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# Quasi-rational solutions of the Hirota equation depending on multi-parameters and rogue waves

Pierre Gaillard,

Université de Bourgogne Franche Comté,  
Institut de mathématiques de Bourgogne,  
9 avenue Alain Savary BP 47870  
21078 Dijon Cedex, France :  
E-mail : Pierre.Gaillard@u-bourgogne.fr

## Abstract

Quasi-rational solutions to the Hirota equation are given. We construct explicit expressions of these solutions for the first orders. As a byproduct, we get quasi-rational solutions to the focusing NLS equation and also rational solutions to the mKdV equation. We study the patterns of these configurations in the  $(x, t)$  plane.

**Key Words :** Hirota equation, quasi-rational solutions.

**PACS numbers :**

33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

## 1 Introduction

We consider the Hirota (H) equation in the following normalization

$$iu_t + \alpha(u_{xx} + 2|u|^2u) - i\beta(u_{xxx} + 6|u|^2u_x) = 0, \quad (1)$$

with as usual the subscript meaning the partial derivatives and  $\alpha, \beta$  real numbers.

Hirota proposed this equation in 1973 in a slightly different formulation

$$iu_t + \rho_1 u_{xx} + \delta_1 |u|^2 u + i\sigma_1 u_{xxx} + 3i\alpha_1 |u|^2 u_x = 0,$$

with  $\alpha_1, \rho_1, \sigma_1, \delta_1$  real numbers satisfying  $\alpha_1 \rho_1 = \sigma_1 \delta_1$  and constructed a kind of soliton in [1].

This last equation can be rewritten as (1) by choosing  $\alpha_1 = -2\beta, \delta_1 = 2\alpha, \rho_1 = \alpha, \sigma_1 = -\beta$ .

Maccari introduced in 1998 a two-dimensional extension of this equation. This equation is used to identify many kinds of nonlinear phenomena in the fields physics, in particular in optical fibers. It describes the evolution of the slowly varying amplitude of a nonlinear train in weakly nonlinear systems. Hirota equation is an integrable equation which has a number of physical applications, such as the propagation of optical pulses [3] or in plasma physics, [4].

A lot of methods to solve this equation have been presented such as general projective Riccati equation method [5], Darboux transformation [6, 7] or the trace method [8].

Here, we construct quasi-rational solutions for the first orders.

## 2 Quasi-rational solutions of order 1 to the Hirota equation

**Theorem 2.1** *The function  $v(x, t)$  defined by*

$$v(x, t) = \left( 1 - 4 \frac{1 + 4i\alpha t}{1 + (2x + 12\beta t)^2 + 16\alpha^2 t^2} \right) e^{2iat} \quad (2)$$

*is a solution to the Hirota equation (1)*

$$iu_t + \alpha(u_{xx} + 2|u|^2u) - i\beta(u_{xxx} + 6|u|^2u_x) = 0.$$

**Proof:** It is sufficient to replace the expression of the solution given by (2) and check that (1) is verified.

We give another type of solution with one real parameter  $a_1$ .

**Theorem 2.2** *The function  $v(x, t)$  defined by*

$$v(x, t) = \left( 1 - 4 \frac{1 + i(4\alpha t - 24a_1)}{1 + (2x + 12\beta t)^2 + (4\alpha t - 24a_1)^2} \right) e^{i(2\alpha t - 6a_1)} \quad (3)$$

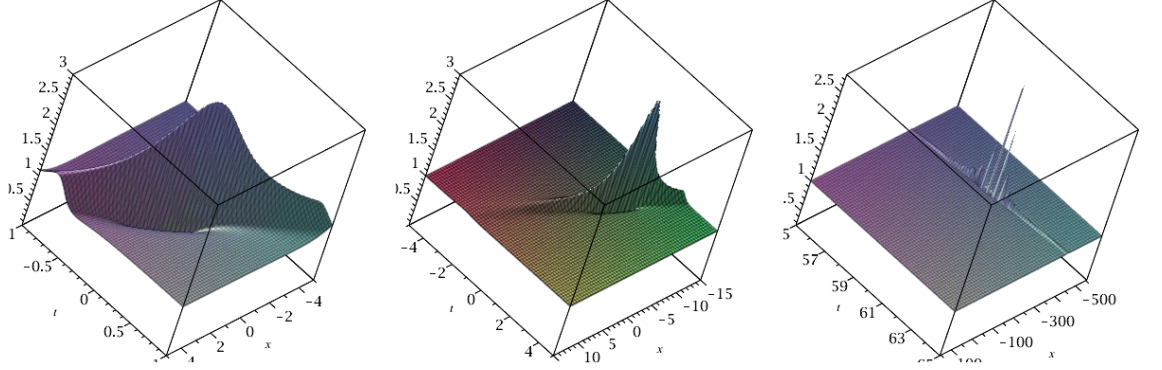
*is a solution to the Hirota equation (1)*

$$iu_t + \alpha(u_{xx} + 2|u|^2u) - i\beta(u_{xxx} + 6|u|^2u_x) = 0,$$

*with  $a_1$  arbitrary real parameter.*

**Proof:** It is sufficient to replace the expression of the solution given by (3) and check that (1) is verified.

**Remark 2.1** *It is possible to add other real parameters but it does not change drastically the structure of the solution as in the case of one parameter  $a_1$ . So, we do not mention these parameters in the following.*



**Figure 1.** Solution of order 1 to (1); to the left  $a_1 = 0$ ; in the center  $a_1 = 0.2$ ; to the right  $a_1 = 10$ ; with  $\alpha = \beta = 1$ .

**Remark 2.2** If we choose,  $\alpha = 1$ ,  $\beta = 0$  and the parameters equal to 0, we recover the classical Peregrine breather solution to the NLS equation

$$iu_t + u_{xx} + 2|u|^2u = 0, \quad (4)$$

$$v(x, t) = \left(1 - 4 \frac{1 + 4it}{1 + 4x^2 + 16t^2}\right) e^{2it}. \quad (5)$$

This solution was first found by Peregrine [9].

It was recovered by the present author by using degeneracy of Riemann theta functions in [18].

**Remark 2.3** If we choose,  $\alpha = 0$ ,  $\beta = 1$  and the parameters equal to 0, we recover a non singular rational solution to the mKdV equation

$$u_t = u_{xxx} + 6|u|^2u_x \quad (6)$$

$$v(x, t) = \left(1 - \frac{4}{1 + (2x + 12t)^2}\right). \quad (7)$$

This solution is different from those constructed by the author in [58].

