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Abstract: In this paper, methods of kinematic synthesis and analysis of the RoboMech class parallel manipulator (PM) with two grippers (end effectors) are presented. This PM is formed by connecting two output objects (grippers) with a base using two passive and one negative closing kinematic chains (CKCs). A PM with two end effectors can be used for reloading operations of stamped products between two adjacent main technologies in a cold stamping line. Passive CKCs represent two serial manipulators with two degrees of freedom, and negative CKC is a three-joined link with three negative degrees of freedom. A negative CKC imposes three geometric constraints on the movements of the two output objects. Geometric parameters of the negative CKC are determined on the basis of the problems of the Chebyshev and least-square approximations. Problems of positions and analogues of velocities and accelerations of the PM with two end effectors have been solved.

Keywords: parallel manipulator; RoboMech; kinematic synthesis and analysis; Chebyshev and least-square approximations

Citation: Baigunchekov, Z.; Laribi, M.A.; Mustafa, A.; Kassinov, A. Kinematic Synthesis and Analysis of the RoboMech Class Parallel Manipulator with Two Grippers. *Robotics* **2021**, *10*, 99. https://doi.org/10.3390/robotics10030099

Academic Editor: Raffaele Di Gregorio

Received: 12 June 2021
Accepted: 30 July 2021
Published: 3 August 2021

1. Introduction

There are technological processes in industry where it is necessary to perform several operations simultaneously or sequentially, for example, in stamping production, in loading and unloading operations. For the simultaneous or sequential execution of several operations, it is advisable to use manipulation robots with many end effectors.

In this paper, a PM with two end effectors is synthesized that can be used to perform reloading operations from one technological equipment to another. This PM with two end effectors replaces two industrial serial robots in the existing production line of cold stamping and it belongs to the RoboMech class PM. The PM, simultaneously setting the laws of motions of the end effectors and actuators, is called the RoboMech class PM [1]. Setting the laws of motion of the actuators monotonously and uniformly but not defining by solving the inverse kinematics problem simplifies the control system and improves dynamics. Replacing two industrial robots with one RoboMech class PM with two end effectors simplifies the control system and increases the productivity and reliability of the technological line.

Since in the RoboMech class PMs simultaneously set the laws of motion of the end effectors and actuators, they work with certain structural schemes and geometric parameters of their links. The existing methods of kinematic analysis and synthesis of mechanisms and manipulators are based on the derivation of loop-closure equations and their study: in kinematic analysis, using known constant geometric parameters of links and variable generalized coordinates, variable parameters characterizing the relative movements of elements of kinematic pairs are determined, and in kinematic synthesis (dimensional or

parametric synthesis) for the given positions of the input and output links, constant geometric parameters of the links are determined. Loop-closure equations are derived on the base of vector and matrix methods [2–10], and the theory of screws [11–13], which are leads to polynomials of higher degrees. Then, examining the resulting polynomials using computers, depending on the assigned tasks, the kinematic analysis or synthesis is performed. McCarthy in his papers [14,15] shows the close relationship between the kinematics, synthesis, polynomials, and computations in the 21st century. In the considered approach of kinematic analysis and synthesis of mechanisms and manipulators, it is rather difficult to obtain the polynomials; moreover, with the complication of the structures of mechanisms and manipulators, the formation of polynomials becomes more complicated and their degree increases. Performance analysis and applications of the PMs and robots are also presented in [16–21].

In this paper, kinematic synthesis of the PM with two end effectors is carried out on the basis of a modular approach [22,23], according to which PMs, regardless of their complexity, are formed by connecting the output objects (end effectors) with a base using closing kinematic chains (CKCs), which are structural modules. CKCs can be active, passive, and negative, which have positive, zero, and negative DOFs, respectively. The active and negative CKCs impose geometric constraints on the motions of the output objects, and passive CKCs do not impose geometric constraints. The representation of PMs from separate structural modules simplifies the methods of their investigation.

2. Kinematic Synthesis of the PM with Two Grippers

A PM with two end effectors can be used in a cold stamping technological line for reloading operations between two hydraulic presses [24].

Figure 1 shows a structural scheme of the PM with two end effectors in two positions.

