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# Efficient Hyper-Reduction of contact problems treated by Lagrange multipliers

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S Le Berre, I Ramière, David Ryckelynck. Efficient Hyper-Reduction of contact problems treated by Lagrange multipliers. Model Reduction and Surrogate Modeling (MORE), Sep 2022, Berlin, Germany. hal-04088319

**HAL Id: hal-04088319**

**<https://hal.science/hal-04088319>**

Submitted on 4 May 2023

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# Efficient Hyper-Reduction of contact problems treated by Lagrange multipliers.

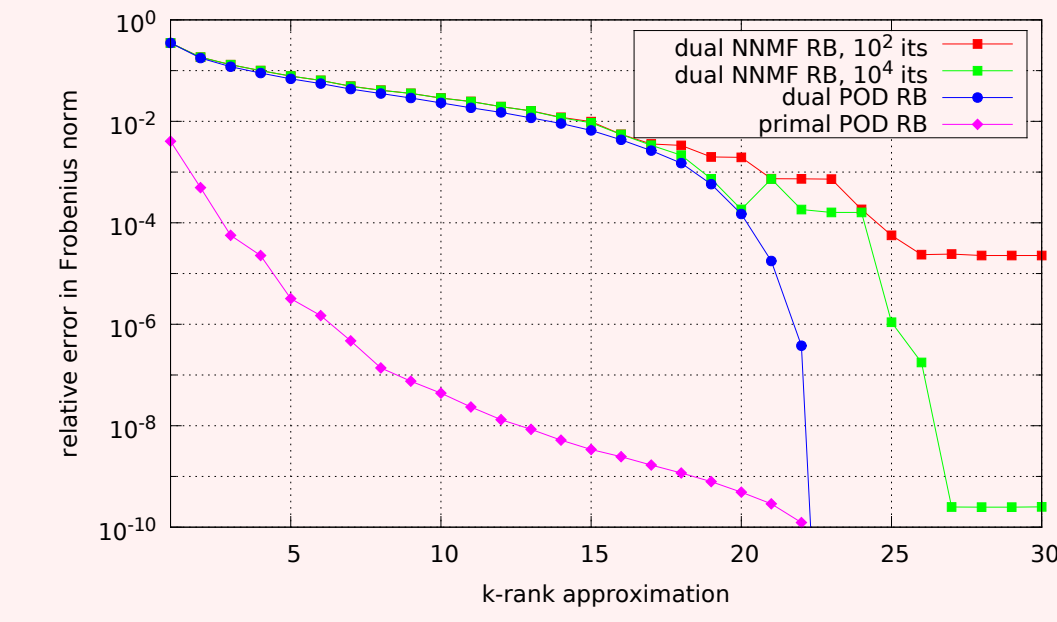
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## Goals

- Propose a reduced order model for Variational inequalities/Saddle-point problems.
- Propose a solution to the challenging task of reducing the dual basis with a positivity constraint (see Figure extracted from [3]).
- Ensure an efficient reduced order model (accuracy and low order)



## Finite element (FE) contact problem model.

$$\begin{cases} \underline{K}\underline{U} + \underline{B}^T \underline{\Lambda} = \underline{F}, \\ \underline{B}\underline{U} \leq \underline{D} \end{cases}$$

$\underline{U} \in \mathbb{R}^N$  displacement,  $\underline{\Lambda} \in \mathbb{R}_+^{N_\lambda}$  Lagrange multipliers,  $\underline{D} \in \mathbb{R}_+^{N_\lambda}$  initial gap

Saddle-point problem  $\Rightarrow$  Inf-sup/ LBB condition (solvability and stability)

Solvability condition :  $\underline{B}$  must be of full row rank

## Hybrid Hyper-reduced (HHR) order model

- Hyper-reduction via Reduced Integration Domain (RID):**  $\mathcal{A}$  primal DOFs of the RID,  $\mathcal{A}_C \subset \mathcal{A}$  primal DOFs on the RID contact surface (see Figure 1).
- Primal reduced basis (RB):**  $\underline{V} \in \mathbb{R}^{N \times l}$  obtained by POD ( $l \ll N$ ).
- Dual RB:**  $\underline{\Delta}[\mathcal{A}_\lambda] \in \mathbb{R}_+^{N_\lambda^C}$  obtained by restricting the FE dual basis to  $\mathcal{A}_\lambda$  dual DOFs of the RID ( $N_\lambda^C \ll N_\lambda$ ).  
 $\Rightarrow$  Dual variable well approximated (see [3]).

