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DESIGN AND OPTIMIZATION OF LONG-RANGE GUIDED AMMUNITION

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ABSTRACT

This paper describes the use of optimization approaches to increase the range of near-future howitzer ammunition. The performance of a gliding ammunition concept, developed at the French-German Research Institute of Saint-Louis, is assessed using an aeroballistic workflow comprising aerodynamic characterization and flight trajectory computation. First, a single-objective optimization is run to find the maximal attainable range for this type of projectile. Results show significant range improvement along the optimization process. Then, a multi-objective formulation of the problem is proposed to consider the compromise between range and time of flight.

1. INTRODUCTION

On the battlefield, howitzer batteries are used for artillery fires on long-distance objectives. As a consequence, their range of fire is of strategic interest and several existing technologies are developed to extend it, such as base-bleed [19, 18, 12] or propulsion using rocket or air-breathing engines [2]. Still, these devices come at high price, which is often limiting for their use in operational situations. In this context, the development of affordable extended-range ammunitions for current and future artillery systems appears particularly relevant. Several studies have been conducted on this matter at the French-German Research Institute of Saint-Louis (ISL) [10, 9, 8]. The present work is part of a project focusing on the design of a long-range guided projectile without base-bleed or rocket propulsion [13, 14]. In particular,

this study aims at introducing an optimization approach in the design process.

In recent decades, optimization methods have known a great development thanks to computer science and the constant increase of computational resources. Today, they regroup various types of algorithms, that can be applied on most engineering processes, including aerodynamic design [17]. In the research field on projectiles, some studies have applied such approaches for ammunition design. For example, Vasile et al. [21] improved the geometry of a long-range guided projectile using a Particle-Swarm Optimizer (PSO). In particular, the performance of the configurations is evaluated with a Semi-Empirical Aerodynamic Prediction (SEAP) code. Another application can be found in the work of Arnoult et al. [4, 3], that performed optimizations of a trajectory control device for spin-stabilized ammunitions. The optimal geometrical parameters of this device, consisting in an aerodynamic spoiler deployed during the flight, are researched with a Surrogate-Based Optimization (SBO) approach. Moreover, this study uses a Multi-Layer Perceptron (MLP) neural network as surrogate model to predict the aerodynamic effect of the spoiler.

In the following, Section 2 introduces the developed long-range projectile and its specific flight scenario. Then, the optimization approach is detailed in Section 3 with its formulation and workflow. Section 4 provides a deeper explanation of aerodynamic characterization and the use of surrogate modelling. Finally, results obtained with single-objective and multi-objective optimizations are discussed in Sections 5 and 6.

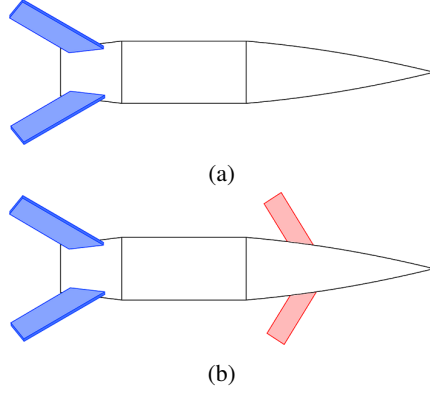


Figure 1: Top-views schematics of the projectile before (a) and after canards deployment (b)

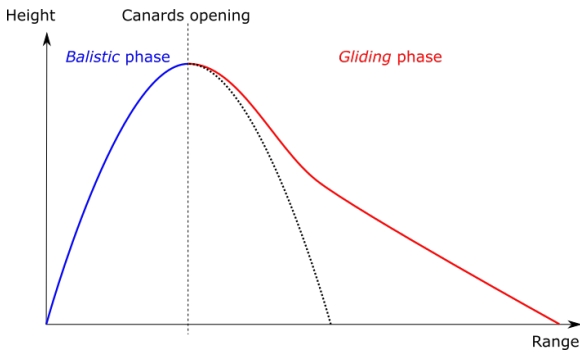


Figure 2: Projectile flight scenario

2. STUDY CASE

The studied configuration is based on a Secant Ogive Cylinder Boat Tail (SOCBT) geometry. The full-scale projectile with a calibre of 155 mm is considered. For long-range applications, the projectile is non-rotating and comprises four tail-fins, unfolded immediately after the muzzle exit. In addition, two canards are deployed during the flight, therefore resulting in the two configurations depicted in Fig. 1.

