



**HAL**  
open science

## Multiscale SPAD modeling: from Monte Carlo to SPICE simulations

Denis Rideau, Rémi Hellebois, Gabriel Mugny, Raul Andres Bianchi, Jean-Robert Manouvrier, Isobel Nicholson, Jeremy Grebot, Elsa Lacombe, Alejandro Lopez, Dominique Golanski, et al.

► **To cite this version:**

Denis Rideau, Rémi Hellebois, Gabriel Mugny, Raul Andres Bianchi, Jean-Robert Manouvrier, et al.. Multiscale SPAD modeling: from Monte Carlo to SPICE simulations. SPIE Defense and Commercial Sensing 2023, 30 April - 4 May 2023 Orlando, Florida, United States, May 2023, Orlando, United States. 10.1117/12.2665099 . hal-04087158

**HAL Id: hal-04087158**

**<https://hal.science/hal-04087158v1>**

Submitted on 2 May 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Multiscale SPAD modeling: from Monte Carlo to SPICE simulations

D. Rideau, R. Helleboid, G. Mugny, R. A. Bianchi, J.R. Manouvrier, I. Nicholson\*, J. Grebot, E. Lacombe, A. Lopez\*, D. Golanski, B. Mamdy, B. Rae\*, S. Pellegrini\*, M. Agnew\*, W. Uhring\*\*, S. Rink\*\* and H. Wehbe Aлаuse\*.  
STMicrоelectronics Crolles, France, \*STMicrоelectronics Edinburgh, UK  
\*\*ICube laboratory, Université de Strasbourg, 1 Pl. de l'Hôpital, 67000 Strasbourg, France.

## Abstract

We present a study of the main SPAD figures of merit using a multiscale approach, from Monte Carlo simulations to SPICE simulations. We explore novel stochastic approaches capable of predicting accurately experimental measurements such as the Breakdown Probability, and the jitter. Additionally, the SPAD avalanche dynamics that is a stochastic process, is discussed within a transient Monte Carlo simulation perspective. We also derived a VerilogA model, making possible the analysis of the stochastic responses of the SPAD, including the buildup of the avalanche but also its quench. This latter quench probability of these diodes once in avalanche, rarely discussed in literature, is related to the dynamics of the voltage change of the floating cathode node. If the cathode voltage recovery (after the debiasing due to the quench circuit) is quicker than the time needed for the carrier evacuation within the avalanche junction, small additional avalanches can occur.

**Keywords:** SPAD avalanche dynamics, Characterization of stochastic responses, Statistical modelling of SPAD transients.

## 1. INTRODUCTION

SPADs, or Single-Photon Avalanche Diodes, play a crucial role in optoelectronics, particularly in medical imaging, camera ranging, and automotive laser imaging detection and ranging (LiDAR) applications. Industrial optimization of SPAD Figures Of Merit (FOMs) is essential to achieve optimal Breakdown Probability (BrP), Photon Detection Efficiency (PDE) [1], and timing statistic response to avalanche (jitter) without compromising SPAD quench probability [3] and power consumption associated with Charge Per Pulse (CPP). However, most of these FOMs are stochastic and strongly influenced by the surrounding quenching circuit. As a result, standard deterministic commercial Technology-Computer Assisted Design (TCAD) tools have limitations in providing outputs for technology optimization. To address this, novel co-optimized strategies based on rigorous multi-scale approaches and interfacing different numerical methods and solvers are urgently needed.

Using advanced Monte Carlo solvers, we present predictions of the BrP and Jitter[1,3]. A more complete modeling view is obtained within a mixed mode approach, and we address the impact of the quench circuit on the avalanche time-dependent dynamics. This novel approach makes possible the prediction of the stochastic time taken by the carrier to reach the avalanche zone by drift-diffusion as well as the time required to establish the avalanche (that is related to the timing jitter [1,3]), but can also predict the avalanche quench probability that depends on the quench circuit itself. We show that when the dynamics of the moving node is faster than the carrier evacuation time within the avalanche junction, secondary avalanches can occur giving rise to longer recovery time, additional power consumption and eventually paralysis of the SPAD (sustained avalanche)

Calculations consist of simulations of random carrier trajectories within the device followed by carrier multiplications by impact ionization in the high field region and the quench of the avalanche by the external circuit. Even though such a mixed mode Monte Carlo approach is usable for a direct comparison with measurements, systematic studies with complete quench circuits involving hundreds of lumped elements would lead to a drastic increase of the CPU simulation time.

