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Correct by Design Coordination of Autonomous Driving Systems

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Abstract. The paper proposes a method for the correct by design coordination of autonomous driving systems (*ADS*). It builds on previous results on collision avoidance policies and the modeling of *ADS* by combining descriptions of their static environment in the form of maps, and the dynamic behavior of their vehicles.

An *ADS* is modeled as a dynamic system involving a set of vehicles coordinated by a *Runtime* that based on vehicle positions on a map and their kinetic attributes, computes free spaces for each vehicle. Vehicles are bounded to move within the corresponding allocated free spaces.

We provide a correct by design safe control policy for an *ADS* if its vehicles and the *Runtime* respect corresponding assume-guarantee contracts. The result is established by showing that the composition of assume-guarantee contracts is an inductive invariant that entails *ADS* safety.

We show that it is practically possible to define speed control policies for vehicles that comply with their contracts. Furthermore, we show that traffic rules can be specified in a linear-time temporal logic, as a class of formulas that constrain vehicle speeds. The main result is that, given a set of traffic rules, it is possible to derive free space policies of the *Runtime* such that the resulting system behavior is safe by design with respect to the rules.

Keywords: Autonomous driving systems · Traffic rule specification · Map specification · Collision avoidance policy · Assume-guarantee contract · Correctness by design.

1 Introduction

Autonomous driving systems (*ADS*) are probably the most difficult systems to design and validate, because the behavior of their agents is subject to temporal and spatial dynamism. They are real-time distributed systems involving components with partial knowledge of their environment, pursuing specific goals while the collective behavior must meet given global goals.

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Development of trustworthy *ADS* is an urgent and critical need. It poses challenges that go well beyond the current state of the art due to their overwhelming complexity. These challenges include, on the one hand, modeling the system and specifying its properties, usually expressed as traffic rules; on the other hand, building the system and verifying its correctness with respect to the desired system properties.

Modeling involves a variety of issues related to the inherent temporal and spatial dynamics as well as to the need for an accurate representation of the physical environment in which vehicles operate. Many studies focus on formalizing and standardizing a concept of map that is central to semantic awareness and decision-making. These studies often use ontologies and logics with associated reasoning mechanisms to check the consistency of descriptions and their accuracy with respect to desired properties [3,2]. Other works propose open source mapping frameworks for highly automated driving [1,16]. Finally, the SOCA method [7] proposes an abstraction of maps called zone graph, and uses this abstraction in a morphological behavior analysis.

There is an extensive literature on *ADS* validation that involves two interrelated problems: the specification of system properties and the application of validation techniques. The specification of properties requires first-order temporal logics because parameterization and genericity are essential for the description of situations involving a varying number of vehicles and types of traffic patterns. The work in [19,17] formalizes a set of traffic rules for highway scenarios in Isabelle/HOL. It shows that traffic rules can be used as requirements to be met by autonomous vehicles and proposes a verification procedure. A formalization of traffic rules for uncontrolled intersections is provided in [12], which shows how the rules can be used by a simulator to safely control traffic at intersections. The work in [10] proposes a methodology for formalizing traffic rules in linear temporal logic; it shows how the evaluation of formalized rules on recorded human behaviors provides insight into how well drivers follow the rules.

Many works deal with the formal verification of controllers that perform specific maneuvers. For example, in [11], a dedicated multi-way spatial logic inspired by interval temporal logic is used to specify safety and provide proofs for lane change controllers. The work in [18] presents a formally verified motion planner in Isabelle/HOL. The planner uses maneuver automata, a variant of hybrid automata, and linear temporal logic to express properties. In [10], runtime verification is applied to check that the maneuvers of a high-level planner conform to traffic rules expressed in linear temporal logic.

Of particular interest for this work are correct by construction techniques where system construction is guided by a set of properties that the system is guaranteed to satisfy. They involve either the application of monolithic synthesis techniques or compositional reasoning throughout a component-based system design process. There is considerable work on controller synthesis from a set of system properties usually expressed in linear temporal logic, see for example [13,27,28,21,26]. These are algorithmic techniques extensively studied in the field of control. They consist of restricting the controllable behavior of a

system interacting with its environment so that a set of properties are satisfied. Nonetheless, their application is limited due to their high computational cost, which depends in particular on the type of properties and the complexity of the system behavior.

An alternative to synthesis is to achieve correctness by design as a result of composing the properties of the system components. Component properties are usually "assume-guarantee" contracts characterizing a causal relationship between a component and its environment: if the environment satisfies the "assume" part of the contract, the state of the component will satisfy the "guarantee" part, e.g. [4,15,8]. The use of contracts in system design involves a decomposition of overall system requirements into contracts that provide a basis for more efficient analysis and validation. In addition, contract-based design is advocated as a method for achieving correctness by design, provided that satisfactory implementations of the system can be found [23]. There are a number of theoretical frameworks that apply mainly to continuous or synchronous systems, especially for analysis and verification purposes [22,14,20]. They suffer computational limitations because, in the general case, they involve the symbolic solution of fixed-point equations, which restricts the expressiveness of the contracts [14]. Furthermore, they are only applicable to systems with a static architecture, which excludes dynamic reconfigurable systems, such as autonomous systems.

The paper builds on previous results [5] on a logical framework for parametric specification of *ADS* combining models of the system's static environment in the form of maps, and the dynamic properties of its vehicles. Maps are metric graphs whose vertices represent locations and edges are labeled with segments that can represent roads at different levels of abstraction, with characteristics such as length or geometric features characterizing their shape and size.

