

# Multilevel Proximal Methods for Image Restoration

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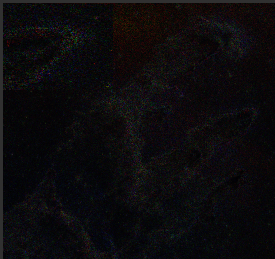
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Optimization and Control in Burgundy, May 9-11, 2023

# Outline of the talk

- 1 A problem of image restoration
- 2 Multilevel algorithm for smooth optimization
- 3 Multilevel algorithm for non-smooth optimization : IML FISTA
- 4 Convergence guarantees of IML FISTA
- 5 Reconstruction of color images
- 6 Reconstruction of hyperspectral images
- 7 Summary

**Our goal:** Recovering  $\hat{x}$  as close as possible to the original image  $\bar{x}$  from a degraded observation  $z$

 $\bar{x}$  $z$  $\hat{x}$ 

**Context:** Restoration of large-scale images ( $N > 10^6$  variables)

# Problem formulation:

Classic degradation model:  $z = A\bar{x} + \epsilon$

- $A \in \mathbb{R}^{M \times N}$  a linear degradation
- $\epsilon \in \mathbb{R}^M$  some gaussian noise

Ill-posed problem, it leads to the following minimization problem:

$$\hat{x} \in \underset{x \in \mathbb{R}^N}{\text{Argmin}} \frac{1}{2} \|Ax - z\|_2^2 + \lambda \|Dx\|_{1,2}$$

- $N$  image size
- $D \in \mathbb{R}^{K \times N}$  linear transform on  $x$  on which we will impose sparsity
- $\lambda > 0$  regularization parameter

# Problem formulation:

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Ill-posed problem, it leads to the following minimization problem:

$$\min_{x \in \mathbb{R}^N} F(x) := \underbrace{(f \circ A)(x)}_{\text{data fidelity}} + \underbrace{(g \circ D)(x)}_{\text{regularization}}$$

$f$  and  $g$  proper, lower semi-continuous and convex.  $f$  is assumed **differentiable** with Lipschitz gradient.  $g$  is not necessarily differentiable.

# First order optimization methods

# First order descent methods

$$x_{k+1} = \Phi(x_k) = x_k - D_k$$

- If  $f$  and  $g$  are differentiable : Gradient method

$$D_k = \tau_k(\nabla(f \circ A)(x_k) + \nabla(g \circ D)(x_k))$$

# First order descent methods

$$x_{k+1} = \Phi(x_k) = x_k - D_k$$

- If  $f$  and  $g$  are differentiable : **Gradient method**

$$D_k = \tau_k(\nabla(f \circ A)(x_k) + \nabla(g \circ D)(x_k))$$

- If  $g$  is not differentiable : **Proximal gradient method**

$$D_k = x_k - \text{prox}_{\tau_k g \circ D}(x_k - \tau_k \nabla(f \circ A)(x_k))$$

$\text{prox}_{\tau g}(\cdot) = \underset{x \in \mathbb{R}^N}{\text{argmin}} \frac{1}{2} \|x - \cdot\|_2^2 + \tau g(x)$  is available explicitly.

- Convergence of the objective function values with rate  $1/k$
- Convergence to a minimizer if it exists



# Main goal : provide acceleration for high dimensional problems

High dimensional problems  $\rightarrow$  high computation time.

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## Alternatives :

- FISTA [Beck & Teboulle, 2009],
- Preconditioning [Donatelli, 2019],
- Blocks methods [Liu, 1996],
- **Exploit the problem structure with a multiresolution strategy.** Idea that comes from the PDE field and developed in particular for non-smooth optimization in [Parpas, 2017], our inspiration.

Common aim of these methods:

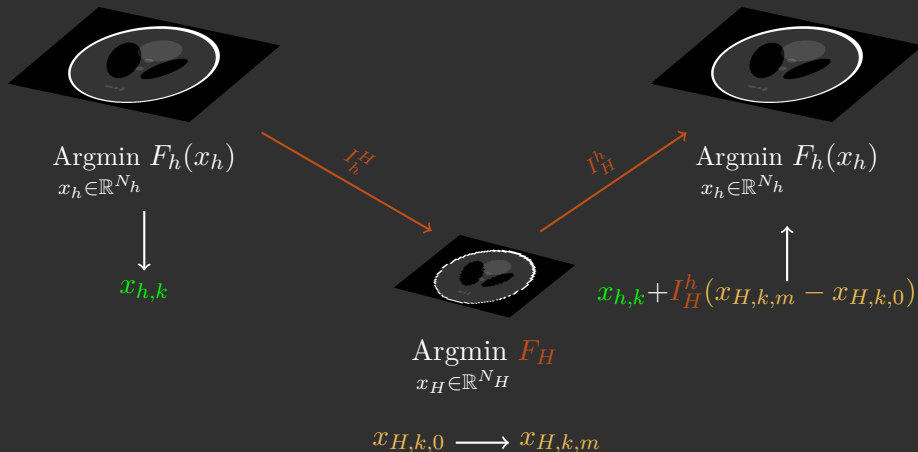
improve the gradient/proximal gradient steps with well chosen rules.

