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# Error analysis of the Hybrid Hyper-Reduction method for frictionless contact problems.

Simon Le Berre<sup>1,2</sup>, Isabelle Ramière<sup>1</sup>, David Ryckelynck<sup>2</sup>

<sup>1</sup> CEA, DES, IRESNE, DEC, France

E-mail: [simon.leberre@cea.fr](mailto:simon.leberre@cea.fr), [isabelle.ramiere@cea.fr](mailto:isabelle.ramiere@cea.fr)

<sup>2</sup> Centre des matériaux, Mines Paris-PSL, France

E-mail: [david.ryckelynck@minesparis.psl.eu](mailto:david.ryckelynck@minesparis.psl.eu)

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The solution of contact problems is often time consuming because of the non-linearity and non-regularity of the contact constraints. For contact problems treated by Lagrange multipliers (ensuring the Signorini contact conditions to be respected), applying reduced-order modeling remains challenging due in particular to the non-negativity constraint on the Lagrange multipliers [1, 2].

We focus here on the Hybrid Hyper-Reduction (HHR) approach for frictionless elastostatic contact problems proposed in [1, 3]. This approach is based on a reduced integration domain (RID) built through a discrete empirical interpolation method as described in [4]. A reliable reduced dual basis is obtained by restricting the full order dual basis to the RID. The HHR model preserves the saddle-point formulation of the Lagrange multipliers method, and allows the verification of the Signorini conditions inside the RID (without projection). Numerical experiments proved the capacity of the HHR method to provide accurate results both for the displacement and the Lagrange multipliers solutions [1, 3], even for large parametric spaces [3].

In this work, we introduce a priori and a posteriori estimators of the HHR method, providing error bounds to quantify the discrepancy between the HHR and FE (Finite Element) solutions. Thanks to the hybrid formulation of the HHR model, a Cea-like primal bound is obtained on the RID (see Figures 1a and 2a). Indeed, the FE-like dual reduced basis hybrid enables the corresponding bounding term to vanish. Classical Brezzi-like analysis leads to bound the dual approximation error by the primal one (see Figure 1b).

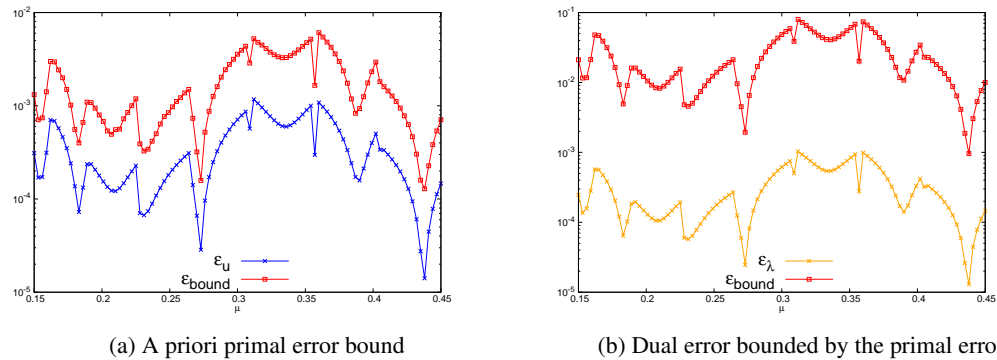


Figure 1: Error bounds on the RID for the Hertz's half disks test case

Hence, the final a priori error estimators for both primal and dual errors, depend on the distance between the primal FE solution and the HHR primal approximation space only. Furthermore, an a priori primal error bound can be derived on the whole computational domain, following Cea's lemma for Petrov-Galerkin formulation (see Figure 2b).

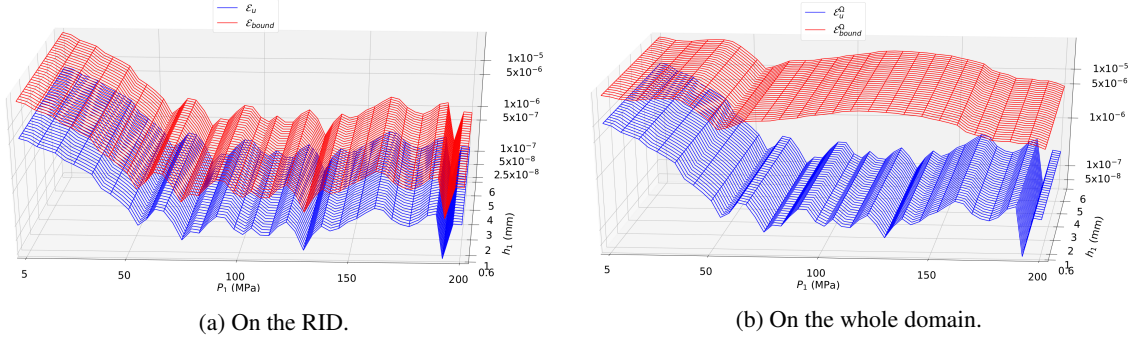


Figure 2: A priori primal error bounds for an industrial test case with 2 parameters.

Primal and dual a posteriori error estimators (depending only on HHR solutions) are also introduced to propose certified HHR error bounds, useful in particular for a efficient greedy snapshot selection. These estimators are based on the HHR equilibrium and constraints equations residuals and will be introduced in the light of those already present in the literature for ROM methods on mixed problems [1, 5, 6].

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