An *ADS* model is a dynamic system consisting of a map and a set of vehicles moving along specific routes. Its state can be conceived as the distribution of vehicles on a map with their positions, speeds and other kinematic attributes. For its movement, each vehicle has a safe estimate of the free space in its neighborhood, according to predefined visibility rules. We assume that vehicle coordination is performed by a *Runtime* that, for given vehicle positions and speeds on the map, can compute the free spaces on each vehicle's itinerary in which it can safely move.

We study a safe control policy for *ADS*, which is correct by design. It results from the combination of two types of assume-guarantee contracts: one contract for each vehicle and another contract for the *Runtime* taking into account the positions of the vehicles on the map. The contract for a vehicle states that, assuming that initially the dynamics of the vehicle allow it to stay in the allocated free space, it will stay in this free space. Note that the details of the contract implementation are irrelevant; only the I/O relationship between free space and vehicle speed matters. The *Runtime* contract asserts that if the free spaces allocated to vehicles are disjoint, then they can be allocated new disjoint free spaces provided they have fulfilled their contract.

We build on this general result by specializing its application in two directions. First, we show that it is possible to define speed policies for vehicles that satisfy their assume-guarantee contract. Second, we show that it is possible to define free space policies for the *Runtime* enforcing safety constraints of a given set of traffic rules. We formalize traffic rules as a class of properties of a linear temporal logic. We provide a method that derives from a given set of traffic rules, constraints on the free spaces chosen by the *Runtime* such that the resulting system behavior is safe with respect to these rules. This is the main result of the paper establishing correctness by design of general *ADS*, provided that their components comply with their respective contracts.

The paper is structured as follows. In Section 2, we establish the general framework by introducing the basic models and concepts for the representation of maps. In Section 3, we introduce the dynamic model of *ADS* involving a set of vehicles and a *Runtime* for their coordination. We show how a correct by design safe control policy is obtained by combining assume-guarantee contracts for the vehicles and the *Runtime*. In Section 4, we study the principle of speed policies respecting the vehicle contract and show its application through an example. In Section 5, we formalize traffic rules as a class of formulas of a linear temporal logic and show how it is possible to generate from a set of traffic rules free space policies such that the system is safe by design. Section 6 concludes with a discussion of the significance of the results, future developments and applications. The proofs of technical results are available in the long version of the paper [6].

2 Map Representation

Following the idea presented in [5], we build contiguous road segments from a set \mathcal{S} equipped with a partial concatenation operator $\cdot : \mathcal{S} \times \mathcal{S} \rightarrow \mathcal{S} \cup \{\perp\}$, a length norm $\|\cdot\| : \mathcal{S} \rightarrow \mathbb{R}_{\geq 0}$ and a partial subsegment extraction operator $.[\cdot, \cdot] : \mathcal{S} \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathcal{S} \cup \{\perp\}$. Thus, given a segment s , $\|s\|$ represents its length and $s[a, b]$ for $0 \leq a < b \leq \|s\|$, represents the sub-segment starting at length a from its origin and ending at length b . Segments can be used to represent roads at different levels of abstraction, from intervals to regions. In this paper, we consider \mathcal{S} as the set of curves obtained by concatenation of line segments and circle arcs, for representing roads of a map as depicted in Fig. 1.

We use metric graphs $G \stackrel{def}{=} (V, \mathcal{S}, E)$ to represent maps, where V is a finite set of *vertices*, \mathcal{S} is a set of segments and $E \subseteq V \times \mathcal{S}^* \times V$ is a finite set of *edges* labeled by *non-zero length* segments (denoted \mathcal{S}^*). For an edge $e = (v, s, v') \in E$ we denote $\bullet e \stackrel{def}{=} v$, $e \bullet \stackrel{def}{=} v'$, $e.s \stackrel{def}{=} s$. For a vertex v , we define $\bullet v \stackrel{def}{=} \{e \mid e \bullet = v\}$ and $v \bullet \stackrel{def}{=} \{e \mid \bullet e = v\}$. We call a metric graph *connected* (resp. *weakly connected*) if a path (resp. an undirected path) exists between any pair of vertices.

We consider the set $Pos_G \stackrel{def}{=} V \cup \{(e, a) \mid e \in E, 0 \leq a \leq \|e.s\|\}$ of *positions* defined by a metric graph. Note that positions $(e, 0)$ and $(e, \|e.s\|)$ are considered equal respectively to positions $\bullet e$ and $e \bullet$. We denote by $p \xrightarrow{s}_G p'$ the existence of an s -labelled *edge ride* between succeeding positions $p = (e, a)$ and $p' = (e, a')$

in the same edge e whenever $0 \leq a < a' \leq \|e.s\|$ and $s = e.s[a, a']$. Moreover, we denote by $p \overset{s}{\rightsquigarrow}_G p'$ the existence of an s -labelled *ride* between arbitrary positions p, p' , that is, $\rightsquigarrow_G \stackrel{\text{def}}{=} (\rightarrow_G)^+$ the transitive closure of edge rides. Finally, we define the distance d_G from position p to position p' as 0 whenever $p = p'$ or the minimum length among all segments labeling rides from p to p' and otherwise $+\infty$ if no such ride exists. Whenever G is fixed in the context, we will omit the subscript G for positions Pos_G , distance d_G , and rides \rightarrow_G or \rightsquigarrow_G .

