

# Development and limits of the mean Continuous Absence Time ( $\overline{CAT}$ )

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## 1 | $\overline{CAT}$ FORMULA

We have,

$$\overline{CAT}_{diff}(\rho) = \frac{a_d}{\rho^{b_d}} \quad (1)$$

$$\overline{CAT}_{return}(\rho) = 1 + \frac{a_r}{\rho^{b_r}} \quad (2)$$

and

$$R(\rho) = a\rho^c \exp(b \times \rho) \quad (3)$$

with  $(a_d, b_d, a_r, b_r, a, b, c) \in \mathbb{R}_+^7$

Hence,

$$\begin{aligned} \overline{CAT}(\rho) &= \frac{A(\rho)\overline{CAT}_{diff}(\rho) + B(\rho)\overline{CAT}_{return}(\rho)}{A(\rho) + B(\rho)} \\ &= \frac{R(\rho)\overline{CAT}_{diff}(\rho) + \overline{CAT}_{return}(\rho)}{R(\rho) + 1} \\ &= \frac{a\rho^c \exp(b\rho) a_d \rho^{-b_d} + 1 + a_r \rho^{-b_r}}{a\rho^c \exp(b\rho) + 1} \end{aligned}$$

## 2 | LIMIT WHEN $\rho \mapsto +\infty$

$$\begin{aligned} \overline{CAT}(\rho) &= \frac{a\rho^c \exp(b\rho) a_d \rho^{-b_d} + 1 + a_r \rho^{-b_r}}{a\rho^c \exp(b\rho) + 1} \\ &= \frac{a_d \rho^{-b_d} + \frac{1}{a} \rho^{-c} \exp(-b\rho) + \frac{a_r}{a} \rho^{-b_r-c} \exp(-b\rho)}{1 + \frac{1}{a} \rho^{-c} \exp(-b\rho)} \end{aligned} \quad (4)$$

We note  $N_{inf} = a_d \rho^{-b_d} + \frac{1}{a} \rho^{-c} \exp(-b\rho) + \frac{a_r}{a} \rho^{-b_r-c} \exp(-b\rho)$   
and  $D_{inf} = 1 + \frac{1}{a} \rho^{-c} \exp(-b\rho)$

We have,

$$\lim_{\rho \rightarrow +\infty} a_d \rho^{-b_d} = 0$$

$$\lim_{\rho \rightarrow +\infty} \frac{1}{a} \rho^{-c} \exp(-b\rho) = 0$$

and

$$\lim_{\rho \rightarrow +\infty} \frac{a_r}{a} \rho^{-b_r-c} \exp(-b\rho) = 0$$

So

$$\lim_{\rho \rightarrow +\infty} N_{inf} = 0$$

and

$$\lim_{\rho \rightarrow +\infty} D_{inf} = 1$$

Hence

$$\lim_{\rho \rightarrow +\infty} \overline{CAT}(\rho) = 0 \quad (5)$$

### 3 | LIMIT WHEN $\rho \mapsto 0$

$$\begin{aligned} \overline{CAT}(\rho) &= \frac{a\rho^c \exp(b \times \rho) \times a_d \rho^{-b_d} + 1 + a_r \rho^{-b_r}}{a\rho^c \exp(b \times \rho) + 1} \\ &= \frac{a \times a_d \times \rho^{c-b_d} \exp(b \times \rho) + 1 + a_r \rho^{-b_r}}{a\rho^c \exp(b \times \rho) + 1} \end{aligned} \quad (6)$$

We note  $N_0 = a \times a_d \times \rho^{c-b_d} \exp(b \times \rho) + 1 + a_r \rho^{-b_r}$   
and  $D_0 = a\rho^c \exp(b \times \rho) + 1$

#### 3.1 | Denominator ( $D_0$ )

We have

$$\lim_{\rho \rightarrow 0} a\rho^c \exp(b\rho) = 0$$

So

$$\lim_{\rho \rightarrow 0} D_0 = \lim_{\rho \rightarrow 0} a\rho^c \exp(b\rho) + 1 = 1$$

#### 3.2 | Numerator ( $N_0$ )

We have

$$\lim_{\rho \rightarrow 0} \left( 1 + a_r \rho^{-b_r} \right) = +\infty$$

If  $c - b_d > 0$

$$\lim_{\rho \rightarrow 0} \left( a a_d \rho^{c-b_d} \exp(b\rho) \right) = 0$$

and

$$\lim_{\rho \rightarrow 0} N_0 = +\infty$$

If  $c - b_d < 0$

$$\lim_{\rho \rightarrow 0} (a a_d \rho^{c-b_d} \exp(b\rho)) = +\infty$$

hence

$$\lim_{\rho \rightarrow 0} N_0 = +\infty$$

If  $c - b_d = 0$

$$\lim_{\rho \rightarrow 0} (a a_d \rho^{c-b_d} \exp(b\rho)) = a a_d$$

hence

$$\lim_{\rho \rightarrow 0} N_0 = +\infty$$

### Conclusion

$\forall (c, b_d) \in \mathbb{R}_+^2 \quad \lim_{\rho \rightarrow 0} N_0 = +\infty$

and  $\lim_{\rho \rightarrow 0} D_0 = 1$ , hence,

$$\lim_{\rho \rightarrow 0} \overline{CAT}(\rho) = +\infty \tag{7}$$