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Anastasiia Sheveleva, Said Hamdi, Aurélien Coillet, Christophe Finot, Pierre Colman. Langevin's model for soliton molecules in ultrafast fiber ring laser cavity: investigating the interplay between noise and inertia. CLEO US (Conference on Laser and ElectroOptic), May 2023, San Jose, United States. 10.1364/CLEO\_FS.2023.FW3B.7. hal-04092065

**HAL Id: hal-04092065**

**<https://hal.science/hal-04092065>**

Submitted on 17 Oct 2023

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# Langevin's model for soliton molecules in ultrafast fiber ring laser cavity: investigating the interplay between noise and inertia

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**Abstract:** We describe the vibration pattern of a soliton-molecule using the Langevin's model, i.e. noise source combined with a deterministic model. This simpler model allows investigating the interplay between fluctuation and dissipation mechanisms at play. © 2022 The Author(s)

Due to the subtle balance between gain and loss mechanisms that coexist within fiber ring laser cavities, very specific temporal localized structures, known as dissipative solitons, propagate inside these systems [1]. A few co-propagating pulses can interact and form a bound state called the soliton molecule (SM). Soliton molecules can exhibit periodic vibration of their constitutive parameters [2] like their atomic counterpart in chemistry (inter-soliton spacing, inter-soliton dephasing, etc. see Fig. 1(a-b) ). The existence of this vibration pattern is often ascribed to the formation inside the laser cavity of a limit-cycle attractor, which results in self-sustained oscillations even in the absence of external excitation of the nonlinear oscillator. Thanks to the Dispersive Fourier Transform technique, numerous vibration patterns and configurations have been investigated experimentally during the last decade.

From a fundamental perspective having a more quantitative description of the different vibration patterns may allow a better investigation of the dynamics. Indeed the dynamics that is observed experimentally results from the interplay between the self-sustained nonlinear oscillator that drives the oscillations and the noise that exists within the laser cavity. As seen in Fig. 1-(a), the vibration pattern of the SM is not exactly periodic. The phase portrait in Fig. 1-(b) reveals indeed that the SM does not stick strictly to the limit cycle and experiences cycle to cycle fluctuations. This vibration pattern has been recorded for a soliton molecule generated in an ultrafast fiber ring laser cavity composed of 1m of  $Er^{3+}$  doped fiber closed by 3m SMF. The mode-locking mechanism comes from nonlinear polarization rotation in the SMF and a subsequent filtering by a polarizing beam splitter. In order to understand better this system, it is important to be able to distinguish what is related to the deterministic motion of the SM (dissipating mechanism, coherent driving and inertia) from the impact of the laser noise. To do so, we first derived a parsimonious deterministic nonlinear model by finding an optimal set of parameters  $\Xi$  and nonlinear function  $f_i$  so that the dynamics of the SM  $\{\tau, \hat{\tau}\}$  can be well reproduced.

$$\frac{d}{dt} \begin{pmatrix} \tau_1 & \hat{\tau}_1 \\ \vdots & \vdots \\ \tau_N & \hat{\tau}_N \end{pmatrix} = \begin{pmatrix} f_1(\tau_1, \hat{\tau}_1) & \dots & f_p(\tau_1, \hat{\tau}_1) \\ \vdots & & \vdots \\ f_1(\tau_N, \hat{\tau}_N) & \dots & f_p(\tau_N, \hat{\tau}_N) \end{pmatrix} \Xi + \sigma_c(\tau, \hat{\tau}, t) \quad (1)$$

In details we used the technique of Sparse Identification of Nonlinear Dynamic (SINDy) [3] in order to model the deterministic evolution of the SM using a seven parameters nonlinear equation, hence :

$$\frac{d^2u}{dt^2} = \xi_1 u + \xi_2 u^2 + \xi_3 u^5 + \dot{u}(\xi_4 u + \xi_5 u^2) + \dot{u}^2(\xi_6 u^2 + \xi_7 u^3) + \sigma_c(t) \quad (2)$$

with  $u = (\tau - \langle \tau \rangle) / \sigma_\tau$ ,  $\Xi = (-0.758, -0.073, -0.01, 0.412, -0.171, -0.078, 0.011)$ , and  $\tau$  and  $\hat{\tau}$  expressed in ps and ps/roundtrip number, respectively. Eq. 2, possesses a stationary solution, the limit cycle, that is shown in black in Fig. 1-(b). The overall dynamics of the SM is well reproduced. Thanks to this deterministic model we can also investigate in Fig. 1-(d) how an initial perturbation would be dissipated by the system, resulting in the SM getting gradually back to its nominal limit-cycle evolution. The relaxation time of the molecule is  $T_{relax} = 98 RTs$ . Considering that the molecule oscillates with a 117 RTs periodicity, this indicates that the system here is very strongly dissipative. We complement then the deterministic model in Eq. 1 by a noise contribution  $\sigma_c(t)$ . We construct therefore a Langevin's model for the SM. The noise properties can be extracted by confronting, roundtrip after roundtrip, the deterministic evolution predicted by Eq. 2 with the one that is observed. Among others, the

noise distribution is shown in Fig. 1-(e); and exhibits a Gaussian distribution, uniform over the whole phase portrait. The result of the full Eq. 1 is shown for comparison with the real system in Fig. 1-(c).