### 3 Quasi-rational solutions of order 2 to the Hirota equation

**Theorem 3.1** The function  $v(x, t)$  defined by

$$v(x, t) = \left(1 - 12 \frac{n(x, t)}{d(x, t)}\right) e^{2i\alpha t} \quad (8)$$

with

$$n(x, t) = (2x + 12\beta t)^4 + 6(16\alpha^2 t^2 + 1)(2x + 12\beta t)^2 + 192\beta t(2x + 12\beta t) + 1280\alpha^4 t^4 + 288\alpha^2 t^2 - 3 + i(4\alpha t(2x + 12\beta t)^4 + 2(64\alpha^3 t^3 - 12\alpha t)(2x + 12\beta t)^2 + 768\alpha t^2\beta(2x + 12\beta t) + 1024\alpha^5 t^5 + 128\alpha^3 t^3 - 60\alpha t)$$

and

$$d(x, t) = ((2x + 12\beta t)^2 + 16\alpha^2 t^2 + 1)^3 - 192\beta t(2x + 12\beta t)^3 - 24(16\alpha^2 t^2 - 1)(2x + 12\beta t)^2 + 576(16\alpha^2 t^2 + 1)\beta t(2x + 12\beta t) + 6144\alpha^4 t^4 + 8 + 1536\alpha^2 t^2 + 9216\beta^2 t^2$$

is a solution to the Hirota equation (1).

**Proof:** Replacing the expression of the solution given by (8), we check that the relation (1) is verified.

We can give also the solution depending on 2 real parameters.

The function  $v(x, t)$  defined by

**Theorem 3.2**

$$v(x, t) = \left(1 - 12 \frac{n(x, t)}{d(x, t)}\right) e^{i(2\alpha t - 6a_1)} \quad (9)$$

with

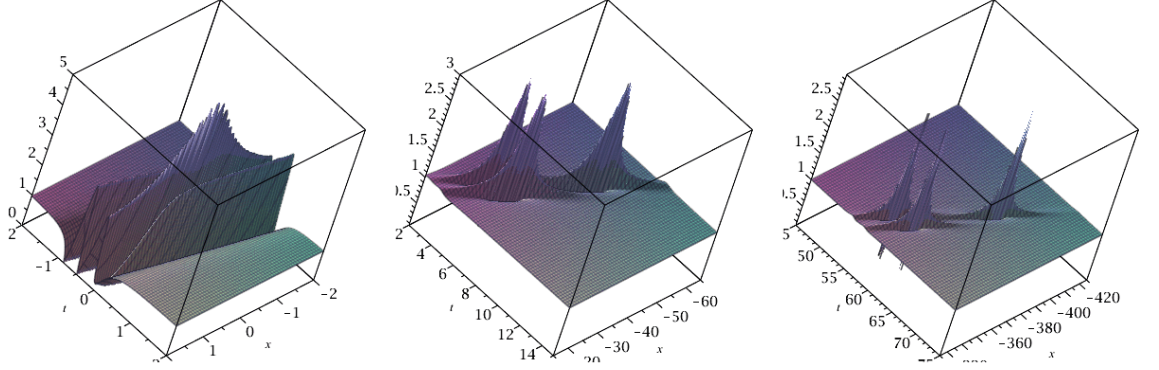
$$n(x, t) = (2x + 12\beta t + 60b_1)^4 + 6((4\alpha t - 24a_1)^2 + 1)(2x + 12\beta t + 60b_1)^2 - 24(-8\beta t - 80b_1)(2x + 12\beta t + 60b_1) + 5(4\alpha t - 24a_1)^4 + 18(4\alpha t - 24a_1)^2 - 384a_1(4\alpha t - 24a_1) - 3 + i((4\alpha t - 24a_1)(2x + 12\beta t + 60b_1)^4 + 2((4\alpha t - 24a_1)^3 - 12\alpha t + 168a_1)(2x + 12\beta t + 60b_1)^2 - 24(4\alpha t - 24a_1)(-8\beta t - 80b_1)(2x + 12\beta t + 60b_1) + (4\alpha t - 24a_1)^5 + 2(4\alpha t - 24a_1)^3 - 192a_1(4\alpha t - 24a_1)^2 - 60\alpha t + 552a_1)$$

and

$$d(x, t) = ((2x + 12\beta t + 60b_1)^2 + (4\alpha t - 24a_1)^2 + 1)^3 + 24(-8\beta t - 80b_1)(2x + 12\beta t + 60b_1)^3 - 24((4\alpha t - 24a_1)^2 - 48a_1(4\alpha t - 24a_1) - 1)(2x + 12\beta t + 60b_1)^2 - 72((4\alpha t - 24a_1)^2 + 1)(-8\beta t - 80b_1)(2x + 12\beta t + 60b_1) + 24(4\alpha t - 24a_1)^4 - 384a_1(4\alpha t - 24a_1)^3 + 96(4\alpha t - 24a_1)^2 - 3456a_1(4\alpha t - 24a_1) + 36864a_1^2 + 144(-8\beta t - 80b_1)^2 + 8$$

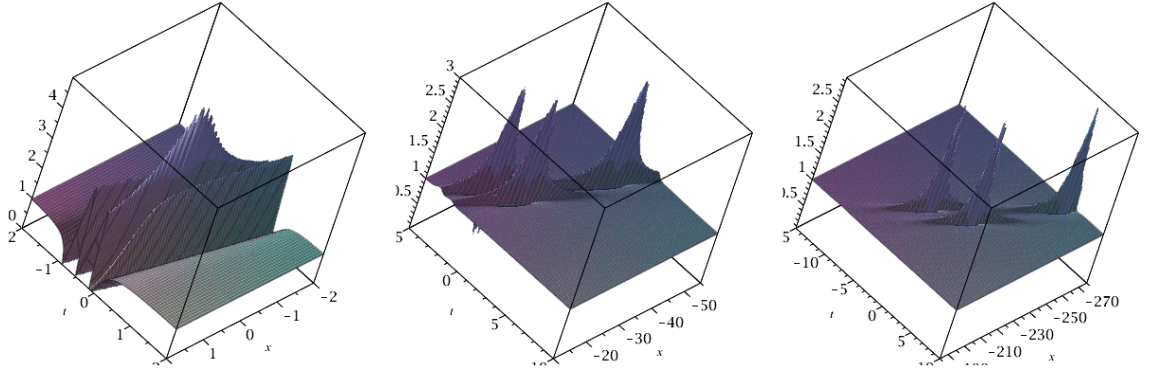
is a solution to the Hirota equation (1).

**Proof:** Replacing the expression of the solution given by (9), we check that the relation (1) is verified.



**Figure 2.** Solution of order 1 to (1); to the left  $a_1 = 0, b_1 = 0$ ; in the center  $a_1 = 1, b_1 = 0$ ; to the right  $a_1 = 10, b_1 = 0$ ; with  $\alpha = \beta = 1$ .

We observe a very fast evolution of the structure of the solution according to the parameter  $a_1$  toward three peaks.



**Figure 3.** Solution of order 1 to (1); to the left  $a_1 = 0, b_1 = 0.01$ ; in the center  $a_1 = 0, b_1 = 0.05$ ; to the right  $a_1 = 0, b_1 = 1$ ; with  $\alpha = \beta = 8$ .