To address this latter issue, we used the rigorous mixed mode Monte Carlo approach, to derive a CPU efficient VerilogA compact model that can be used to simulate larger quench circuits (including notably the layout parasitic supplementary capacitances). Simulating a large number of avalanches and quenches, the predictions of the distributions of pulse width and inter pulse time are possible. We show that the predictions obtained with this physics-based SPICE model are in line with extensive series of transient measurements.

## 2. MULTISCALE SIMULATION APPROACH

### 2.1 Full Band Monte Carlo

Full band Monte Carlo solvers are well suited for breakdown probability and jitter calculations [1,2,3,4]. Calculations consist of a relatively large number of simulations of random carrier trajectories within the device followed by carrier multiplications by Impact Ionization (II) in the high field region. Figure 1-a shows the carrier trajectories (starting from ~1000 different positions within the device) as well as the carrier energy in a diode biased 5V above breakdown voltage (BV). This highlights the main avalanche region (exhibiting a larger energy due to the carrier acceleration by the electric field), but also the lateral electrostatic guard (circled region) introduced to prevent lateral avalanches near the surface of the device ( $y=0$ ). A breakdown criterion is assessed if a given threshold (typically several hundreds of II per initial particle) is achieved (Figure 1-b). More results of the PDE will be presented at the conference, coupling Monte Carlo BrP predictions to numerical optical absorption calculations.

The distribution of the time taken by the carrier to reach the avalanche zone by drift-diffusion as well as the time required to establish the avalanche is related to the timing jitter [1,3]. This latter build-up (that also depends on the quench circuit itself) typically occurs within less than 100 ps. It contrasts with the carrier drift and diffusion which can be much longer, especially for depleted SPADs where it can exceed 1 ns. In Figure 2, the jitter tail is obtained from the Monte Carlo drift-diffusion timing distribution  $P2(t)$ , while the build-up distribution  $P1(t)$  (that can also include additional parasitic effects, such as the laser photon arrival statistics) has been fitted on measurements. The jitter distribution is obtained using a convolution:  $P(t) = \int dt' P1(t-t')P2(t')$ .