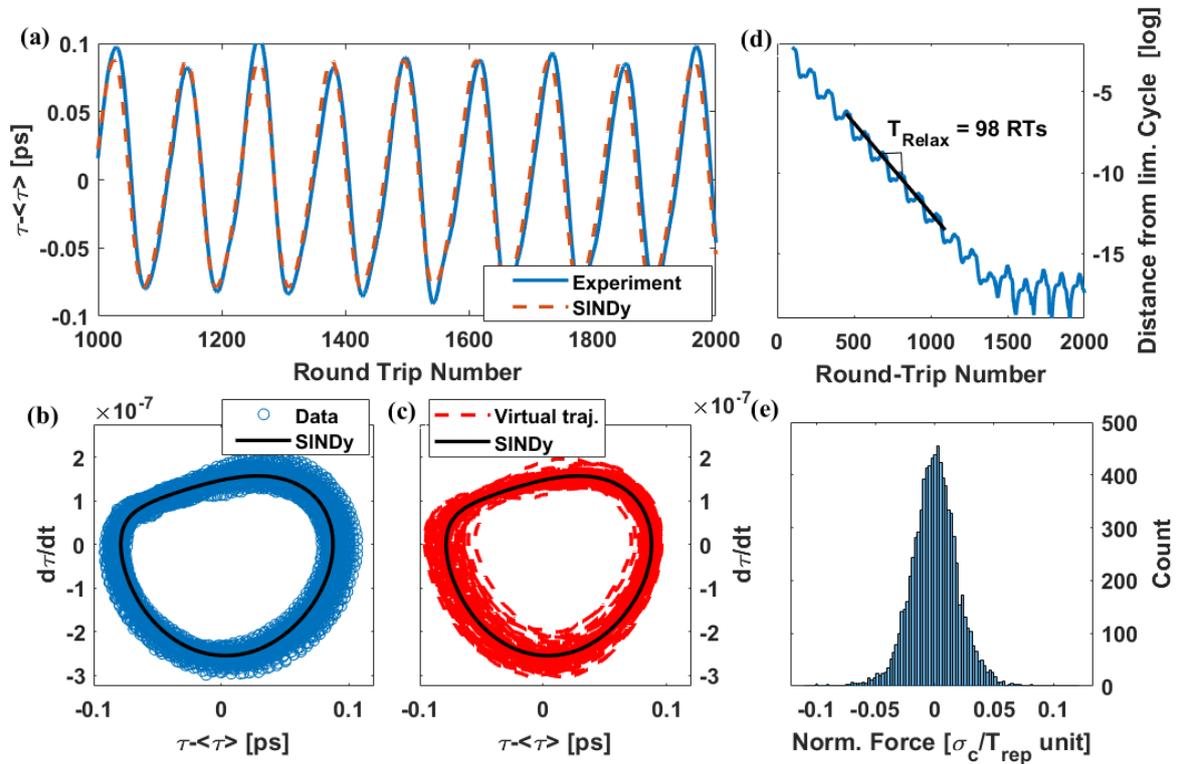


Fig. 1. a) Evolution of the inter-molecules spacing  $\tau$  (experimental and reconstructed curves are presented in blue and red, respectively). b) The phase diagrams of the experimental (blue dots) data. black: limit cycle attractor as defined by eq. 2. c) Same as b), but with simulated data, including noise. d) Experimental distance from the limit cycle evolving with the round trip number. e) Distribution of the noise extracted experimentally.

To conclude, we constructed a Langevin model for the accurate description of the vibration pattern of a soliton molecule (SM). Thanks to this model, it is possible to extract from the overall dynamics the contributions from noise and from the deterministic evolution (inertia). This allows first a more quantitative description of the laser properties. It is also possible to infer how the SM would respond to various external perturbations, and how stable the limit cycle attractor is. In turn, by looking how the parameters governing the differential equation are influenced by the laser control parameters (e.g. gain level) it would be possible to get a better and deterministic control.

### Acknowledgement

This work was supported by the French ANR funded CoMuSim (ANR-20-CE24-0007) and OPTIMAL projects (ANR-20-CE30-0004).

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