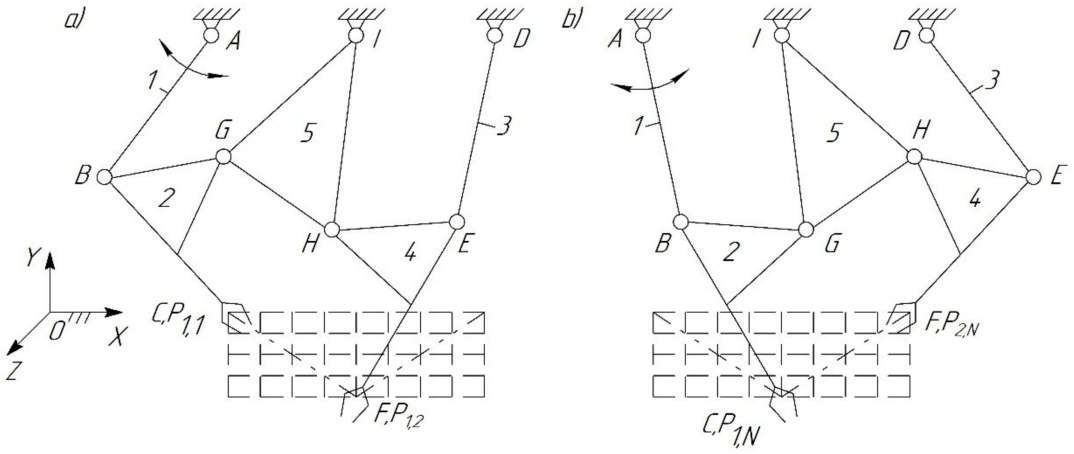

Figure 1. PM with two end effectors in two positions: (**a**) the first position (**b**) the second position.

In the first position (Figure 1a), the first gripper P_1 in position $P_{1,1}$ takes the workpiece after processing in the first hydraulic press for delivery to the store. At this time, the second gripper P_2 in position $P_{2,1}$ takes the previous workpiece processed in the first hydraulic press for delivery to the second hydraulic press for further processing.

In the second position (Figure 1b), the first gripper P_1 in position $P_{1,N}$ delivers the workpiece to the store and the second gripper P_2 in position $P_{2,N}$ delivers the previous workpiece to the second hydraulic press. The cycle is then repeated.

The considered positioning PM with two end effectors is formed by connecting two output objects (grippers P_1 and P_2) with a base using two passive and one negative CKC in the following sequence. First, the grippers P_1 and P_2 are connected to the base using passive CKCs ABC and DEF with revolute kinematic pairs, respectively, which have two degrees of freedom. Since passive CKCs ABC and DEF have two degrees of freedom, they

can reproduce the given laws of motion of the output points P_1 and P_2. Then, to form a single movable PM with two end effectors, we connect the links BC and EF of the passive CKCs ABC and DEF with the base using a negative CKC GHI with three negative degrees of freedom. Figure 2 shows a block structure of the formed PM with two end effectors.

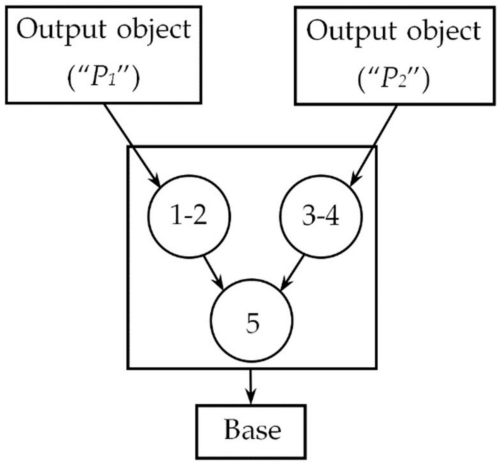

Figure 2. Block structure of the PM with two end effectors.

According to the block structure (Figure 2), the parametric synthesis of the PM with two end effectors (Figure 3) consists of the parametric synthesis of two passive CKCs—ABC and DEF—and one negative CKC—GHI. The parameters of the synthesis of two passive CKCs—ABC and DEF—are X_A, Z_A, l_{AB}, l_{BC} and X_D, Z_D, l_{DE}, l_{EF}, respectively, where X_A, Z_A and X_D, Z_D are the coordinates of the fixed joints A and D in the absolute coordinate system $OXYZ$; $l_{AB}, l_{BC}, l_{DE}, l_{EF}$ are the length of the links AB, BC, DE, EF. Let denote these parameters by the vectors $\mathbf{p}_1$ and $\mathbf{p}_2$, where $\mathbf{p}_1 = [X_A, Z_A, l_{AB}, l_{BC}]^T$ and $\mathbf{p}_2 = [X_D, Z_D, l_{DE}, l_{EF}]^T$.

