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CEREBRAL VENOUS BLOOD FLOW MODEL ENHANCED WITH FLOW-MRI DATA ASSIMILATION

Olivier Balédent², Guillaume Dollé¹, Pierre Mollo¹, and Stéphanie Salmon¹

¹Laboratoire de Mathématiques de Reims, CNRS UMR 9008

²Équipe CHIMERE, CNRS UR 7516

SUMMARY

In this paper a framework is presented to enhance numerical model of cerebral venous blood flow using flow-MRI data assimilation. The goal of the technique developed here is to use a parametric PDE model to catch the physical behavior and use observations to correct the model and obtain more realistic simulations.

Key words: *computational fluids dynamics, data assimilation, cerebral venous blood flow*

1 INTRODUCTION

In the context of a project HANUMAN supported by the French National Agency of Research (ANR-18-CE45-0014), we want to study interaction between blood and cerebrospinal fluid in the brain. In this work, we focus on venous flows, that are less studied and present features different from the arterial ones. In particular, venous flows are recently said to be related to high intra-cranial pressure [1], which plays an important role in the proper functioning of the brain.

As the venous flows are very variable from one individual to the other, we need to work on the patient specific geometry of the network. On the one hand, in Amiens Hospital, we acquire MRI images of the main venous vessels consisting in the superior sagittal sinus, the straight sinus and the transverse sinuses (right and left) that end up in the jugulars (see Fig. 1). Moreover, we are able to obtain measures of the flux in different plane sections of the network during the same acquisition process, by means of a contrast-phase sequence that links the intensity of the pixel to the velocity of the flow [2].

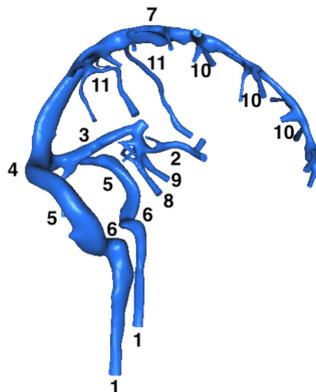


Figure 1: Cerebral Venous Network; 1) internal jugular veins, 2) inferior sagittal sinus, 3) straight sinus, 4) confluence of sinuses, 5) lateral sinus (transverse portion), 6) lateral sinus (sigmoid portion), 7) superior sagittal sinus, 8) internal cerebral vein, 9) basilar vein, 10) superior cerebral veins, 11) superior anastomotic veins.

On the other hand, we have developed open source computational fluid dynamics finite element codes to solve bloodflows in the cerebral networks [3]. The data obtained allow us first to compute the bloodflow in the patient geometry, then to calibrate our inflow parameters to correspond to the measures acquired and finally to validate our numerical results.

As the model used which is based on the Navier-Stokes equations can suffer from simplifications (*e.g.* rigid tubes, or linearization of the equations), we want then to fully exploit the "physiological"

measures we have on the patient's network to correct our in silico results by using data assimilation and the PBDW framework [4, 5, 6].

In this paper, we present a complete versatile pipeline allowing to compute the venous bloodflow in the cerebral veins, then reducing the computations with a Proper Orthogonal Decomposition (POD) and finally using the PBDW framework. We present in the next section the different bricks of the pipeline, the preliminary results obtained and the perspectives that are opened.

2 METHODOLOGY

2.1 Data acquisition and pre-processing

In order to numerically solve our computational fluid dynamic (CFD) model by a finite element method (FEM), a first step consists in extracting the anatomical geometry of the cerebral vascular network from Magnetic Resonance Imaging (MRI) acquisitions. Image segmentation tools have been developed specifically to take into account the morphology of vessels in MRI images [7]. Segmentations were performed on different MRI sequences. The spatial discretization for the resolution of the PDEs of the fluid model is constructed from the generation of a mesh from the surface of the vessel wall extracted from the 3D segmentations.