The evolution previously observed is the same as in the case of the parameter  $a_1$ .

**Remark 3.1** If we choose,  $\alpha = 1, \beta = 0$  and the parameters equal to 0, we recover solutions to the NLS equation

$$iu_t + u_{xx} + 2|u|^2u = 0, \tag{10}$$

$$u_k(x, t) = \left(1 - 12 \frac{n(x, t)}{d(x, t)}\right) e^{2it} \tag{11}$$

with  $n(x, t) = 16x^4 + 24(16t^2 + 1)x^2 - 3 + 1280t^4 + 288t^2 + i(64tx^4 + 8(64t^3 -$

$$12t)x^2 + 1024t^5 + 128t^3 - 60t)$$

and

$$d(x, t) = (4x^2 + 16t^2 + 1)^3 + 8 - 96(16t^2 - 1)x^2 + 6144t^4 + 1536t^2.$$

We recover the solutions given by the author in [18].

**Remark 3.2** If we choose,  $\alpha = 0$ ,  $\beta = 1$  and the parameters equal to 0, we recover rational solutions to the mKdV equation

$$u_t = u_{xxx} + 6|u|^2u_x \quad (12)$$

$$u(x, t) = \left(1 - 12 \frac{n(x, t)}{d(x, t)}\right) \quad (13)$$

with

$$n(x, t) = (2x + 12t)^4 + 6(2x + 12t)^2 + 192t(2x + 12t) - 3$$

$$\text{and } v(x, t) = ((2x + 12t)^2 + 1)^3 - 192t(2x + 12t)^3 + 24(2x + 12t)^2 + 576t(2x + 12t) + 8 + 9216t^2.$$

These solutions are different from these presented in [64].

## 4 Quasi-rational solutions of order 3 to the Hirota equation

**Theorem 4.1** The function  $v(x, t)$  defined by

$$v(x, t) = \left(1 - 24 \frac{n(x, t)}{d(x, t)}\right) e^{2i\alpha t} \quad (14)$$

with

$$\begin{aligned} n(x, t) = & (2x + 12\beta t)^{10} + 15(1 + 16\alpha^2 t^2)(2x + 12\beta t)^8 + (210 - 960\alpha^2 t^2 + \\ & 12800\alpha^4 t^4)(2x + 12\beta t)^6 + (92160\alpha^2 t^3 \beta + 5760\beta t)(2x + 12\beta t)^5 + (7200\alpha^2 t^2 - \\ & 38400\alpha^4 t^4 + 286720\alpha^6 t^6 - 450 + 345600\beta^2 t^2)(2x + 12\beta t)^4 + (4915200\alpha^4 t^5 \beta - \\ & 57600\beta t + 1843200\alpha^2 t^3 \beta)(2x + 12\beta t)^3 + (1728000\alpha^4 t^4 + 1720320\alpha^6 t^6 + 2949120t^8 \alpha^8 - \\ & 43200\alpha^2 t^2(5 + 256\beta^2 t^2) - 675 - 691200\beta^2 t^2)(2x + 12\beta t)^2 + (55050240\alpha^6 t^7 \beta + \\ & 1382400\alpha^2 t^3 \beta - 86400\beta t - 22118400\beta^3 t^3 - 7372800\alpha^4 t^5 \beta)(2x + 12\beta t) + 8970240\alpha^6 t^6 + \\ & 32440320t^8 \alpha^8 + 11534336t^{10} \alpha^{10} + 675 + 115200\alpha^4 t^4(-17 + 1792\beta^2 t^2) + 10800\alpha^2 t^2(-3 + \\ & 1024\beta^2 t^2) + 1728000\beta^2 t^2 + i((-60\alpha t + 320\alpha^3 t^3)(2x + 12\beta t)^8 + (-600\alpha t - \\ & 8960\alpha^3 t^3 + 10240\alpha^5 t^5)(2x + 12\beta t)^6 + (122880\alpha^3 t^4 \beta - 23040\alpha t^2 \beta)(2x + 12\beta t)^5 + \\ & (-28800\alpha^3 t^3 - 215040\alpha^5 t^5 + 163840\alpha^7 t^7 + 1800\alpha t(-3 + 768\beta^2 t^2))(2x + 12\beta t)^4 - \\ & 14254080\alpha^7 t^7 + (3932160\alpha^5 t^6 \beta - 230400\alpha t^2 \beta + 2457600\alpha^3 t^4 \beta)(2x + 12\beta t)^3 + \\ & (1751040\alpha^5 t^5 - 983040\alpha^7 t^7 + 1310720t^9 \alpha^9 - 57600\alpha^3 t^3(7 + 256\beta^2 t^2) + 2700\alpha t(7 + \\ & 1024\beta^2 t^2))(2x + 12\beta t)^2 + 6553600t^9 \alpha^9 + (31457280\alpha^7 t^8 \beta + 43200\alpha t(-24\beta t - \\ & 2048\beta^3 t^3) - 5529600\alpha^3 t^4 \beta - 41287680\alpha^5 t^6 \beta)(2x + 12\beta t) + 4194304t^{11} \alpha^{11} + \\ & 4\alpha t(2x + 12\beta t)^{10} + 92160\alpha^5 t^5(-107 + 1792\beta^2 t^2) - 14400\alpha^3 t^3(11 + 5120\beta^2 t^2) - \\ & 2700\alpha t(-7 + 3584\beta^2 t^2) \end{aligned}$$

