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Review of "Reduced-order modeling of geometrically nonlinear rotating structures using the direct parametrisation of invariant manifolds"

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Authors We thank the reviewers for their positive and constructive comments. Below is given a point-by-point answer to their concerns. The paper has been fully revised according to the raised comments, the changes in the new version of the text are highlighted using red fonts.

Reviewer 1 (Olivier Bauchau)

Reviewer The paper is very well written. Clearly, the paper builds on developments that have appeared in the literature. The authors did an excellent job at reviewing the literature and highligting their own contributions.

In section 2.1, the authors state that "It is assumed that, due to the configuration considered, Coriolis forces can be neglected such that only the centrifugal effect is considered." This restriction should be explained much more clearly: for the examples shonw in the paper, yes gyroscopic effects are negligible. So I guess I want to see a more precise definition of "due to the configuration considered..."

Authors We thank the reviewer for this remark. The presentation has been improved in section 2.1, in order to explain more clearly in which general cases one can safely neglect Coriolis force. Besides, an appendix (now numbered Appendix A in the new version) has been added in order to give a more quantitative assessment. The new sentences in Section 2.1 read:

The rotation brings about two different effects that can be separately analyzed: the centrifugal effect and the Coriolis force (Rao 1991). The expressions of these forces show that the Coriolis effect can be safely neglected in numerous situations, in particular for structures rotating along an axis that is orthogonal to the main vibrations, which is the case considered here. In order to give a more quantitative assessment, Appendix A details the expression of the Coriolis term and highlights its negligible effect in the numerical cases that are considered in the present study. As

a consequence, only the centrifugal force will be considered in the main text.

A quantitative comparison on the eigenfrequencies of the beam under study has been added to Appendix A, showing that in the range of rotation speed considered and for the first nine eigenfrequencies of the structure, the maximum difference with and without Coriolis effect is 0.8%, see Fig. 13.

Reviewer The authors mention several times helicopter and turbine blades as being potential applications of this technology. For these structures, the applied aerodynamic loads play a crucial role and must be included in the model. While I am not asking to see modal reduction techniques of full Navier-Stokes codes, the conclusion section could provide hint of how this could be achieved, if at all.

Authors We thank the reviewer for pointing this out and we agree that for applications to helicopter or turbine blades, aerodynamic loads are of prime importance. We have added at the very end of the conclusion the following paragraph:

For applications to engineering structures like helicopter or turbine blades, the proposed reduction method can thus be directly applied to deal with the structural nonlinearities. However, in such problems, nonlinearities arising from the aerodynamic loads need also to be taken into account. While this is yet out of the scope of the present study, one can see that numerous different options are available to extend the results presented here. A first option would be to consider heuristic models for the aerodynamics effect, that can be appended to the equations of the ROM. The opposite option would be to consider the fully coupled fluid-structure interaction problem and introduce the parametrization method in such framework, which asks for very important developments.

Reviewer 2 (Anonymous)

Reviewer This paper presents reduced-order modeling of geometrically nonlinear rotating structures via direct parameterization of invariant manifolds. In particular, the paper focus on the effects of centrifugal effects, and parametric ROMs are constructed via Chebyshev interpolation. Detailed results and discussion are provided. However, there are some issues that need to be addressed before publication. They are listed below.

Reviewer For rotating structures, Coriolis and centrifugal effects play an essential role. However, only centrifugal effects are taken into account in this study. This simplification needs to be justified further, especially for large rotation speeds. The author claims that the ROMs work 'for any rotation speed' in the abstract. This is confusing because Coriolis effects can be significant for large rotation speeds. The authors are encouraged to provide justification for the simplification, e.g., by comparing the natural frequencies or forced response curves of the structures with and without the consideration of Coriolis effects or providing relevant references.

Authors We thank the reviewer for this important remark. The point has also been raised by reviewer 1. To answer the concerns, an Appendix has been added to the paper (Appendix A). In this appendix, the general formulation of the Coriolis force is given. It is underliend that based on its analytical expression, it is awaited to bring about negligible effect in the cases considered in the present study. To provide a more quantitative assessment, the natural frequencies of the studied beam are compared with and without the Coriolis effect, showing that it is indeed negligible in our case and for the range of rotating speeds covered. Also the sentences at the beginning of Section 2.1 have been rewritten. We think that all these added materials give sufficient data to the readership for a better understanding of this point. Finally, the claim 'for any rotation speed' in the abstract has been simply removed, thus proposing a lighter and more efficient presentation.