### 2.2 Empirical Monte Carlo within a mixed mode approach

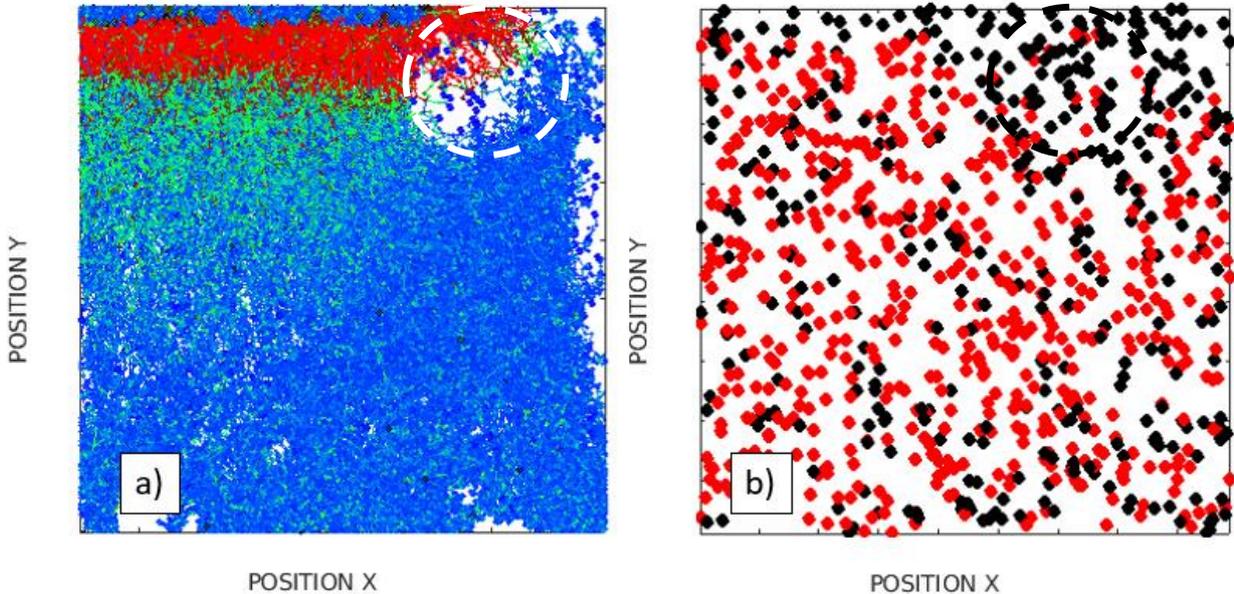


Figure 1: a) Carrier trajectories (red segments correspond to energies larger than 1eV). b) starting carrier positions leading (red) or not (black) to avalanches. Circled region highlights a lower field region optimized to reduce lateral breakdown. The right-half of the device is shown,  $T=333K$ , diode biased 5V above breakdown voltage; all dimensions given in arbitrary units.

In this study, we used an empirical version of the Monte Carlo solver, in which the carrier motion is randomly assessed according to the drift-diffusion distribution probability [5], and the impact ionization probability computed using ionization integrals along the carrier trajectory. This empirical solver has been preliminarily calibrated to accurately reproduce the predictions of the Full Band one. One notes incidentally that this approach accounts for random carrier diffusion and is

therefore more accurate than the previously used field-lines-based McIntyre model [6] as exemplified in Figure 3. In contrast to the McIntyre results, the empirical Monte Carlo predicts non-vanishing BrP in the lateral lower field region, as it is the case with the Full Band Monte Carlo predictions (see Figure 1-b). The averaged BrP (over the X-and-Y-positions) are shown in Figure 4. Although some differences are visible in Figure 3, one notices that averaged BrP predictions are relatively similar for both models even when only the central part of the device (extending up to the lateral region) is considered.

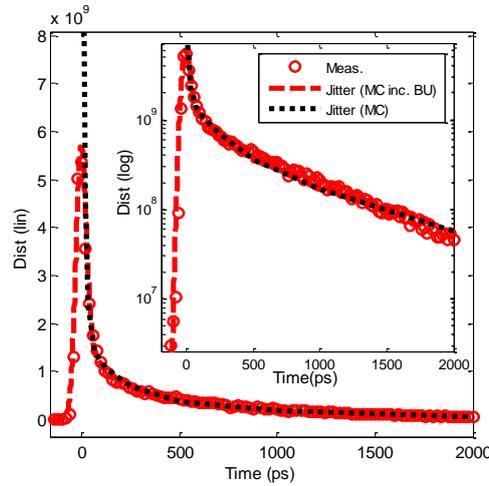


Figure 2: Jitter measurements compared to Monte Carlo predictions. Same device as in Figure 1.

The dynamics of the quench after the avalanche has been recently studied using an Effective-Mass-Approximation-based Monte Carlo solver coupled to a Poisson equation solver [3,4] on smaller diodes. Performing the same approach in larger and more realistic SPAD diodes (the diameter of which can be several tens of microns), and considering few hundreds of such mixed-mode Monte Carlo simulations would lead to a drastic increase of the CPU simulation time. For that purpose, the empirical Monte Carlo offers an interesting alternative as demonstrated in Figure 5. One can see some slight fluctuations of the number of carriers in the junction and in the FOM of the diode voltage transients. These fluctuations, related to the stochastic number of impact ionizations and carriers in the junction, not only impact the jitter itself (see previous), but can also have a dramatic impact on the SPAD power consumption when secondary small avalanches occur during the SPAD recharge phase [3,4,7]. A careful co-optimization of the SPAD diode, but also of the quench circuit itself (simplified in Figure 5-a) is key to minimize such undesired stochastic effects.