# Multilevel algorithm for smooth optimization

# Classical scheme for two levels

**Goal:** Exploit a hierarchy of approximations of the objective function.

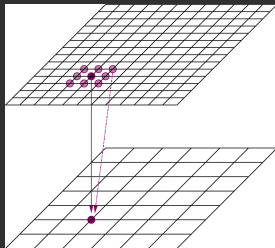
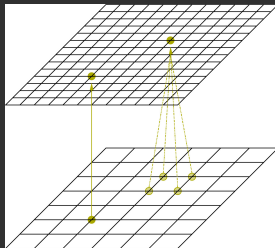
**Two levels case:** fine ( $h$ ) and coarse ( $H$ )



# Ingredients

## Information transfer operators

- $I_h^H \in \mathbb{R}^{N_H \times N_h}$ : transfer from fine to coarse scales ( $N_H < N_h$ )
- $I_H^h \in \mathbb{R}^{N_h \times N_H}$ : transfer from coarse to fine scales
- Coherence between operators:  $I_H^h = \nu(I_h^H)^T$
- Example: multiresolution analysis (orthogonal wavelets)


 $I_h^H$ 

 $I_H^h$

# Ingredients

Objective function at coarse level:

$$f_H \circ A_H + g_H \circ D_H$$

Image restoration context

$$\forall x_h \in \mathbb{R}^{N_h} \quad f_h(x_h) = \frac{1}{2} \|x_h - z\|_2^2 \quad g_h(x_h) = \lambda \|x\|_2$$

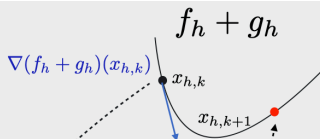
$$\forall x_H \in \mathbb{R}^{N_H} \quad f_H(x_H) = \frac{1}{2} \|x_H - I_h^H z\|_2^2 \quad g_H(x_H) = \lambda \|x_H\|_2$$

where:

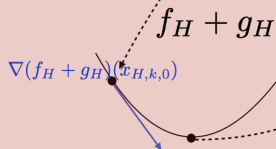
- $A_H$  is a reduced order version of  $A_h$  (e.g. Galerkin approximation, decimation)
- $D_H$  is a reduced order version of  $D_h$  (e.g. TV at fine and coarse levels)

# First order coherence

Fine level  $h$



Coarse level  $H$

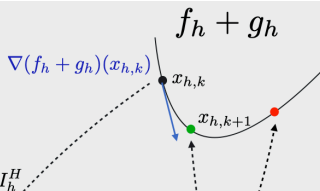


$I_h^H$

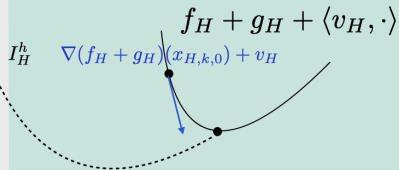
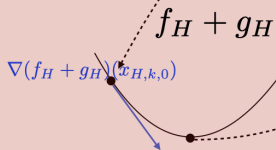
$I_H^h$

# First order coherence

Fine level  $h$



Coarse level  $H$





# Guarantees associated with this construction

Coarse model  $F_H$  for smooth functions

$$F_H = f_H \circ A_H + g_H \circ D_H + \langle v_H, \cdot \rangle$$

The first order coherence term involves

$$\begin{aligned} v_H = & I_h^H (\nabla(f_h \circ A_h)(x_h) + \nabla(g_h \circ D_h)(x_h)) \\ & - (\nabla(f_H \circ A_H)(I_h^H x_h) + \nabla(g_H \circ D_H)(I_h^H x_h)) \end{aligned}$$

which guarantees that if

$$\langle x_{H,m} - x_{H,0}, \nabla F_H(x_{H,0}) \rangle \leq 0$$

then

$$\langle I_H^h(x_{H,m} - x_{H,0}), \nabla F_h(x_h) \rangle \leq 0$$

# Multilevel algorithm for smooth optimization

for  $k$  do

  if *Descent condition* then

    Coarse model:

$$x_{H,k,0} = I_h^H x_{h,k} \quad \text{Projection}$$

$$x_{H,k,m} = \Phi_{H,k,m-1} \circ \dots \circ \Phi_{H,k,0}(x_{H,k,0}) \quad \text{Coarse decrease}$$