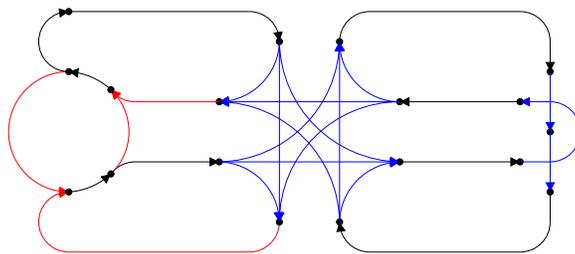


Fig. 1: A map with junctions (blue edges) and a merger vertices (red edges)

A connected metric graph $G = (V, \mathcal{S}, E)$ can be interpreted as a map, structured into roads and junctions, subject to additional assumptions:

- we restrict to metric graphs which are 2D-consistent [5], meaning intuitively they can be drawn in the 2D-plane such that the geometric properties of the segments are compatible with the topological properties of the graph. In particular, if two distinct paths starting from the same vertex v , meet at another vertex v' , the coordinates of v' calculated from each path are identical. For the sake of simplicity, we further restrict to graphs where distinct vertices are located at distinct points in the plane, and moreover, where no edge is self-crossing (meaning actually that distinct positions (e, a) of the same edge e are located at distinct points).
- the map is equipped with a symmetric *junction* relationship \times on edges E which abstracts the geometric crossing (or the proximity) between edges at positions other than the edge end points. This relationship is used to define the *junctions* of the map, that is, as any non-trivial equivalence class in the transitive closure of \times . Actually, junctions need additional signalisation to regulate the traffic on their edges (e.g., traffic lights, stop signs, etc). In addition, we assume a partial ordering $<_j$ on the set of vertices to reflect their static priorities as junction entries.
- to resolve conflicts at merger vertices, i.e., vertices with two or more incident segments which do not belong to a junction, we assume that the map is equipped with a static priority relationship. Specifically, for a vertex v , there is a total priority order $<_v$ on the set of edges $\bullet v$. This order reflects an abstraction of the static priority rules associated with each of the merging edges (e.g., right-of-way, yield-priority, etc).
- every edge e is associated with a maximal speed limit $e.v \in \mathbb{R}_{\geq 0}$.

In the remainder of the paper, we consider a fixed metric graph $G = (V, \mathcal{S}, E)$ altogether with the junction relationship \bowtie , static priorities \prec_v and edge speed limits as discussed above. Also, we extend the junction and priority relationships from edges to their associated positions, that is, consider $(e_1, a_1) \sim (e_2, a_2) \stackrel{def}{=} e_1 \sim e_2$ for any relation $\sim \in \{\bowtie, (\prec_v)_{v \in V}\}$. Finally, we denote by $r_1 \uplus r_2$ the property that rides r_1, r_2 in G are *non-crossing*, that is, their sets of positions are disjoint and moreover not belonging to the same junction(s), except for endpoints.

3 The ADS Dynamic Model

3.1 General ADS Architecture

Given a metric graph G representing a map, the state of an ADS is a tuple $\langle st_o \rangle_{o \in \mathcal{O}}$ representing the distribution of a finite set of objects \mathcal{O} with their relevant dynamic attributes on the map G . The set of objects \mathcal{O} includes a set of vehicles \mathcal{C} and fixed equipment such as lights, road signs, gates, etc. For a vehicle c , its state $st_c \stackrel{def}{=} \langle c.p, c.\delta, c.v, c.wt, c.it \dots \rangle$ includes respectively its *position* on the map (from Pos), its *displacement* traveled since $c.p$ (from $\mathbb{R}_{\geq 0}$), its *speed* (from $\mathbb{R}_{\geq 0}$), the *waiting time* (from $\mathbb{R}_{\geq 0}$) which is the time elapsed since the speed of c became zero, its *itinerary* (from the set of segments \mathcal{S}) which labels a ride starting at $c.p$, etc. For a traffic light lt , its state $st_{lt} \stackrel{def}{=} \langle lt.p, lt.cl, \dots \rangle$ includes respectively its *position* on the map (from Pos), and its *color* (with values *red* and *green*), etc.

The general ADS model is illustrated in Fig. 2 and consists of a set of vehicle models \mathcal{C} and a *Runtime* that interact cyclically with period Δt . The *Runtime* calculates free space values for each vehicle c which are lengths $c.f$ of initial rides on their itineraries $c.it$ whose positions are free of obstacles. In turn, the vehicles adapt their speed to stay within the allocated free space. Specifically, the interaction proceeds as follows:

- each vehicle c applies a *speed policy* for period Δt respecting its free space $c.f$ received from the *Runtime*. During Δt , it travels a distance $c.\delta'$ to some new position $c.p'$, and at the end of the period its speed is $c.v'$, its itinerary $c.it'$, etc. The new state is then communicated to the *Runtime*.
- the *Runtime* updates the system state on the map taking into account the new vehicle states and time-dependent object attributes. Then it applies a *free space policy* computing the tuple $\langle c.f' \rangle_{c \in \mathcal{C}}$, the new free space for all vehicles based on the current system state. The corresponding free spaces are then communicated to vehicles and the next cycle starts.

Note that the coordination principle described is independent of the type of segments used in the map, e.g. intervals, curves or regions. For simplicity, we take the free spaces to measure the length of an initial ride without obstacles on the vehicle itinerary. This abstraction is sufficient to state the basic results. We discuss later how they can be generalized for richer interpretations of the map.

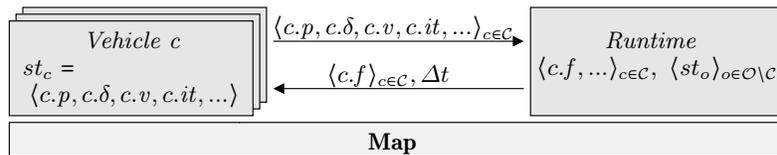


Fig. 2: General ADS architecture

3.2 Assume-Guarantee for Safe Control Policies

We give below the principle of a safe control policy for vehicles, which respects their allocated free space, applying assume-guarantee reasoning.

