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Efficient Hyper-Reduction of contact problems treated by Lagrange multipliers.

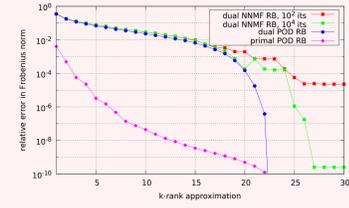
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Goals

- Propose a reduced order model for Variational inequalities/Saddle-point problems.
- Propose a solution to the challenging task of reducing the dual basis with a positivity constraint (see Figure extracted from [3]).
- Ensure an efficient reduced order model (accuracy and low order)



Finite element (FE) contact problem model.

$$\begin{cases} \underline{K}\underline{U} + \underline{B}^T \underline{\Lambda} = \underline{F}, \\ \underline{B}\underline{U} \leq \underline{D} \end{cases}$$

$\underline{U} \in \mathbb{R}^N$ displacement, $\underline{\Lambda} \in \mathbb{R}_+^{N_\lambda}$ Lagrange multipliers, $\underline{D} \in \mathbb{R}_+^{N_\lambda}$ initial gap

Saddle-point problem \Rightarrow Inf-sup/ LBB condition (solvability and stability)

Solvability condition : \underline{B} must be of full row rank

Hybrid Hyper-reduced (HHR) order model

- Hyper-reduction via Reduced Integration Domain (RID):** \mathcal{A} primal DOFs of the RID, $\mathcal{A}_C \subset \mathcal{A}$ primal DOFs on the RID contact surface (see Figure 1).
- Primal reduced basis (RB):** $\underline{V} \in \mathbb{R}^{N \times l}$ obtained by POD ($l \ll N$).
- Dual RB:** $\underline{\Delta}[\mathcal{A}_\lambda] \in \mathbb{R}_+^{N_\lambda \times N_\lambda^C}$ obtained by restricting the FE dual basis to \mathcal{A}_λ dual DOFs of the RID ($N_\lambda^C \ll N_\lambda$).
 \Rightarrow Dual variable well approximated (see [3]).

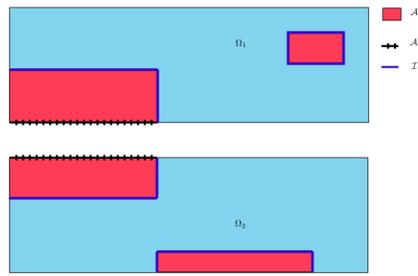


Figure 1. Sketch of a RID with contact.

$$\begin{cases} \underline{V}[\mathcal{A}, :]^T \underline{K}[\mathcal{A}, \mathcal{A} \cup \mathcal{I}] \underline{V}[\mathcal{A} \cup \mathcal{I}, :] \underline{\gamma} + \underline{V}[\mathcal{A}_C, :]^T \underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C]^T \underline{\Delta}[\mathcal{A}_\lambda] = \underline{V}[\mathcal{A}, :]^T \underline{F}[\mathcal{A}], \\ \underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C] \underline{V}[\mathcal{A}_C, :] \underline{\gamma} \leq \underline{D}[\mathcal{A}_\lambda] \end{cases}$$

$\underline{\gamma} \in \mathbb{R}^l$ and $\underline{\Delta}[\mathcal{A}_\lambda] \in \mathbb{R}_+^{N_\lambda^C}$ the reduced unknown.

The HHR order model verifies the contact conditions on the RID

This model is also a saddle-point problem \Rightarrow Inf-sup/ LBB condition

Solvability condition: $\underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C] \underline{V}[\mathcal{A}_C, :]$ must be of full row rank

Enrichment of the primal RB

FE enrichment:

- If solvability cond. not verified initially
- $\underline{V}_2 = (\underline{V} \mid \underline{I}[\cdot, \mathcal{L}_C]) \in \mathbb{R}^{N \times \bar{l}}$
 $\mathcal{L}_C \subset \mathcal{A}_C$ nodes of the FE shape functions to be added. $\bar{l} = l + \text{card}(\mathcal{L}_C) \ll N$

POD modes + FE enrichment:

- $\underline{V} \in \mathbb{R}^{N \times l} \Rightarrow$ RID construction
- $\underline{V}_2 \in \mathbb{R}^{N \times (l+l_2)}$ larger POD basis than \underline{V}_1
- FE enrichment applied to \underline{V}_2 with
 $\bar{l} = l + l_2 + \text{card}(\mathcal{L}_C) \ll N$

\Rightarrow Strong relation between the condition number κ of $\underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C] \underline{V}[\mathcal{A}_C, :]$ and the dual relative error.

$\Rightarrow \mathcal{L}_C$ is built through a greedy algorithm based on κ (see [1])

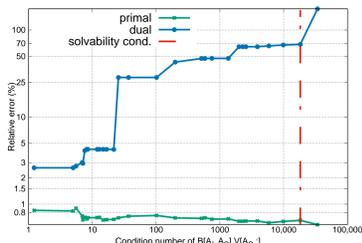


Figure 2. Impact of the condition number of $\underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C] \underline{V}[\mathcal{A}_C, :]$ on the HHR errors

Results of the enrichment methods on a large 2D parametric space.