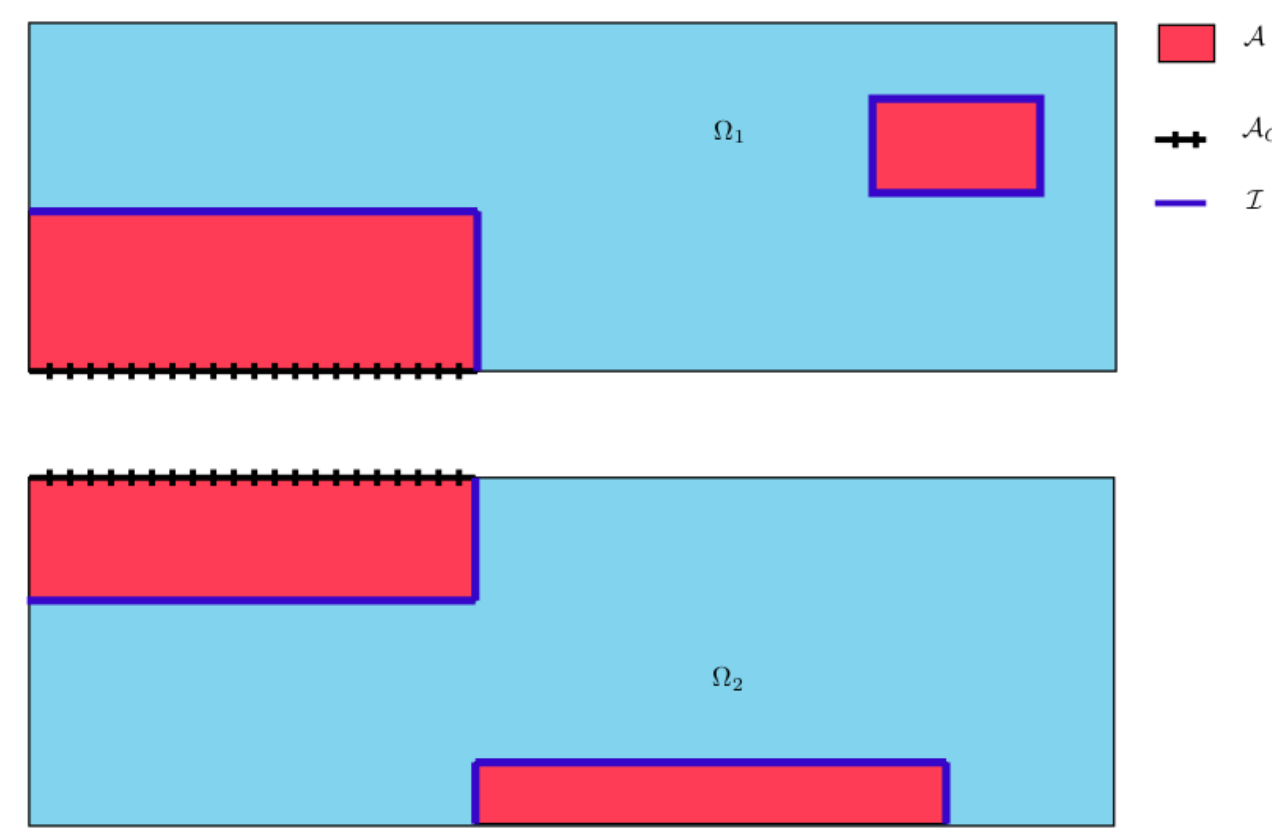


Figure 1. Sketch of a RID with contact.

$$\begin{cases} \underline{V}[\mathcal{A}, :]^T \underline{K}[\mathcal{A}, \mathcal{A} \cup \mathcal{I}] \underline{V}[\mathcal{A} \cup \mathcal{I}, :] \underline{\gamma} + \underline{V}[\mathcal{A}_C, :]^T \underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C]^T \underline{\Delta}[\mathcal{A}_\lambda] = \underline{V}[\mathcal{A}, :]^T \underline{F}[\mathcal{A}], \\ \underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C] \underline{V}[\mathcal{A}_C, :] \underline{\gamma} \leq \underline{D}[\mathcal{A}_\lambda] \end{cases}$$

$\underline{\gamma} \in \mathbb{R}^l$  and  $\underline{\Delta}[\mathcal{A}_\lambda] \in \mathbb{R}_+^{N_\lambda^C}$  the reduced unknown.

The HHR order model verifies the contact conditions on the RID

This model is also a saddle-point problem  $\Rightarrow$  Inf-sup/ LBB condition

Solvability condition:  $\underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C] \underline{V}[\mathcal{A}_C, :]$  must be of full row rank

## Enrichment of the primal RB

### FE enrichment:

- If solvability cond. not verified initially
- $\underline{V}_2 = (\underline{V} \mid \underline{I}[\cdot, \mathcal{L}_C]) \in \mathbb{R}^{N \times \bar{l}}$   
 $\mathcal{L}_C \subset \mathcal{A}_C$  nodes of the FE shape functions to be added.  $\bar{l} = l + \text{card}(\mathcal{L}_C) \ll N$

### POD modes + FE enrichment:

- $\underline{V} \in \mathbb{R}^{N \times l} \Rightarrow$  RID construction
- $\underline{V}_2 \in \mathbb{R}^{N \times (l+l_2)}$  larger POD basis than  $\underline{V}_1$
- FE enrichment applied to  $\underline{V}_2$  with  
 $\bar{l} = l + l_2 + \text{card}(\mathcal{L}_C) \ll N$

$\Rightarrow$  Strong relation between the condition number  $\kappa$  of  $\underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C] \underline{V}[\mathcal{A}_C, :]$  and the dual relative error.

