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Pre-service teachers' difficulties with the semiotic representation of the decimal number line

Macarena Flores González¹ and Elann Lesnes²

¹LDAR, CY Cergy Paris Université, France; macarena.flores-gonzalez@cyu.fr

²CREAD, Univ Brest, France

As a semiotic representation, the numerical line is often used in school as a tool. However, it is also considered an epistemological obstacle, which can be problematic if one is not trained to use it. In this paper, we investigate the difficulties encountered by primary and secondary pre-service teachers in representing the number line of decimal numbers when numbers are given. Using a methodology with qualitative analyses, we analysed the written productions of 103 pre-service teachers when constructing these number lines. This allowed us to identify five categories of difficulties.

Keywords: Decimal numbers, number line, teacher education.

Introduction

The number line is frequently used from the first grade to represent natural numbers in mathematics instruction. A few years later, rational numbers are the first new types of numbers that students meet. Different representations are used and mostly a so-called “regions” model (Bright et al., 1988) which may take the form of colored regions, fraction strips, fraction circles, etc. However, the numerical line is also reinvested to represent this type of numbers. Decimal numbers are usually introduced in the same year from decimal fractions and placed on a numerical line. However, numerical lines are mostly seen only as tools (Lemonidis & Gkolfos, 2020), there seems to be no specific teaching on this object.

In turn, pre-service teachers will have to teach number lines as representations of numbers. This is problematic considering the result that they make errors when placing unfamiliar decimal numbers on a previously drawn number line (Thipkong & Davis, 1991). In France, it seems that there is no specific teaching of number lines in the initial training of primary school teachers either, even if they are used as tools of representation. Therefore, our question is: how do pre-service teachers perceive the number line?

Didactic point of view on the numerical line

There is a lot of research on problems that students from grade 4 to 12 have with decimal numbers. These difficulties can be found when students read or construct number lines. However, the numerical line brings its own difficulties and can be an *epistemological obstacle* (Lemonidis & Gkolfos, 2020). Indeed, numbers are abstract objects to which we have access only through their representations. One of these semiotic representations is the numerical line (Adjage & Pluvineau, 2000). On this line, a number is considered as the abscissa of a point. But Bright et al. (1988) explain that unlike other visual representations such as the “regions” models which can be understood without the need to use any symbols, the number line requires the use of symbols, at least to define a unit. Using a numerical line therefore requires combining two types of information: visual and symbolic (Bright et al., 1988, p. 215).

On a numerical line, the difference between two numbers is represented by the length of the segment whose extremities are the abscissae of the two corresponding points. Although the numerical line can be useful for relating decimal numbers and fractions, for developing the relative magnitude of those numbers, and for representation for the density property (Widjaja, Stacey, & Steinle, 2011). Nevertheless, it can also be seen as an inadequate representation for differentiating decimal numbers from rationals (Durand-Guerrier, 2016), which could be a problem for the students and create didactic problems for teachers. This is especially true when trying to visualise the density property in a static line. Moreover, as Castela (1996) shows, for students the number line for reals seems to be confused with the decimal line, and not all students make the link between the geometric field (in reference to the points of the line) and the numerical field (in reference to the numbers).

Pre-service primary teachers have more experience with mathematics, and in particular with decimal numbers, than school students (because they continued to use them at school and in their daily lives). Thus, studies show that we find some similar difficulties in the conception of decimal numbers with this audience. For example, many of the pre-service teachers consider that whole numbers are not decimal or confuse the set of decimal numbers with the decimal writing of numbers (Flores González & Lesnes, 2023). The density property seems particularly hard to conceive (Alatorre & Sáiz, 2008). Widjaja et al. (2008) show that it is not recognised by about half of the pre-service teachers (in Indonesia) who incorrectly apply the “discretisation” of natural numbers to the set of decimal numbers. In the context of teacher training Putra (2019) shows that the mathematical and didactic use of the number line depends on the mathematical knowledge of the trainee teachers. He shows that Danish pre-service teachers are able to convert between fractional and decimal notation, they understand the density property of rationals and they can use the number line to investigate these properties with (fictitious) students. In contrast, Indonesian pre-service teachers treat fractional and decimal notation separately. When they try to use the number line, they construct two lines (one for fractions, one for decimals) and do not use the density property. Girit and Akyuz (2016) compare the strategies of students and pre-service teachers in Turkey when estimating the value of a decimal number on a segment of the graduated number line (with a given unit). Their results show that the two groups approach this problem in different ways. However, both tended to think of decimals in succession (i.e. ignore the density property) and to give precise answers rather than specifying a range of possible values. In addition, they investigated pre-service teachers' anticipation of students' misconceptions and difficulties with the same task. The result was that pre-service teachers could only partially anticipate them.