    Set  $\bar{\tau}_{h,k} > 0$ , Update by coarse step

$$\bar{x}_{h,k} = x_{h,k} + \bar{\tau}_{h,k} I_H^h (x_{H,k,m} - x_{H,k,0})$$

  else

$$\bar{x}_{h,k} = x_{h,k}$$

  end

Fine level :

$$x_{h,k+1} = \bar{x}_{h,k} - \tau (\nabla(f_h \circ A_h) + \nabla(g_h \circ D_h))(\bar{x}_{h,k}) \quad \text{Gradient step}$$

end

S. G. Nash, **A Multigrid Approach to Discretized Optimization Problems**, Optimization Methods and Software (2000)

# Multilevel algorithm for smooth optimization

In the context of image processing

Some references:

- A. Javaherian and S. Holman, **A Multi-Grid Iterative Method for Photoacoustic Tomography**, IEEE Transactions on Medical Imaging, (2017)
- S. W. Fung and Z. Wendy, **Multigrid Optimization for Large-Scale Ptychographic Phase Retrieval**, SIAM Journal on Imaging Sciences, 13 (2020)
- J. Plier, F. Savarino, M. Kočvara, and S. Petra, **First-Order Geometric Multilevel Optimization for Discrete Tomography**, in Scale Space and Variational Methods in Computer Vision, A. Elmoataz, J. Fadili, Y. Quéau, J. Rabin, and L. Simon, eds., vol. 12679, Springer International Publishing, Cham, (2021)

→ Successful attempts of accelerating minimization in imaging problems

→ Smooth optimization: not very good for image reconstruction problems

# Multilevel algorithm for non-smooth optimization: IML FISTA

# What has been done:

Extension of the multilevel framework to non-smooth optimization:

- ✓ Handles non-smooth  $g$
- ✓ Smoothing to define the first order coherence
- ✗ Requires **explicit form** of  $\text{prox}_{g \circ D}$  :  $D$  must be orthogonal
- ✗ No guarantee of convergence to a minimizer

V. Hovhannisyán, P. Párpas, and S. Zafeiriou, **MAGMA: Multilevel Accelerated Gradient Mirror Descent Algorithm for Large-Scale Convex Composite Minimization**, SIAM Journal on Imaging Sciences (2016)

P. Párpas, **A Multilevel Proximal Gradient Algorithm for a Class of Composite Optimization Problems**, SIAM Journal on Scientific Computing, 39 (2017)

# Our contribution

# Motivations and contribution

In a multilevel setting we want to:

- include **inexact proximal** steps to handle **state-of-the-art regularization**: Total Variation (TV) and Non-Local Total Variation (NLTV) based semi-norm
- obtain **state-of-the-art** convergence guarantees

In this context we provide IML FISTA a **convergent multilevel inexact and inertial proximal gradient algorithm** that works when:

- the proximity operator of  $g$  is explicit
- the proximity operator of  $g \circ D$  is **not known under closed form**

# Framework

Inertial convergent proximal algorithm that can handle **inexact steps**:

$$x_{k+1} \approx_{\epsilon_k} \text{prox}_{\tau g \circ D} (y_k - \tau \nabla (f \circ A)(y_k) + e_k)$$

$$y_{k+1} = x_{k+1} + \alpha_k (x_{k+1} - x_k)$$

where  $\alpha_k$  is chosen with  $t_{k+1} = \left(\frac{k+a}{a}\right)^d$ ,  $\alpha_k = \frac{t_k - 1}{t_{k+1}}$ .

**Our idea** (inspired by [Parpas, 2016-2017]): update  $y_k$  through a multilevel step.

- How to construct such multilevel update ?
- How to guarantee convergence ?

J.-F. Aujol and C. Dossal, **Stability of Over-Relaxations for the Forward-Backward Algorithm, Application to FISTA**, SIAM Journal on Optimization, (2015)



# Smoothing of $F_h$ and $F_H$ with the Moreau envelope

Moreau envelope of  $g_H$ :  $\gamma g_H = \inf_{y \in \mathcal{H}} g_H(y) + \frac{1}{2\gamma} \|\cdot - y\|^2$

Properties of the Moreau envelope:

- $\nabla \gamma g_H = \gamma^{-1}(\text{Id} - \text{prox}_{\gamma g_H})$
- $\nabla \gamma g_H$   $\gamma^{-1}$  - Lipschitz
- $\nabla (\gamma g_H \circ D_H)(\cdot) = \gamma_H^{-1} D_H^* (D_H \cdot - \text{prox}_{\gamma_H g_H}(D_H \cdot))$