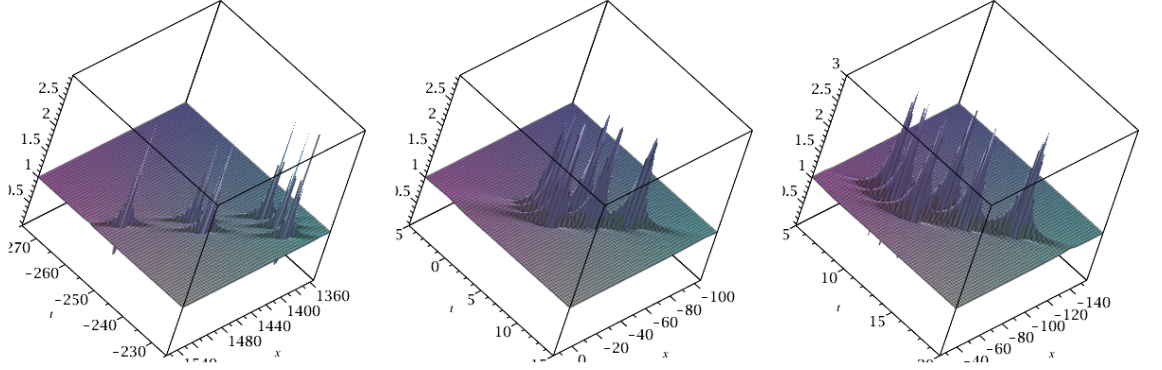
and

$$\begin{aligned}
d(x, t) = & 2024 + (-1920 \alpha^2 t^2 + 120)(2x + 12\beta t)^8 + (7680 \alpha^2 t^2 - 61440 \alpha^4 t^4 + 2320 + \\
& 138240 \beta^2 t^2)(2x + 12\beta t)^6 + (1474560 \alpha^4 t^5 \beta - 552960 \alpha^2 t^3 \beta + 51840 \beta t)(2x + \\
& 12\beta t)^5 + (-368640 \alpha^4 t^4 + 3360 + 3840 \alpha^2 t^2(56 + 8640 \beta^2 t^2) + 2073600 \beta^2 t^2)(2x + \\
& 12\beta t)^4 + (31457280 \alpha^6 t^7 \beta + 16588800 \alpha^2 t^3 \beta + 88473600 \alpha^4 t^5 \beta - 345600 \beta t + \\
& 44236800 \beta^3 t^3)(2x + 12\beta t)^3 + (55050240 \alpha^6 t^6 + 15728640 t^8 \alpha^8 - 61440 \alpha^4 t^4(-326 + \\
& 2880 \beta^2 t^2) + 7680 \alpha^2 t^2(-76 + 8640 \beta^2 t^2) + 12144 - 6220800 \beta^2 t^2)(2x + 12\beta t)^2 + \\
& (188743680 \alpha^8 t^9 \beta + 47185920 \alpha^6 t^7 \beta - 66355200 \alpha^4 t^5 \beta + 1036800 \alpha^2 t^2(40 \beta t - \\
& 2048 \beta^3 t^3) + 648000 \beta t - 132710400 \beta^3 t^3)(2x + 12\beta t) + 22809600 \beta^2 t^2 + 243793920 t^8 \alpha^8 + \\
& 125829120 t^{10} \alpha^{10} + 2123366400 \beta^4 t^4 - 960 \beta t(2x + 12\beta t)^9 + 327680 \alpha^6 t^6(191 + \\
& 4032 \beta^2 t^2) + 61440 \alpha^4 t^4(599 + 8640 \beta^2 t^2) + 384 \alpha^2 t^2(3881 + 777600 \beta^2 t^2) + (1 + \\
& (2x + 12\beta t)^2 + 16 \alpha^2 t^2)^6
\end{aligned}$$

is a solution to the Hirota equation (1).

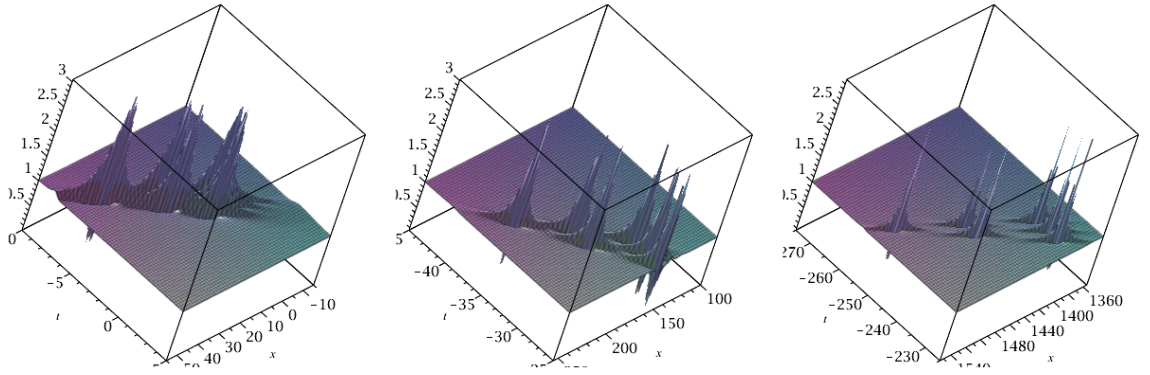
**Proof:** We have to check that the relation (1) is verified when we replace the expression of the solution given by (14).

We can give also the solutions to the Hirota depending on 4 real parameters. Because of the length of the expression, we only give it in the appendix. We give patterns of the modulus of the solutions in function of the parameters.

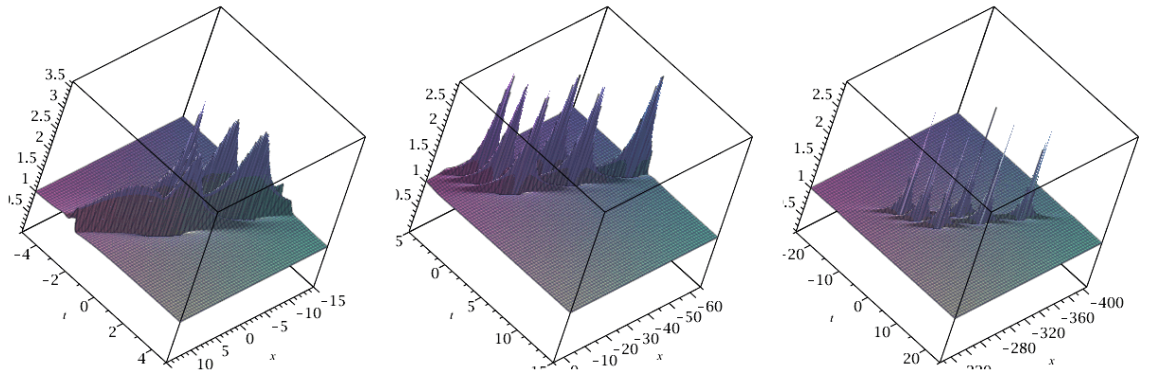


**Figure 4.** Solution of order 3 to (1); to the left  $a_1 = 0, b_1 = 0, a_2 = 0, b_2 = 0$ ; in the center  $a_1 = 1, b_1 = 0, a_2 = 0, b_2 = 0$ ; to the right  $a_1 = 2, b_1 = 0, a_2 = 10, b_2 = 0$ ; with  $\alpha = \beta = 1$ .