Research question

It seems that the numerical line is not always specifically studied in a systematic way in pre-service teacher education. However, according to the previous section, problems with the number line have been identified among pre-service teachers. Thus, we ask: what difficulties do future teachers have in spontaneously representing the decimal number line?

Methods

This research is part of a wider research that seeks to find out the conceptions of pre-service primary teachers about decimal numbers (Flores González & Lesnes, 2023). To do this, our methodology is

close to the qualitative methodology described in (Fraenkel et al., 2012, pp. 429–431). The analysis of the data is based on description, this does not preclude some quantification when it comes to studying in more detail a phenomenon that is being observed.

We tested the knowledge of pre-service teachers from four different National Higher Institutes of Professorship and Education (called INSPÉ) in France. We have developed a ten-question test on decimal numbers, but in order to answer the research question we are asking in this paper, we will focus on the one question that dealt with the number line: “*Place the following numbers on a graduated line: $\frac{1}{2}$; 1.51 ; 0.155 ; 1.5 ; 1.05.*”¹. The graduated scale was not allowed. The data collected are the written responses of the pre-service teachers, which gave us direct access to the semiotic representations used. They were able to answer the question without time constraint.

The test was completed only once in each institute (the four institutes were coded as ANT, MSA, SGL and REN), but it was performed at two different times (for the first time, the number 0.155 was not present in the test). These are detailed below:

- the first one (without the value 0.155) took place during the first quarter of the year 2022, about two months after the lesson on decimal and rational numbers with primary school pre-service teachers from two INSPÉ (34 from ANT and 30 from MSA) ;
- the second one took place in the last quarter of the year 2022 before the lesson on decimal and rational numbers. The students of the second run are primary school pre-service teachers (13 from SGL) and middle and high schools pre-service teachers (26 from REN).

All the pre-service teachers mentioned in this paper are in the first year of the master’s degree for teaching.

Analysis of the question

We want to know the semiotic elements used spontaneously by pre-service teachers on the *register of semiotic representation* of the numerical line as defined by Duval (2017) (Adjage & Pluvinaige, 2000; Teppo & Van den Heuvel-Panhuizen, 2014). *According to Duval's theory, we can say that the number line results from the articulation of two different registers of representation, more precisely the geometric register (Euclidean line) and the arithmetic register (numbers)* (Lemonidis & Gkolfos, 2020, p. 37). Therefore, we choose decimal numbers up to the thousandth with essentially 3 different digits (0, 1 and 5), and we vary the positioning of these digits which changes the value of the number. Only 0.5 is given in fractional writing ($\frac{1}{2}$) because it’s a number that pre-service teachers should know, and one that research shows pupils have problems with. The numbers were chosen so that they could be placed on the same graduated line but with some spatial constraints (see Figure 1). Moreover, this is not a traditional task for pre-service teachers because they are used to placing numbers on an already drawn graded line but not to having to place the graduations themselves.

¹ Traduction from French, the decimal commas used in French have been replaced by decimal points. French version: “Placer sur une droite graduée les nombres suivants : $\frac{1}{2}$; 1,51 ; 0,155 ; 1,5 ; 1,05”.

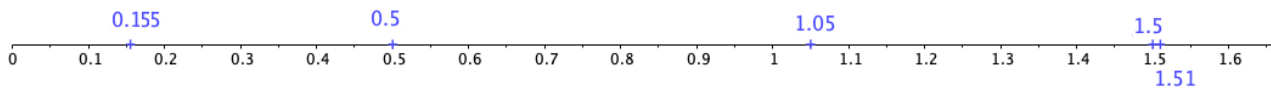


Figure 1: a possible number line with the numbers $\frac{1}{2}$; 1.51; 0.155; 1.5 and 1.05

Indeed, in a static line, the differences between tenths, hundredths and thousandths become difficult to represent and observe: how do pre-service teachers cope with this difficulty? We will identify the semiotic elements of the line considered by the pre-service teachers to represent the given numbers.