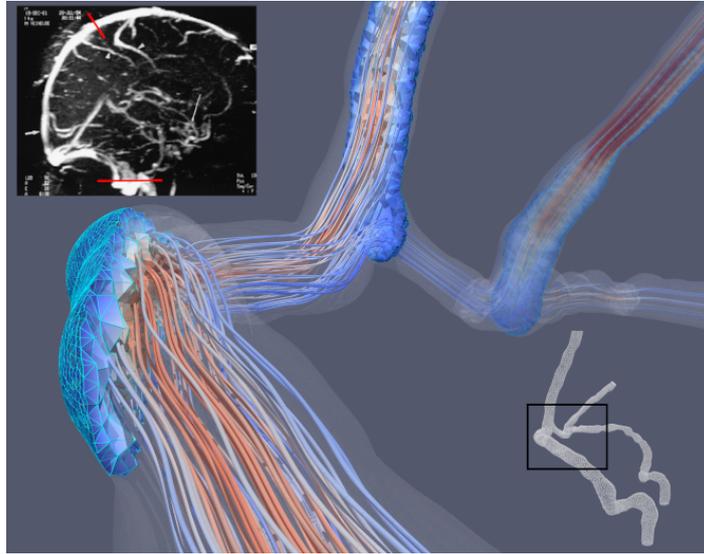


Figure 2: Example of the pipeline : from MRI acquisitions to computed bloodflow.

2.2 Fluid flows model

At the macroscopic level corresponding to the size of the concerned vessels, the blood can then be seen as an incompressible Newtonian fluid. The model considered is then the Navier-Stokes equations.

We present here only the simpler model of the parametric steady Stokes equations which describes a fluid flow using \mathbf{u} the velocity field and p the pressure field. We use a parameter $\mu \in \mathcal{P}$ to encode a range of viscosity, inflow boundary conditions, outflow resistances respectively denoted by η_μ , ρ_μ , \mathbf{g}_μ and \mathbf{h}_μ . In this way, the problem statement is as follows: for $\mu \in \mathcal{P}$, find $(\mathbf{u}(\mu), p(\mu))$ such that;

$$\left\{ \begin{array}{ll} -\nu_\mu \Delta \mathbf{u}(\mu) + \nabla p(\mu) = 0 & \text{in } \Omega \\ \nabla \cdot \mathbf{u}(\mu) = 0 & \text{on } \Omega \\ \mathbf{u}(\mu) = \mathbf{g}_\mu & \text{on } \Gamma_{\text{input}} \\ \mathbf{u}(\mu) = 0 & \text{on } \Gamma_{\text{wall}} \\ \nu_n \mu \frac{\partial \mathbf{u}(\mu)}{\partial \mathbf{n}} + p(\mu) \mathbf{n} = \mathbf{h}_\mu & \text{on } \Gamma_{\text{output}} \end{array} \right. \quad (1)$$

with $\nu_\mu = \frac{\eta_\mu}{\rho_\mu}$ the kinematic viscosity of the fluid, \mathbf{g}_μ a given inflow function and a partition of the boundary $\Gamma_{\text{input}} \cup \Gamma_{\text{wall}} \cup \Gamma_{\text{output}} = \partial\Omega$.

We introduce the following spaces: $\mathcal{X} = \{\mathbf{v} \in [H^1(\Omega)]^3, v|_{\Gamma_{\text{input}} \cup \Gamma_{\text{wall}}} = 0\}$, with the inner product $(\cdot, \cdot)_{\mathcal{X}}$ and norm $\|\cdot\|_{\mathcal{X}} = \sqrt{(\cdot, \cdot)_{\mathcal{X}}}$, for the velocity and $\mathcal{Q} = L^2(\Omega)$, with standard inner product and norm, for the pressure. The mixed problem is then solved using Taylor-Hood ($\mathbb{P}^2 - \mathbb{P}^1$) finite element to ensure the inf-sup condition and then well-posedness. Some specific solvers for the complete Navier-Stokes system are also developed, using parallel computing to increase computations speed. All codes in Freefem++ [8] are open-source and freely available on a dedicated web page.

In what follows, we focus on the velocity for the sake of simplicity, but the pressure field and the link between both velocity and pressure are also studied. The information embedded by the parametric model can be described using the manifold \mathcal{M} spanned by the parameter μ , denoted as follows:

$$\mathcal{M} = \{\mathbf{u}(\mu) \in \mathcal{X}, \mu \in \mathcal{P}\}. \quad (2)$$

If the model is well suited, we expect the real velocity field \mathbf{u}^{true} to be close to the manifold \mathcal{M} . But In fact, even with parameter μ as close as possible to the true parameter, denoted μ^{true} , i.e.:

$$\mu^{\text{true}} = \arg \inf_{\mu \in \mathcal{P}} \|\mathbf{u}(\mu) - \mathbf{u}^{\text{true}}\|_{\mathcal{X}}, \quad (3)$$

the model will never catch perfectly the reality. One can express the model error as follows:

$$\mathbf{u}^{\text{true}} = \mathbf{u}(\mu^{\text{true}}) + \eta, \quad (4)$$

with the difference η called the unparametric uncertainty, which depends on the chosen model. Some ideas have been proposed in recent works [4, 5, 6] to provide estimation of η using measures with the Parametric Background Data-Weak method (PBDW).

