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# Turbo Equalization for Underwater Communication Systems using Rotated Constellations

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**Abstract**—The available bandwidth of the underwater acoustic channel is severely limited. Therefore, it is of interest to use modulations with high spectral efficiency, such as rotated constellations. These constellations offer better theoretical performance than conventional constellations, without reducing the spectral or energy efficiency of the system. However, in practice, when conventional equalization algorithms are employed for a system using rotated constellations, the theoretical gains are not fully achieved. To address this issue, this paper proposes a new turbo equalization algorithm that inherently considers the characteristics of rotated signals and allows for a significant gain compared to conventional methods, as demonstrated by simulations, in particular over a real underwater channel.

**Index Terms**—Rotated constellations, Signal Space Diversity, Inter-Symbol Interference (ISI), Turbo equalization, phase estimation.

## I. INTRODUCTION

Underwater acoustic communication rates are very limited due to the low available bandwidth of the Underwater Acoustic (UWA) channel [1]. Therefore, it is interesting for UWA systems to use high spectral efficiency modulations such as high-order modulation, faster-than-Nyquist modulation, and rotated constellations. In particular, rotated constellations offer better theoretical performance than conventional constellations on fading channels thanks to its inherent diversity [2]–[7]. This technique consists first in correlating the in-phase (I) and quadrature (Q) components of the conventional constellation through rotation and then sending the obtained rotated components separately over two independent channels.

As the UWA channel is doubly selective [1], using rotated constellations is clearly a promising technique to improve the performance of underwater communication systems. However, in practice, the theoretical gain of rotated constellations is not always obtained due to synchronization issues. In particular, the frequency selectivity of the UWA channel induces inter-symbol interference (ISI) that can seriously limit system performance. Following the discovery of turbo codes, turbo equalization was proposed: it allows a significant improvement in Bit Error Rate (BER) compared to previous classical approaches by iteratively considering the correlations between bits introduced by channel coding through exchange of information between the equalizer and

channel decoder [8]–[14].

In this article, we propose an original equalization algorithm for rotated constellations that takes into account not only the correlation between bits, but also the inherent correlation between rotated symbols. Our algorithm uses a feedforward-feedback temporal equalizer that works in conjunction with a phase estimator, given the sensitivity of reception quality to phase synchronization errors. Due to the sparsity of the UWA channel and to reduce the complexity of our algorithm, about 90% of the equalizer coefficients are set to zero once convergence over the preamble is obtained. In order to have a robust algorithm on UWA channels with low coherence times, we propose a completely adaptive algorithm where the step size is self-adapting to channel variations continuously and not just on the preamble.

This article is organized as follows. The next section introduces the system model. The proposed equalizer is detailed in section 3. Section 4 presents simulation results, and section 5 concludes this article.

## II. SYSTEM MODEL

The information bits  $b$  are first coded [15], then interleaved and finally transformed into a sequence of symbols of an M-QAM constellation  $s_n$ , where  $n$  indicates the time index:

$$s_n = s_n^I + j s_n^Q, \quad (1)$$

with  $s_n^I$  (resp.  $s_n^Q$ ) is the in-phase (resp. quadrature) component. Then, the symbols  $s_n$  are rotated by a rotation angle  $\alpha$ . In our simulations, we use rotation angles  $\alpha = \text{atan}(\frac{1}{\sqrt{M}})$  as they offer good performance in terms of Bit Error Rate (BER) and for these rotation angles, a low-complexity soft demapper has been proposed [2], [4]. The rotated symbols can be written as follows:

$$x_n = x_n^I + j x_n^Q. \quad (2)$$

Then, the transmitter cyclically delays the Q components of  $x(n)$  as:

$$z_n = x_n^I + j x_{n+\Delta_T}^Q, \quad (3)$$

where  $\Delta_T$  is an integer time delay; to fully benefit from the time diversity of the ASM channel,  $\Delta_T$  should be greater than the channel's coherence time. Finally, a preamble is added at

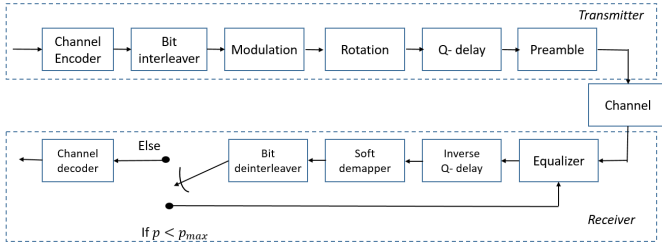


Fig. 1. System model.

the beginning of each frame to enable good synchronization performance at the receiver side [16], [17].