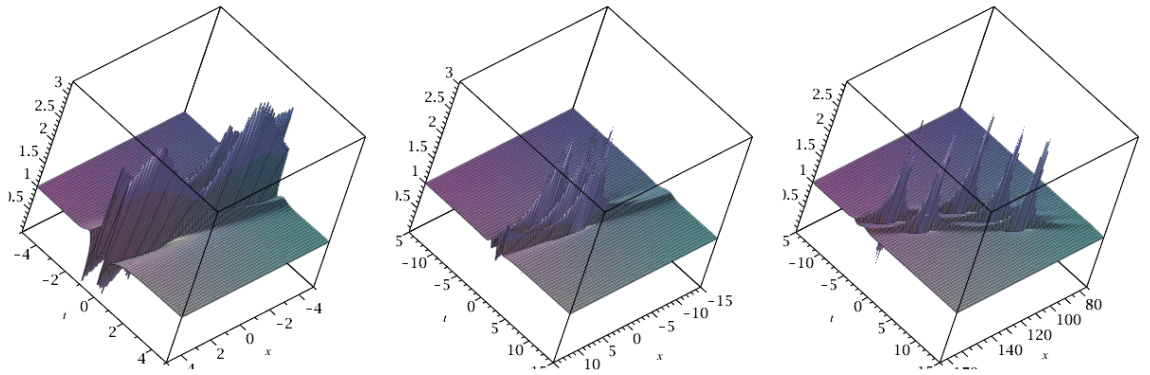




**Figure 5.** Solution of order 3 to (1); to the left  $a_1 = 0, b_1 = 0, a_2 = 0.1, b_2 = 0$ ; in the center  $a_1 = 0, b_1 = 10, a_2 = 1, b_2 = 0$ ; to the right  $a_1 = 10, b_1 = 0, a_2 = 8, b_2 = 0$ ; with  $\alpha = \beta = 1$ .



**Figure 6.** Solution of order 3 to (1); to the left  $a_1 = 0, b_1 = 0.1, a_2 = 0, b_2 = 0$ ; in the center  $a_1 = 0, b_1 = 1, a_2 = 0, b_2 = 0$ ; to the right  $a_1 = 2, b_1 = 10, a_2 = 0, b_2 = 0$ ; with  $\alpha = \beta = 1$ .



**Figure 7.** Solution of order 3 to (1); to the left  $a_1 = 0, b_1 = 0, a_2 = 0, b_2 = 0.01$ ; in the center  $a_1 = 0, b_1 = 10, a_2 = 0, b_2 = 0.1$ ; to the right  $a_1 = 10, b_1 = 0, a_2 = 0, b_2 = 1$ ; with  $\alpha = \beta = 1$ .

In the evolution of the structure of the solutions according to the different parameters, we note the appearance of triangles with six peaks with more at least speed according to the parameters.

**Remark 4.1** *If we choose,  $\alpha = 1, \beta = 0$  and the parameters equal to 0, we recover solutions to the NLS equation*

$$iu_t + u_{xx} + 2|u|^2u = 0, \quad (15)$$

$$u_k(x, t) = \left(1 - 24 \frac{n(x, t)}{d(x, t)}\right) e^{2i\alpha t} \quad (16)$$

with

$$n(x, t) = 16x^4 + 24((4t - 24000)^2 + 1)x^2 + 9215999997 + 5(4t - 24000)^4 + 18(4t - 24000)^2 - 1536000t + i(16(4t - 24000)x^4 + 8((4t - 24000)^3 - 12t + 168000)x^2 + (4t - 24000)^5 + 2(4t - 24000)^3 - 192000(4t - 24000)^2 - 60t + 552000)$$

and

$$d(x, t) = (4x^2 + (4t - 24000)^2 + 1)^3 + 119808000008 - 96((4t - 24000)^2 - 192000t + 1151999999)x^2 + 24(4t - 24000)^4 - 384000(4t - 24000)^3 + 96(4t - 24000)^2 - 13824000t.$$

We recover the solutions constructed in [24].

**Remark 4.2** *If we choose,  $\alpha = 0, \beta = 1$  and the parameters equal to 0, we recover rational solutions to the mKdV equation*

$$u_t = u_{xxx} + 6|u|^2u_x \quad (17)$$

$$u_k(x, t) = \left(1 - 24 \frac{n(x, t)}{d(x, t)}\right) e^{2i\alpha t} \quad (18)$$

with

$$n(x, t) = (2x + 12t)^4 + 6(2x + 12t)^2 + 192t(2x + 12t) - 3$$

$$\text{and } d(x, t) = ((2x + 12t)^2 + 1)^3 - 192t(2x + 12t)^3 + 24(2x + 12t)^2 + 576t(2x + 12t) + 8 + 9216t^2.$$

These solutions are different from these given in [58, 64].

## 5 Conclusion

Quasi-rational solutions to the Hirota equation have been given for the first orders. As a byproduct, we have recover quasi-rational solutions to the NLS equation and rational solutions to the mKdV equation.

We can mention some other recent works about this equation. For example,

in [10], by using the inverse scattering transform an explicit soliton solution formula for the Hirota equation has been constructed. This formula allows to get, as a particular case, the N-soliton solution, the breather solution and, most relevantly, a new class of solutions called multi-pole soliton solutions.

By using the Hirota direct method, lump-soliton to the Hirota equation were given in [11].

In [12], the mixed localized wave solutions of the Hirota equation are constructed through the modified Darboux transformation. One of them is the mixed 1-breather and 1-rogue wave solution and the other two are the mixed 1-breather and 2-rogue wave solution, and the mixed 2-breather and 1-rogue wave solution. These mixed localized wave solutions are presented graphically by choosing proper parameters and we discuss their dynamic behavior and control briefly.

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### Appendix

Solution of order 3 to the Hirota equation depending on 4 real parameters :  
The function  $v(x, t)$  defined by

$$v(x, t) = \left( 1 - 24 \frac{n(x, t)}{d(x, t)} \right) e^{2i\alpha t} \quad (19)$$

with

$$\begin{aligned} n(x, t) = & 675 + 15 (1 + (4\alpha t - 24a_1 + 120a_2)^2)(2x + 12\beta t + 60b_1 - 280b_2)^8 + \\ & (210 - 60(4\alpha t - 24a_1 + 120a_2)^2 + 50(4\alpha t - 24a_1 + 120a_2)^4 + 480(4\alpha t - 24a_1 + \\ & 120a_2)(16a_1 - 160a_2))(2x + 12\beta t + 60b_1 - 280b_2)^6 + (-720(4\alpha t - 24a_1 + \\ & 120a_2)^2(-8\beta t - 80b_1 + 560b_2) + 5760\beta t - 11520b_1 + 564480b_2)(2x + 12\beta t + \\ & 60b_1 - 280b_2)^5 + (450(4\alpha t - 24a_1 + 120a_2)^2 - 150(4\alpha t - 24a_1 + 120a_2)^4 + \\ & 70(4\alpha t - 24a_1 + 120a_2)^6 + 1200(4\alpha t - 24a_1 + 120a_2)^3(16a_1 - 160a_2) - 450 + \\ & 5400(-8\beta t - 80b_1 + 560b_2)^2 - 1800(16a_1 - 160a_2)^2 + 3600(4\alpha t - 24a_1 + \\ & 120a_2)(16a_1 - 224a_2))(2x + 12\beta t + 60b_1 - 280b_2)^4 + (-2400(4\alpha t - 24a_1 + \\ & 120a_2)^4(-8\beta t - 80b_1 + 560b_2) + 28800(4\alpha t - 24a_1 + 120a_2)(-8\beta t - 80b_1 + \\ & 560b_2)(16a_1 - 160a_2) - 57600\beta t - 806400b_1 + 7257600b_2 - 7200(4\alpha t - 24a_1 + \\ & 120a_2)^2(-16\beta t - 128b_1 + 672b_2))(2x + 12\beta t + 60b_1 - 280b_2)^3 + (6750(4\alpha t - \\ & 24a_1 + 120a_2)^4 + 420(4\alpha t - 24a_1 + 120a_2)^6 + 45(4\alpha t - 24a_1 + 120a_2)^8 - \\ & 2700(4\alpha t - 24a_1 + 120a_2)^2(5 + 4(-8\beta t - 80b_1 + 560b_2)^2 - 12(16a_1 - 160a_2)^2) - \\ & 675 - 10800(-8\beta t - 80b_1 + 560b_2)^2 - 10800(16a_1 - 160a_2)^2 + 21600(4\alpha t - \\ & 24a_1 + 120a_2)(32a_1 - 384a_2) - 7200(4\alpha t - 24a_1 + 120a_2)^3(32a_1 - 128a_2))(2x + \\ & 12\beta t + 60b_1 - 280b_2)^2 + (-1680(4\alpha t - 24a_1 + 120a_2)^6(-8\beta t - 80b_1 + 560b_2) - \\ & 28800(4\alpha t - 24a_1 + 120a_2)^3(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) - 10800(4\alpha t - \\ & 24a_1 + 120a_2)^2(-8\beta t - 272b_1 + 3248b_2) - 86400\beta t - 1209600b_1 + 10886400b_2 + \\ & 43200(-8\beta t - 80b_1 + 560b_2)^3 + 43200(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2)^2) + \end{aligned}$$