$\Rightarrow \mathcal{L}_C$  is built through a greedy algorithm based on  $\kappa$  (see [1])

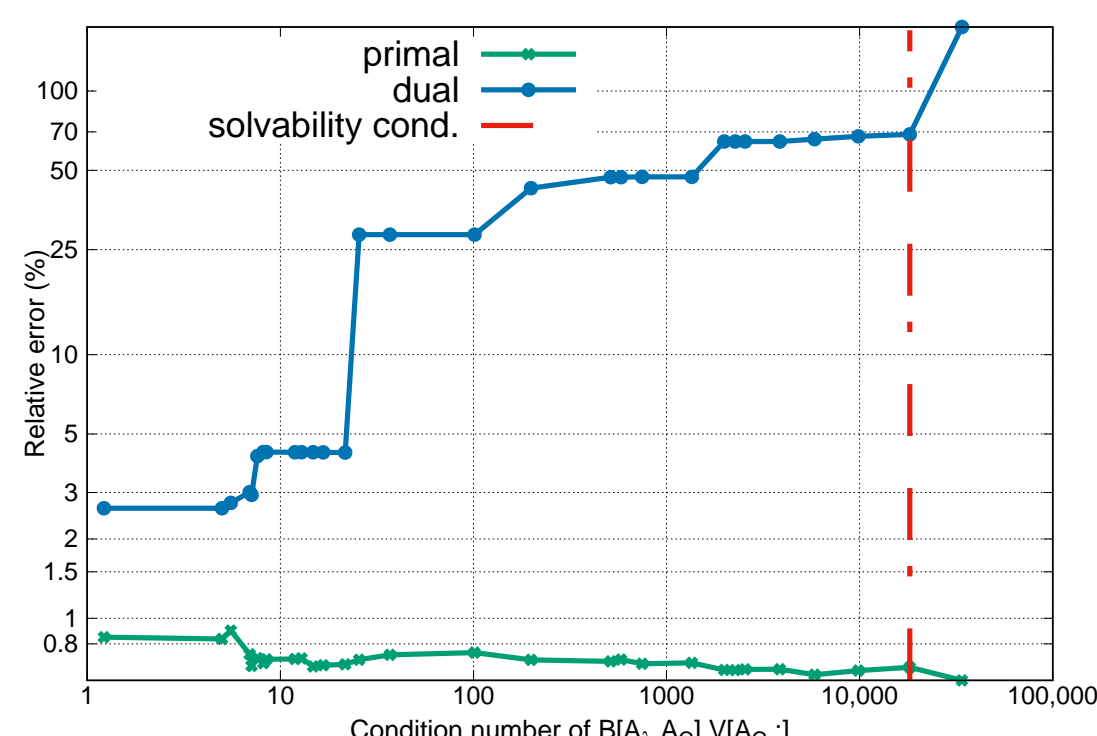


Figure 2. Impact of the condition number of  $\underline{B}[\mathcal{A}_\lambda, \mathcal{A}_C] \underline{V}[\mathcal{A}_C, :]$  on the HHR errors