Assuming perfect time synchronization [18], [19], the received signal can be written as:

$$r_n = \sum_l h_l z_{n-l} + w_n, \quad (4)$$

where  $w_n$  represents the additive white Gaussian noise and  $h_l$  denotes the coefficient of the channel's impulse response.

The receiver must effectively suppress ISI as the system's performance and complexity depend heavily on the equalizer; in particular, phase error can lead to a serious performance degradation [20]–[23]. Similar to the conventional turbo equalizer, the extrinsic information provided by the decoder is used by our equalizer iteratively; at the  $p$ -th iteration, we obtain the following equalized symbols:

$$\tilde{z}_n^p = \beta_0 z_n + v_n, \quad (5)$$

where  $\beta_0$  is an attenuation factor and  $v_n$  includes both the noise and the residual ISI [14].

Then, the receiver deinterleaves the quadrature components and computes the Log-Likelihood Ratio (LLR) for each received bit [2], [4]. The channel decoding is then performed. The extrinsic information obtained by the decoder is used by the Bit to Symbol Converter (BSC) to obtain soft estimates on the transmitted symbols  $\tilde{z}_k^{(p-1)}$  (for  $p > 1$ ); additionally, the symbols  $\tilde{s}_k^{(p-1)}$  are used at the next iteration  $p$  of our algorithm to improve equalization performance and phase synchronization. Finally, after  $p_{max}$  iterations (i.e. channel equalization/decoding), the receiver estimates the information bits from the LLRs obtained at the output of the decoder.

### III. PROPOSED TURBO EQUALIZATION AND PHASE SYNCHRONIZATION ALGORITHM

We now briefly present the outline of our proposed joint phase and equalizer estimator, as illustrated in Figure 2. Its output can be expressed as follows:

$$\tilde{z}_n^p = (F_n^p R_n - Q_n^p Y_n^p) e^{-j\phi_n^p}, \quad (6)$$

where:  $R_n = [r_{n+L_1}, \dots, r_n, \dots, r_{n-L_1}]^T$  represents the received symbols,  $F_n^p = [f_{-L_1}^p(n), \dots, f_0^p(n), \dots, f_{L_1}^p(n)]$  and

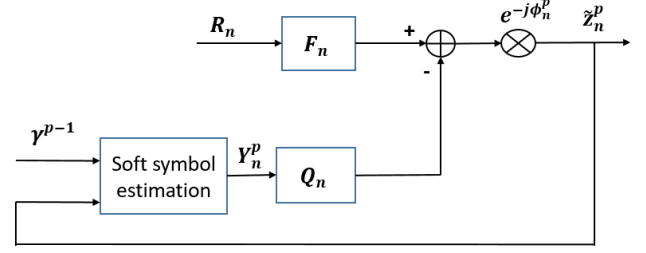


Fig. 2. The proposed joint equalizer and phase estimator.

$Q_n = [q_{-L_2}(n), \dots, 0, \dots, q_{L_2}(n)]^T$  are transversal filters of respective lengths  $2L_1+1$  and  $2L_2+1$ ,  $\phi_n^p$  is the estimated phase error, and  $Y_n^p = [y_{n+L_2}^p, \dots, y_n^p, \dots, y_{n-L_2}^p]$  with  $y_n$  can be either a known symbol  $z_n$  during the preamble or an estimated symbol as follows:

$$y_n^p = \sum_{y \in \chi_z} y \Pr(y | \gamma^{p-1}, \tilde{z}_n - \Delta_T^p, \tilde{z}_{n+\Delta_T}^p), \quad (7)$$

where  $\chi_z$  is the set of all possible constellation points  $z_n$ .