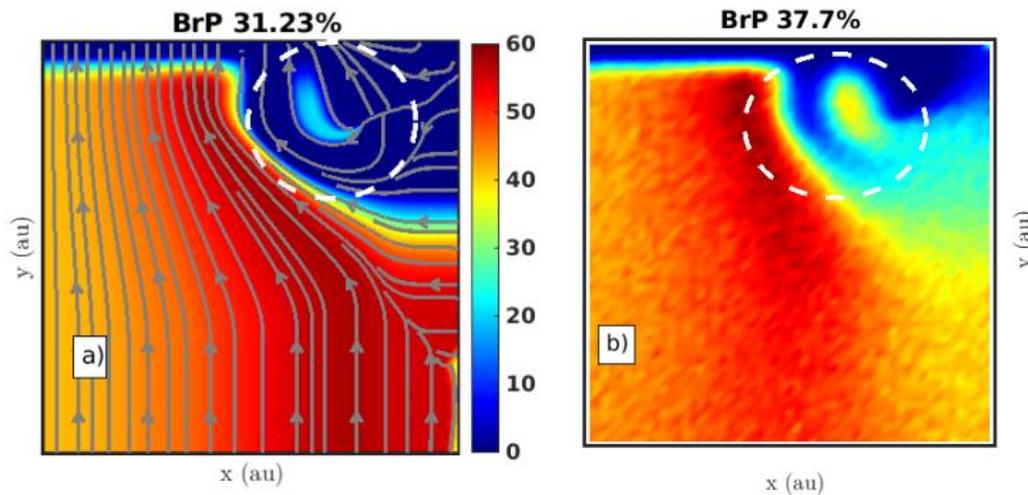


Figure 3: McIntyre a) and Empirical Monte Carlo b) breakdown probability calculated for the same device as Figure 1 (engineered to have a defective guard ring);  $\sim 5e6$  Empirical simulations are used to generate the map shown in b).

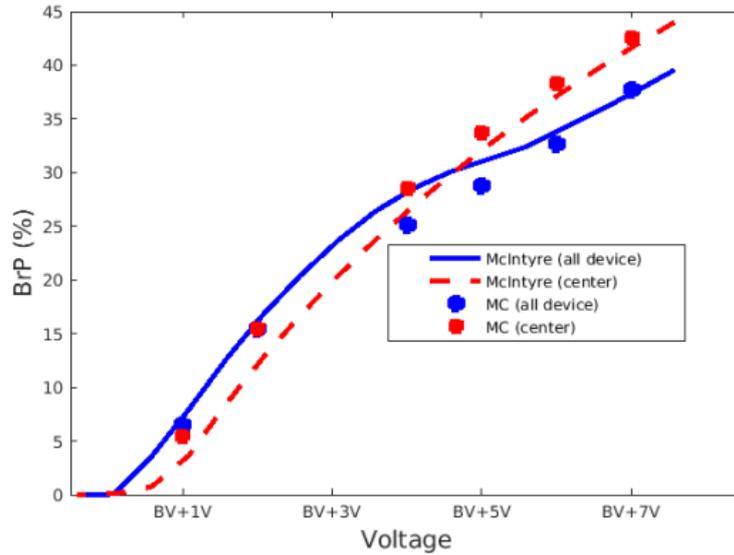


Figure 4: McIntyre a) and Empirical Monte Carlo b) breakdown probability averaged over the device or its central region only (see Figure 3).