## Results of the enrichment methods on a large 2D parametric space.

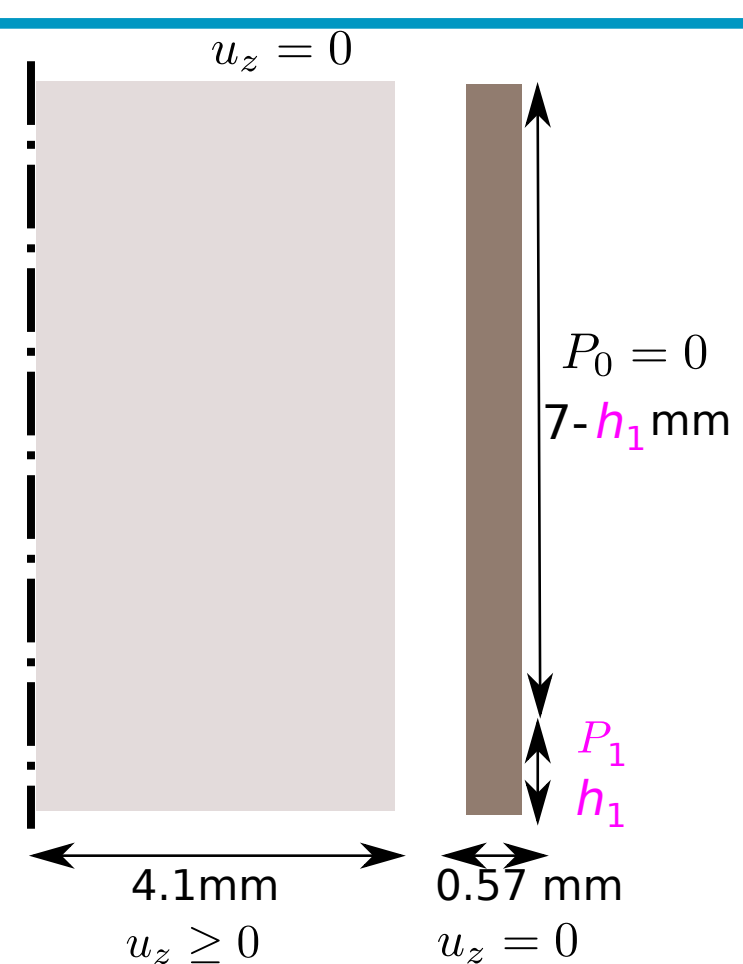


Figure 3. Test case sketch.  $P_1 \in [5; 200]$ MPa and  $h_1 \in [0.6; 6]$ mm

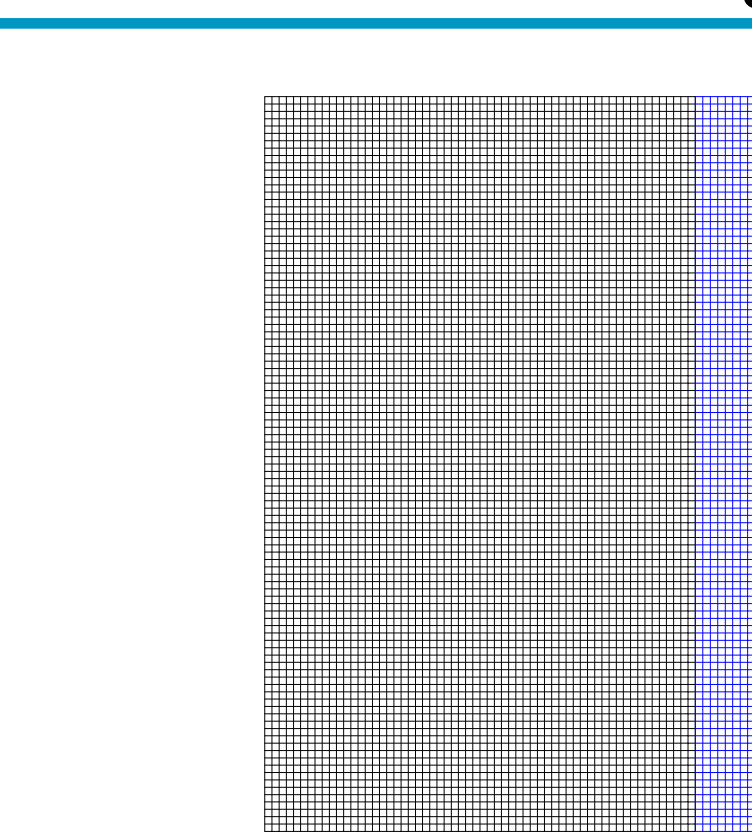


Figure 4. FE mesh.  $N = 7070$ ,  $N_\lambda = 101$

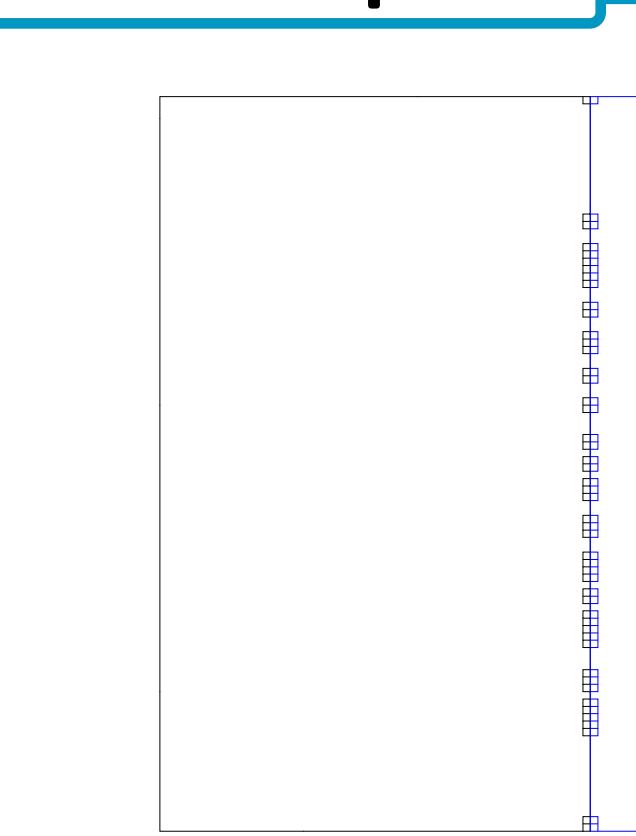


Figure 5. RID.  $l = 33$ ,  $N_\lambda^C = 57$ .

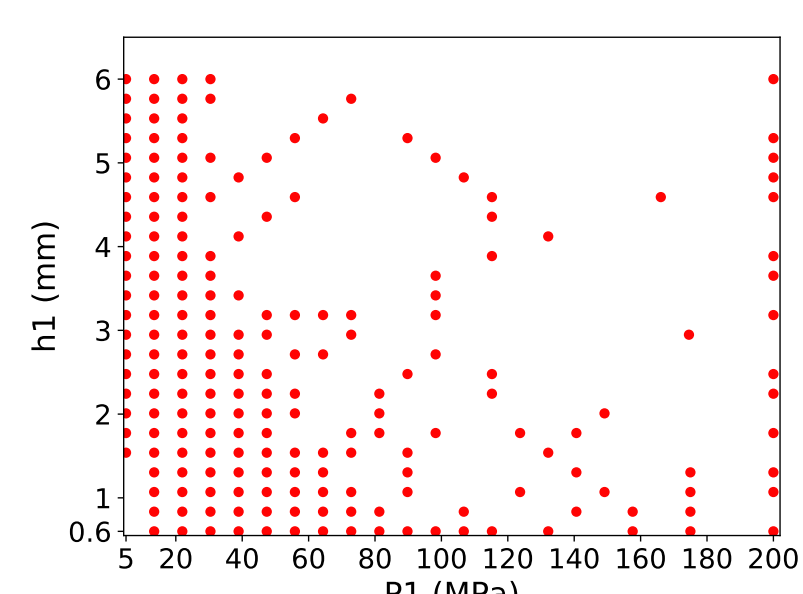


Figure 6. Snapshots positions.