Method for analysing the semiotic representations

In order to analyse the difficulties in the written productions, in this article we set aside the answers that present expected semiotic representations (about 16% of answers collected) and we are focusing on responses that show signs of a struggle against the obstacle of the numerical line. Then, we establish two big categories of errors in relation to the difficulties already identified in the first section. First, there are difficulties relating to decimal numbers in general which may result here in a wrong order of numbers on the numerical line, or by an incorrect translation of $\frac{1}{2}$ into decimal writing (if applicable). Second, there also are difficulties relating to the representation of the difference between two numbers by the length of a segment, especially with numbers that do not have the same orders of magnitude (for example 0.155 and 1.5). The density property, which is a major difficulty for students and pre-service teachers, is not really visible here. This research is exploratory, so we expect other categories to emerge in the future.

Results of the first experiments

A summary of the errors identified is presented in table 1. In the following, we present the different categories we identified by grouping together errors that appeared several times in the students' answers and make some hypotheses explaining them. A single answer may include several errors.

Table 1: Summary of the errors

Pre-service teachers	Error in the decimal writing of $\frac{1}{2}$	$\frac{1}{2}$ in the middle of the segment	Just ordering the numbers	No clearly defined unit	Difficulties with 1/100 and 5/100
After the lesson on decimal and rational numbers					
ANT (34)	7	5	3	7	15
MSA (30)	2	1	2	4	14
Before the lesson on decimal and rational numbers					
SGL (13)	2	1	2	5	3
REN (26 secondary pre-service teachers)	1	1	1	7	12
Total (103)	12	8	8	23	44

Error in the decimal writing of $\frac{1}{2}$

We find the confusion between $\frac{1}{2}$ and 1.5 in 9 primary pre-service teachers' answers (even after the lesson on decimal and rational numbers) and 1 secondary pre-service teacher (see figure 2). 2 other primary school pre-service teachers (1 from ANT, 1 from SGL) place $\frac{1}{2}$ on the scale corresponding to 1 (see figure 3). A part of these errors (4/12) appear to be related to the transition from fractional

to decimal writing. However, we can explain most of these errors (8/12) by a confusion between the part-whole and the number aspects of fractions as we will see below.



Figure 2: “ $\frac{1}{2} = 1.5$ ” in two answers, students n°98 (ANT) and n°14 (SGL)

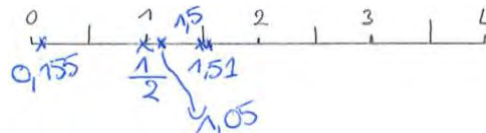


Figure 3: “ $\frac{1}{2} = 1$ ”, student n°11 (SGL)

$\frac{1}{2}$ in the middle of the segment

The number $\frac{1}{2}$ can be a difficulty as we saw in the description of the first category. If the unit chosen is the whole traced segment, the confusion with the part-whole aspect of fractions leads some students to place $\frac{1}{2}$ in the middle of the segment and not in the middle of the interval $[0 ; 1]$ (see figure 4). In this category we find 6/64 primary pre-service teachers after the lesson and 2/39 pre-service teachers before the lesson.

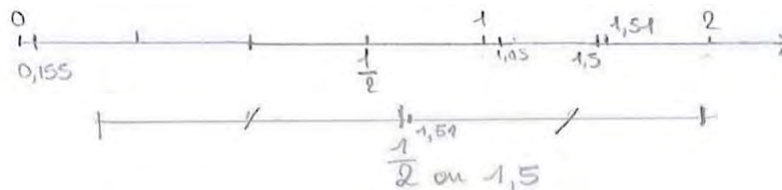


Figure 4: $\frac{1}{2}$ in the middle of the segment, n°7 (SGL) and n°109 (ANT)

Just ordering the numbers

In this category, we find representations of the number line that do not take into account either the magnitude of the number or a unit of length that makes it possible to assign a position to the number on the line. Thus, pre-service teachers seem to redefine the given task as “arranging numbers in ascending (or descending) order on a line” as we can see on the figure 5. There are 3 answers in this category before the lesson (1 from a secondary pre-service teacher) and 5 answers after the lesson.



Figure 5: numbers in ascending order, student n°5 (SGL) or in descending order, student n°53 (MSA)

No clearly defined unit

We observe that some number lines do not have a reference unit for placing the numbers requested. Although in some cases the 0 or 1 is placed, this causes the difficulty of locating $\frac{1}{2}$, as we saw with the “ $\frac{1}{2}$ in the middle of the segment” category, but also the difficulty of locating the numbers on a line with an adequate order of magnitude. This use of the number line occurs before (11/64 pre-

service teachers) or after (12/39 pre-service teachers) the lesson (see figure 6). The answers from the previous category are included in this category (see figure 5).