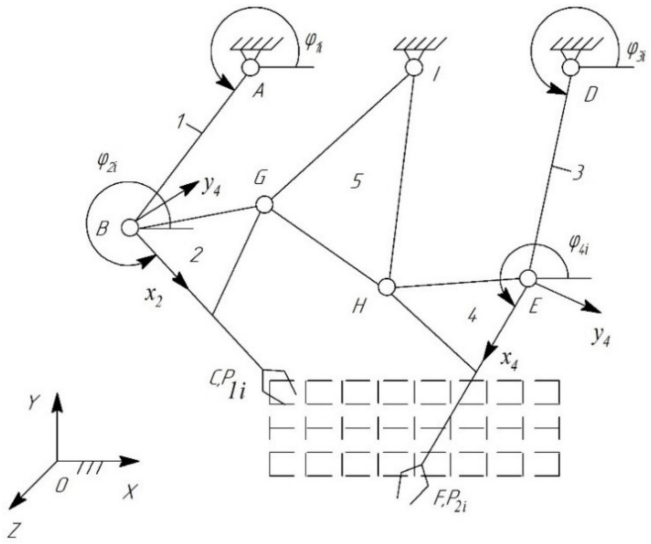

Figure 3. PM with two end effectors in the first position.

Since the passive CKCs do not impose geometric constraints on the movements of the output points C and F, the vectors of the synthesis parameters $\mathbf{p}_1$ and $\mathbf{p}_2$ are varied by the

generator of LP_τ sequence [25] to satisfy the constraints of the negative CKC *IDH*. In this case, the following conditions should be fulfilled:

$$|l_{AB} - l_{BC}| \le \rho_{1\min}, \quad l_{AB} + l_{BC} \ge \rho_{1\max}, \tag{1}$$

$$|l_{DE} - l_{EF}| \le \rho_{2\min}, \quad l_{DE} + l_{EF} \ge \rho_{2\max}, \tag{2}$$

where:

$$\rho_{1\min} = \min_{X_{P_1}, Y_{P_1}} |l_{AP_{1i}}|, \ \rho_{1\max} = \max_{X_{P_1}, Y_{P_1}} |l_{AP_{1i}}|, \tag{3}$$

$$\rho_{2\min} = \min_{X_{P_2}, Y_{P_2}} |l_{DP_{2i}}|, \ \rho_{2\max} = \max_{X_{P_2}, Y_{P_2}} |l_{DP_{2i}}|. \tag{4}$$

The variable distances $l_{AP_{1i}}$ and $l_{CP_{2i}}$ in the Expressions (1)–(4) are determined by the equations:

$$l_{AP_{1i}} = \left[\left(X_{P_{1i}} - X_A \right)^2 + \left(Y_{P_{1i}} - Y_A \right)^2 \right]^{\frac{1}{2}}, (i = 1, 2, \ldots, N), \tag{5}$$

$$l_{DP_{2i}} = \left[\left(X_{P_{2i}} - X_D \right)^2 + \left(Y_{P_{2i}} - Y_D \right)^2 \right]^{\frac{1}{2}}. \tag{6}$$

Let us consider the parametric synthesis of the negative CKC *GHI* with three negative degrees of freedom, determined by the Chebyshev formula [26]:

$$W = 3n - 2p_5 = 3 \cdot 1 - 2 \cdot 3 = -3, \tag{7}$$

where n is number of links, p_5 is the kinematic pairs of the fifth class.