$$\begin{aligned}
& 3600(4\alpha t - 24a_1 + 120a_2)^4(-8\beta t + 80b_1 - 1680b_2) - 86400(4\alpha t - 24a_1 + 120a_2)((-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) + (16a_1 - 160a_2)(32b_1 - 448b_2) - 64(-8\beta t - 80b_1 + 560b_2)a_2))(2x + 12\beta t + 60b_1 - 280b_2) - 720(4\alpha t - 24a_1 + 120a_2)^7(16a_1 - 160a_2) + 450(4\alpha t - 24a_1 + 120a_2)^4(-17 + 28(-8\beta t - 80b_1 + 560b_2)^2 + 12(16a_1 - 160a_2)^2) - 3600(4\alpha t - 24a_1 + 120a_2)^3(48a_1 - 1376a_2) - 720(4\alpha t - 24a_1 + 120a_2)^5(272a_1 - 3168a_2) + 10800(4\alpha t - 24a_1 + 120a_2)(-16a_1 + 224a_2 + 4(-8\beta t - 80b_1 + 560b_2)^2(16a_1 - 160a_2) + 4(16a_1 - 160a_2)^3) + 675(4\alpha t - 24a_1 + 120a_2)^2(-3 + 16(-8\beta t - 80b_1 + 560b_2)^2 + 16(16a_1 - 160a_2)^2 - 128(-8\beta t - 80b_1 + 560b_2)(32b_1 - 448b_2) - 8192(16a_1 - 160a_2)a_2) - 86400(-8\beta t - 80b_1 + 560b_2)(32b_1 - 448b_2) - 11059200(16a_1 - 160a_2)a_2 + (2x + 12\beta t + 60b_1 - 280b_2)^{10} + 2190(4\alpha t - 24a_1 + 120a_2)^6 + 27000(-8\beta t - 80b_1 + 560b_2)^2 + 91800(16a_1 - 160a_2)^2 + 495(4\alpha t - 24a_1 + 120a_2)^8 + 11(4\alpha t - 24a_1 + 120a_2)^{10} + 86400(32b_1 - 448b_2)^2 + 353894400a_2^2 + i(-21600(-8\beta t - 80b_1 + 560b_2)^2(16a_1 - 160a_2) + (4\alpha t - 24a_1 + 120a_2)(2x + 12\beta t + 60b_1 - 280b_2)^{10} + (-60\alpha t + 840a_1 - 6600a_2 + 5(4\alpha t - 24a_1 + 120a_2)^3)(2x + 12\beta t + 60b_1 - 280b_2)^8 + (-600\alpha t - 240a_1 + 58800a_2 - 140(4\alpha t - 24a_1 + 120a_2)^3) + 10(4\alpha t - 24a_1 + 120a_2)^5 + 240(4\alpha t - 24a_1 + 120a_2)^2(16a_1 - 160a_2))(2x + 12\beta t + 60b_1 - 280b_2)^6 + (-240(4\alpha t - 24a_1 + 120a_2)^3(-8\beta t - 80b_1 + 560b_2) - 1440(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) + 720(4\alpha t - 24a_1 + 120a_2)(-8\beta t - 176b_1 + 1904b_2))(2x + 12\beta t + 60b_1 - 280b_2)^5 + (-450(4\alpha t - 24a_1 + 120a_2)^3 - 210(4\alpha t - 24a_1 + 120a_2)^5 + 10(4\alpha t - 24a_1 + 120a_2)^7 + 300(4\alpha t - 24a_1 + 120a_2)^4(16a_1 - 160a_2) + 450(4\alpha t - 24a_1 + 120a_2)(-3 + 12(-8\beta t - 80b_1 + 560b_2)^2 - 4(16a_1 - 160a_2)^2) - 14400a_1 + 259200a_2 + 1800(4\alpha t - 24a_1 + 120a_2)^2(16a_1 - 224a_2))(2x + 12\beta t + 60b_1 - 280b_2)^4 + (-480(4\alpha t - 24a_1 + 120a_2)^5(-8\beta t - 80b_1 + 560b_2) + 14400(4\alpha t - 24a_1 + 120a_2)^2(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) + 7200(4\alpha t - 24a_1 + 120a_2)(-8\beta t - 48b_1 + 112b_2) - 2400(4\alpha t - 24a_1 + 120a_2)^3(-16\beta t - 128b_1 + 672b_2) - 14400(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) - 14400(16a_1 - 160a_2)(32b_1 - 448b_2) + 921600(-8\beta t - 80b_1 + 560b_2)a_2)(2x + 12\beta t + 60b_1 - 280b_2)^3 + (1710(4\alpha t - 24a_1 + 120a_2)^5 - 60(4\alpha t - 24a_1 + 120a_2)^7 + 5(4\alpha t - 24a_1 + 120a_2)^9 - 900(4\alpha t - 24a_1 + 120a_2)^3(7 + 4(-8\beta t - 80b_1 + 560b_2)^2 - 12(16a_1 - 160a_2)^2) + 675(4\alpha t - 24a_1 + 120a_2)(7 + 16(-8\beta t - 80b_1 + 560b_2)^2 + 16(16a_1 - 160a_2)^2) - 345600a_1 + 4492800a_2 - 21600(-8\beta t - 80b_1 + 560b_2)^2(16a_1 - 160a_2) - 21600(16a_1 - 160a_2)^3 + 691200(4\alpha t - 24a_1 + 120a_2)^2a_2 - 1800(4\alpha t - 24a_1 + 120a_2)^4(64a_1 - 448a_2))(2x + 12\beta t + 60b_1 - 280b_2)^2 + (-240(4\alpha t - 24a_1 + 120a_2)^7(-8\beta t - 80b_1 + 560b_2) - 7200(4\alpha t - 24a_1 + 120a_2)^4(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) + 10800(4\alpha t - 24a_1 + 120a_2)(-24\beta t - 400b_1 + 3920b_2 + 4(-8\beta t - 80b_1 + 560b_2)^3 + 4(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2)^2) + 3600(4\alpha t - 24a_1 + 120a_2)^3(-24\beta t - 176b_1 + 784b_2) + 720(4\alpha t - 24a_1 + 120a_2)^5(-56\beta t - 400b_1 + 1680b_2) + 21600(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) + 43200(16a_1 - 160a_2)(32b_1 - 448b_2) - 2764800(-8\beta t - 80b_1 + 560b_2)a_2 - 43200(4\alpha t - 24a_1 + 120a_2)^2((-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) + (16a_1 - 160a_2)(32b_1 - 448b_2) - 64(-8\beta t - 80b_1 + 560b_2)a_2))(2x + 12\beta t + 60b_1 - 280b_2) - 90(4\alpha t - 24a_1 + 120a_2)^8(16a_1 - 160a_2) + 90(4\alpha t - 24a_1 + 120a_2)^5(-107 + 28(-8\beta t - 80b_1 + 560b_2)^2 + 12(16a_1 - 160a_2)^2) + 5400(4\alpha t - 24a_1 + 120a_2)^2(176a_1 - 2464a_2 + 4(-8\beta t - 80b_1 + 560b_2)^2(16a_1 - 160a_2) + 4(16a_1 - 160a_2)^3) -
\end{aligned}$$

