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# Verification of a CFD benchmark solution of transient low Mach number flows with Richardson extrapolation procedure

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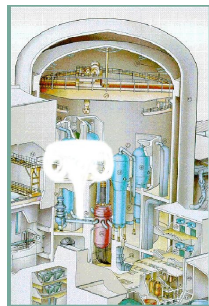
**ASME 2012 Verification & Validation Symposium**

May 2-4, 2012

## Motivation & Physical problem

**Nuclear reactor safety:** containment pressurization due to loss-of-coolant accident in nuclear reactor

- Fluid injection at low Mach number into an axisymmetric cavity
- Mixed convection flow (hot fluid injected in cold atmosphere)
- **H. Paillère et al.**, "Modelling of natural convection flows with large temperature differences : A Benchmark problem for low Mach number solvers. *ESAIM: Math. Mod. & Num. Anal.*, 2005; 39:617-621
- **A. Beccantini et al.**, Numerical simulations of transient injection flow at low Mach number regime. *Int. J. Numer. Meth. Engng* 2008; 76:662-696.
- **S. Benteboula**, Modeling of ideal gas injection into a closed domain at low Mach number: Fractional step and pressure based solvers. Tech. Report: CEA - SFME/LTMF/RT08-034/A 2008.



## Objectives

- 1 Modeling flow injection with large density gradients caused by thermal effects
- 2 Computing numerical benchmark solution of the flow with two different CFD codes
- 3 Estimating numerical errors and the method convergence order
- 4 Applying Richardson extrapolation (R.E) to determine a reference solution

# Low Mach number model

Compressibility effects are characterized by the Mach number  $M = \frac{v}{c}$

- 1 **Compressible flow** ( $M > 0.3$ ): evolution equation for  $\rho$ ,  $\rho(p, T)$
- 2 **Incompressible flow** ( $M \rightarrow 0$ ):  $\nabla \cdot v = 0$  constraint,  $\rho$  constant
- 3 **Low Mach flow**  $M < 0.3$

**Low Mach formulation** (A. Majda and J.Sethian (1985))

Asymptotic analysis  $\gamma M^2 \ll 1 \rightarrow$  Navier-Stokes equations of order zero

## Asymptotic results

- Pressure decomposition: thermodynamic  $P(t)$ , dynamic  $p''(x, t)$
- Density variations decoupled from pressure gradients,  $\rho(T)$ ,  $\nabla \cdot v \neq 0$

## Advantages

- Allows large variations of  $\rho$  due to temperature gradients
- Removes acoustic waves

# CFD codes modeling

## Model 1: Conservative unsteady axisymmetric N-S equations

**S. Benteboula, G. Lauriat,**  
Numerical simulations of anisothermal laminar vortex rings with large density variations.

*Int. J. Heat & Fluid Flow* 2009; 30:186-197.

$$\left\{ \begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) &= 0 \\ \frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) &= -\nabla p'' + \frac{1}{Re} \nabla \cdot \underline{\underline{\tau}} \\ &+ \frac{1}{Fr} (\rho - \rho_0) \vec{z} \\ \frac{\partial T}{\partial t} + \gamma T \nabla \cdot \vec{u} - \frac{T}{\rho} \nabla \cdot (\rho \vec{u}) &= \frac{\gamma}{RePr} \nabla \cdot (\lambda \nabla T) \\ P &= \rho T \end{aligned} \right.$$

### Numerical method 1: FD1

- Finite Difference  $2^{nd}$  centered
- Staggered grid (MAC)
- Continuous projection method
- Time integration: Explicit  $2^{nd}$  order A-B/A-M predictor-corrector scheme

## Model 2: Non-conservative unsteady axisymmetric N-S equations

**A. Beccantini, et al,**  
Numerical simulations of transient injection flow at low Mach number regime.

*Int. J. Numer. Meth. Engng* 2008; 76:662-696.

$$\left\{ \begin{aligned} \frac{1}{\gamma P} \frac{dP}{dt} + \nabla \cdot \vec{u} &= \frac{\gamma - 1}{\gamma P} \nabla \cdot (\lambda \nabla T) \\ \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} &= \frac{RT}{P} \left( -\nabla p'' + \nabla \cdot \underline{\underline{\tau}} \right) \\ &+ \vec{g} \left( 1 - \frac{T P_0}{T_0 P} \right) \\ \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T &= \frac{\gamma - 1}{\gamma} \frac{T}{P} \frac{dP}{dt} \\ &+ \frac{\gamma - 1}{\gamma} \frac{T}{P} \nabla \cdot (\lambda \nabla T) \\ P &= r \rho T \end{aligned} \right.$$

### Numerical method 2: FE2

- Finite Element Q2/P1
- Regular mesh
- Algebraic projection (Quarteroni)
- Time integration: Implicit BDF2 scheme