The projectile is shot from a standard howitzer gun with an initial spin rate supposed to be null. The flight strategy associated to such ammunition concept is depicted in Fig. 2 and can be decomposed in two phases, namely a *ballistic* phase with tail-fins only, and a *gliding* phase after the canards deployment. The canards are then used as lifting surfaces but also as flight control steering devices during the gliding flight.

3. OPTIMIZATION APPROACH

3.1 Formulation

The main objective of this study is to improve the range of the gliding ammunition concept, denoted as L_{flight} . This

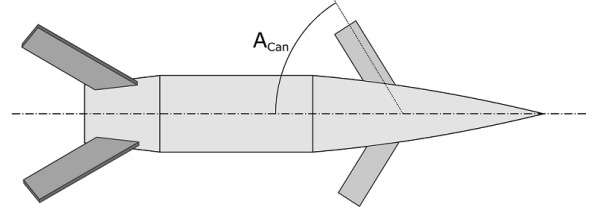


Figure 3: Definition of the canards design parameter

Table 1: Recapitulative table of optimization parameters

Decision parameters	Lower - upper boundaries
A_{Can}	$30,0^\circ - 90,0^\circ$
$Projectile_AOA_{max}$	$0,0^\circ - 16,0^\circ$
$Gun_elevation_{angle}$	$30,0^\circ - 80,0^\circ$
$Opening_time_{canards}$	$20,0\text{ s} - 80,0\text{ s}$

function is to be maximized by the optimization process. In addition, attention is given to the total flight time of the projectile, which can be of equal interest as the range from the operational perspective. Thus, it is considered as an additional function of interest, and minimizing t_{flight} is the second objective defined for optimizations.

The decision parameters considered in this work consist in both geometrical and flight parameters. The geometrical design parameter is the sweep angle of the canards after deployment, represented by the angle between the SOCBT revolution axis and the canards and named A_{Can} (as depicted on Fig. 3). Among flight parameters, the gun elevation angle, the maximum angle of attack of the projectile and the time of canards deployment can be modified by the optimizer. A summary of the decision parameters and objective functions is given in Tab. 1 and 2 respectively.

3.2 Workflow and tools

The functions of interest (summarized in Tab. 2) imply that the optimization workflow has to be aeroballistic, i.e. include aerodynamic projectile characterization and full-flight trajectory computations. The process defined for this purpose is depicted in Fig. 4.

First, the aerodynamic characterization consists in computing the projectile aerodynamic force and moment coefficients with respect to flight conditions and design parameters. Due to the flight scenario, a wide range of Mach numbers is expected. In this situation, Computational Fluid Dynamics (CFD) simulations appear well-suited, because they allow to study subsonic, transonic and supersonic flows. Still, a large number of simulations is required to characterize one geometry. This step has to be repeated when decision parameters are modified, which results in a significant overall computational cost. For this reason, the CFD simulations cannot be performed in-line during the optimization process. As

Table 2: Recapitulative table of functions of interest

Functions of interest	Objective
L_{flight}	Maximization
t_{flight}	Minimization

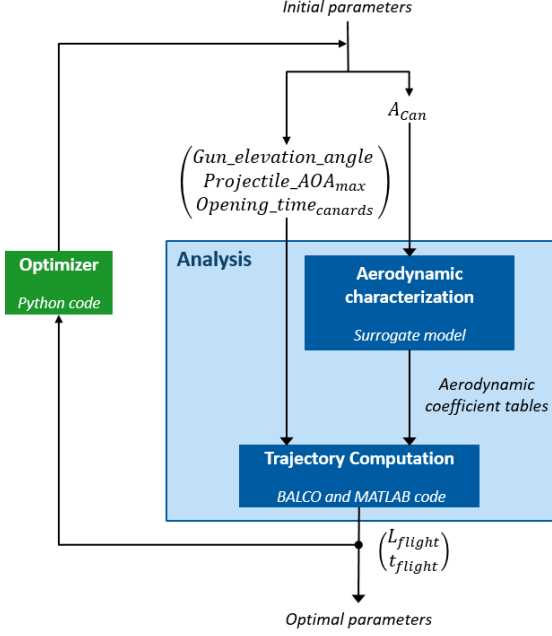


Figure 4: Optimization workflow for long range ammunition

an alternative, an approach based on surrogate models is proposed. The models aim at returning the aerodynamic coefficients of the projectile for given flight conditions and design parameters. A database of CFD computations performed prior to the optimization process, is used as training dataset for the surrogate model. The database computation, as well as model construction and training are further detailed in Section 4.