$$\begin{aligned}
& 120(4\alpha t - 24a_1 + 120a_2)^6(80a_1 - 1248a_2) + 900(4\alpha t - 24a_1 + 120a_2)^4(464a_1 - \\
& 4000a_2) - 225(4\alpha t - 24a_1 + 120a_2)^3(11 + 80(-8\beta t - 80b_1 + 560b_2)^2 + 80(16a_1 - \\
& 160a_2)^2) + 128(-8\beta t - 80b_1 + 560b_2)(32b_1 - 448b_2) + 8192(16a_1 - 160a_2)a_2 - \\
& 675(4\alpha t - 24a_1 + 120a_2)(-7 + 56(-8\beta t - 80b_1 + 560b_2)^2 + 88(16a_1 - 160a_2)^2 - \\
& 128(-8\beta t - 80b_1 + 560b_2)(32b_1 - 448b_2) - 128(32b_1 - 448b_2)^2 - 524288a_2^2) + \\
& 5529600(-8\beta t - 80b_1 + 560b_2)^2a_2 - 5529600(16a_1 - 160a_2)^2a_2 - 172800(-8\beta t - \\
& 80b_1 + 560b_2)(16a_1 - 160a_2)(32b_1 - 448b_2) + 64800(16a_1 - 160a_2)^3 - 870(4\alpha t - \\
& 24a_1 + 120a_2)^7 + 25(4\alpha t - 24a_1 + 120a_2)^9 + (4\alpha t - 24a_1 + 120a_2)^{11} - 151200a_1 + \\
& 1857600a_2)
\end{aligned}$$

and

$$\begin{aligned}
d(x, t) = & 2024 - 777600(-8\beta t - 80b_1 + 560b_2)(32b_1 - 448b_2) - 82944000(16a_1 - \\
& 160a_2)a_2 + (-120(4\alpha t - 24a_1 + 120a_2)^2 + 360(4\alpha t - 24a_1 + 120a_2)(16a_1 - \\
& 160a_2) + 120)(2x + 12\beta t + 60b_1 - 280b_2)^8 + (480(4\alpha t - 24a_1 + 120a_2)^2 - \\
& 240(4\alpha t - 24a_1 + 120a_2)^4 + 960(4\alpha t - 24a_1 + 120a_2)^3(16a_1 - 160a_2) + 2320 + \\
& 2160(-8\beta t - 80b_1 + 560b_2)^2 + 5040(16a_1 - 160a_2)^2 - 1440(4\alpha t - 24a_1 + \\
& 120a_2)(64a_1 - 960a_2))(2x + 12\beta t + 60b_1 - 280b_2)^6 + (-720(4\alpha t - 24a_1 + \\
& 120a_2)^4(-8\beta t - 80b_1 + 560b_2) - 17280(4\alpha t - 24a_1 + 120a_2)(-8\beta t - 80b_1 + \\
& 560b_2)(16a_1 - 160a_2) + 4320(4\alpha t - 24a_1 + 120a_2)^2(-8\beta t - 176b_1 + 1904b_2) + \\
& 51840\beta t + 103680b_1 + 2177280b_2)(2x + 12\beta t + 60b_1 - 280b_2)^5 + (-1440(4\alpha t - \\
& 24a_1 + 120a_2)^4 + 720(4\alpha t - 24a_1 + 120a_2)^5(16a_1 - 160a_2) + 240(4\alpha t - 24a_1 + \\
& 120a_2)^2(56 + 135(-8\beta t - 80b_1 + 560b_2)^2 - 45(16a_1 - 160a_2)^2) + 32400(4\alpha t - \\
& 24a_1 + 120a_2)(16a_1 - 288a_2) + 7200(4\alpha t - 24a_1 + 120a_2)^3(48a_1 - 544a_2) + \\
& 3360 + 32400(-8\beta t - 80b_1 + 560b_2)^2 - 54000(16a_1 - 160a_2)^2 + 86400(-8\beta t - \\
& 80b_1 + 560b_2)(32b_1 - 448b_2) + 5529600(16a_1 - 160a_2)a_2)(2x + 12\beta t + 60b_1 - \\
& 280b_2)^4 + (-960(4\alpha t - 24a_1 + 120a_2)^6(-8\beta t - 80b_1 + 560b_2) + 57600(4\alpha t - \\
& 24a_1 + 120a_2)^3(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) - 43200(4\alpha t - 24a_1 + \\
& 120a_2)^2(-24\beta t - 272b_1 + 2128b_2) - 7200(4\alpha t - 24a_1 + 120a_2)^4(-48\beta t - \\
& 448b_1 + 2912b_2) - 345600\beta t - 5529600b_1 + 53222400b_2 - 86400(-8\beta t - 80b_1 + \\
& 560b_2)^3 - 86400(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2)^2 + 172800(4\alpha t - 24a_1 + \\
& 120a_2)((-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) - (16a_1 - 160a_2)(32b_1 - 448b_2) + \\
& 64(-8\beta t - 80b_1 + 560b_2)a_2))(2x + 12\beta t + 60b_1 - 280b_2)^3 + (13440(4\alpha t - \\
& 24a_1 + 120a_2)^6 + 240(4\alpha t - 24a_1 + 120a_2)^8 - 240(4\alpha t - 24a_1 + 120a_2)^4(-326 + \\
& 45(-8\beta t - 80b_1 + 560b_2)^2 - 135(16a_1 - 160a_2)^2) + 480(4\alpha t - 24a_1 + 120a_2)^2(-76 + \\
& 135(-8\beta t - 80b_1 + 560b_2)^2 + 1215(16a_1 - 160a_2)^2) - 129600(4\alpha t - 24a_1 + \\
& 120a_2)^3(32a_1 - 256a_2) - 12960(4\alpha t - 24a_1 + 120a_2)^5(32a_1 - 256a_2) - 64800(4\alpha t - \\
& 24a_1 + 120a_2)(-96a_1 + 1280a_2 + 4(-8\beta t - 80b_1 + 560b_2)^2(16a_1 - 160a_2) + \\
& 4(16a_1 - 160a_2)^3) + 12144 - 97200(-8\beta t - 80b_1 + 560b_2)^2 + 32400(16a_1 - \\
& 160a_2)^2 + 518400(32b_1 - 448b_2)^2 - 33177600(16a_1 - 160a_2)a_2 + 2123366400a_2^2)(2x + \\
& 12\beta t + 60b_1 - 280b_2)^2 + (-360(4\alpha t - 24a_1 + 120a_2)^8(-8\beta t - 80b_1 + 560b_2) - \\
& 17280(4\alpha t - 24a_1 + 120a_2)^5(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) - 1440(4\alpha t - \\
& 24a_1 + 120a_2)^6(-8\beta t - 240b_1 + 2800b_2) + 32400(4\alpha t - 24a_1 + 120a_2)^4(-8\beta t + \\
& 112b_1 - 2128b_2) + 64800(4\alpha t - 24a_1 + 120a_2)^2(40\beta t + 752b_1 - 7728b_2 + \\
& 4(-8\beta t - 80b_1 + 560b_2)^3 + 4(-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2)^2) - 777600(4\alpha t - \\
& 24a_1 + 120a_2)((-8\beta t - 80b_1 + 560b_2)(16a_1 - 160a_2) + 2(16a_1 - 160a_2)(32b_1 - \\
& 448b_2) - 128(-8\beta t - 80b_1 + 560b_2)a_2) - 172800(4\alpha t - 24a_1 + 120a_2)^3(3(-8\beta t - \\
& 80b_1 + 560b_2)(16a_1 - 160a_2) + (16a_1 - 160a_2)(32b_1 - 448b_2) - 64(-8\beta t -
\end{aligned}$$