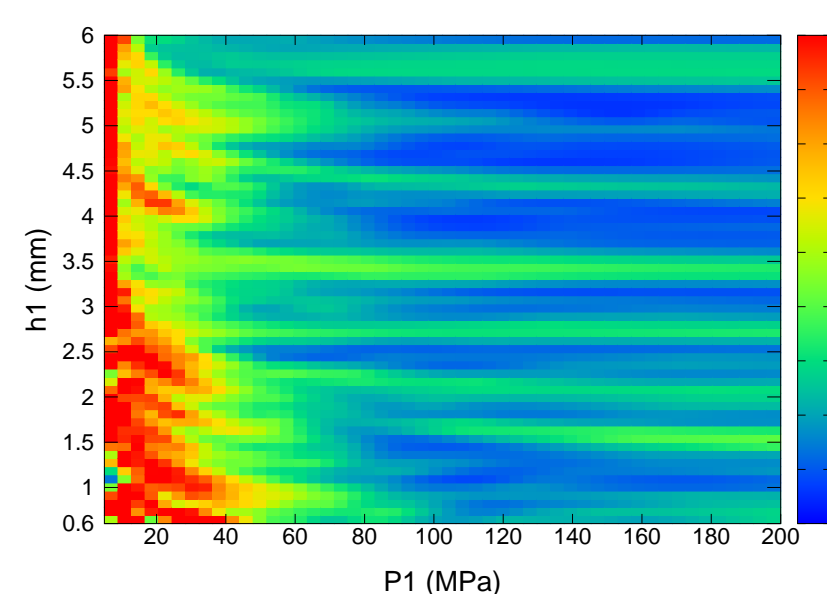


Figure 7. FE enrichment.  $\bar{l} = 60$ .

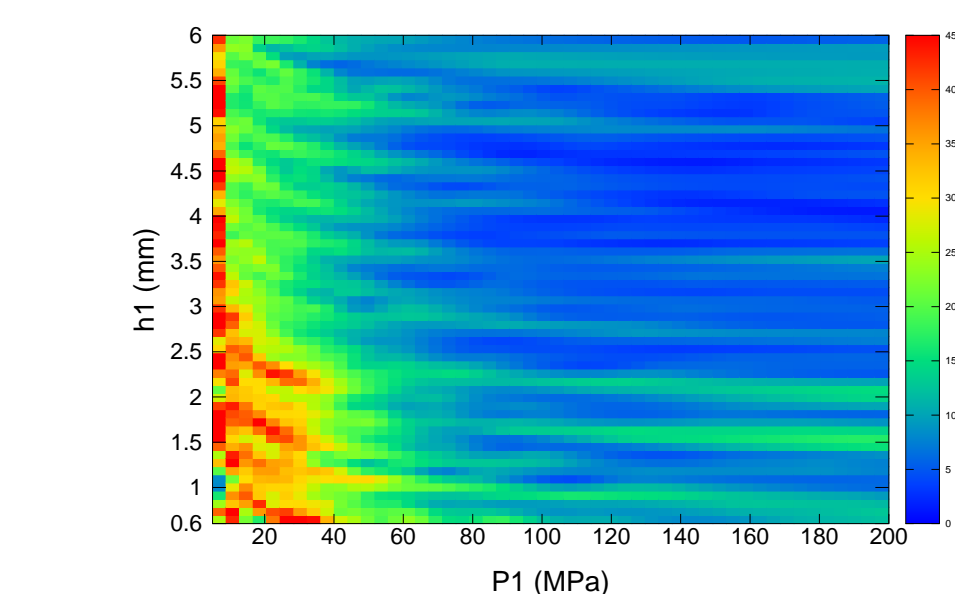


Figure 8. POD+FE enrich.  $l_2 = 5$ ,  $\bar{l} = 82$ .

Dual relative errors maps (in %).  $\kappa_{max} = 10$ .

- $\Rightarrow$  Robust strategies for large parametric spaces
- $\Rightarrow$  Better results with POD+FE enrichment but higher primal RB size
- $\Rightarrow$  Still high relative errors for low contact pressure parametric zones

## Clustering on the parametric space

For large parametric spaces, reaching low errors on the whole parametric space could lead to low data-compression (increasement of the RBs size)  $\Rightarrow$  interest of clustering the parametric space

The nonlinear solution is searched in a piecewise linear manner

Methodology ([2]):

- Create clusters on the parametric space: Grassmann distance, k-medoids algo.
- Build local low-rank ROMs on each cluster
- Train a supervised classifier: Random forest algo.
- Solve the problem with the local ROM selected by the classifier

$\Rightarrow$  increase the solution's (primal and dual) precision without increasing the RB order.