# Test case - Hot fluid injection



## Physical parameters

- **Computational domain**

$$R_j = 0.5 \text{ m}, L_z = 2 \text{ m}, L_r = 7 \text{ m}.$$

- **Injected fluid**

$$T_j = 600 \text{ K},$$

$$\dot{m}_j = 1. \text{ kg/m}^2/\text{s},$$

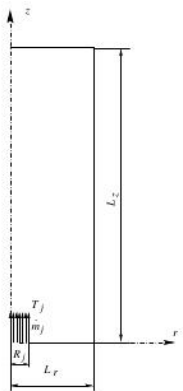
$$V_j = \frac{\dot{m}_j}{\rho_j} = 1.722 \text{ m/s}.$$

$$(\text{Re} = 200, \text{Pe} = 142 \text{ and } \text{Fr} = 0.302)$$

$$t_{\text{injection}} = 6. \text{ s}$$

- **Cavity atmosphere**

$$T_a = 300 \text{ K}, P = 10^5 \text{ Pa}.$$



## Boundary conditions

- **Inlet:**  $T = T_j, \rho u_z = \frac{6\dot{m}_j}{R_j^2} (R_j^2 - r^2).$

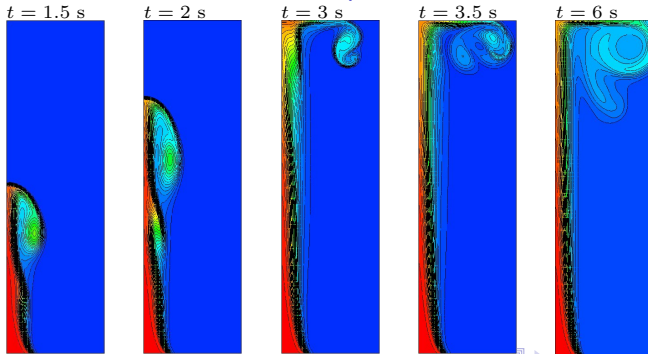
- **Walls:**  $\frac{\partial T}{\partial n} = 0, \rho \vec{u} = 0.$

# Numerical parameters & Results

## Numerical parameters

| Method     | $N_r \times N_z$ | $\Delta t$ | Consistency order                | User time  |
|------------|------------------|------------|----------------------------------|------------|
| <b>FD1</b> | 100 × 350        | 8.610E-04  | 2 for $\rho u_r, \rho u_z, T, p$ | 0.51 hour  |
|            | 200 × 700        | 4.305E-04  |                                  | 5.45 hours |
|            | 400 × 1400       | 2.152E-04  |                                  | 54.53hours |
| <b>FE2</b> | 108 × 378        | 1.075E-03  | 3 for $u_r, u_z, T$<br>2 for $p$ | 528 hours  |
|            | 144 × 504        | 8.065E-04  |                                  |            |
|            | 192 × 672        | 6.049E-04  |                                  |            |

## Instantaneous temperature fields



## Richardson Extrapolation (RE): Assumptions

- 1  $f_{exact}$  regular enough to apply Taylor expansion

$$f_h = f_{exact} + C_\alpha h^\alpha + O(h^{\alpha+1})$$

- 2  $h_i$  small enough to get asymptotic convergence

$$C_\alpha h_i^\alpha \gg O(h^{\alpha+1})$$

- 3 Uniform space and time refinement, so for 3 grids  $\frac{h_1}{h_2} = \frac{h_2}{h_3}$ :

$$\tilde{\alpha} = \ln\left(\frac{f_{h_1} - f_{h_2}}{f_{h_2} - f_{h_3}}\right) / \ln\left(\frac{h_1}{h_2}\right), \quad \tilde{C}_\alpha = \frac{f_{h_2} - f_{h_3}}{h_2^{\tilde{\alpha}} - h_3^{\tilde{\alpha}}}, \quad \tilde{f}^{ex} = f_{h_3} - \tilde{C}_\alpha h_3^{\tilde{\alpha}}$$

## R.E advantages

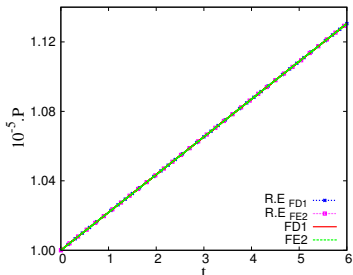
- **Increase of accuracy** by eliminating the truncation errors
- **Error estimate** corresponding to  $\tilde{C}_\alpha h_{fine}^{\tilde{\alpha}}$
- **Method verification** by estimating the convergence orders



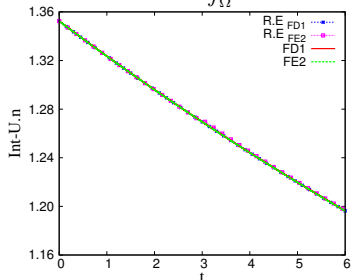
# Results & Verifications: Time dependent quantities