Reviewer It looks to the reviewer that the reduction method the authors used is not able to handle Coriolis effects, which makes this study focuses on the centrifugal effects. In contrast, the spectral submanifold (SSM) based reduction method can handle gyroscopic and even follower forces (see Nonlinear Dyn (2022) 110:1005–1043 for instance). The authors are encouraged to comment on this fact in the paper. This can be helpful to readers who want to construct ROMs with both Coriolis and centrifugal effects considered.

Authors As noted by the reviewer in later points, SSMtool and MORFE are based on the same mathematical method, the parametrisation method for invariant manifold. Consequently both softwares are theoretically able to deal with Coriolis effect. We first focus on centrifugal force since it already provides important physical effects to analyze. We are now working on integrating Coriolis force in the MORFE suite. This has been specified in the conclusion where the following sentences have been added:

The next targeted development is to incorporate the Coriolis force in the reduction process and implement it in the software MORFE. No theoretical limitation hinders this development, and results have already been reported in the literature where the parametrisation method is used with Coriolis effect in (Li et al. 2022).

Reviewer This paper constructs 2D ROMs in the whole paper and it has been discussed in some sections that one must take internal resonance into account for some choices of parameters. This again contradicts the claim of 'accurate ROMs for any rotation speed' in the abstract. The authors are encouraged to make edits to the claim to make it rigorous. In addition, for systems with both internal resonance and gyroscopic force, the SSM-based ROMs work well, as demonstrated in Nonlinear Dyn (2022) 110:1005–1043. Please comment on this fact in the paper given this can be informative to some readers.

Authors The claim in the abstract has been simply removed thus offering a more direct presentation. Citation to the paper by Li, Jain and Haller has been added.

Reviewer In the second paragraph of page 2, the authors are encouraged to make a proper citation to Haller et al 2016, which gives the first adaption of Haro's abstract results to the vibration community.

Authors The word "first" has been added to the sentence which makes reference to the paper by Haller 2016, in order to address the reviewer's comment.

Reviewer In the third paragraph of page 2, when the authors mention the direct version of parameterization for invariant manifold, they are encouraged to talk about SSMTool 2.x when they cite MORFE. This is because SSMTool 2.x also supports direct parameterization of invariant manifolds, which was released earlier than MORFE.

Authors The citation has been added, thus also giving the opportunity to underline that MORFE has 3D finite elements built-in, whereas SSMtool proposes beam and plate elements.

Reviewer In section 2.2.2, please consider citing Nonlinear Dynamics (2022) 110:1045-1080 at the end of the first paragraph when quasi-periodic solutions and internal resonances were discussed. The reference presented a detailed study of this case. It would be helpful to readers who would like to know more about this topic.

Authors We did not add the proposed reference since the sentence at the end of the first paragraph of section 2.2.2 is very general and only underlines a well-known fact from nonlinear dynamics. A meaningful reference to be added here should then consist of a general textbook on nonlinear dynamics, not a particular and recent paper. We prefer to keep the text as is.

Reviewer Please fix the typo in equation (18): $\bar{\Phi}$ should be used for the last several entries.

Authors There is no typo in Eq. (18) which is correct. We are working with real modes.

Reviewer At the end of the paragraph following equation (18), please consider citing Nonlinear Dyn (2019) 98:2755–2773 given this paper also computed higher-orders of whiskers.

Authors The citation has been added.

Reviewer In the second last paragraph of Section 2.2, please comment on the paper Jain and Haller 2022, where direct solutions are also computed.

Authors We thank the reviewer for pointing this out. Indeed the citation to the paper by Jain and Haller was needed here. The last sentences of the paragraph have been revised, allowing us to be more accurate in the description of the techniques used by the two groups. It now reads:

The second main problem in deriving arbitrary order solutions of the homological equations,

is to compute direct solutions, in order to bypass the expression in the modal coordinates. In (Vizzaccaro, Opreni, et al. 2022; Opreni, Vizzaccaro, Touzé, et al. 2022), this is achieved by using a bordering technique to solve for the homological equations. On the other hand, a norm-minimizing solution is adopted in (Jain and Haller 2022) to cope with the same problem.