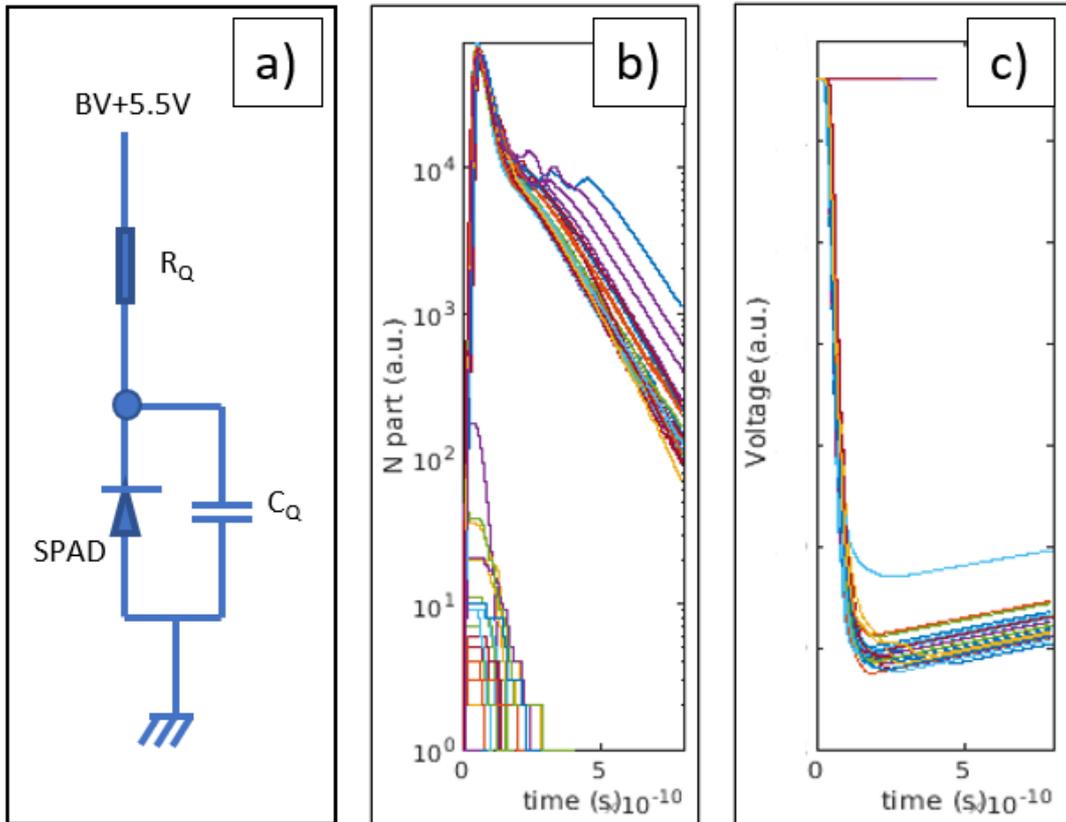


Figure 5: Simplified quench circuit (a). Empirical Mixed-mode Monte Carlo SPAD predictions for the number of carriers in the junction (b), and the diode voltage as a function of time (c). ( $C_Q$  is typically a few tens of fF and is an equivalent quench capacitance.  $R_Q$  is a typical quench resistor, the value of which is several hundred of  $k\Omega$ ). The SPAD is operating at  $BV+5.5V$ . Many mixed-mode Monte Carlo simulations are performed with the initial electron generated at the same position.

### 2.3 SPICE Simulations

To further describe the statistical behaviors observed in the SPAD, we utilized the VerilogA model from reference [7]. For convenience, we provide the equations for  $I_e$  and  $I_h$  currents below:

$$\begin{aligned} \frac{dI_e}{dt} &= [M_e \cdot I_e + M_h \cdot I_h - I_e] / \tau_e, \text{ for electrons,} \\ \frac{dI_h}{dt} &= [M_e \cdot I_e + M_h \cdot I_h - I_h] / \tau_h, \text{ for holes.} \end{aligned} \quad (\text{Eq. 1})$$