Note that during the first iteration, the extrinsic LLR values  $\gamma^{p-1}$  are not available and are therefore not taken into account in Eq. (7). Furthermore,  $\tilde{z}_{n-\Delta_T}^p$  and  $\tilde{z}_{n+\Delta_T}^p$  may also not be available, in which case they are replaced for  $p > 1$  by  $\tilde{z}_{n-\Delta_T}^{p-1}$  and  $\tilde{z}_{n+\Delta_T}^{p-1}$ , respectively. To fully take into account and leverage the time correlation between symbols (i.e.,  $\tilde{z}_{n-\Delta_T}^p$  and  $\tilde{z}_{n+\Delta_T}^p$ ), we propose to perform equalization twice at each iteration  $p$ .

Furthermore, the filter coefficients are updated to minimize the soft Mean Squared Error defined as follows:

$$J(F_n^p, Q_n^p, \phi_n^p) = \mathbf{E}_n \mathbf{E}_{x \in \chi_z} (\Pr(x | \tilde{z}_n^p) | x - \tilde{z}_n^p|^2). \quad (8)$$

where  $\mathbb{E}(d)$  denotes the expectation of  $d$  and:

$$\Pr(x | \tilde{z}_n^p) = \frac{\Pr(x) \Pr(\tilde{z}_n^p | x)}{\sum_{x' \in \chi} \Pr(x') \Pr(\tilde{z}_n^p | x')}, \quad (9)$$

and:

$$\Pr(\tilde{z}_n^p | \tilde{x}) = \frac{1}{\pi \sigma_v^2} \exp\left(-\frac{|\tilde{z}_n^p - \beta_0 \tilde{x}|^2}{\sigma_v^2}\right). \quad (10)$$

Using a gradient descent algorithm, we obtain an iterative algorithm to jointly update the phase error  $\phi_n^p$  and filter coefficients  $F_n^p$  and  $Q_n^p$ :

$$F_{n+1}^p = F_n^p - \mu \nabla_{F_n^p} J(F_n^p, Q_n^p, \phi_n^p) \quad (11)$$

$$Q_{n+1}^p = Q_n^p - \mu \nabla_{Q_n^p} J(F_n^p, Q_n^p, \phi_n^p) \quad (12)$$

$$\phi_{n+1}^p = \phi_n^p - \gamma \nabla_{\phi_n^p} J(F_n^p, Q_n^p, \phi_n^p), \quad (13)$$

where  $\mu$  and  $\gamma$  are appropriate steps.

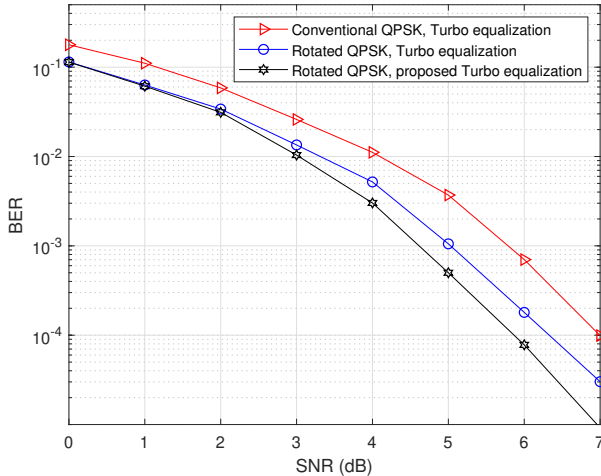


Fig. 3. BER comparison over the KAU1 channel.

Moreover, we also propose to optimize the step-size  $\mu$  and  $\gamma$  along the coefficients' trajectory to minimize the mean square error (see eq. (8)). Using the gradient descent algorithm, the step-sizes  $\mu$  and  $\gamma$  are updated as follows:

$$\begin{cases} \mu_{k+1} = \mu_k - \frac{\alpha}{2} \frac{\partial J}{\partial \mu} \\ \gamma_{k+1} = \gamma_k - \frac{\beta}{2} \frac{\partial J}{\partial \gamma} \end{cases} \quad (14)$$

where  $\frac{\partial J}{\partial \mu}$  and  $\frac{\partial J}{\partial \gamma}$  can be easily obtained with some basic mathematical derivations. Note that, unlike conventional methods, the proposed step-size optimization can be carried out over all received symbols, not just over the preamble thanks to chosen definition of the mean square error (8).