After aerodynamic characterization, the trajectory simulation computes the two-phase flight path. First, the BALCO trajectory computation tool [23] is used to simulate the ballistic flight phase and integrate the aerodynamic coefficients of the projectile with folded canards. After canards deployment, the flight is assumed to be planar and a Three-Degrees-Of-Freedom (3DoF) step-by-step computation of the gliding trajectory is performed. In addition, a basic guidance law is implemented in the simulation during the gliding phase. The canards operate as control surfaces and their steering is updated at each iteration in the code. The complete trajectory computation process is managed with MATLAB [15].

The last element in the workflow is the optimizer, which uses algorithms to find the best set of decision parameters with respect to the formulation described in Section 3.1. Two types of optimization problems are consid-

Table 3: Recapitulative table of optimization algorithms

Formulation	Algorithm	Python library
Single objective	Differential evolution	SciPy.optimize
Multi-objective	NSGA II	Platypus

ered in this work. First, a single-objective optimization is implemented, in order to maximize the flight range. Having a surrogate model instead of CFD computations inside the analysis workflow highly reduces the cost of the function evaluations. Therefore, it allows the use of algorithms that require thousands of evaluations to reach convergence. Genetic algorithms are efficient when it comes to finding an optimum in an unknown complex design space [11]. For this study, the choice is made to use a Differential Evolution (DE) algorithm [20], and more particularly its implementation proposed in the SciPy Python library [22]. This algorithm has the advantage of managing boundaries on decision parameters, as specified in Tab. 1. However, considering range maximization as a single-objective problem does not consider the induced flight time increase. In order to take it into account, a multi-objective optimization approach is introduced in a second time. Instead of looking for maximal range only, this method focuses on finding the best compromises between flight distance and time. The Non-Sorted Genetic Algorithm II (NSGA-II) [7] appears well suited in this case and is used as multi-objective optimizer. The implementation provided by the Platypus [1] Python library is chosen to perform the optimization. A summary of the algorithms and the associated libraries is presented in Tab. 3.

4. AERODYNAMIC CHARACTERIZATION AND SURROGATE MODEL

This section is dedicated to providing more explanation on the computation of aerodynamic coefficients and the use of surrogate modelling. Depending on the flight phase, the presence or absence of canards influences the behaviour of the projectile. The aerodynamic characterization as well as the trajectory computation method change accordingly.

4.1 Ballistic configuration

During the ballistic phase, the BALCO trajectory simulation uses a table of aerodynamic coefficients for the projectile without canards. Considering that this tool performs a Six-Degrees-Of-Freedom (6DoF) computation, the required aerodynamic coefficients consist in the axial, lateral and normal force coefficients denoted as (C_A, C_Y, C_N) as well as the associated moments computed at the center of gravity (C_l, C_m, C_n) . The coefficient table

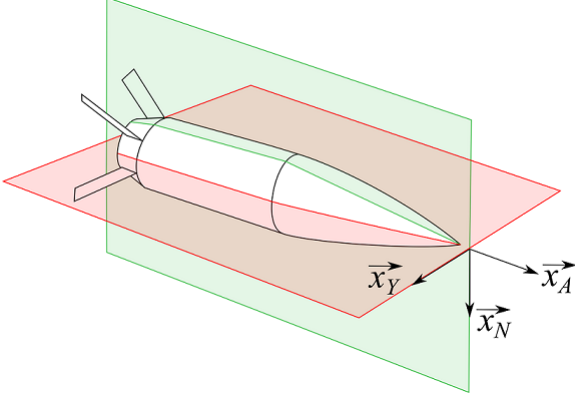


Figure 5: Sketch of the ballistic projectile geometry with its vertical and horizontal symmetry planes

consists in their evaluation at various flight conditions, i.e. for discrete values of the Mach number M , the angle of attack α and the angle of sideslip β . Under some assumptions, the set of CFD simulations required to generate this table can be simplified.