Here  $\tau_{e,h}$  are the transit escape times, while  $M_{e,h}$  are the multiplication factors for electrons and holes, respectively.  $M_{e,h} = \alpha_{e,h} \cdot w_{e,h}$  are dimensionless coefficients where  $w_{e,h}$  are the effective widths of the multiplication region [7]. The coefficients also depend on the maximum electric field  $F$  in the junction, which is defined by  $\alpha_{e,h} = a_{e,h} \cdot \exp(-b_{e,h}/F)$ . The constants  $a_{e,h}$  and  $b_{e,h}$  are determined by the Van Overstraeten-de-Man model [8]. Whenever an electron-hole pair is produced in the SPAD collection region, the electron or hole can drift towards the avalanche region and generate a current  $I_0 \triangleq q \cdot v_s / w$ , where  $q$  is the elementary charge,  $v_s$  is the carriers' saturation velocity, and  $w$  is an effective width of the depleted region. This may initiate an avalanche and lead to the sudden increase of the total current  $I_T(t) = I_e(t) + I_h(t)$ .

The *rhs.* of Eq. 1 is a balance equation between additional carriers  $\Delta_{e,h} = M_{e,h} \cdot I_{e,h} / I_0$  introduced due to impact ionization and carriers escaping the depleted region ( $-I_e / I_0$ ) during a time step of  $\tau_{e,h}$ . The intrinsic stochastic fluctuations of carriers in the junction are accounted for using a Poisson distribution applied to  $\Delta_{e,h}$ .

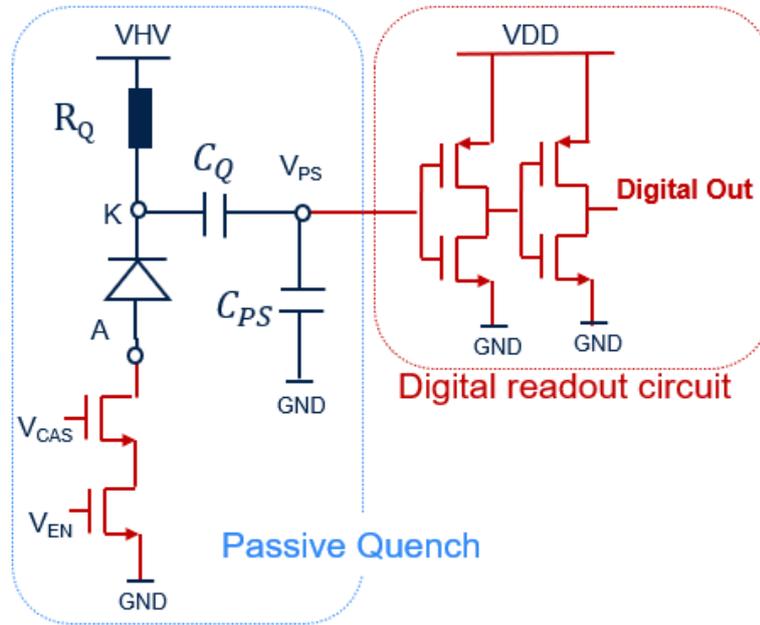


Figure 6: Realistic passive quench circuit, including transistor and the digital output stage.

A more realistic circuit includes transistors and layout parasitic capacitances as shown in Figure 6. Depending on the quench capacitance  $C_Q$  and resistance  $R_Q$  used ( $C_{PS}$  being an inevitable small parasitic layout capacitance), a large variety of situations can occur as shown in Figure 7: from a perfectly quenching diode to nearly paralyzed diodes exhibiting high power consumption (due to additional small avalanches).