To do this, we preliminarily determine the positions of links 2 and 4 of the passive CKCs *ABC* and *DEF* by the equations:

$$\varphi_{2i} = \mathrm{tg}^{-1} \frac{Z_{P_{1i}} - Z_{B_i}}{X_{P_{1i}} - X_{B_i}}, \tag{8}$$

$$\varphi_{4i} = \mathrm{tg}^{-1} \frac{Z_{P_{2i}} - Z_{E_i}}{X_{P_{2i}} - X_{E_i}}, \tag{9}$$

where:

$$\begin{bmatrix} X_{Bi} \\ Z_{Bi} \end{bmatrix} = \begin{bmatrix} X_A \\ Z_A \end{bmatrix} + l_{AB} \begin{bmatrix} \cos \varphi_{1i} \\ \sin \varphi_{1i} \end{bmatrix}, \tag{10}$$

$$\begin{bmatrix} X_{Ei} \\ Z_{Ei} \end{bmatrix} = \begin{bmatrix} X_D \\ Z_D \end{bmatrix} + l_{DE} \begin{bmatrix} \cos \varphi_{3i} \\ \sin \varphi_{3i} \end{bmatrix}, \tag{11}$$

$$\varphi_{1i} = \varphi_{AP_{1i}} - \cos^{-1} \frac{l_{AB}^2 + l_{AP_{1i}}^2 - l_{BC}^2}{2 l_{AB} l_{AP_{1i}}}, \tag{12}$$

$$\varphi_{3i} = \varphi_{DP_{2i}} + \cos^{-1} \frac{l_{DE}^2 + l_{DP_{2i}}^2 - l_{EF}^2}{2 l_{DE} l_{DP_{2i}}}, \tag{13}$$

$$\varphi_{AP_{1i}} = \mathrm{tg}^{-1} \frac{Z_{P_{1i}} - Z_A}{X_{P_{1i}} - X_A}, \tag{14}$$

$$\varphi_{DP_{2i}} = \mathrm{tg}^{-1} \frac{Z_{P_{2i}} - Z_D}{X_{P_{2i}} - X_D}. \tag{15}$$

Let us attach the coordinate systems Bx_2z_2 and Ex_4z_4 with the links *BC* and *EF* of the passive CKCs *ABC* and *DEF*, where the axes Bx_2 and Ex_4 are directed along the links *BC* and *EF*, respectively (Figure 3). Then, the synthesis parameters of the negative CKC *GHI* are $x_G^{(2)}, z_G^{(2)}, x_H^{(4)}, z_H^{(4)}, l_{GH}, X_I, Z_I, l_{GI}, l_{HI}$ where $x_G^{(2)}, z_G^{(2)}, x_H^{(4)}, z_H^{(4)}, X_I, Z_I$ are the coordinates of the joints *G*, *H*, *I* in the coordinate systems Bx_2z_2, Dx_4z_4, *OXYZ*, respectively;

l_{GH}, l_{GI}, l_{HI} are the lengths of the sides *GH*, *GI*, *HI* of the link *GHI*. Let us denote these synthesis parameters through the vector $\mathbf{p}_3 = [x_G^{(2)}, z_G^{(2)}, x_H^{(4)}, z_H^{(4)}, l_{GH}, X_I, Z_I, l_{GI}, l_{HI}]$.

Since the three-joined link *GHI* imposes three geometric constraints on the movements of the links of two passive CKCs *ABC* and *DEF*, we derive three functions of weighted differences:

$$\Delta q_{1i} = (x_{Hi}^{(2)} - x_G^{(2)})^2 + (z_{Hi}^{(2)} - z_G^{(2)})^2 - l_{HG}^2, \tag{16}$$

$$\Delta q_{2i} = (X_{Gi} - X_I)^2 + (Z_{Gi} - Z_I)^2 - l_{GI}^2, \tag{17}$$

$$\Delta q_{3i} = (X_{Hi} - X_I)^2 + (Z_{Hi} - Z_I)^2 - l_{HI}^2, \tag{18}$$

where $x_{Hi}^{(2)}$ and $z_{Hi}^{(2)}$ are the coordinates of the joint *H* in the local coordinate system Bx_2z_2; Z_{Gi}, X_{Gi} and X_{Hi}, Z_{Hi} are the coordinates of the joints *G* and *H* in the absolute coordinate system *OXYZ*, which are determined by the equations:

$$\begin{bmatrix} x_{Hi}^{(2)} \\ z_{Hi}^{(2)} \end{bmatrix} = \begin{bmatrix} \cos\varphi_{2i} & \sin\varphi_{2i} \\ -\sin\varphi_{2i} & \cos\varphi_{2i} \end{bmatrix} \cdot \begin{bmatrix} X_{Hi} - X_{Bi} \\ Z_{Hi} - Z_{Bi} \end{bmatrix}, \tag{19}$$