## Clustering of a 2D parameter space and classifier.

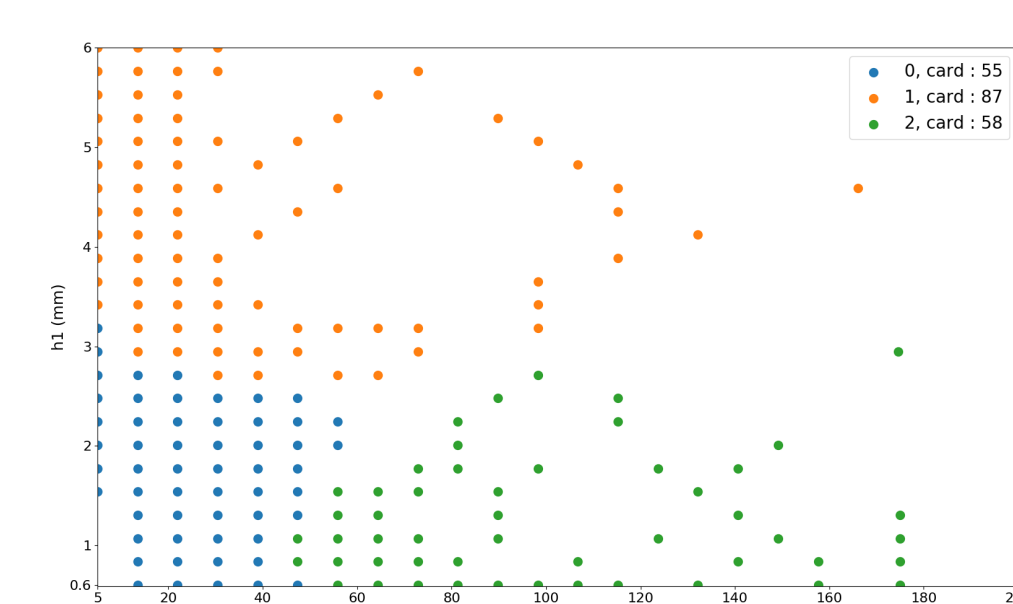


Figure 9. Snapshot clusters

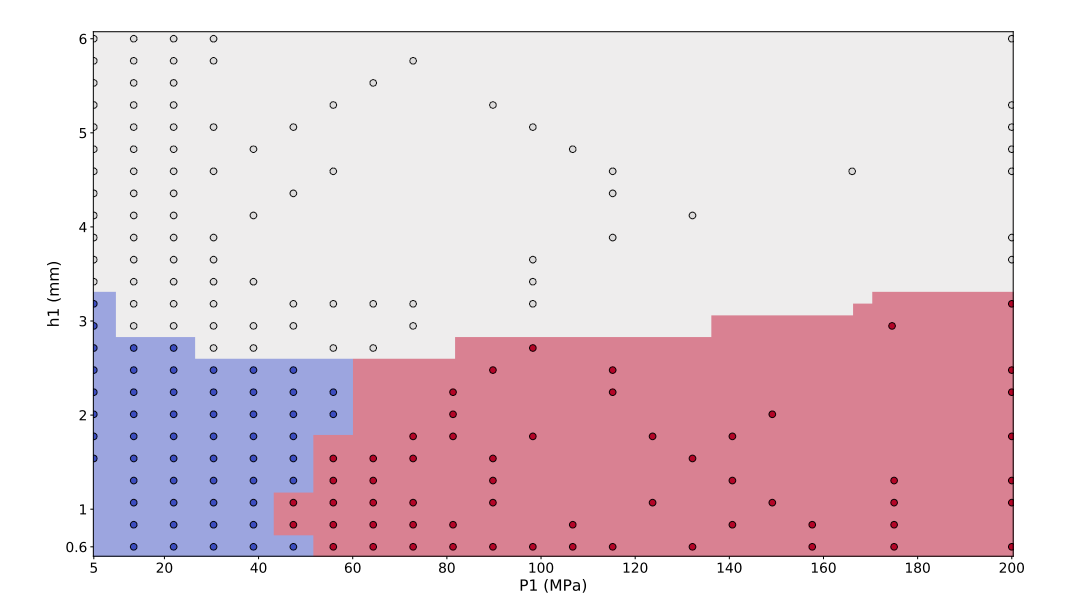


Figure 10. Classifier trained through random forest classification

## Clustering impact on the RB and RID.

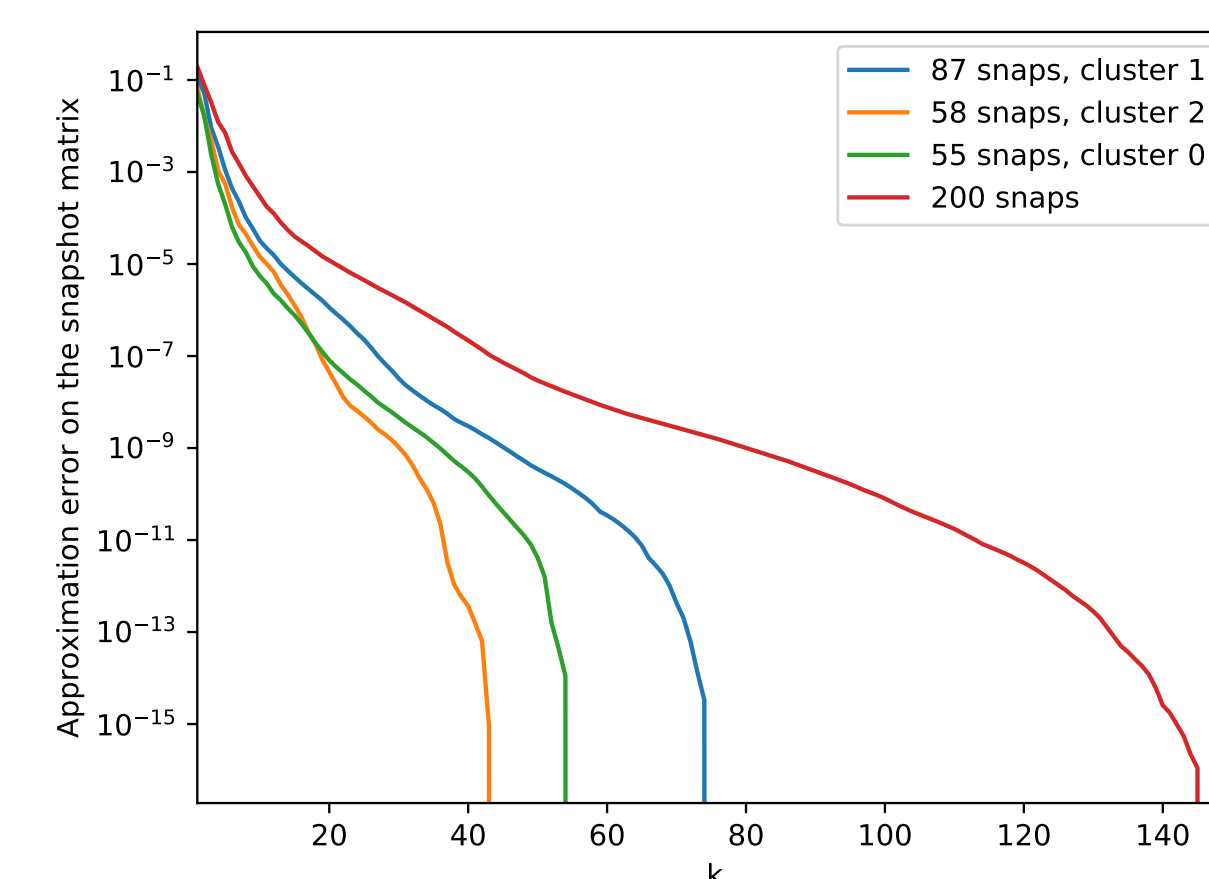