We consider the following two hypotheses. For a vehicle c , there exists a function $B_c : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ that gives the minimum braking distance c needs to stop from speed v , in case of emergency. Furthermore, for a non-negative distance f , let $Ahead_c(f)$ denote the ride consisting of the positions reachable on the itinerary $c.it$ from the current vehicle position $c.p$ within distance f , formally $Ahead_c(f) \stackrel{def}{=} \{p' \in Pos \mid \exists \delta \leq f. c.p \stackrel{c.it[0, \delta]}{\rightsquigarrow} p'\}$.

The following definition specifies a safe control policy using assume-guarantee reasoning on the components of the ADS architecture. We consider assume-guarantee contracts on components defined as pairs of properties A/G specifying respectively the input-output component behavior for a cycle, i.e., respectively, what the component guarantees (G) provided its environment conforms to given assumption (A).

Definition 1 (safe control policy). *A control policy is safe if*

- each vehicle $c \in \mathcal{C}$ respects the assume-guarantee contract:

$$0 \leq c.v, B_c(c.v) \leq c.f \mid 0 \leq c.v', 0 \leq c.\delta', c.\delta' + B_c(c.v') \leq c.f, \\ c.p \stackrel{c.it[0, c.\delta']}{\rightsquigarrow} c.p', c.it' = c.it[c.\delta', -]$$

- the Runtime respects the assume-guarantee contract:

$$\bigwedge_c 0 \leq c.\delta \leq c.f, \bigcup_c Ahead_c(c.f - c.\delta) \mid \bigwedge_{c \in \mathcal{C}} c.f' \geq c.f - c.\delta, \bigcup_{c \in \mathcal{C}} Ahead_c(c.f')$$

The policy is the joint enforcement of safe speed policies for vehicles and safe free space policies for the Runtime. Vehicle safe speed policies require that if a vehicle can brake safely by moving forward within its allocated free space at the beginning of a cycle, then it can adapt its speed moving forward within this space. Runtime safe free space policies require that if the free spaces of the vehicles are non-crossing at the beginning of a cycle, then it is possible to find new non-crossing free spaces for the vehicles provided they move forward in their allocated free space.

Theorem 1. *Safe control policies preserve the following invariants:*

- the speed is positive and compliant to the free space, for all vehicles, that is, $\bigwedge_{c \in \mathcal{C}} 0 \leq c.v \wedge B(c.v) \leq c.f$,

– the free spaces are non-crossing, that is, $\biguplus_{c \in C} Ahead_c(c.f)$.

Note that this theorem guarantees the safety of the coordination insofar as the vehicles respecting their contracts remain in their allocated free spaces which are non-crossing by construction. Nevertheless, the result leaves a lot of freedom to vehicles and the *Runtime* to choose speeds and non-crossing free spaces. In particular, two questions arise concerning these choices. The first question is whether the system can reach states where no progress is possible. One can imagine traffic jam situations, for example when vehicles do not have enough space to move. The second question is whether free space choices can be determined by traffic rules that actually enforce fairness in resolving conflicts between vehicles. This question is discussed in detail in Section 5.

4 Speed Policies abiding by the Vehicle Contract

In this section, we show that it is possible for vehicles to compute speed policies in accordance with their contract.

The behavior of each vehicle is defined by a controller, which given its current speed and its free space, computes the displacement for Δt so that it can safely move in the free space. Such safe speed policies have been studied in [24,25].

We illustrate the principle of safe speed policy with respect to f considering that each vehicle is equipped with a controller that receives a free space value and adjusts its speed adequately. For the sake of simplicity, assume the controller can select among three different constant acceleration values $\{-b_{max}, 0, a_{max}\} \in \mathbb{R}$ respectively, the negative value $-b_{max}$ for decreasing, the zero value for maintaining and the positive value a_{max} for increasing the speed. At every cycle, the controller will select the highest acceleration value for which the vehicle guarantee holds as defined by its contract in Def. 1. Nonetheless, an exception applies for the very particular case where the vehicle stops within the cycle, which cannot be actually handled with constant acceleration.

The proposed speed policy defines the new speed v' and displacement δ' using a region decomposition of the safe $v \times f$ space (that is, where $v \geq 0$ and $f \geq B(v)$) as follows:

$$v', \delta' \stackrel{def}{=} \begin{cases} 0, f & \text{if } f \geq B(v), f - v\Delta t < B(v), v - b_{max}\Delta t < 0 \\ v - b_{max}\Delta t, v\Delta t - b_{max}\Delta t^2/2 & \text{if } f \geq B(v), f - v\Delta t < B(v), v - b_{max}\Delta t \geq 0 \\ v, v\Delta t & \text{if } f - v\Delta t \geq B(v), f - v\Delta t - a_{max}\Delta t^2/2 < B(v + a_{max}\Delta t) \\ v + a_{max}\Delta t, v\Delta t + a_{max}\Delta t^2/2 & \text{if } f - v\Delta t - a_{max}\Delta t^2/2 \geq B(v + a_{max}\Delta t) \end{cases} \quad (1)$$

Intuitively, the regions are defined such that, when the corresponding acceleration is constantly applied for Δt time units, the guarantee on the vehicle is provable given the assumptions and the region boundary conditions.

Moreover, the vehicle position and the itinerary are updated according to the travelled distance by taking $c.p'$ such that $c.p \xrightarrow{c.it[0,c.\delta']} c.p'$ and $c.it' = c.it[c.\delta', -]$. Furthermore, the waiting time $c.wt$ is updated by taking $c.wt' \stackrel{def}{=} c.wt + \Delta t$ if $c.v = c.v' = 0$ and $c.wt' \stackrel{def}{=} 0$ otherwise.

