

Mirror symmetry and colored graph duality

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Abstract

A general approach to Mirror symmetry (string theory), universalizing the Batyrev-Borisov construction, is proposed. For this (multi)graph duality and the fact that compact n -manifolds are $(n+1)$ -colored graphs are used.

The way

Besides the Batyrev-Borisov construction [1] and the double branched covers construction [2], mainly we are not having other examples for Mirror symmetry. Moreover, a classification of Calabi-Yau threefolds has not been known.

Paper [3] shows that compact n -manifolds are $(n+1)$ -colored graphs, where colored graphs mean regular multigraphs. Hence, for planar (multi)graphs we have notion of duality, see, for example, 9.1.5 Definition in [4]. The term dual is used because the property of being a dual graph is symmetric.

Remark 1. *Graph duality is a topological generalization of the geometric concepts of dual polytope.*

Remark 2. *The same planar graph can have non-isomorphic dual graphs, but if a graph is 3-connected, then the dual graph is unique, see Theorem 10.28 in [5]. Moreover, the dual of a simple graph need not be simple.*

Hence, the paper [1] is about combinatorial manifolds, i.e. with PL-structure. Concepts of Kahler metric [6] and Ricci curvature [7] are defined in discrete settings, see also Proposition 3.15 in [3]. However, there is no difference between piecewise linear and smooth manifolds in dimension < 7 [8] (PL-manifolds do not always have smooth structures). Thus, due to the Hodge decomposition theorem, at least it is possible to deal with threefolds.

Remark 3. *The concept of duality can be extended on two-dimensional manifolds other than the plane. The definition is the same: there is a dual vertex for each connected component of the complement of the graph in the manifold, and a dual edge for each graph edge connecting the two dual vertices on either side of the edge. However, it is restricted to embeddings with the property that each face is a topological disk; this constraint generalizes the requirement for planar graphs that the graph be connected. With this constraint, the duality is symmetric.*

References

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