It is worth noting that at the first iteration, only the Feedforward equalizer is used. Furthermore, the extrinsic information provided by the channel decoding at iteration (p-1) is used at iteration  $p$ , both to improve the channel response estimation and to feed the Feedback equalizer to better neutralize ISI interferences. After  $p_{max}$  iterations, the receiver estimates the information bits from the LLRs obtained from the channel decoding.

Finally, it should be noted that the response of the underwater channel may be particularly important; thus,  $L_1$  and  $L_2$  should also be sufficiently large, which has a significant impact on the receiver's architecture. Therefore, once convergence during the preamble is established, we propose to set the coefficients of filters  $F_n^p$  and  $Q_n^p$  that are below a predefined threshold to zero; in practice, only about 10% of these transversal filter coefficients need to be updated, allowing a considerable reduction of the system complexity.

#### IV. SIMULATIONS RESULTS

This section compares the performance of the proposed algorithm with that of the Turbo equalization method proposed for conventional constellations [8]. In our simulations, each

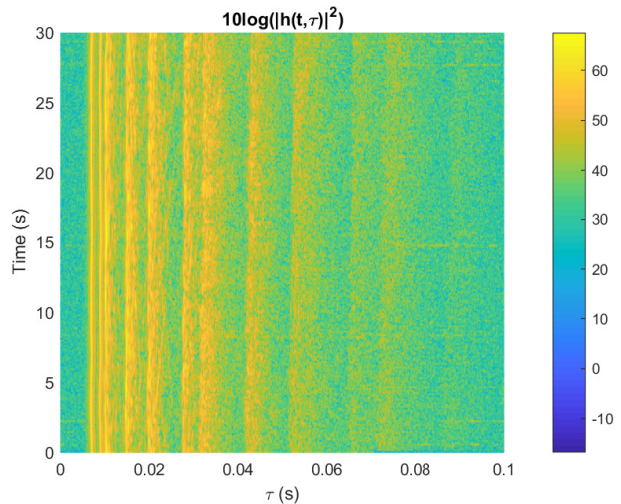


Fig. 4. The multi-path KAU1 channel.

block of 1024 bits is encoded with a turbo code of rate 1/3. The coded symbols are then transformed into a sequence of QPSK constellation symbols. The length of the preamble is fixed at 128 symbols, and  $p_{max}$  is set to 5.

We first tested our proposal on a synthetic channel. Similar to [8], [9], [14], the channel response is modeled as follows:

$$h_l(k) = \sqrt{\frac{P_l}{I}} \sum_{i=1}^I e^{j(k\epsilon\pi \cos \psi_{l,i} + \zeta_{l,i})}, \quad (15)$$

where the Doppler shift is characterized by  $\epsilon$ ,  $P_l$  denotes the average power of the  $l$ -th path,  $\psi_{l,i}$  and  $\zeta_{l,i}$  are uniform random variables on  $[0, 2\pi[$ ;  $I$  is set to 10, and (15) includes three paths of equal average power, with  $\epsilon$  set to 0.002.

Figure 3 compares the Bit Error Rate (BER) performance of the conventional QPSK with the Turbo equalization algorithm [8], the rotated QPSK with the conventional Turbo equalization algorithm [8], and the rotated QPSK with the proposed equalization algorithm on the channel (15). We can observe that, on this channel and with the chosen coding rate, rotated constellations provide significantly better performance in terms of BER, with a gain of about 0.7 dB compared to conventional QPSK when the same Turbo equalization algorithm is used. In addition, the proposed equalization algorithm provides a gain of more than 0.6 dB compared to the reference algorithm when rotated constellations are used.