First, the ballistic projectile geometry comprises several symmetry planes, as illustrated in Fig. 5. Thanks to these symmetries, the aerodynamic behaviour can be only considered in the vertical plane, and the remaining coefficients necessary for 6DoF can be deduced from it. Similarly, the dependency on the sideslip angle is obtained from the angle of attack. This implies that only $(C_A, C_N, C_m)(M, \alpha)$ have to be considered in the following. In addition, the symmetries imply that the normal force and pitching moment are null at zero angle of attack (Eqs. 1 and 2).

$$C_N(\alpha = 0^\circ) = 0 \quad (1)$$

$$C_m(\alpha = 0^\circ) = 0 \quad (2)$$

Then, the projectile stability has been assessed previously in [14] and showed that the ammunition remains stable during the complete ballistic flight. Consequently, the angles of attack and of sideslip are supposed to vary in a limited interval around zero, and a small-angle approximation can be introduced. This hypothesis translates to a constant axial force coefficient in Eq. 3. Moreover, the lift and pitching moment coefficients are assumed to follow a linear evolution as written in Eqs. 4 and 5.

$$C_A(\alpha) \approx C_A(\alpha = 1^\circ) \quad (3)$$

$$C_N(\alpha) \approx C_N(\alpha = 0^\circ) + \alpha \times \frac{\partial C_N}{\partial \alpha}(\alpha = 0^\circ) \quad (4)$$

$$C_m(\alpha) \approx C_m(\alpha = 0^\circ) + \alpha \times \frac{\partial C_m}{\partial \alpha}(\alpha = 0^\circ) \quad (5)$$

By approximating the coefficient derivatives using their values at $\alpha = 0^\circ$ and $\alpha = 1^\circ$, then using Eqs. 1 and 2, the final formulation is obtained in Eq. 6.

$$\begin{pmatrix} C_A \\ C_N \\ C_m \end{pmatrix}(\alpha, M) \approx \begin{pmatrix} C_A(\alpha = 1^\circ) \\ \alpha \times C_N(\alpha = 1^\circ) \\ \alpha \times C_m(\alpha = 1^\circ) \end{pmatrix}(M) \quad (6)$$

To summarize, under the preceding assumptions, only one value of the angle of attack is required at each Mach number to evaluate the aerodynamic coefficients of the projectile. Simulations are performed at $\alpha = 1^\circ$, and the total cost solely depends on the investigated Mach numbers, ranging from $M = 0.3$ to $M = 3.0$.

Finally, the decision parameters detailed in Tab.1 do not affect the projectile aerodynamics before canards opening. Therefore, the coefficient table is computed once and used in the ballistic flight phase for every trajectory simulation.

In order to perform the aerodynamic computations of the projectile, a mesh comprising 5×10^6 cells is generated. Reynolds Averaged Navier-Stokes (RANS) simulations are performed with the implicit density-based solver implemented in ANSYS Fluent. The $k - \omega$ Shear Stress Transport (SST) turbulence model [16] is chosen, as well as second-order spatial discretization.

Each CFD simulation needs between 1 and 2 hours to converge on 224 cores of the supercomputer at ISL. Consequently, the computational cost associated to the ballistic projectile characterization is acceptable, with a duration of several days at most.

4.2 Gliding configuration

After canards deployment, the projectile trajectory is supposed to be contained in a vertical plane, allowing a 3DoF simulation. In this case, only 3 aerodynamic coefficients are required as inputs for the trajectory computation, namely (C_A, C_N, C_m) . Moreover, the sweep angle of the deployed canards, A_{Can} , has an impact on the aerodynamic behaviour of the projectile. Each time this design parameter is modified, the coefficient tables have to be updated accordingly. If the aerodynamic characterization is done using in-line CFD simulations in the optimization workflow, the set of computations has to be performed at each iteration.

The aerodynamics of the projectile with deployed canards not only depend on M and α , but also on the deflection angle of the canards δ . Assuming that N_α , N_M , N_δ are the numbers of discrete values taken by each parameter for the characterization, usually around a dozen, then the total cost can be expressed as $N_\alpha \times N_M \times N_\delta \sim 10^3$. Consequently, the full CFD characterization of a gliding projectile geometry represents a significantly increased cost in comparison with the ballistic configuration. This

one-time computational cost can hardly be afforded on the supercomputer at ISL, and as this is to be repeated at each optimizer iteration, this solution is not feasible.

4.2.1 Superposition hypothesis

In order to mitigate computational cost and enable the characterization with CFD nevertheless, a superposition hypothesis is proposed. This assumption considers that the aerodynamic effect of canards deflection can be *superposed* with the effect of the angle of attack. The resulting equations are developed in the following.