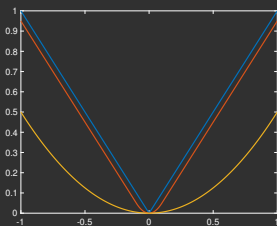


Figure: Moreau envelope of  $l_1$ -norm for  $\gamma = 0.1$  and  $\gamma = 1$

# First order coherence for $g$ non-smooth

*Coarse model  $F_H$  for non-smooth functions*

$$F_H = f_H \circ A_H + (\gamma^H g_H \circ D_H) + \langle v_H, \cdot \rangle$$

where  $\gamma g$  is the **Moreau envelope** of  $g$   
and the coherence term involves

$$\begin{aligned} v_H &= I_h^H (\nabla(f_h \circ A_h)(y_h) + \nabla(\gamma^h g_h \circ D_h)(y_h)) \\ &\quad - (\nabla(f_H \circ A_H)(I_h^H y_h) + \nabla(\gamma^H g_H \circ D_H)(I_h^H y_h)) \end{aligned}$$

and the **minimization operator at coarse level** as:

$$\Phi_H := \nabla(f_H \circ A_H) + \nabla(\gamma^H g_H \circ D_H)$$

# Decrease at fine level

With this first order coherence, any algorithm that decreases  $F_H$  implies

$$F_h(y_h + \bar{\tau} I_H^h(x_{H,m} - x_{H,0})) \leq F_h(y_h) + \eta \gamma_h$$

where :

- $\eta, \gamma_h > 0$  depend on the Moreau approximation
- $\bar{\tau}$  is some step size.

→ Bound on one multilevel step but not enough for global convergence guarantees. Similar as [Parpas, 2017]

# Multilevel steps interpreted as errors

FISTA steps allow **errors** on the computation of the backward and on the forward steps:

$$x_{h,k+1} \simeq_{\epsilon_{h,k}} \text{prox}_{\tau_h g_h \circ D_h} (y_{h,k} - \tau_h \nabla (f_h \circ A_h) (y_{h,k}) + e_{h,k})$$

$$y_{h,k+1} = x_{h,k+1} + \alpha_{h,k} (x_{h,k+1} - x_{h,k})$$

Rewriting coarse corrections:

$$e_{h,k} = \tau_h (\nabla (f_h \circ A_h) (y_{h,k}) - \nabla (f_h \circ A_h) (\bar{y}_{h,k}))$$

$$+ \frac{\bar{\tau}_{h,k}}{\tau_h} I_H^h (x_{H,k,m} - x_{H,k,0})$$

→ **Multilevel steps are bounded errors on the gradient**

# Estimation of the proximity operator

At each iteration of fine level minimization we need to compute  $\text{prox}_{\gamma g_h \circ D_h}(x) = x - D_h^* \hat{u}$  with:

$$\hat{u} \in \arg \min_{u \in \mathbb{R}^K} \frac{1}{2} \|D_h^* u - x\|^2 + \gamma g_h^*(u)$$

which can be solved iteratively with accuracy  $\epsilon$  so that:

$$x - D_h^* \hat{u}_\epsilon \simeq_\epsilon \text{prox}_{\gamma g_h \circ D_h}(x)$$

Equivalent to having:

$$\frac{D_h^* \hat{u}_\epsilon}{\gamma} \in \partial_\epsilon (g_h \circ D_h)(x - D_h^* \hat{u}_\epsilon)$$

S. Villa, S. Salzo, L. Baldassarre, and A. Verri, **Accelerated and Inexact Forward-Backward Algorithms**, SIAM Journal on Optimization, 23 (2013)

# Convergence of the algorithm

## Convergence theorem

Assume that:

- we compute accurate estimation of the proximity operators so that:  
$$\sum_k k^{2d} \epsilon_{h,k} < +\infty$$
- we compute at most  $p$  coarse corrections so that:  $\sum_k k e_{h,k} < +\infty$

Then:

- $(k^{2d} (F_h(x_{h,k}) - F_h(\hat{x})))_{k \in \mathbb{N}}$  belongs to  $\ell_\infty(\mathbb{N})$
- $(x_{h,k})_{k \in \mathbb{N}}$  converges to a minimizer of  $F_h$

# Inexact Multilevel FISTA

**for**  $k$  **do**

**if** *Descent condition and*  $r < p$  **then**

Coarse model ( $r = r+1$ ):

$$x_{H,k,0} = I_h^H y_{h,k} \text{ Projection}$$

$$x_{H,k,m} = \Phi_{H,k,m-1} \circ \dots \circ \Phi_{H,k,0}(x_{H,k,0}) \text{ Coarse decrease}$$