Figure 6: no unit clearly defined, students n°28 (REN) and n°60 (MSA)

It is possible that, according to the student, the unit chosen is clear, in particular when 0 or 1 is placed (see figure 7). However, we often find other errors on the number line, which leads us not to favour this hypothesis, which also does not correspond to a classical use of the regularly graduated line as it may be presented at INSPÉ or, in primary or secondary school.



Figure 7: 0 placed but no unit clearly defined, students n°10 (SGL) and n°81 (ANT)

Difficulties to perceive and/or construct the difference between one hundredth and five hundredths

This category includes responses that reveal a certain difficulty in semiotically representing differences in terms of length between $1/100$ and $5/100$. Thus, the distance between the points corresponding to the numbers 1.5 and 1.51 (one hundredth) is equal to or greater than that between the points corresponding to the numbers 1 and 1.05 (five hundredths) as we can see on figure 8. We assume that the digit 0 of the number 1.05 is associated with a small number and therefore "just next to" 1, regardless of the number of hundredths.

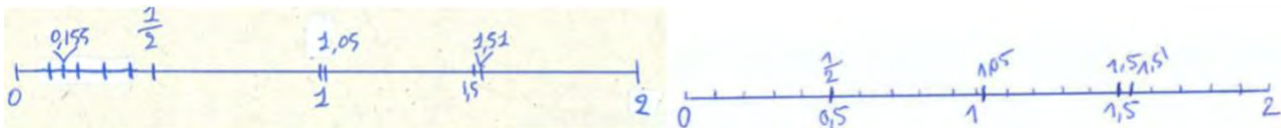


Figure 8: $1/100$ equal or greater than $5/100$, n°42 (REN) and n°90 (ANT)

In addition, 12 pre-service teachers did not place the numbers 1.05, 1.5 or 1.51 (mostly in REN and ANT), which makes it impossible to know whether they perceive this difference on the number line. If we take into account the students who do not clearly define a unit and those who misplace or do not place 1.05, 1.5 or 1.51, we find that 75% of the constructed number lines do not correctly place these points in such a way as to highlight the relative differences between these numbers.

The position of the dot corresponding to the number 0.155 may also be an indication of a misperception of the relationships between tenths, hundredths and thousandths. Thus, 14 pre-service teachers (out of 39 because during the run before the lesson, 0.155 was not included) positioned the dot incorrectly. Either because of confusion with another number like 1.155 or 1.55 ($5/14$), or by placing it too close to 0 ($6/14$) as we can see on figure 9. On figure 8 (student n° 42), there is another error identified with the position of 0.155. However, the vast majority of students do not scale the number line finely enough to accurately locate the position of the dot. It might have been interesting to have them place another number close by, such as 0.15.



Figure 9: 0.155 too close to 0, student n°12 (SGL)

Conclusion

The aim of this research was to study the difficulties of future teachers when constructing the semiotic representation of the numerical decimal line. Thanks to the methodology used, we were able to describe and identify categories for these difficulties. First, confusion between $\frac{1}{2}$ and 1.5 is common among students beginning to learn rational and decimal numbers, but it was also present in 13% of answers from pre-service teachers after the lesson. Some answers use only $\frac{1}{2}$ to divide a segment into two equal parts without taking into account the location of $\frac{1}{2}$ in the number line. This may be a consequence of the current teaching progression (fractions then decimals), or a problem with the status of the number $\frac{1}{2}$. As well, it is not insignificant that pre-service teachers answer the question by giving a list of numbers arranged in ascending or descending order. For them, there seems to be a lack of understanding of the number line and the graduations in order to be able to represent these numbers in terms of length measures. Furthermore, the construction of the number line with decimals does not seem to be based on a unit of reference according to several pre-service teachers: without a reference, how do they place the requested numbers on the number line? Moreover, one of the striking results of this research is the large number of student teachers (almost half) who have difficulty representing the difference between one hundredth and five hundredths in terms of measuring lengths (before and after the lesson). What transposition of the number line will they make to their future students?

We agree with Teppo & Van den Heuvel-Panhuizen (2014) on the complexity of the number line as a semiotic representation of numbers, and that therefore we argue that specific teaching of the number line is needed in teacher education to understand the issues of this representation. Finally, a more detailed analysis in search of links between the categories found in this paper, as well as interviews with teachers in training, are two prospects for our short-term research in order to better understand the origins of the difficulties identified in this work.

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