$$\begin{bmatrix} X_{Gi} \\ Z_{Gi} \end{bmatrix} = \begin{bmatrix} X_{Bi} \\ Z_{Bi} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{2i} & -\sin\varphi_{2i} \\ \sin\varphi_{2i} & \cos\varphi_{2i} \end{bmatrix} \cdot \begin{bmatrix} x_G^{(2)} \\ z_G^{(2)} \end{bmatrix}, \tag{20}$$

$$\begin{bmatrix} X_{Hi} \\ Z_{Hi} \end{bmatrix} = \begin{bmatrix} X_{Ei} \\ Z_{Ei} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{4i} & -\sin\varphi_{4i} \\ \sin\varphi_{4i} & \cos\varphi_{4i} \end{bmatrix} \cdot \begin{bmatrix} x_H^{(4)} \\ z_H^{(4)} \end{bmatrix}, \tag{21}$$

where:

$$\begin{bmatrix} X_{Bi} \\ Z_{Bi} \end{bmatrix} = \begin{bmatrix} X_A \\ Z_A \end{bmatrix} + l_{AB} \begin{bmatrix} \cos\varphi_{1i} \\ \sin\varphi_{1i} \end{bmatrix}, \tag{22}$$

$$\begin{bmatrix} X_{Ei} \\ Z_{Ei} \end{bmatrix} = \begin{bmatrix} X_D \\ Z_D \end{bmatrix} + l_{DE} \begin{bmatrix} \cos\varphi_{3i} \\ \sin\varphi_{3i} \end{bmatrix}. \tag{23}$$

The geometric meanings of Functions (16)–(18) are the deviations of the coordinates of the joints *H* and *G* from circles with radiuses l_{HG}, l_{GI}, l_{HI} in the relative motion of the plane Ex_4z_4 and in the absolute motion of link 5.

After replacing the synthesis parameters of the form:

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} x_G^{(2)} \\ y_G^{(2)} \end{bmatrix}, \begin{bmatrix} p_4 \\ p_5 \end{bmatrix} = \begin{bmatrix} x_H^{(4)} \\ y_H^{(4)} \end{bmatrix},$$

$$p_3 = \frac{1}{2}(x_G^{(2)^2} + z_G^{(2)^2} + x_H^{(4)^2} + z_H^{(4)^2} - l_{GH}^{(2)}),$$

$$\begin{bmatrix} p_6 \\ p_7 \end{bmatrix} = \begin{bmatrix} X_I \\ Z_I \end{bmatrix}, p_8 = \frac{1}{2}(x_G^{(2)^2} + z_G^{(2)^2} + X_I^2 + Z_I^2 - l_{GI}^2),$$

$$p_9 = \frac{1}{2}(x_H^{(4)^2} + z_H^{(4)^2} + X_I^2 + Z_I^2 - l_{HI}^2) \tag{24}$$

Functions (16)–(18) are expressed linearly in the following vectors of synthesis parameters $\mathbf{p}_3^{(1)} = [p_1, p_2, p_3]^T$, $\mathbf{p}_3^{(2)} = [p_4, p_5, p_3]^T$, $\mathbf{p}_3^{(3)} = [p_6, p_7, p_8]^T$, $\mathbf{p}_3^{(4)} = [p_1, p_2, p_8]^T$, $\mathbf{p}_3^{(5)} = [p_6, p_7, p_9]^T$, $\mathbf{p}_3^{(6)} = [p_4, p_5, p_9]^T$ as:

$$\Delta q_{1i}^{(k)} = 2(\mathbf{g}_{1i}^{(k)^T} \cdot \mathbf{p}_3^{(k)} - g_{0i}^{(k)}),\ k = 1, 2, \tag{25}$$

$$\Delta q_{2i}^{(k)} = 2(\mathbf{g}_{2i}^{(k)^T} \cdot \mathbf{p}_3^{(k)} - g_{0i}^{(k)}),\ k = 3, 4, \tag{26}$$