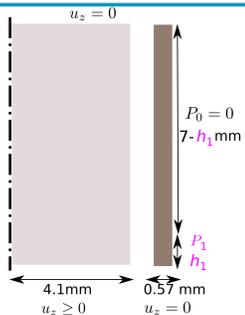


Figure 3. Test case sketch. $P_1 \in [5; 200]$ MPa and $h_1 \in [0.6; 6]$ mm

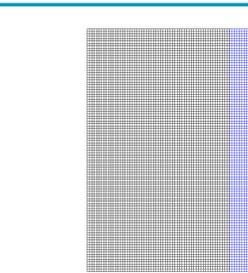


Figure 4. FE mesh. $N = 7070$, $N_\lambda = 101$

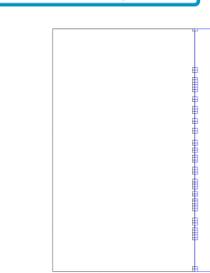


Figure 5. RID. $l = 33$, $N_\lambda^C = 57$.

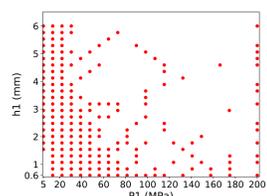


Figure 6. Snapshots positions.

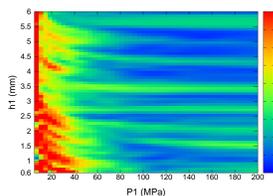


Figure 7. FE enrichment. $\bar{l} = 60$.

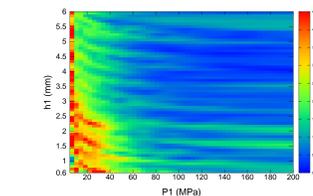


Figure 8. POD+FE enrich. $l_2 = 5$, $\bar{l} = 82$.

Dual relative errors maps (in %). $\kappa_{max} = 10$.

- \Rightarrow Robust strategies for large parametric spaces
- \Rightarrow Better results with POD+FE enrichment but higher primal RB size
- \Rightarrow Still high relative errors for low contact pressure parametric zones

Clustering on the parametric space

For large parametric spaces, reaching low errors on the whole parametric space could lead to low data-compression (increasement of the RBs size) \Rightarrow interest of clustering the parametric space

The nonlinear solution is searched in a piecewise linear manner

Methodology ([2]):

- Create clusters on the parametric space: Grassmann distance, k-medoids algo.
- Build local low-rank ROMs on each cluster
- Train a supervised classifier: Random forest algo.
- Solve the problem with the local ROM selected by the classifier

\Rightarrow increase the solution's (primal and dual) precision without increasing the RB order.

Clustering of a 2D parameter space and classifier.

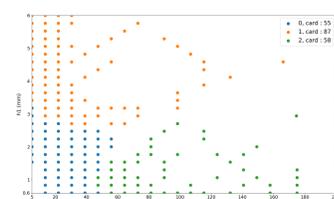


Figure 9. Snapshot clusters

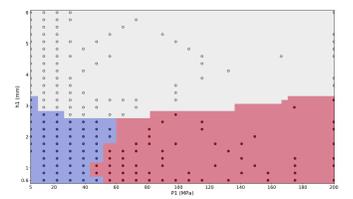


Figure 10. Classifier trained through random forest classification

Clustering impact on the RB and RID.

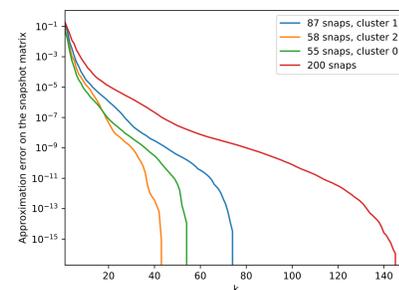


Figure 11. Approximation error on the primal snapshot matrices with and without clustering

\Rightarrow Enhancement of the data compression

\Rightarrow Leads to primal RB of lower ranks

\Rightarrow FE enrichment only to avoid overfitting

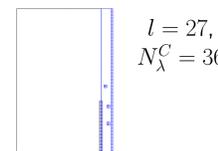


Figure 12. RID cluster 0.

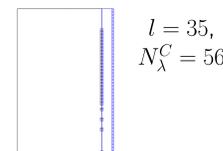


Figure 13. RID cluster 1.

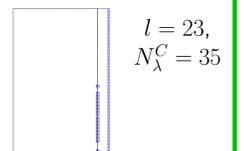


Figure 14. RID cluster 2.

\Rightarrow Piecewise linear treatment visible on the RID.

\Rightarrow Similar sizes of RB

Results of the clustering on a large parametric space.

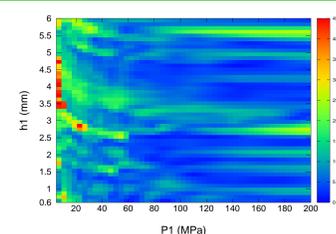


Figure 15. Dual relative errors map (in %) with FE enrichment set with $\kappa_{max} = 10$. $\bar{l} = \{52, 77, 48\}$.

\Rightarrow Clustering strategy helps smoothing the dual errors

\Rightarrow Errors considerably reduced for local RBs compared to a global RB

\Rightarrow High accuracy on the whole parametric space, even for low contact pressures

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