2.3 Model reduction and data assimilation

If the parametric model has a simple dependence on the parameter μ , one can use the Reduced Basis Method (RBM) widely documented in [9, 10], even in the case on non-affine dependence using Empirical Interpolation Method, also described in [9, 10]. In this way, one can have an exhaustive description of the manifold \mathcal{M} spanned by the parametric model with only a few resolutions. This description is exhaustive in the sense that the built reduced space Z_N can approach any elements of the manifold with a user-prescribed tolerance:

$$\forall \mu \in \mathcal{P}, \quad \|\mathbf{u}(\mu) - \Pi_{Z_N} \mathbf{u}\|_{\mathcal{X}} \leq \varepsilon_N. \quad (5)$$

This strong *a posteriori* error control, fully monitored by the user, is the best advantage of this approach which can be very expensive in the case where the parameter space \mathcal{P} is high dimensional.

An alternative option is to use Proper Orthogonal Decomposition (POD) [9, 10] to compress data contained in K snapshots $\{u(\mu_k)\}_{k=1}^K$ which are generated using the model and some random μ_k . This technique has two advantages: 1) it does not require any assumption on the parameter dependence and 2) it can compress data regardless of the model used to generate them. Unlike the RBM, the user can not control the *a posteriori* error since the snapshots used may not explore the whole space. Here one can only control level of compression, in a sense that we build a reduced space Z_N such that:

$$\|\mathbf{u}(\mu_k) - \Pi_{Z_N} \mathbf{u}\|_{\mathcal{X}} \leq \varepsilon_N, \quad 1 \leq k \leq K. \quad (6)$$

As an example, with $K = 100$ snapshots generated by the parametric model shown above we can compress 99.9999% of the information with only $N = 3$ modes.

2.4 Flow-MRI data

In Amiens hospital, we consider Phase-Contrast Velocimetry (PC) flow acquisition sequences in several adapted slices, at the level of the sinuses (straight and sagittal), lateral sinuses and jugulars in order to calibrate our numerical model using data assimilation.

These measures can be seen mathematically as follows:

$$l_m(\mathbf{v}) = \int_{\omega_m} \mathbf{v} \cdot \mathbf{n}_m d\sigma, \quad (7)$$

where ω_m is a neighbourhood of a pixel on a slice, \mathbf{n} its normal and \mathbf{v} the velocity field. This gives us M measurements to manage: one for each pixel in each slice. These measures provide reliable data on the real velocity field \mathbf{u}^{true} and motivate their use to improve the model. One way suggested in the PBDW framework to deal with these linear measures, is to seek the model error η , in (4), in the space U_M spanned by the Riesz representer $\{\mathbf{q}_1, \dots, \mathbf{q}_M\}$ of measure functions:

$$U_M = \text{span}(\mathbf{q}_1, \dots, \mathbf{q}_M) \subset \mathcal{X}, \text{ s.t. } (\mathbf{q}_m, \mathbf{v})_{\mathcal{X}} = l_m(\mathbf{v}), \quad \forall \mathbf{v} \in \mathcal{X}. \quad (8)$$

It has been shown in [4] that U_M is a reasonable candidate to find an approximation of η . One issue here is that measure functions l_m do not belong to the correct space since they act on a slice with a lower dimension (2D) than the full model (3D). We then propose alternative formulations to approximate l_m with volumetric measures in order to deploy the full PBDW framework.

3 PERSPECTIVES: COUPLING REAL DATA AND MODEL

Preliminary results were obtained numerically on an arbitrary number of observations using assimilated synthetic data. Real data from MRI slices are incorporated in the current framework to fit the reality and compared to ideal case. New MRI sequences recently developed in Amiens Hospital give access to real 3D+t data of the velocity on plane sections of the network [2] and lead us to considerate the extension of the framework to the unstationary case.

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