First, the effect of canards deflection on the overall aerodynamic coefficients is assumed to be obtained from a set of simulations at zero angle of attack, as expressed in Eq. 7.

$$\begin{pmatrix} \delta C_A \\ \delta C_N \\ \delta C_m \end{pmatrix}(\delta, M) = \begin{pmatrix} C_A \\ C_N \\ C_m \end{pmatrix}_{\alpha=0^\circ}(\delta, M) - \begin{pmatrix} C_A \\ C_N \\ C_m \end{pmatrix}_{\alpha=0^\circ}(\delta=0, M) \quad (7)$$

Then, this contribution is summed with coefficients from simulations at $\delta = 0^\circ$, in order to approximate the total aerodynamic coefficients for any flight condition. This leads to the expression given in Eq. 8.

$$\begin{pmatrix} C_A \\ C_N \\ C_m \end{pmatrix}(\alpha, \delta, M) \approx \begin{pmatrix} C_A \\ C_N \\ C_m \end{pmatrix}_{\delta=0^\circ}(\alpha, M) + \begin{pmatrix} \delta C_A \\ \delta C_N \\ \delta C_m \end{pmatrix}(\delta, M) \quad (8)$$

The superposition hypothesis allows in fact a decomposition of the coefficients dependency on α and δ . Thanks to this formulation, the aerodynamic characterization of a projectile geometry can be estimated with two sets of CFD simulations. Considering that each variable is discretized over a dozen of values for the characterization, the resulting total number of computations is $(N_\alpha \times N_M) + (N_\delta \times N_M) \sim 10^2$. Although the overall computational cost appears now acceptable for a one-time characterization, it remains too significant to be operated inside the optimization process. For that reason, the cost mitigation brought by the superposition hypothesis appears interesting but not sufficient.

4.2.2 Surrogate models

To further reduce the cost of aerodynamic characterization during the gliding flight, a surrogate modelling approach is introduced. The role of the models is to replace in the workflow the time-consuming sets of CFD simulations required to evaluate projectile aerodynamic

coefficients. In particular, it should be able to predict the coefficient table depending on the value of the decision parameter A_{Can} .

Under the superposition assumption, two CFD coefficient tables at $\alpha = 0^\circ$ and $\delta = 0^\circ$ are required to characterize the ammunition. Thus, two extensive databases of CFD simulations have been performed to evaluate $(C_A, C_N, C_m)_{\delta=0^\circ}(\alpha, M, A_{Can})$ and $(C_A, C_N, C_m)_{\alpha=0^\circ}(\delta, M, A_{Can})$. A mesh comprising 12×10^6 cells is generated for the gliding geometry, and the CFD parameters detailed in Section 4.1 are re-used. Each database comprises hundreds of CFD simulations and the overall computational time was approximately of two weeks on the supercomputer at ISL.

The computations are post-processed according to Eqs. 7 and 8 to generate training datasets. An approach using Radial-Basis Functions (RBF) interpolation method [6] is chosen for the surrogate models. More precisely, the RBF process is operated using Gaussian basis functions, as provided in the Surrogate Modelling Toolbox (SMT) [5] Python framework. The two surrogate models are then trained on the training databases to generate $(C_A, C_N, C_m)_{\alpha=0^\circ}(\alpha, M)$ and $(\delta C_A, \delta C_N, \delta C_m)(\delta, M)$ for a given value of A_{Can} .

Finally, the quality of the surrogate models is assessed before using the models for optimization. They appear to have a satisfying accuracy over the complete range of flight conditions required for trajectory simulation and parameters for optimization.

5. SINGLE-OBJECTIVE OPTIMIZATION

The first optimization is conducted with a single-objective formulation. Therefore, the genetic algorithm aims at maximizing the range of the projectile. The differential evolution process explores the design space and stops after 47 generations of 40 samples, totalizing 1880 evaluations.