Finally, we tested our method on a real underwater channel measured at Kauai (KAU1), in the United States, over a distance of 1080 m [24]. Figure 4 shows the multi-path delay dispersion on this channel. In these simulations, the size of the Feedforward (respectively Feedback) filter is set to 41 (respectively 21). Figure 5 compares the BER performance between the same three configurations as in Figure 3. We can again observe that the performance obtained with the QPSK constellation for any equalization algorithm surpasses

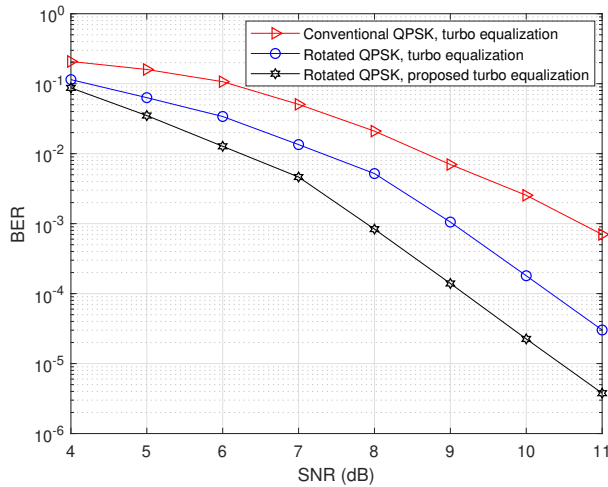


Fig. 5. BER comparison over the KAU1 channel.

that of the conventional QPSK. Moreover, the proposed Turbo equalization algorithm provides an improvement in BER of about 1 dB compared to the reference Turbo equalization algorithm [8].

## V. CONCLUSION

Rotated constellations improves the performance of underwater communication systems thanks to their inherent diversity. However, on the doubly selective UWA channel, interference between symbols and phase synchronization errors can prevent to achieve the full theoretical gain of rotated constellations. To address this problem, we proposed a new equalization algorithm for rotated constellations. This algorithm uses a feedforward-feedback equalizer together with a phase estimator to take into account both the correlation between bits and the inherent correlation between rotated symbols. Our simulation results have shown that our algorithm provides a significant gain in terms of bit error rate compared to conventional Turbo equalization, which could promote the adoption of rotated constellations by underwater communication systems.

## REFERENCES

- [1] D. B. Kilfoyle and A. B. Baggeroer, "The state of the art in underwater acoustic telemetry," *IEEE Journal of Oceanic Engineering*, 25(1) : 4–27, Jan. 2000.
- [2] T. Arbi, Z. Ye and B. Geller, "Low-Complexity Blind PAPR Reduction for OFDM Systems With Rotated Constellations," *IEEE Transactions on Broadcasting*, vol. 67, no. 2, pp. 491-499, June 2021.
- [3] Z. Ye, T. Arbi, F. X. Socheleau and B. Geller, "Fast Soft Demapping for Underwater Acoustic Communications With Signal Space Diversity," *OCEANS 2018 MTS/IEEE Charleston*, 2018, pp. 1-6, doi: 10.1109/OCEANS.2018.8604747.
- [4] T. Arbi, B. Geller, J. Yang, C. Abdel Nour and O. Rioul, "Uniformly Projected RCQD QAM: A Low-Complexity Signal Space Diversity Solution Over Fading Channels With or Without Erasures," *IEEE Transactions on Broadcasting*, vol. 64, no. 4, pp. 803-815, Dec. 2018.