$80 b_1 + 560 b_2)a_2) + 648000 \beta t + 8553600 b_1 - 74390400 b_2 + 259200 (-8 \beta t - 80 b_1 + 560 b_2)^3 + 1296000 (-8 \beta t - 80 b_1 + 560 b_2)(16 a_1 - 160 a_2)^2 - 1036800 (-8 \beta t - 80 b_1 + 560 b_2)^2(32 b_1 - 448 b_2) + 1036800 (16 a_1 - 160 a_2)^2(32 b_1 - 448 b_2) - 132710400 (-8 \beta t - 80 b_1 + 560 b_2)(16 a_1 - 160 a_2)a_2(2x + 12 \beta t + 60 b_1 - 280 b_2) + 120 (-8 \beta t - 80 b_1 + 560 b_2)(2x + 12 \beta t + 60 b_1 - 280 b_2)^9 + 1440 (32 b_1 - 448 b_2)(2x + 12 \beta t + 60 b_1 - 280 b_2)^7 - 120 (4 \alpha t - 24 a_1 + 120 a_2)^9(16 a_1 - 160 a_2) + 80 (4 \alpha t - 24 a_1 + 120 a_2)^6(191 + 63 (-8 \beta t - 80 b_1 + 560 b_2)^2 + 27 (16 a_1 - 160 a_2)^2) - 2160 (4 \alpha t - 24 a_1 + 120 a_2)^5(240 a_1 - 4576 a_2) + 21600 (4 \alpha t - 24 a_1 + 120 a_2)^3(-368 a_1 + 3488 a_2 + 4 (-8 \beta t - 80 b_1 + 560 b_2)^2(16 a_1 - 160 a_2) + 4 (16 a_1 - 160 a_2)^3) - 1440 (4 \alpha t - 24 a_1 + 120 a_2)^7(80 a_1 - 864 a_2) + 240 (4 \alpha t - 24 a_1 + 120 a_2)^4(599 + 135 (-8 \beta t - 80 b_1 + 560 b_2)^2 - 225 (16 a_1 - 160 a_2)^2 - 360 (-8 \beta t - 80 b_1 + 560 b_2)(32 b_1 - 448 b_2) - 23040 (16 a_1 - 160 a_2)a_2) - 16200 (4 \alpha t - 24 a_1 + 120 a_2)(496 a_1 - 6240 a_2 + 80 (-8 \beta t - 80 b_1 + 560 b_2)^2(16 a_1 - 160 a_2) + 16 (16 a_1 - 160 a_2)^3 + 128 (-8 \beta t - 80 b_1 + 560 b_2)(16 a_1 - 160 a_2)(32 b_1 - 448 b_2) - 4096 (-8 \beta t - 80 b_1 + 560 b_2)^2 a_2 + 4096 (16 a_1 - 160 a_2)^2 a_2) + 24 (4 \alpha t - 24 a_1 + 120 a_2)^2(3881 + 12150 (-8 \beta t - 80 b_1 + 560 b_2)^2 + 28350 (16 a_1 - 160 a_2)^2 + 21600 (-8 \beta t - 80 b_1 + 560 b_2)(32 b_1 - 448 b_2) + 21600 (32 b_1 - 448 b_2)^2 + 88473600 a_2^2) + 1036800 (-8 \beta t - 80 b_1 + 560 b_2)^2(16 a_1 - 160 a_2)^2 + 356400 (-8 \beta t - 80 b_1 + 560 b_2)^2 + 874800 (16 a_1 - 160 a_2)^2 + 3720 (4 \alpha t - 24 a_1 + 120 a_2)^8 + 120 (4 \alpha t - 24 a_1 + 120 a_2)^{10} + 518400 (32 b_1 - 448 b_2)^2 + 2123366400 a_2^2 + (1 + (2x + 12 \beta t + 60 b_1 - 280 b_2)^2 + (4 \alpha t - 24 a_1 + 120 a_2)^2)^6 + 518400 (-8 \beta t - 80 b_1 + 560 b_2)^4 + 518400 (16 a_1 - 160 a_2)^4$

is a solution to the Hirota equation (1).