A representation of the function evaluations can be observed in Fig. 6. In particular, it shows how the first generations of samples are widespread over the design space, while the last generations are evolving towards sets of parameters resulting in maximal range values. The algorithm evolution, in terms of the functions of interest, is presented in Fig. 7. The global trend of the graph depicts the strong correlation between the distance attained and the time of flight. Despite this expected dependency, it can also be observed that the decision variables allow to reach a given distance in a significant time interval, leaving room for future optimizations. Between the first and last sample generations, the mean range has been improved by 57%. As a consequence, the time of flight is increased by 52%. The optimal set of decision parameters is summarized in Tab. 4. The optimizer has found the best

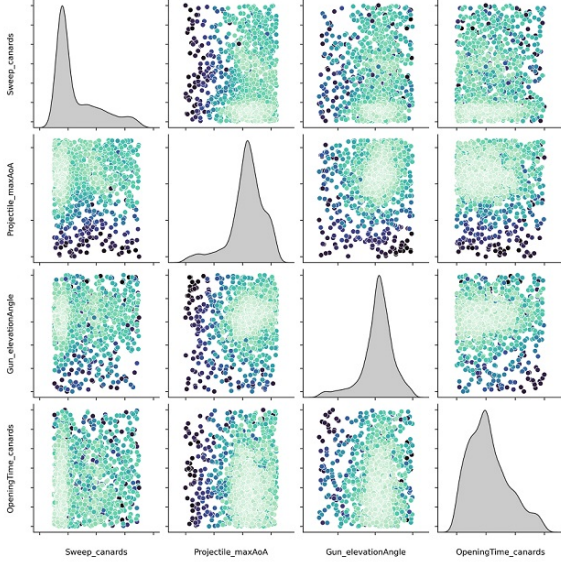


Figure 6: Evaluations performed by the genetic algorithm (coloured by flight range)

Table 4: Resulting optimal configuration, obtained with single-objective formulation

Decision parameter	Optimal value
A_{Can}	36.2 °
$Projectile_AOA_{max}$	10.7 °
$Gun_elevation_angle$	61.8 °
$Opening_time_{canards}$	39.9 s

compromise between canards-induced lift and drag over the entire gliding path by setting a low value of A_{Can} . The maximum angle of attack is limited at 10,7°, probably in order to avoid the interaction of the canards wake with the tail-fins. A significant elevation angle is retained, in order to provide a sufficient altitude before starting the glide. Moreover, the canards opening happens shortly after the apogee of the ballistic flight in order to take advantage of this altitude.

In conclusion, this first application of optimization for the design of long-range guided projectiles achieves a significant improvement of the flight range. However, a single-optimization formulation, only focusing on maximizing range without time constraint, is not representative of operational reality. Thus, the next step consists in considering multi-objective optimization problems.

6. MULTI-OBJECTIVE OPTIMIZATION

Multi-objective optimizations consider multiple functions of interest, which can be of great benefit for the long-range guided ammunition design. Such approach is implemented in the following, with a research of the best

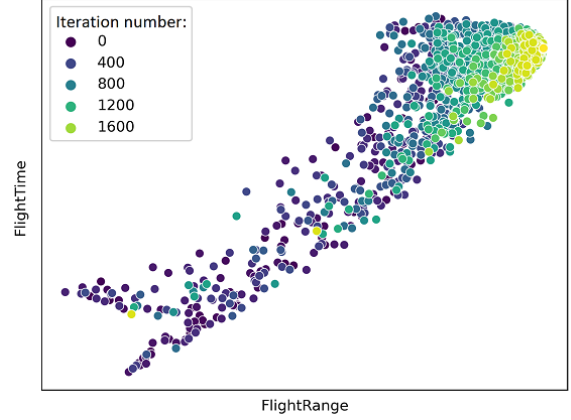


Figure 7: Evolution of flight range and time throughout genetic algorithm iterations

compromise between a minimal time of flight and maximal range. The representation of this compromise, called “Pareto front”, is generated in this study by a NSGA-II algorithm. This genetic algorithm is specially developed in order to research non-dominated solutions, i.e. to progressively populate, refine and improve this front.

The research process is launched over 100 generations of 50 samples, therefore leading to a total of 5000 analysis evaluations. All evaluated points can be observed on the visualization matrix displayed in Fig. 8. The evolution of range and time of flight over the successive samples generations is plotted in Fig. 9, as well as the resulting Pareto front. The successive generations appear to be progressively converging towards the final front. On the Pareto border, the evolution of optimal flight time with respect to the range appears to be non-linear. Due to the projectile deceleration during the gliding phase, increasing the range at long distance is more time-demanding than at shorter distances.

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In practice, the Pareto front can be of great interest for operational use and tactical decision making. It indicates the maximal reachable distance for a given flight time, or on the contrary the trajectory of minimal flight time when considering an objective at a certain range.