- [5] T. Arbi, I. Nasr and B. Geller, "Near Capacity RCQD Constellations for PAPR Reduction of OFDM Systems," *Proc. of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 5110-5114, May 2020.
- [6] T. Arbi and B. Geller, "Joint BER Optimization and Blind PAPR Reduction of OFDM Systems With Signal Space Diversity," *IEEE Communications Letters*, vol. 23, no. 10, pp. 1866-1870, Oct. 2019.
- [7] J. Yang, K. Wan, B. Geller, C. Abdel Nour, O. Rioul and C. Douillard, "A low-complexity 2D signal space diversity solution for future broadcasting systems," *IEEE International Conference on Communications (ICC)*, pp. 2762-2767, June 2015.
- [8] Z. Ye, T. Arbi and B. Geller, "Adaptive Step-size Iterative Equalization for Underwater Communication," *IEEE OCEANS 2021*, San Diego, USA, 2021.
- [9] Z. Ye, T. Arbi, and B. Geller, "Low complexity adaptive turbo equalization for multipath underwater communication," *Proceedings of IEEE Oceans 19*, Marseille, June 2019.
- [10] L. Zhou, B. Zheng, A. Wei, B. Geller and J. Cui, "A Robust Resolution-Enhancement Scheme for Video Transmission Over Mobile Ad-Hoc Networks," in *IEEE Transactions on Broadcasting*, vol. 54, no. 2, pp. 312-321, June 2008, doi: 10.1109/TBC.2008.917746.
- [11] B. Geller, V. Capellano, J.-M. Brossier, A. Essebbar and G. Jourdain, "Equalizer for video rate transmission in multipath underwater communications," *IEEE Journal of Oceanic Engineering*, vol. 21, no. 2, pp. 150-155, April 1996, doi: 10.1109/48.486790.
- [12] B. Geller, V. Capellano and G. Jourdain, "Equalizer for real time high rate transmission in underwater communications," *1995 International Conference on Acoustics, Speech, and Signal Processing*, 1995, pp. 3179-3182 vol.5, doi: 10.1109/ICASSP.1995.479560.
- [13] B. Geller, V. Capellano, J.M. Brossier, "Equalizer for High Rate Transmission," *Proc. of OCEANS IEEE Oceanic engineering*, pp. 302-306, Brest, Sept. 1994.
- [14] C. Laot, A. Glavieux, and J. Labat, "Turbo equalization: adaptive equalization and channel decoding jointly optimized," *IEEE Journal on Selected Areas in Communications*, 19, pp. 1744–1752, Sept. 2001.
- [15] B. Geller, I. Diatta, J. P. Barbot, C. Vanstracele and F. Rambeau, "Block Turbo Codes: From Architecture to Application," *IEEE International Symposium on Information Theory, ISIT 2006*, pp. 1813-1816, July 2006, doi: 10.1109/ISIT.2006.261747.
- [16] T. Arbi, J. Yang, B. Geller, "Récepteur multi-normes pour les réseaux de capteurs de l'iot médical," *Internet des objets*, 1 (Numéro 1), 2017.
- [17] J. Yang, B. Geller and T. Arbi, "Proposal of a multi-standard transceiver for the WBAN Internet of Things," *2016 International Symposium on Signal, Image, Video and Communications (ISIVC)*, pp. 369-373, Nov. 2016.
- [18] I. Nasr, B. Geller, L. Najjar Atallah, S. Cherif, "Performance Study of a Near Maximum Likelihood Code-Aided Timing Recovery Technique," *IEEE Transactions on Signal Processing*, 64(3), pp. 799-811, Feb. 2016.
- [19] I. Nasr, L. N. Atallah, S. Cherif and B. Geller, "Near MAP Dynamical Delay Estimator and Bayesian CRB for Coded QAM Signals," *IEEE Transactions on Wireless Communications*, vol. 17, no. 1, pp. 636-651, Jan. 2018.
- [20] J. Yang, B. Geller and S. Bay, "Bayesian and Hybrid Cramér-Rao Bounds for the Carrier Recovery Under Dynamic Phase Uncertain Channels," *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 667-680, Feb. 2011.
- [21] J. Yang, B. Geller and A. Wei, "Approximate Expressions for Cramer-Rao Bounds of Code Aided QAM Dynamical Phase Estimation," *2009 IEEE International Conference on Communications*, June 2009, pp. 1-5.
- [22] J. Yang and B. Geller, "Near Optimum Low Complexity Smoothing Loops for Dynamical Phase Estimation—Application to BPSK Modulated Signals," *IEEE Transactions on Signal Processing*, vol. 57, no. 9, pp. 3704-3711, Sept. 2009.
- [23] J.M. Brossier, P.O. Amblard, B. Geller, "Self-adaptive PLL for general QAM constellations," *2002 11th European Signal Processing Conference, Proc. EUSIPCO*, pp. 631-635, Sept. 2002.
- [24] P. A. van Walree, F. X. Socheleau, R. Otnes, and T. Jensenud, "The "watermark" benchmark for underwater acoustic modulation schemes," *IEEE Journal of Oceanic Engineering*, 42(4), pp. 1007–1018, Oct. 2017.