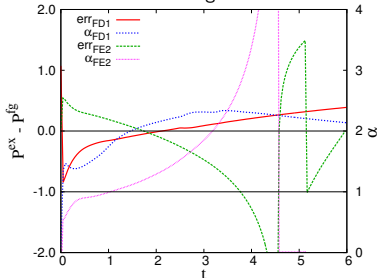
Thermodynamic pressure:  $P = \rho r T$



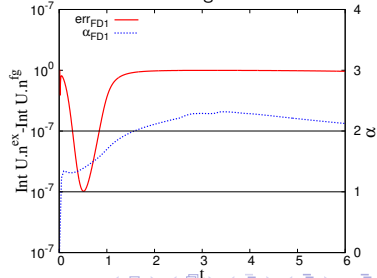
Divergence integral:  $\int_{\Omega} (\nabla \cdot \mathbf{u}) dV$



Errors & convergence order



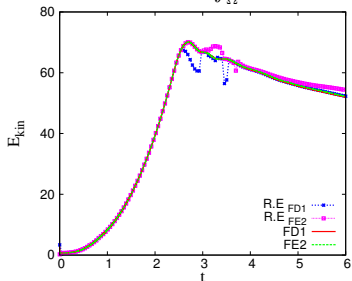
Errors & convergence order



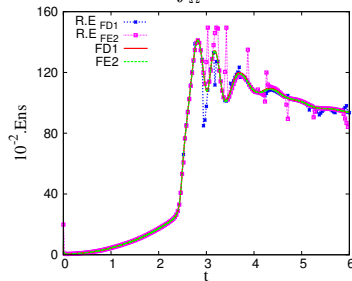
# Results & Verifications: Time dependent quantities



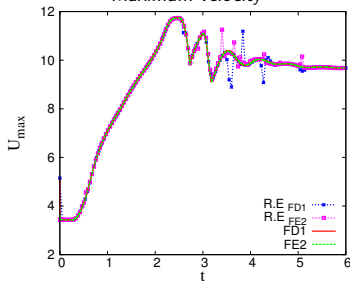
Kinetic energy:  $\frac{1}{2} \int_{\Omega} \rho \mathbf{u}^2 dV$



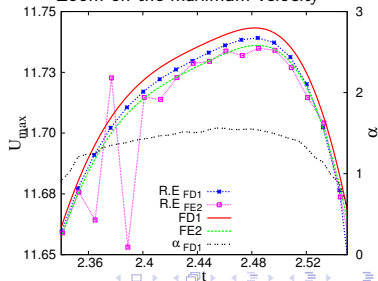
Enstrophy:  $\int_{\Omega} (\nabla \mathbf{u})^2 dV$



Maximum velocity



Zoom on the maximum velocity





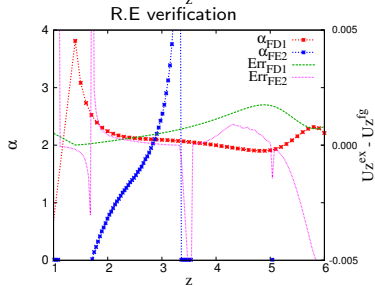
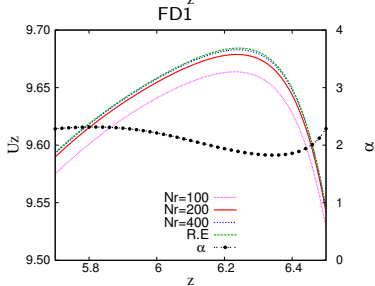
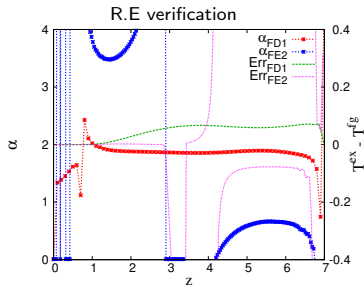
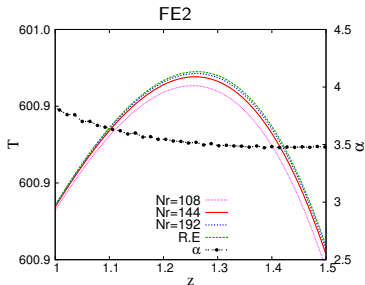
At  $t = 2s$