Reviewer In equation (24), the En(z) could be normalized by the number of points. It seems this error metric depends on the number of sampling points.

Authors The fact that the error here does not depend on the number of points is not problematic since we are comparing series having the same number of points. Hence adding such a normalization to the metric would not change the result. We have thus preferred to keep the text as is.

Reviewer In section 3.2.4, how the linear term with the forcing is interpolated with respect to the rotation speed is not clear to the reviewer. It is true that this linear term is proportional to the amplitude factor α, however, it is a nonlinear function of the rotation speed, as indicated by equation (13), where the eigenvector Φ_j depends on the rotation speed Ω in a nonlinear fashion. It would be helpful if more details are given. The same comments hold for the first paragraph of section 4.4.

Authors The reviewer is right to point out that the mode shape depends on the rotating speed, such that the dependence highlighted through Eq. (13) might have some consequences. However, as shown in Fig. 2(c-d), this dependence is very mild for the beam structure. Besides, at the points where the ROM is computed to perform the extrapolation, this dependence is taken into account. The main consequence is that the possible nonlinear dependence underlined by the reviewer is so weak that it is completely unobservable in the presented results. Consequently it has been found much more direct to keep the presentation as is, instead of adding putative effects and dependence that might obscured the presentation of the results.

Reviewer Figure 10 is very interesting but also a little bit confusing. It seems that the 2D ROM is able to predict correctly most parts of the forced response curves, except the bifurcated branches. Can these bifurcated branches be predicted by higher-dimensional ROMs? The authors are encouraged to make comments on this. If possible, some numerical results of higher-dimensional ROMs can be provided.

Authors The comments on the bifurcated branches for the second mode of the beam, Fig. 8, and the twisted plate, Fig. 10, have been revised in order to give more precisions to the reader. It is more clearly underlined that the bifurcations found by the full-order model are difficult to analyze since they do not display any simple relationships between the modal coordinates. A systematic study of low-order resonance have been made and ROMs with up to four interacting NNMs have been tested. However, all of them fail to found back those very sharp bifurcated branch. We think that they thus involve a high-order internal resonance. Besides, we also think that their physical meaning is mild since occurring on very small range of parameters.

Reviewer 3 (Anonymous)

Reviewer The manuscript under review concerns the application of the so-called parameterization method of invariant manifolds for reduced order modeling in huge size systems of odes coming from finite-element discretizations of evolutionary pds describing the motion (vibration) of geometrically nonlinear structures. These are rotating mechanical systems, like a rotating cantilever beam and a twisted plate.

The authors explain the basics of the method, and the (not at all straightforward) application to the problems at hand. The paper is pedagogical and present techniques that, even though are well-know in the dynamical systems community, are relatively new in the Applied Mechanics community. The problems are first discretized using finite-elements and then the huge systems of ordinary differential equations are studied (and reduced) in the light of the parameterization method. The team has proposed recently a public open code (MORFE), a Julia package for the directed parameterization method, that is used in the present study. As such, this paper complements several relatively recent papers by some of the authors of the manuscript, and also by Haller and his collaborators.

The paper opens the range of applicability of the parameterization method to complex

problems in Applied Mechanics. Going from theory to practice is not a trivial step. The examples are meticulously detailed, and the gain in computational efficiency with respect to the previous methods is simply extraordinary (in time, from one day to one minute, let's say).

I have some questions and comments that I would like the authors to consider. Some questions are asked to make the paper more selfcontained.

Reviewer In (1), I guess matrices M, C, K, N are symmetric, and M is definite positive. It is not explicitly mentioned.

Authors Indeed, the reviewer is right. We have added this information to the text.

Reviewer The fact that the nonlinearities are consider up to third order, is fundamental or technical? I think that mathematically is irrelevant, but in practice this is related with the computational cost. Please, clarify a bit.

Authors Having only quadratic and cubic polynomial dependence for geometric nonlinearity directly stems from the modelling assuming linear elasticity and large amplitude displacements. This is embedded in the equations of motion and is true as long as three-dimensional elastic continua are considered. Since these relationships are embedded in 3D finite elements for solids, the property still holds for the semi-discrete problem. Note that this is not true anymore for beams, plate or shell nonlinear models where assumptions need to be made in the kinematics. A sentence pointing to dedicated references has been added to the text.

