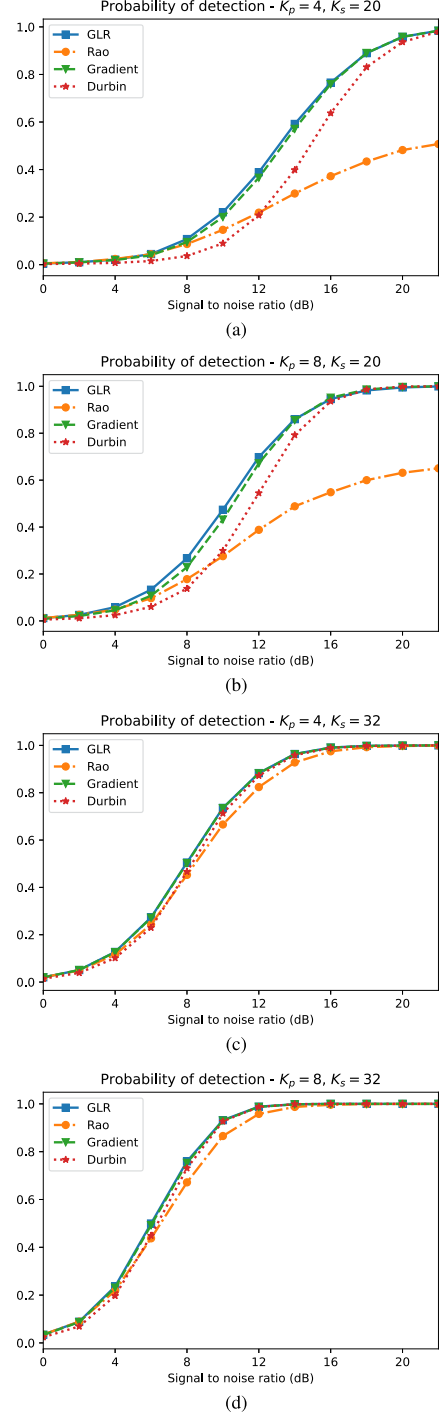


Fig. 1. Probability of detection versus SNR. $\mathbf{v} = \bar{\mathbf{v}}$. (a) $K_p = 4, K_s = 20$ (b) $K_p = 8, K_s = 20$ (c) $K_p = 4, K_s = 32$ (d) $K_p = 8, K_s = 32$.

$s(1)$. Rao's test, based on $s(1)$ only, has the worst performance, especially when $K_s = 20$: otherwise for $K_s = 32$, it is not very far from the three other detectors.

We now consider a mismatch $\bar{f}_t = f_t + \Delta f$ and we plot in Fig. 2 P_d versus $N\Delta f$. Again, one can see the same behaviour between the gradient test and the GLRT. Rao's test is seen to be the most selective while Durbin's test offers an interesting trade-off between them.

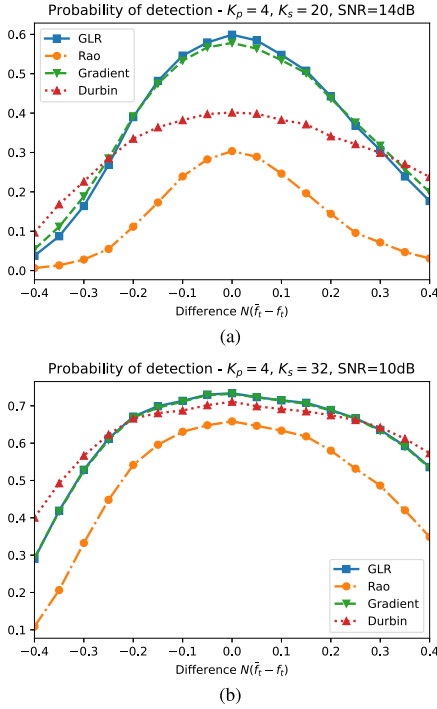


Fig. 2. Probability of detection versus $N(\bar{f}_t - f_t)$. (a) $K_p = 4$, $K_s = 20$, SNR = 14 dB (b) $K_p = 4$, $K_s = 32$, SNR = 10 dB.

IV. CONCLUSION

In this letter we considered adaptive detection of Swerling I targets using adaptively whitened data. As an alternative to the GLRT derived in [23] we investigated somehow simpler approaches, namely Rao, gradient and Durbin tests. These tests depend either on the score function (Rao's test), the maximum likelihood estimate of the signal to noise ratio (Durbin's test) or both (gradient test). All three enable to obtain a constant false alarm rate. We showed that the gradient test has a performance close to that of the GLRT, that Rao's test is selective but incurs some loss in the matched case, and that Durbin's test has an in-between behaviour.

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