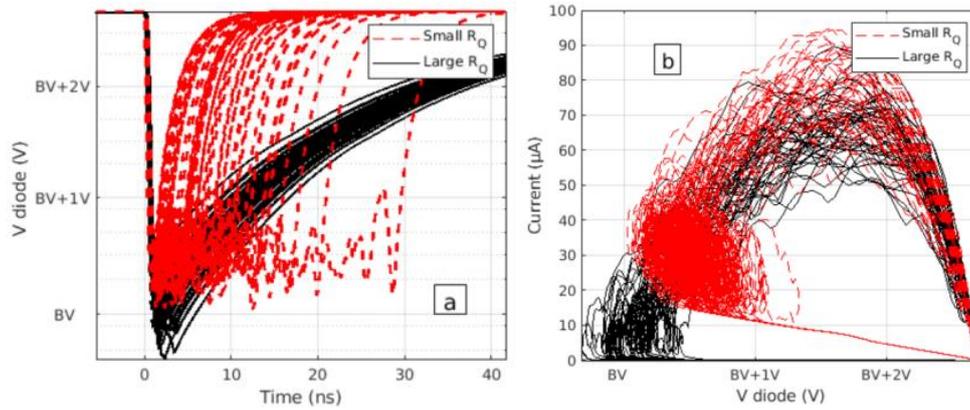


Figure 7: VerilogA model predictions for the statistical SPAD Diode voltage a) as a function of time and b) as a function of diode current, for a large  $R_Q=R_0$  and a small  $R_Q=R_0/5.7$  values and identical  $C_Q$  and  $C_{PS}$  values for both quench circuits ( $R_0$  is a typical quench resistor, the value of which is several hundred of  $k\Omega$ ). The SPAD is operating at  $BV+2.7V$ .

### 3. CONCLUSION

The introduction of new modeling techniques for predictions of the transient responses of SPADs has provided a better understanding of their stochastic transient behavior. The impact of the quench circuit on the SPAD performance is important and can be modelled using a Monte-Carlo mixed-mode approach. Our findings demonstrate that if the change of the floating cathode node is faster than the time needed for the carrier evacuation within the avalanche junction, there is a possibility of secondary avalanches, which can lead to a longer recovery time, additional power consumption, and ultimately, the paralysis of the SPAD. Consequently, a more CPU efficient VerilogA model has been developed (based on the Mixed-mode Monte Carlo approach) which accurately account for the whole quench circuit.

This statistical modeling approach enables the optimization of SPAD pixel performance, particularly for low-power and high-count rate applications. Such a multi-scale modeling approach has the potential to shorten the development time and enhance the performances of future SPAD diodes for various fields, such as time-resolved 3D imaging, lidar systems, and quantum communication.

### REFERENCES

- [1] A. Spinelli and A. L. Lacaita, "Physics and numerical simulation of single photon avalanche diodes," IEEE Trans. Electron Devices, vol. 44, no. 11, 1997, pp. 1931–1943.
- [2] D. Dolgos, H. Meier, A. Schenk and B. Witzigmann, "Fullband Monte Carlo simulation of single photon avalanche diodes," 2013 IEEE Photonics Conference, Bellevue, WA, 2013, pp. 360-361.
- [3] D. Rideau et al., "Single Photon Avalanche Diode with Monte Carlo Simulations: PDE, Jitter and Quench Probability," 2021 International Conference on Simulation of Semiconductor Processes and Devices (SISPAD), 2021, pp. 293-296.
- [4] T. Cazimajou, M. Pala, J. Saint-Martin, R. Helleboid, J. Grebot, D. Rideau, P. Dollfus, Quenching Statistics of Silicon Single Photon Avalanche Diodes, IEEE Journal of the Electron Devices Society 9, 2021, pp 1098-1102.
- [5] C. Jacoboni, P. Lugli, "Monte Carlo Method for Semiconductor Device Simulation", Springer Vienna, ISBN-10: 3211821104, 1989.
- [6] R. Helleboid et al., "Comprehensive modeling and characterization of Photon Detection Efficiency and Jitter in advanced SPAD devices," ESSDERC 2021 - IEEE 51st European Solid-State Device Research Conference (ESSDERC), 2021, pp. 271-274.
- [7] Y. oussaiti, et al. "Verilog-A model for avalanche dynamics and quenching in Single-Photon Avalanche Diodes", 2020 International Conference on Simulation of Semiconductor Processes and Devices (SISPAD), 2020.
- [8] R. van Overstraeten and H. de Man, "Measurement of the Ionization Rates in Diffused Silicon p-n Junctions," Solid-State Electronics, vol. 13, no. 1, pp. 583–608, 1970.