$$\Delta q_{3i}^{(k)} = 2(\mathbf{g}_{3i}^{(k)^T} \cdot \mathbf{p}_3^{(k)} - g_{0i}^{(k)}),\ k = 5, 6, \tag{27}$$

where:

$$\mathbf{g}_{1i}^{(1)} = -\begin{bmatrix} \mathbf{\Gamma}^{-1}(\varphi_{2i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{Ei} - X_{Bi} \\ Z_{Ei} - Z_{Bi} \\ 1 \end{bmatrix} - \begin{bmatrix} \mathbf{\Gamma}(\varphi_{4i} - \varphi_{2i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_4 \\ p_5 \\ 0 \end{bmatrix}, \quad (28)$$

$$\mathbf{g}_{1i}^{(2)} = \begin{bmatrix} \mathbf{\Gamma}^{-1}(\varphi_{4i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{Ei} - X_{Bi} \\ Z_{Ei} - Z_{Bi} \\ 1 \end{bmatrix} - \begin{bmatrix} \mathbf{\Gamma}^{-1}(\varphi_{4i} - \varphi_{2i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ 0 \end{bmatrix}, \quad (29)$$

$$g_{0i}^{(1)} = -\frac{1}{2}[(X_{Ei} - X_{Bi})^2 + (Z_{Ei} - Z_{Bi})^2] + [X_{Ei} - X_{Bi}, Z_{Ei} - Z_{Bi}] \cdot \mathbf{\Gamma}(\varphi_{4i}) \cdot \begin{bmatrix} p_4 \\ p_5 \end{bmatrix}, \quad (30)$$

$$g_{0i}^{(2)} = -\frac{1}{2}[(X_{Ei} - X_{Bi})^2 + (Z_{Ei} - Z_{Bi})^2] - [X_{Ei} - X_{Bi}, Z_{Ei} - Z_{Bi}] \cdot \mathbf{\Gamma}(\varphi_{2i}) \cdot \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}, \quad (31)$$

$$\mathbf{g}_{2i}^{(3)} = -\begin{bmatrix} X_{Bi} \\ Z_{Bi} \\ 1 \end{bmatrix} - \begin{bmatrix} \mathbf{\Gamma}(\varphi_{2i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ 0 \end{bmatrix}, \quad (32)$$

$$\mathbf{g}_{2i}^{(4)} = \begin{bmatrix} \mathbf{\Gamma}^{-1}(\varphi_{2i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{Bi} \\ Z_{Bi} \\ 1 \end{bmatrix} - \begin{bmatrix} \mathbf{\Gamma}(\varphi_{2i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_6 \\ p_7 \\ 0 \end{bmatrix}, \quad (33)$$

$$g_{0i}^{(3)} = -\frac{1}{2}[{X_{Bi}}^2 + {Z_{Bi}}^2] + [X_{Bi}, Z_{Bi}] \cdot \mathbf{\Gamma}(\varphi_{2i}) \cdot \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}, \quad (34)$$

$$g_{0i}^{(4)} = -\frac{1}{2}[{X_{Bi}}^2 + {Z_{Bi}}^2] - [X_{Bi}, Z_{Bi}] \cdot \begin{bmatrix} p_6 \\ p_7 \end{bmatrix}, \quad (35)$$

$$\mathbf{g}_{3i}^{(5)} = -\begin{bmatrix} X_{Ei} \\ Z_{Ei} \\ 1 \end{bmatrix} - \begin{bmatrix} \mathbf{\Gamma}(\varphi_{4i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_4 \\ p_5 \\ 0 \end{bmatrix}, \quad (36)$$

$$\mathbf{g}_{3i}^{(6)} = \begin{bmatrix} \mathbf{\Gamma}^{-1}(\varphi_{4i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{Ei} \\ Z_{Ei} \\ 1 \end{bmatrix} + \begin{bmatrix} \mathbf{\Gamma}(\varphi_{4i}) & 0 \\ 0 \quad 0 & 0 \\ 0 \quad 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_6 \\ p_7 \\ 0 \end{bmatrix}. \quad (37)$$

$$g_{0i}^{(5)} = -\frac{1}{2}[{X_{Ei}}^2 + {Z_{Ei}}^2] + [X_{Ei}, Z_{Ei}] \cdot \mathbf{\Gamma}(\varphi_{4i}) \cdot \begin{bmatrix} p_4 \\ p_5 \end{bmatrix}, \quad (38)$$

$$g_{0i}^{(6)} = -\frac{1}{2}[{X_{Ei}}^2 + {Z_{Ei}}^2] - [X_{Ei}, Z_{Ei}] \cdot \begin{bmatrix} p_6 \\ p_7 \end{bmatrix}. \quad (39)$$

Furthermore, the synthesis parameters of the negative CKC *GHI* are determined on the basis of the problems of Chebyshev and least-square approximations [16,17].