Figure 11. Approximation error on the primal snapshot matrices with and without clustering

$\Rightarrow$  Enhancement of the data compression

$\Rightarrow$  Leads to primal RB of lower ranks

$\Rightarrow$  FE enrichment only to avoid overfitting

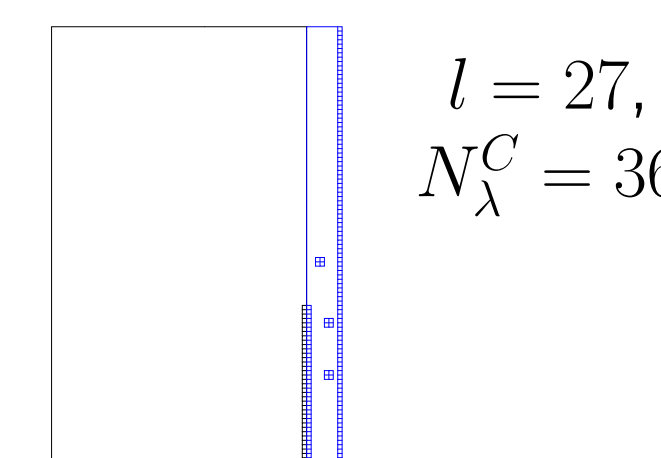


Figure 12. RID cluster 0.

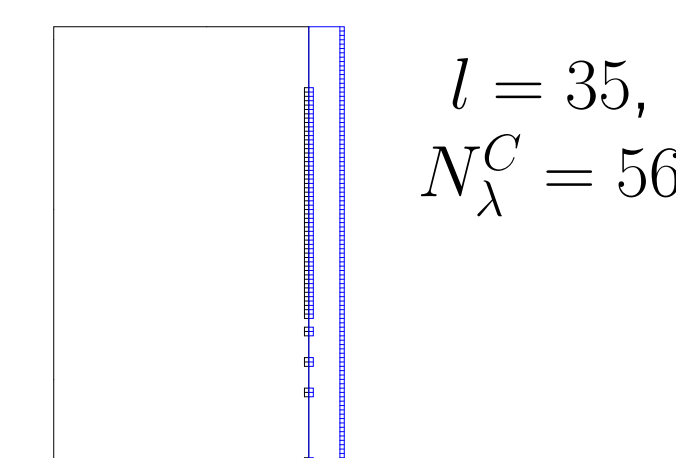


Figure 13. RID cluster 1.