Proposition 1. *The region-based speed policy respects the safety contract for vehicles if the braking function is $B(v) = v^2/2b_{max}$.*

Note that the speed policy works independently of the value of the parameter Δt , which is subject only to implementation constraints, e.g., it must be large enough to allow the controlled electromechanical system to realize the desired effect. A large Δt may imply low responsiveness to changes and jerky motion, but will never compromise the safety of the system.

The proposed implementation of the speed policy is "greedy" in the sense that it applies maximum acceleration to move as fast as possible in the available space. We could have "lazy" policies that do not move as fast as possible, and simply extend the travel time. We have shown in [24] that the region-based speed policy approaches the optimal safety policy, i.e., the one that gives the shortest travel time, when we refine the choice of acceleration and deceleration rates in the interval $[-b_{max}, a_{max}]$.

5 Free Space Policies implied by Traffic Rules

In this section, we study free space safety policies for a given set of global system properties describing traffic rules. We formalize traffic rules as a class of linear temporal logic formulas and provide a method for computing free space values for vehicles that allow them to meet a given set of traffic rules.

5.1 Writing Specifications of Traffic Rules

Given a map G and a set of objects \mathcal{O} , we specify traffic rules as formulas of a linear time logic of the following form, where \square is the *always* time modality and \mathbf{N} is the *next* time modality:

$$\square \forall c_1. \forall o_2 \dots \forall o_k. \phi(c_1, o_2, \dots, o_k) \implies \mathbf{N} \psi(c_1, o_2, \dots, o_k) \quad (2)$$

A rule says that for any run of the system, the satisfaction of the precondition ϕ implies that the postcondition ψ holds at the next state. Both ϕ and ψ are boolean combinations of state predicates as defined below. Furthermore, we assume that ψ constrains the speed of a single vehicle c_1 for which the property is applicable, and which we call for convenience the *ego* vehicle.

The rules involve state predicates ϕ in the form of first-order assertions built from variables and object attributes (denoting map positions, segments, reals, etc) using available primitives on map positions (e.g., rides \rightsquigarrow , edge rides

\rightarrow , distance d , equality $=$), on segments (e.g., concatenation and subsegment extraction), in addition to real arithmetic and boolean operators.

Moreover, we define auxiliary non-primitive *location* and *itinerary* predicates proven useful for the expression of traffic rules. For a vehicle $c \in \mathcal{C}$ and x either an object $o \in \mathcal{O}$, a vertex u or an edge e of the map, we define the predicates $c@x$ (c is at x), $c \rightarrow x$ (c meets x along the same edge), $c \rightsquigarrow x$ (c meets x) as in Table 1. Furthermore, for a vehicle $c \in \mathcal{C}$ and non-negative δ let $c.p \oplus_c \delta$ denote the future position of c after traveling distance δ , that is, either $c.p$ if $\delta = 0$ or the position p' such that $c.p \xrightarrow{c.it[0,\delta]} p'$. We extend \oplus_c to arbitrary future positions of c by taking $(c.p \oplus_c \delta) \oplus_c \delta' \stackrel{def}{=} c.p \oplus_c (\delta + \delta')$ and we consider the total ordering \leq_c defined as $c.p \oplus_c \delta \leq_c c.p \oplus_c \delta'$ if and only if $\delta \leq \delta'$.

	$c@x$	$c \rightarrow x$	$c \rightsquigarrow x$
$x = o$	$c.p = o.p$	$\exists \delta. c.p \xrightarrow{c.it[0,\delta]} o.p$	$\exists \delta. c.p \xrightarrow{c.it[0,\delta]} o.p$
$x = u$	$c.p = u$	$\exists \delta. c.p \xrightarrow{c.it[0,\delta]} u$	$\exists \delta. c.p \xrightarrow{c.it[0,\delta]} u$
$x = e$	$\exists a. c.p = (e, a)$	$\exists \delta. \exists a > 0. c.p \xrightarrow{c.it[0,\delta]} \bullet e \wedge c.p \xrightarrow{c.it[0,\delta+a]} (e, a)$	$\exists \delta. \exists a > 0. c.p \xrightarrow{c.it[0,\delta]} \bullet e \wedge c.p \xrightarrow{c.it[0,\delta+a]} (e, a)$

Table 1: Location and itinerary predicates.

We define the semantics of state predicates ϕ in the usual way, by providing a satisfaction relation $\sigma, st \vdash \phi$, where σ is an assignment of free variables of ϕ and st is a system state. A complete formal definition can be found in [5]. The semantics of rules is defined on pairs $\sigma, [st^{(t_i)}]_{i \geq 0}$ consisting of a function σ assigning objects instances to object variables of the formulas and a run $[st^{(t_i)}]_{i \geq 0}$ for a finite set of objects \mathcal{O} . For initial state $st^{(t_0)}$ we define *runs* as sequences of consecutive states $[st^{(t_i)}]_{i \geq 0}$ obtained along the cyclic *ADS* execution as described in section 3.1 and parameterized by the sequence of time points $t_i \stackrel{def}{=} t_0 + i \cdot \Delta t$, that is, equal to the time for reaching the i^{th} system state.