Finally, a deeper analysis of the set of points belonging to the Pareto front is proposed. The evolution of the decision parameters along the front is plotted in Fig. 10. Some tendencies appear clearly on this plot and give a better understanding of the points constituting the frontier. First, it seems that increasing the projectile angle of attack and the gun elevation angle are mandatory conditions to reach long-distance targets. Then, the optimal value of canards sweep angle is barely modified along the optimal line; thus, this parameter could be fixed in future studies. In addition, the optimal moments for canards deployment seem to have a less critical influence on

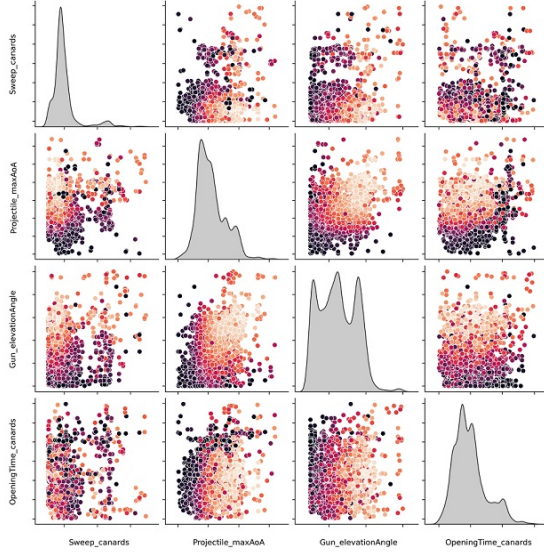


Figure 8: Evaluations performed by the NSGA-II algorithm (coloured by flight range)

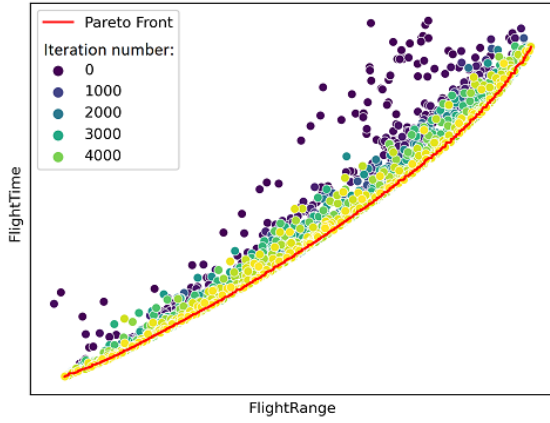


Figure 9: Evolution of flight range and time throughout NSGA-II iterations, and resulting Pareto front

the result than other parameters, as long as they remain in a certain interval.

7. CONCLUSIONS

In this paper, an alternative is presented to usual technical devices extending the fire range of howitzers. The proposed solution consists in a guided ammunition concept with tail-fins and unfoldable canards. Accordingly, the flight scenario of such projectile is decomposed in two phases, with a ballistic phase followed by canards deployment and a steered gliding phase.

Then, an optimization approach is introduced, in order to find the most-suited design and flight parameters. The workflow developed for this purpose comprises an aero-

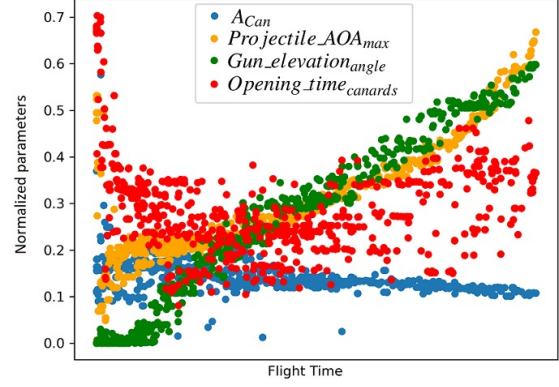


Figure 10: Evolution of normalized decision parameters along the Pareto front, depending on the associated flight time

dynamic characterization and a complete trajectory simulation. Applying single-objective range maximization results in a great improvement of the reachable distance of the projectile, at the expense of the flight duration. Therefore, a multi-objective optimization is implemented in a second time, to search sets of parameters resulting in an optimal compromise between fire range and flight time. The resulting Pareto front has the advantage of being directly interpretable, and can be used for operational decisions.

To conclude, the proposed solution has the advantage of using additional lifting surfaces and guidance, instead of requiring base-bleed or propulsion device. Hence, this long-range guided concept is expected to come at reduced cost. As a perspective, a hybrid ammunition configuration combining propulsion, additional lifting surfaces and guidance could be considered to reach very-long-range targets.

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