| Method |                     | $P$      | $\int \nabla \cdot \mathbf{u}$ | $E_{kin}$ | $E_{ns}$ | $U_{max}$ |
|--------|---------------------|----------|--------------------------------|-----------|----------|-----------|
| FD1    | fine grid           | 104356.1 | 1.296001778                    | 39.33128  | 1728.252 | 10.29879  |
|        | R.E                 | 104356.8 | 1.296001779                    | 39.33629  | 1727.218 | 10.29893  |
|        | $ \Delta f /f_{fg}$ | 6.71 E-6 | 7.72 E-10                      | 1.27E-4   | 5.98E-4  | 1.36E-5   |
|        | $\alpha$            | 2.49     | 2.49                           | 1.91      | 1.45     | (4.11)    |
| FE2    | fine grid           | 104356.6 | 1.295949                       | 39.37172  | 1726.572 | 10.29514  |
|        | R.E                 | 104357.6 | 1.295972                       | 39.35677  | 1726.390 | 10.29805  |
|        | $ \Delta f /f_{fg}$ | 3.83E-6  | 2.31E-5                        | 3.80E-4   | 1.05E-4  | 1.82E-4   |
|        | $\alpha$            | 1.29     | 1.10                           | 1.65      | 2.16     | 1.83      |

## Remarks

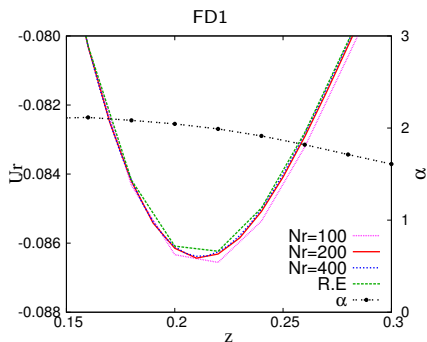
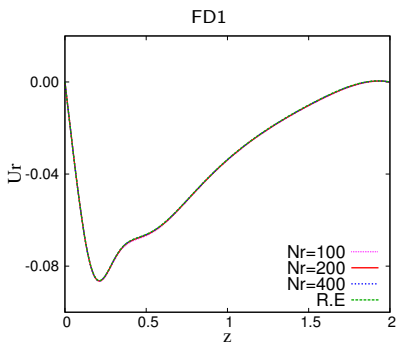
- Relative errors of the same order of magnitude for both **FD1** and **FE2** solver solutions
- **FD1** solver: convergence order given by R.E close to the consistency order of the scheme (2) obtained for  $P$ ,  $\int \nabla \cdot \mathbf{u}$  and  $E_{kin}$
- **FE2** solver: convergence order given by R.E different from the consistency order of the scheme (3)

## Temperature and axial velocity on the axis



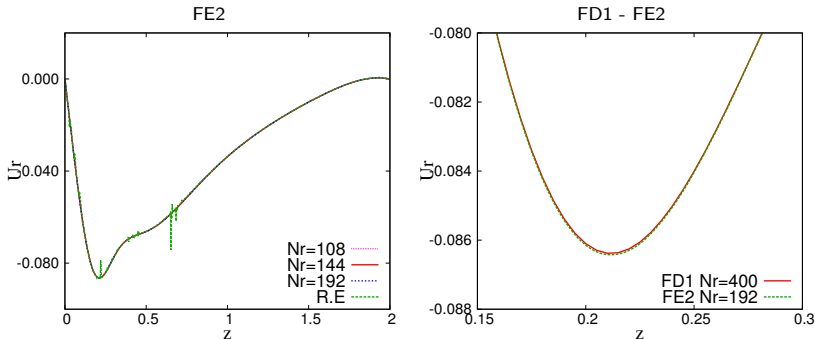


### Radial velocity in the crosswise direction at $z = 3$



- R.E works better for variable profiles far enough from the inlet and the impact area (cavity ceiling)

### Radial velocity in the crosswise direction at $z = 3$



- Fine grid solutions obtained with FD1 and FE2 solvers are in good agreement even if the R.E doesn't work evrywhere in the computational domain



- 1 Numerical solution of low Mach number fluid injection is computed with two CFD codes using:
  - Two different formulations of the low Mach number Navier-Stokes equations
  - Two different numerical methods
- 2 Very good agreement is obtained for local and global quantities characterizing the flow computed with **FD1** and **FE2** solvers
- 3 Difficulties in applying Richardson extrapolation on the whole computing domain since the physical problem doesn't satisfy the R.E assumptions:
  - Stiffness in time: impulsive injection s
  - Stiffness in space: large gradients in particular at the jet impact regions
  - Discontinuity of the boundary conditions on temperature at the inlet (Dirichlet for  $0 < r < R_j$ , Neumann for  $R_j < r < L_r$ )
  - Intersection of the variable profiles resulting from the successively refined meshes
- 4 **FD1** solver: in it's validity domain, R.E gives convergence order about 2 (formal order) for primitive variables,  $P$  and  $\int \nabla \cdot \mathbf{u}$
- 5 **FE2** solver: even if R.E results are not relevant, the computed solution is quite accurate