We provide examples of traffic rules in Table 2. We restrict ourselves to safety rules that characterize boundary conditions that should not be violated by the driver controlling the vehicle speed. Therefore, the preconditions characterize potential conflict situations occurring at intersections as well as other constraints implied by the presence of obstacles or speed rules, e.g., traffic lights or speed limit signals. The preconditions may involve various itinerary and location predicates and constraints on the speed of the ego vehicle. Moreover, the latter are limited to constraints maintained by the vehicle and involving braking functions in the form $B_c(c.v) \# k$ where k is a distance with respect to a reference position on the map and $\#$ is a relational symbol $\# \in \{<, \leq, =, \geq, >\}$. Furthermore, the postconditions involve two types of constraints on the speed of the ego vehicle: either speed regulation constraints that limit the distance to full stop, that is

1	enforcing safety distance between following vehicles c_1 and c_2 : $\square \forall c_1. \forall c_2. c_1 \rightsquigarrow c_2 \implies \mathbf{N} B_{c_1}(c_1.v) \leq d(c_1.p, c_2.p)$
2	coordination within all-way-stop junctions: <i>(i)</i> safe braking of vehicle c_1 approaching a stop so_1 $\square \forall c_1. \forall so_1. c_1 \rightarrow so_1 \implies \mathbf{N} B_{c_1}(c_1.v) \leq d(c_1.p, so_1.p)$ <i>(ii)</i> vehicle c_1 obeys a stop sign when another vehicle c_2 crosses the junction $\square \forall c_1. \forall so_1. \forall c_2. c_1 @ so_1 \wedge c_1.v = 0 \wedge c_2.v > 0 \wedge c_1.p \times c_2.p \implies \mathbf{N} c_1.v = 0$ <i>(iii)</i> if two vehicles c_1, c_2 are waiting before the respective stops so_1, so_2 and c_2 waited longer than c_1 then c_1 has to stay stopped $\square \forall c_1. \forall so_1. \forall c_2. \forall so_2. c_1 @ so_1 \wedge c_1.v = 0 \wedge c_2 @ so_2 \wedge c_2.v = 0 \wedge$ $c_1.p \times c_2.p \wedge c_1.wt < c_2.wt \implies \mathbf{N} c_1.v = 0$ <i>(iv)</i> if two vehicles c_1, c_2 are waiting before the respective stops so_1, so_2 the same amount of time and c_2 is at an entry with higher priority then c_1 has to stay stopped $\square \forall c_1. \forall so_1. \forall c_2. \forall so_2. c_1 @ so_1 \wedge c_1.v = 0 \wedge c_2 @ so_2 \wedge c_2.v = 0 \wedge$ $c_1.p \times c_2.p \wedge c_1.wt = c_2.wt \wedge so_1.p <_j so_2.p \implies \mathbf{N} c_1.v = 0$
3	coordination using traffic-lights: if vehicle c_1 meets a red traffic light lt_1 , it will remain in safe distance $\square \forall c_1. \forall lt_1. c_1 \rightarrow lt_1 \wedge lt_1.color = red \wedge B_{c_1}(c_1.v) \leq d(c_1.p, lt_1.p)$ $\implies \mathbf{N} B_{c_1}(c_1.v) \leq d(c_1.p, lt_1.p)$
4	priority-based coordination of two vehicles c_1 and c_2 whose itineraries meet at merger vertex u : <i>(i)</i> if c_2 cannot stop at u then c_1 must give way $\square \forall c_1. \forall c_2. \forall u. c_1 \rightarrow u \wedge B_{c_1}(c_1.v) \leq d(c_1.p, u) \wedge$ $c_2 \rightarrow u \wedge B_{c_2}(c_2.v) > d(c_2.p, u) \implies \mathbf{N} B_{c_1}(c_1.v) \leq d(c_1.p, u)$ <i>(ii)</i> if c_1, c_2 are reaching u and c_1 has less priority than c_2 then c_1 must give way $\square \forall c_1. \forall c_2. \forall u. c_1 \rightarrow u \wedge B_{c_1}(c_1.v) = d(c_1.p, u) \wedge c_1.p <_u c_2.p \wedge$ $c_2 \rightarrow u \wedge B_{c_2}(c_2.v) = d(c_2.p, u) \implies \mathbf{N} B_{c_1}(c_1.v) \leq d(c_1.p, u)$
5	enforcing speed limits for vehicle c_1 : <i>(i)</i> if c_1 is traveling in an edge e then its speed should be lower than the speed limit $\square \forall c_1. \forall e. c_1 @ e \implies \mathbf{N} c_1.v \leq e.v$ <i>(ii)</i> if c_1 is approaching an edge e then it controls its speed so that it complies with the speed limit at the entrance of e $\square \forall c_1. \forall e. c_1 \rightarrow e \implies \mathbf{N} B_{c_1}(c_1.v) \leq d(c_1.p, \bullet e) + B_{c_1}(e.v)$

Table 2: Traffic rules

$B_{c_1}(c_1.v)$, or speed limitation constraints requiring that the speed $c_1.v$ does not exceed a given limit value.

Note the difference with other approaches using unrestricted linear temporal logic, with "eventually" and "until" operators, to express traffic rules, e.g. [5]. We have adopted the above restrictions because they closely characterize the vehicle safety obligations in the proposed model. Furthermore, as we show below, traffic rules of this form can be translated into free space rules that can reinforce the policy managed by the *Runtime*.

5.2 Deriving Free Space Rules from Traffic Rules

We show that we can derive from traffic rules limiting the speed of vehicles, rules on free space variables controlled by the *Runtime* such that both the traffic rules and the free space contract hold.

To express constraints on the free space variables $c.f$, we use, for vehicles c , auxiliary *limit position* variables $\langle c.\pi \rangle_{c \in \mathcal{C}}$ such that $c.\pi = c.p \oplus_c c.f$. In other words, the limit position $c.\pi$ defines the position beyond which a vehicle should not be according to its contract. It is clear that for given $c.\pi$ and $c.p$, $c.f$ is defined as the distance from $c.p$ to $c.\pi$.