Reviewer The maps g and h, are gradient of other functions?

Authors Indeed, in the mechanical context of geometric nonlinearity, the considered forces are conservative, in the sense that their work do not depend on the trajectory, but only on the starting and ending point. Consequently all forces derive from a potential, which means that the quadratic and cubic nonlinearities can be written as gradients of other functions. This point is not recalled nor expanded in this paper for the sake of conciseness.

Reviewer There is not a formal definition of what is a frequency-response curve, although one can deduce it from the examples. The authors could introduce the concept.

Authors Frequency-response curves or FRCs are routinely used in a mechanical context, such that we forgot to recall the definition. A short sentence has been added in Section 2.2.1 when the acronym FRC is introduced.

Reviewer When considering (small) periodic forcing (14a,14b), it is not clear to me how is computed the substitutive of the equilibrium (the periodic solution that appears when perturbing). It is a continuation method combined with Newton? Or a perturbation method (expansion in powers of a certain perturbation parameter)?

Authors The periodic solutions are computed thanks to the numerical continuation method. As a matter of fact, the computed periodic orbits are solutions lying in the computed time-dependent invariant manifold and are thus some special solutions that we continue as branches with the numerical path-following method.

Reviewer For the periodic solution, the linear dynamics can be reduced to a constant linear system with Floquet transformations. Or, presumably, one could only reduce to constant coefficients the linear part corresponding to the whisker. I do not see any mention to that. Is this done?

Authors Indeed, Floquet transformation can be used in such context, and has been used in the applied mechanics community to deal with time-dependent problems, see e.g. the following references where this is achieved:

Sinha, S., Pandiyan, R.: Analysis of quasilinear dynamical systems with periodic coefficients via Liapunov–Floquet transformation. Int. J. Non-Linear Mech. 29(5), 687–702 (1994)

Sinha, S.C., Redkar, S., Butcher, E.A.: Order reduction of nonlinear systems with time periodic coefficients using invariant manifolds. J. Sound Vib. 284(3), 985–1002 (2005)

Wooden, S.M., Sinha, S.: Analysis of periodic-quasiperiodic nonlinear systems via Lyapunov–Floquet transformation and normal forms. Nonlinear Dyn. 47, 687–702 (2007)

This technique is however not used in the present paper, since it appears to be meaningful and computationally efficient for systems with time-varying periodic linear parts, which is not the case considered here.

Reviewer Since meshing is so important, which is the finite-element software for obtaining the odes in the formulation (1)?

Authors We are using classical three-dimensional finite elements for elastic bodies, which are directly implemented as built-in capacities in the code MORFE.

Reviewer How far are the examples of getting at the end that the damping matrix C is a constant multiple β of the identity?

Note: In some circles, this case is referred to be as a conformally symplectic. The flow does not preserve the symplectic form because of the damping effect, but is is multiplied by $exp(-\beta t)$ at a time t. It would be interesting to explore the consequences of this property in the properties of the computed geometrical objects, and if these properties could lead to improvements of the computational methods. In particular, It is known that the reduced manifolds are isotropic (the symplectic form vanishes on them).

Authors We thank the reviewer for these remarks and the clarifications. In a mechanical context, accurate and problem-specific description of the damping forces is a very difficult task in general, such that in the vast majority of studies where damping is not the key factor, it is considered under the form of a Rayleigh-type damping matrix, with a term proportional to the mass matrix and another one proportional to the stiffness matrix. Since the losses are not key in the present study, we restrict ourselve to the simple case given by Eq. (10), i.e. a damping matrix proportional to the mass.

Editor's assessment (Alexander Popp)

The given contribution applies and extends the direct parametrization method for invariant manifolds to reduced order modeling of geometrically nonlinear structures, in particular rotating mechanical systems. Three independent reviews by expert referees have been written for the first version of the submitted manuscript, two of which were immediately very positive and only provided minor comments for improvement, e.g. a request to the authors to add comments on Coriolis force effects and aerodynamic loads. The third review in the first round has asked the authors more closely and critically about their work and consequently demanded a major revision. Important remarks of this referee addressed (again) the influence of Coriolis force effects and also higher-dimensional reduced order models. Within a revised version of the manuscript, the referees' concerns were adequately resolved and all three referees unanimously recommended publication.