Set  $\bar{\tau}_{h,k} > 0$ , **Update by coarse step**

$$\bar{y}_{h,k} = y_{h,k} + \bar{\tau}_{h,k} I_H^h (x_{H,k,m} - x_{H,k,0})$$

**else**

$$\bar{y}_{h,k} = y_{h,k}$$

**end**

Fine level :

$$x_{h,k+1} \approx_{\epsilon_k} \text{FB}(\bar{y}_{h,k}) \text{ Forward-backward step}$$

$$t_{h,k+1} = \left(\frac{k+a}{a}\right)^d, \alpha_{h,k} = \frac{t_{h,k-1}}{t_{h,k+1}}$$

$$y_{h,k+1} = x_{h,k+1} + \alpha_{h,k} (x_{h,k+1} - x_{h,k}) \text{ Inertial step}$$

**end**

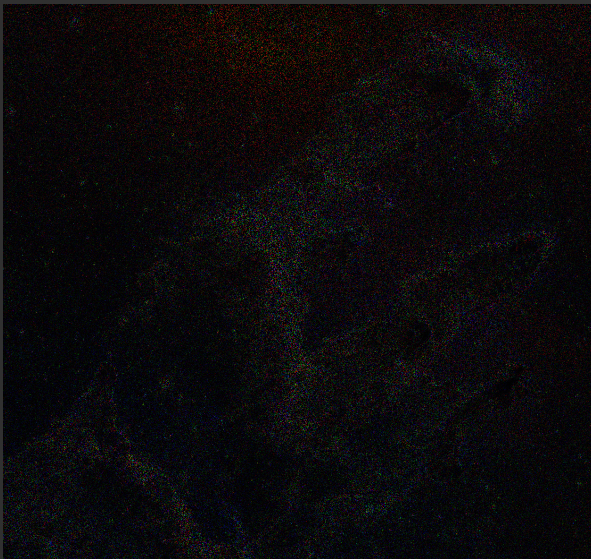
# Numerical experiments on color images



# Image reconstruction with NLTV prior



# Image reconstruction with NLTV prior



# Image reconstruction with NLTV prior



# Problem formulation:

- $N = 2048 \times 2048 \times 3$  image size
- A inpainting operator: 90% along all channels
- D a Non Local Total Variation operator on  $x$
- Gaussian noise  $\sigma(\text{noise}) = 5e - 2$
- $\lambda = 3e - 2$
- Parameters:  $\tau = 1, \gamma = 1.1, l = 5, p = 2, m = 5, \lambda_H = \lambda/4$

$$\hat{x} \in \underset{x \in \mathbb{R}^N}{\text{Argmin}} \frac{1}{2} \|Ax - z\|_2^2 + \lambda \|Dx\|_{1,2}$$

# Information transfer operators : orthogonal wavelets

For a generic quadrature mirror filter  $\mathbf{q} = (q_1, \dots, q_m)$ :

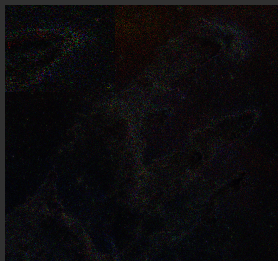
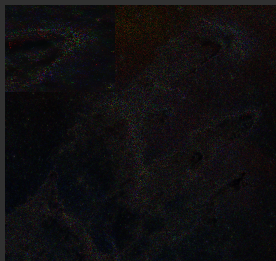
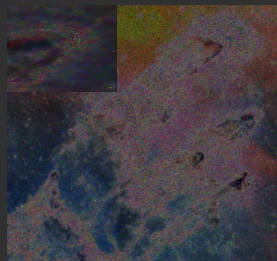
$$I_h^H := (\mathbf{R}_{\mathbf{q},r} \otimes \mathbf{R}_{\mathbf{q},c})$$

$$T_{\mathbf{q}} = \begin{pmatrix} q_1 & q_2 & \dots & q_m & 0 & \dots & 0 \\ 0 & q_1 & q_2 & \dots & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ 0 & \dots & 0 & 0 & q_1 & q_2 & \dots \end{pmatrix}$$

- $\mathbf{R}_{\mathbf{q},c}$  size  $N_{H,r}$ -by- $N_{h,r}$ : dyadic decimated version of  $T_{\mathbf{q}}$
- $\mathbf{R}_{\mathbf{q},r}$  size  $N_{H,c}$ -by- $N_{h,c}$ : dyadic decimated version of  $T_{\mathbf{q}}$