Using the limit position variables $\langle c.\pi \rangle_{c \in \mathcal{C}}$ we can transform structurally any state formula ϕ into a free space formula ϕ_π by replacing constraints on speeds by induced constraints on limit positions as follows, for $\# \in \{<, \leq, =, \geq, >\}$ and t a non-negative real constant:

$$\begin{array}{ll} B_c(c.v) \# d(c.p, x) + t & \mapsto c.\pi \#_c x \oplus_c t \\ c.v \# t & \mapsto c.\pi \#_c c.p \oplus_c B_c(t) \end{array}$$

The first case concerns speed regulation constraints bounding the limit position $c.\pi$ relatively to the position x of a fixed or moving obstacle ahead of c , that is, a stop or traffic light sign, a vehicle, etc. The second case concerns speed limitation constraints bounding $c.\pi$ relatively to the current vehicle position $c.p$ and the allowed speed.

Given a state formula ϕ , let ϕ_π be the derived formula obtained by replacing constraints on speeds by constraints on limit positions. The following theorem guarantees preservation between properties involving speed constraints and properties involving limit positions, in relation to the vehicle speed contracts.

Theorem 2. *The following equivalences hold:*

$$\begin{array}{l} (i) \quad \phi \iff (\exists c.\pi)_c \phi_\pi \wedge \bigwedge_c B_c(c.v) = d(c.p, c.\pi) \\ (ii) \quad \not\prec\phi \iff (\exists c.\pi)_c \phi_\pi \wedge \bigwedge_c B_c(c.v) \leq d(c.p, c.\pi) \end{array}$$

where $\not\prec\phi$ is the speed-lower closure of ϕ , that is, ϕ where speed constraints of the form $c.v \# t$, $B_c(c.v) \# d(c.p, x) + t$ for $\# \in \{\geq, >\}$ are removed.

Re-calling Thm. 1 in section 3.2, notice that $B_c(c.v) \leq d(c.p, c.\pi)$ is enforced by safe control policies as $d(c.p, c.\pi) = c.f$. Therefore, any property ϕ is preserved through equivalence only when all the vehicles run with the maximal allowed speed by the distance to their limit positions. Otherwise, the speed-lower closure $\not\prec\phi$ is preserved through equivalence, that is, only the upper bounds on speeds as derived from corresponding bounds on limit positions.

Therefore, all traffic rules of form (2) which, for states satisfying the precondition ϕ , constrain the speed of vehicle c_1 at the next cycle according to constraint ψ , are transformed into free space rules on limit positions of the form:

$$\square \forall c_1. \forall o_2 \dots \forall o_k. \phi_\pi(c_1, o_2, \dots, o_k) \implies \mathsf{N} \psi_\pi(c_1, o_2, \dots, o_k) \quad (3)$$

Notice that the postcondition ψ_π is of the form $c_1.\pi \leq_{c_1} b_\psi(c_1, o_2, \dots, o_k)$ for a position term b_ψ obtained by the transformation of ψ .

For example, the traffic rule 1 is transformed into the free space rule: $\Box \forall c_1. \forall c_2. c_1 \rightsquigarrow c_2 \implies \mathbf{N} c_1.\pi \leq_{c_1} c_2.p$. The traffic rule 4(ii) is transformed into the free space rule: $\Box \forall c_1. \forall c_2. \forall u. c_1 \rightarrow u \wedge c_2 \rightarrow u \wedge c_1.\pi = u \wedge c_2.\pi = u \wedge c_1.p <_u c_2.p \implies \mathbf{N} c_1.\pi \leq_{c_1} u$.

We are now ready to define the *Runtime* free space policy based on traffic rules. Let \mathcal{R} denotes the set of traffic rules of interest e.g., the ones defined in Table 2. For a current *ADS* state st and current limit positions and free spaces $\langle c.\pi, c.f \rangle_{c \in \mathcal{C}}$ the policy computes new limit positions and new free spaces $\langle c.\pi', c.f' \rangle_{c \in \mathcal{C}}$ as follows:

$$c.\pi' \stackrel{def}{\leq_c} \min_{\leq_c} \{ \sigma b_\psi \mid [\forall c_1. \forall o_2 \dots \forall o_k. \phi \implies N\psi] \in \mathcal{R}, \sigma[c/c_1], st \vdash \phi_\pi \} \cup \{ e^\bullet \mid \exists a < \|e\|, c.\pi = (e, a), c \rightsquigarrow e \} \quad (4)$$

$$c.f' \stackrel{def}{=} \delta \text{ such that } c.p \oplus_c \delta = c.\pi' \quad (5)$$

Actually, that means computing for every vehicle c the new limit position $c.\pi'$ as the nearest position with respect to \leq_c from two sets of bounds. The first set contains the bounds σb_ψ computed for all the free space rules derived from the traffic rules in \mathcal{R} and applicable for c at the given state st . The second set contains the endpoint e^\bullet of the edge e where the current limit position $c.\pi$ is located. It is needed to avoid "jumping" over e^\bullet , even though this is allowed by application of the rules, as e^\bullet may be a merger node and should be considered for solving potential conflicts. Then, we define the new free space $c.f'$ as the distance δ from the current position $c.p$ to the new limit position $c.\pi'$ measured along the itinerary of c .

Note that if the free space policy respects the assume-guarantee contract of the *Runtime* from Def. 1 then it will moreover guarantees the satisfaction of all traffic rules from \mathcal{R} where both the pre- and the postcondition ϕ and ψ are speed-lower closed formulas. First, conformance with respect to the contract is needed to obtain the invariants $B_c(c.v) \leq c.f = d(c.p, c.\pi)$ according to Thm. 1. Second, these invariants ensure preservation through equivalence between speed-lower closed formula and derived formula on limit positions, according to Thm. 2. Third, the free space policy ensures the satisfaction of the derived free space rules, that is, by construction it chooses limit positions ensuring postconditions ψ_π hold whenever preconditions ϕ_π hold. As these formulas are preserved through equivalence, it leads to the satisfaction of the original traffic rule.