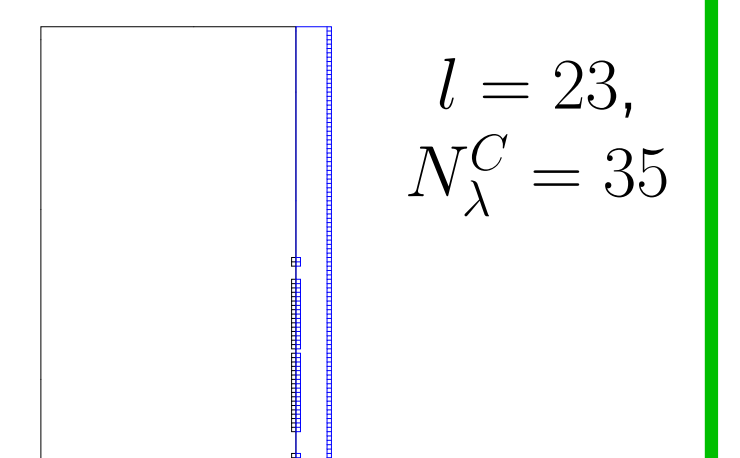


Figure 14. RID cluster 2.

$\Rightarrow$  Piecewise linear treatment visible on the RID.

$\Rightarrow$  Similar sizes of RB

## Results of the clustering on a large parametric space.

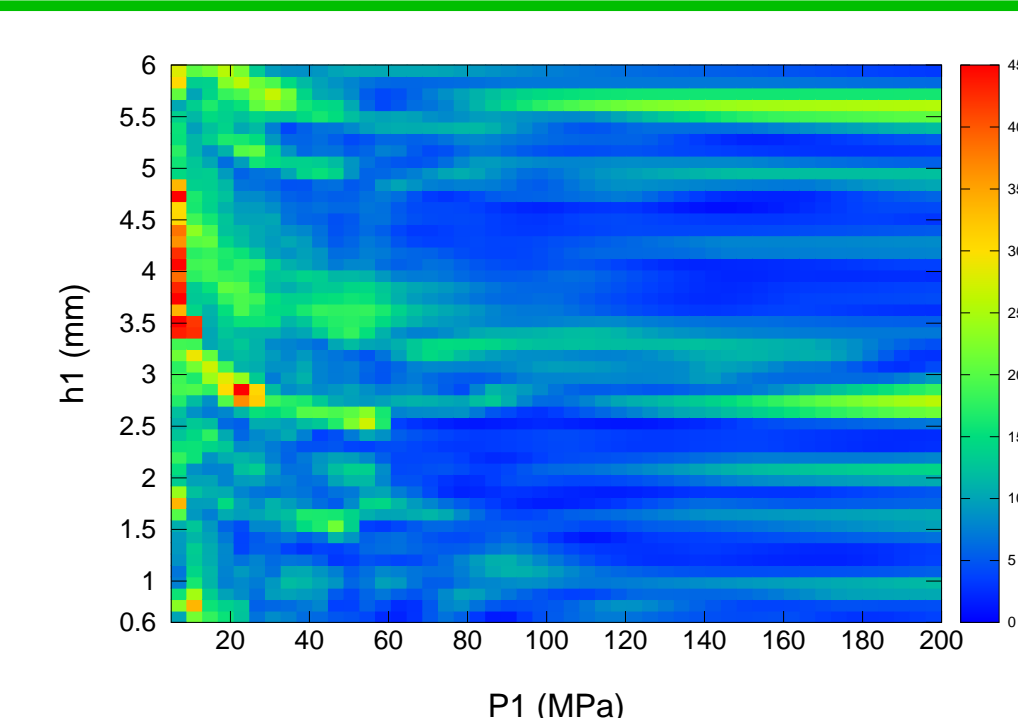


Figure 15. Dual relative errors map (in %) with FE enrichment set with  $\kappa_{max} = 10$ .  $\bar{l} = \{52, 77, 48\}$ .

$\Rightarrow$  Clustering strategy helps smoothing the dual errors

$\Rightarrow$  Errors considerably reduced for local RBs compared to a global RB

$\Rightarrow$  High accuracy on the whole parametric space, even for low contact pressures

## References

- S. Le Berre, I. Ramière, J. Fauque, and D. Ryckelynck. "Condition Number and Clustering-Based Efficiency Improvement of Reduced-Order Solvers for Contact Problems Using Lagrange Multipliers". In: *Mathematics* (2022). DOI: 10.3390/math10091495.
- T. Daniel, F. Casenave, N. Akkari, and D. Ryckelynck. "Model order reduction assisted by deep neural networks (ROM-net)". In: *Advanced Modeling and Simulation in Engineering Sciences* (2020). DOI: 10.1186/s40323-020-00153-6.
- J. Fauque, I. Ramière, and D. Ryckelynck. "Hybrid hyper-reduced modeling for contact mechanics problems". In: *International Journal for Numerical Methods in Engineering* 115.1 (2018). DOI: 10.1002/nme.5798.