M. I. Espanol, **Multilevel methods for discrete ill-posed problems: Application to deblurring**, PhD thesis, 2009

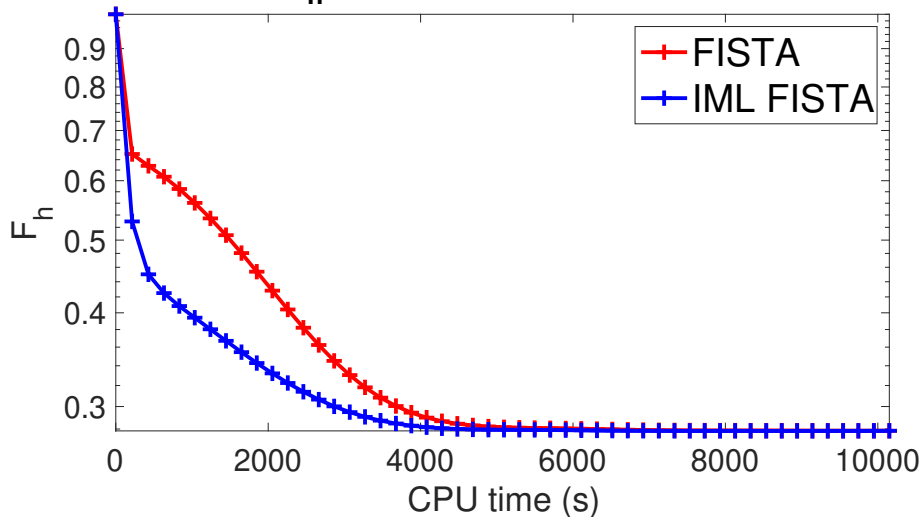
# Reconstruction after 2 iterations with NLTV

 $z$  $x_2$  FISTA $x_2$  IML FISTA

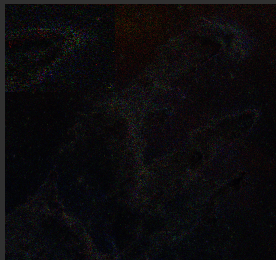
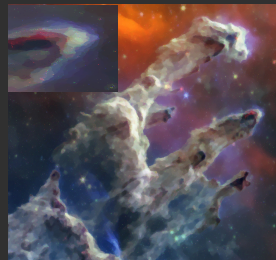
$\hat{x}$  estimated after 2 iterations of FISTA vs 2 iterations of IML FISTA.

Evolution of  $F_h$  for a  $N_h = 2048 \times 2048 \times 3$  image

### $F_h$ evolution with CPU time



# Reconstruction after 50 iterations with NLTV

 $\bar{x}$  $z$  $\hat{x}$ 

$\hat{x}$  estimated after 50 iterations of IML FISTA.



# Additional results on deblurring problems

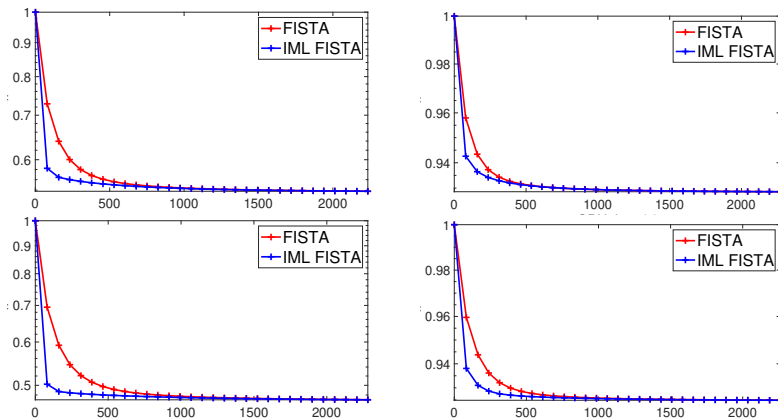
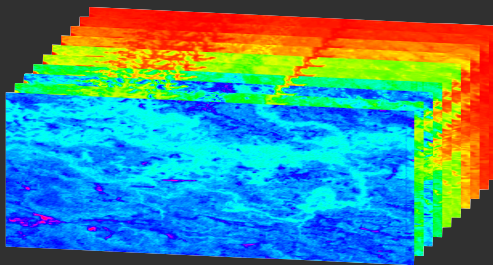


Figure: Deblurring  $\ell_{1,2}$ -TV. Objective function (normalized w.r.t. the initial value) vs CPU time (sec). First column:  $\sigma(\text{noise}) = 0.01$ ; second column:  $\sigma(\text{noise}) = 0.05$ . First row:  $\dim(\text{PSF}) = 20$ ,  $\sigma(\text{PSF}) = 3.6$ ; second row:  $\dim(\text{PSF}) = 40$ ,  $\sigma(\text{PSF}) = 7.3$ .