5.3 Correctness with respect to the Free Space Contract

We prove correctness, that is, conformance with the assume-guarantee contract of Def. 1, of the free space policy obtained by the application of the traffic rules from Table. 2 excluding the one concerning traffic lights. For this rule we

need additional assumptions taking into account the light functioning and the behavior of the crossing vehicles.

First, we assume that the vehicle braking dynamics are compatible with the speed limits associated with the map segments, that means:

- for any edge e leading to a junction (and henceforth a stop sign) or a merger vertex holds $B_c(e.v) \leq \|e\|$, for any vehicle $c \in \mathcal{C}$ (see Figure 3(a)),
- for any consecutive edges e_1, e_2 holds $B_c(e_1.v) \leq \|e_1\| + B_c(e_2.v)$, for any vehicle $c \in \mathcal{C}$ (see Figure 3(b)) i.e., between two consecutive speed limit changes, there is sufficient space to adapt the speed.

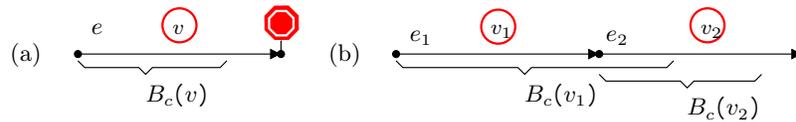


Fig. 3: Explaining restrictions on speed limits

Second, we call an *ADS* state $\langle st_o \rangle_{o \in \mathcal{O}}$ consistent with limit positions $\langle c.\pi \rangle_{c \in \mathcal{C}}$ iff for every vehicle $c \in \mathcal{C}$:

- the limit position is ahead of the current vehicle position, that is, $c.p \leq c.\pi$,
- there is no stop sign located strictly between the current vehicle position and the limit position, that is, $c.p < so.p < c.\pi$ does not hold for any stop so ,
- the limit position conforms to the speed limits of the current edge (e_1) and next edge (e_2) on the itinerary of c , that is, $d(c.p, c.\pi) \leq B_c(e_1.v)$ and $d(c.p, c.\pi) \leq d(c.p, e_2) + B_c(e_2.v)$.

The next proposition states the correctness of the free space policy constructed from traffic rules. The proof is provided in [6].

Proposition 2. *The free space policy respects the safety contract for the Runtime provided the initial ADS state is consistent with initial limit positions.*

6 Discussion

The paper studies results for the correct by design coordination of *ADS* based on assume-guarantee contracts. The coordination follows a two-step synchronous interaction protocol between vehicles and a *Runtime* that, based on the distribution of vehicles on a map, computes the corresponding free spaces. A first result characterizes safe control policies as the combination of assume-guarantee contracts for vehicles and the *Runtime*. This result is then specialized by showing how policies consistent with their respective contracts can be defined for vehicles and the *Runtime*. In particular, for vehicles, we provide a principle for defining speed policies and, for the *Runtime*, we compute free space policies that conform to a set of traffic rules. The results are general and overcome the limitations of

a posteriori verification. They can be applied to *ADS* involving a dynamically changing number of vehicles. In addition, they rely on a general map-based environment model, which has been extensively studied in [5]. Control policies for vehicles and the *Runtime* can be implemented efficiently. In particular, the speed policy has been tested in various implementations [24,25] and found to be not only safe, but also closer to the optimum when refining the space of possible accelerations.

Note that the results can be extended with slight modifications to maps where the segments are curves or regions to express traffic rules involving properties of two-dimensional space, for example for passing maneuvers. For example, if we consider region maps, their segments will be regions of constant width centered on curves. Itineraries, free spaces and $B(v)$ will be regions. The relationship $B(v) \leq f$ becomes $B(v) \subseteq f$ and the addition of lengths of segments should be replaced by the disjoint union of the regions they represent. The speed control policy will remain unchanged in principle but will need a function computing the distance travelled in a region. Finally, the runtime verification of the disjointness of free spaces may incur a computational cost depending on the accuracy of the region representation.

The presented results provide a basis for promising developments in several directions. One direction is to extend the results to achieve correctness by design for general properties. We have shown that traffic rules, which are declarative properties of vehicles, can be abstracted into safety constraints on free spaces. In this way, we solved a simple synthesis problem by transforming a “static” constraint on vehicle speed into a “dynamic” constraint on shared resources.

An interesting question that should be further investigated, is whether the method can be extended to more general properties involving the joint obligation of many vehicles. For example, we can require that for any pair of vehicles c_1 and c_2 that are sufficiently close, the absolute value of the difference between their speeds is less than a constant k , i.e., $|c_1.v - c_2.v| \leq k$. This can be achieved by a free space constraint that gives more free space to the vehicle with the lower speed, assuming that vehicle speed policies are not “lazy” and use as soon as possible the available space.

For general properties involving more than one vehicle, it seems realistic to translate them directly into free space constraints that will enforce the constraints processed by the *Runtime* to ensure the safe control policy. In particular, in addition to safety properties, we could devise free space policies that optimize criteria such as road occupancy and uniform separation for a given group of vehicles e.g. platoon systems studied in [9]. Note that achieving non-blocking control is such a property that involves the application of occupancy criteria.

Another direction is to move from centralized to distributed coordination with many runtimes. It seems possible to partition traffic rules according to the geometric scope of their application, e.g., a specific runtime could control access to each junction. Finally, the *Runtime* can be used as a monitor to verify that the vehicle speed policies of an *ADS* are safe and respect the given traffic rules.

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