# Numerical experiments on hyperspectral images

# Hyperspectral images



$x^{(i,b)} = x^{(i_1,i_2,b)}$  the pixel  $i = (i_1, i_2) \in \{1, \dots, N_r\} \times \{1, \dots, N_c\}$  of the band  $b \in \{1, \dots, L\}$  of the hypercube  $x$ .  $N = N_r \times N_c \times L$

# Inpainting problem for Hyperspectral Images

Degradation model:  $z = A\bar{x} + \epsilon$ . Associated minimization problem:

$$\hat{x} \in \underset{x \in \mathbb{R}^N}{\text{Argmin}} \frac{1}{2} \|Ax - z\|_2^2 + \lambda g(Dx)$$

- $A \in \mathbb{R}^{M \times N}$  an inpainting operator which applies independant masks on each spectral band
- $D \in \mathbb{R}^{(N \times K \times L) \times N}$  such that:

$$(Dx)^{(i)} = \left[ \omega^{(i,j)} (x^{(i,b)} - x^{(j,b)}) \right]_{j \in \mathcal{N}_i, 1 \leq b \leq L} \in \mathbb{R}^{\tilde{K} \times L}$$

$$(\forall x \in \mathbb{R}^N) \quad g(Dx) = \sum_{i=1}^N \|(Dx)^i\|_*$$

# IML FISTA for hyperspectral images

Optimization scheme of IML FISTA:

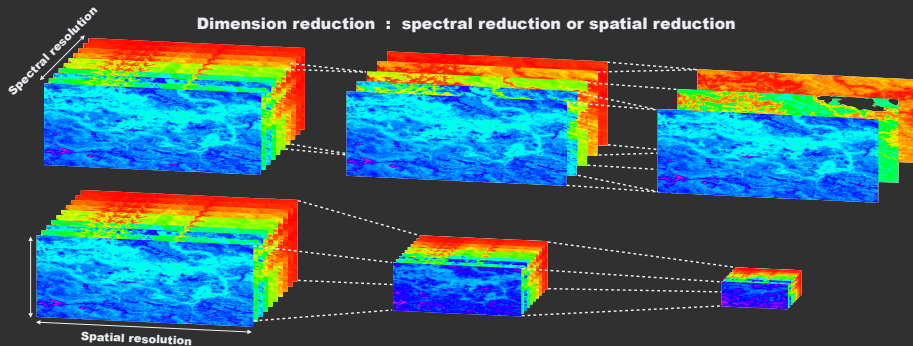
$$\begin{aligned}\bar{y}_{h,k} &= \text{ML}(y_{h,k}) \\ x_{h,k+1} &\approx_{\epsilon_{h,k}} \text{prox}_{\tau g_h(\mathcal{D}_h \cdot)}(\bar{y}_{h,k} - \tau \nabla(f_h \circ \mathbf{A}_h)(\bar{y}_{h,k})), \\ y_{h,k+1} &= x_{h,k+1} + \alpha_{h,k}(x_{h,k+1} - x_{h,k}),\end{aligned}$$

The MultiLevel (ML) step encompasses the coarse correction.

How to construct ML to exploit the high correlation in the data ?

# Dimension reduction

**Dimension reduction : spectral reduction or spatial reduction**



# Coarse model construction

## Spectral IML FISTA

```

b, ℓ = 1
while  $b \leq L_h - 1$  do
  if  $\mu(\lambda) - \sigma(\lambda) \leq \lambda^{(b+1)} - \lambda^{(b)} \leq$ 
     $\mu(\lambda) + \sigma(\lambda)$  then
    |  $x_H^{(:,\ell)} = \frac{1}{2}(x_h^{(:,b)} + x_h^{(:,b+1)})$ ,
    |  $\ell = \ell + 1$ 
  else
    |  $x_H^{(:,\ell)} = x_h^{(:,b)}$ ,
    |  $x_H^{(:,\ell+1)} = x_h^{(:,b+1)}$ ,
    |  $\ell = \ell + 2$ 
  end
   $b = b + 2$ 
end

```

The blocks of  $A_h$  are passed through this procedure to construct the blocks of  $A_H$ .

## Spatial IML FISTA

$$(\forall b = \{1, \dots, L_h\})$$

$$x_H^{(:,b)} = I_h^H(x_h^{(:,b)})$$

where:

$$I_h^H := (\mathbf{R}_{\mathbf{q},r} \otimes \mathbf{R}_{\mathbf{q},c})$$

$T_{N_{H,c}}^{\text{dec}}(\mathbf{q})$  decimated Toeplitz matrix engendered by  $\mathbf{q}$ .

The blocks of  $A_h$  are decimated to construct the blocks of  $A_H$ .

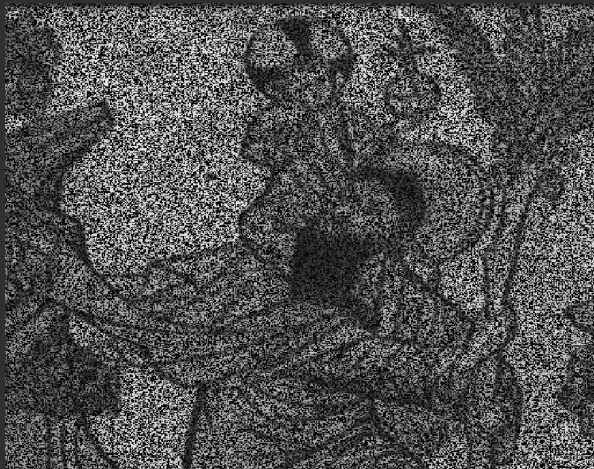
# Results with Spectral IML FISTA



$$\bar{x} \in \mathbb{R}^{512 \times 512 \times 33}$$



# Results with Spectral IML FISTA

 $\approx$

# Results with Spectral IML FISTA



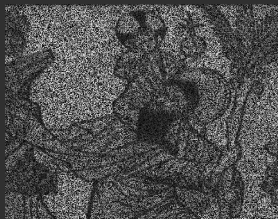
$x_{\text{end},\text{FISTA}}$

# Results with Spectral IML FISTA



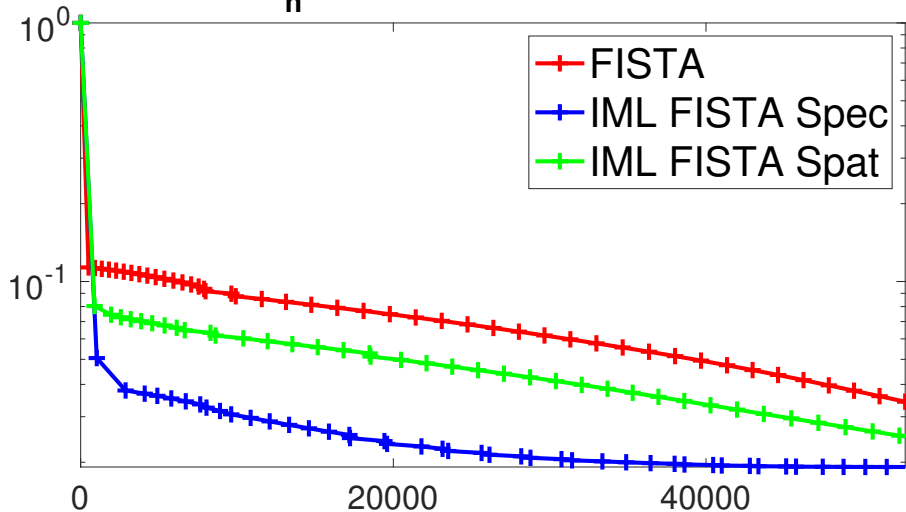
$x_{\text{end,IML FISTA}}$

## Results with Spectral IML FISTA

 $\bar{x}$  (SNR) $x_{2,\text{FISTA}}$  (7 dB) $x_{\text{end},\text{FISTA}}$  (21 dB) $z$  (3 dB) $x_{2,\text{IML FISTA}}$  (19 dB) $x_{\text{end},\text{IML FISTA}}$  (35 dB)

# Objective function evolution

## $F_h$ evolution with CPU time



# Summary

# Summary

We have developed a *convergent multilevel algorithm* with:

- same convergence rates as FISTA
- efficient construction of coarse models
- good experimental performances on large scale images
- embed-able state-of-the-art regularizations for inverse problems

# Future work

- **Hyperspectral imaging** : possible application : fast restoration available to improve segmentation tasks (e.g. identification of crop types).
- **Reconstruction of circumstellar environments** : direct application of IML FISTA to a real problem.
- **Radio interferometric imaging** : possible extension of multilevel methods to primal-dual approaches and application to real data.
- Better guarantees for the multilevel steps



## References

- **IML FISTA: Inexact MuLtilevel FISTA for Image Restoration**, preprint and code available
- **Méthodes multi-niveaux et restauration d'images hyperspectrales**, preprint
- **Multilevel FISTA for Image Restoration**, ICASSP 2023
- **Méthodes proximales multi-niveaux pour la restauration d'images**, GRETSI 2022



<https://laugaguillaume.github.io/>