3. Kinematic Analysis of the PM with Two Grippers

In the kinematic analysis of the PM with two end effectors (Figure 4) for the given geometric parameters of the links and the input angle φ_{1i}, it is necessary to determine the positions and analogues of the velocities and accelerations of the links, including the output points *C* and *F*.

The considered PM with two end effectors has the structural formula:

$$\mathrm{I}(1) \rightarrow \mathrm{II}(2,5), \rightarrow \mathrm{II}(3,4), \quad (40)$$

i.e., it contains two dyads $\mathrm{II}(2,5)$ and $\mathrm{II}(3,4)$.

According to the structural Formula (40), first, a kinematic analysis of the dyad $\mathrm{II}(2,5)$ is carried out, and then of the dyad $\mathrm{II}(3,4)$.

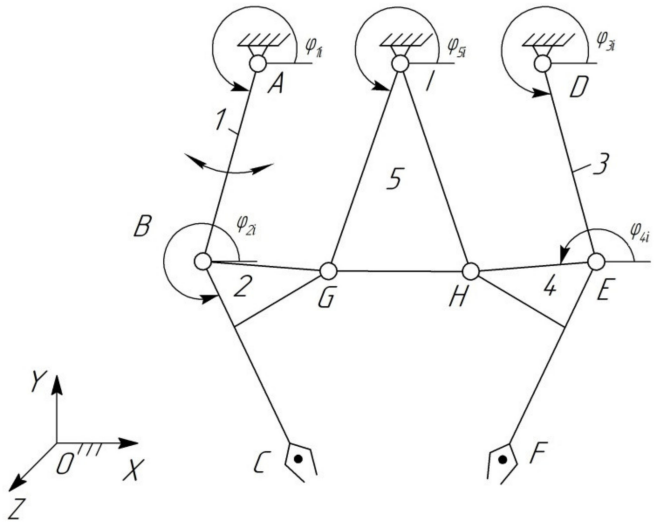

Figure 4. PM with two end effectors.

3.1. Kinematic Analysis of the PM with Two Grippers

Let us derive a vector *BGI* loop-closure equation:

$$l_{BG}\mathbf{e}_{2i} - l_{IG}\mathbf{e}_{5i} + l_{(IB)i}\mathbf{e}_{(IB)i} = 0, \tag{41}$$

where:

$$l_{(IB)i} = \left[\left(X_{Bi} - X_I \right)^2 + \left(Z_{Bi} - Z_I \right)^2 \right]^{\frac{1}{2}}, \tag{42}$$

$$\varphi_{(IB)i} = \mathrm{tg}^{-1} \frac{Z_{Bi} - Z_I}{X_{Bi} - X_I}, \tag{43}$$

$$\begin{bmatrix} X_{Bi} \\ Z_{Bi} \end{bmatrix} = \begin{bmatrix} X_A \\ Z_A \end{bmatrix} + l_{AB} \begin{bmatrix} \cos\varphi_{1i} \\ \sin\varphi_{1i} \end{bmatrix}. \tag{44}$$

Transfer $l_{BG}\mathbf{e}_{2i}$ to the right side of Equation (41) and square both sides. As a result, we obtain:

$$\varphi_{5i} = \varphi_{(IB)i} + \cos^{-1} \frac{l_{(IB)i}^2 + l_{IG}^2 - l_{BG}^2}{2 l_{(IB)i} l_{IG}}. \tag{45}$$

Next, we define:

$$\begin{bmatrix} X_{Gi} \\ Z_{Gi} \end{bmatrix} = \begin{bmatrix} X_I \\ Z_I \end{bmatrix} + l_{IG} \begin{bmatrix} \cos\varphi_{5i} \\ \sin\varphi_{5i} \end{bmatrix}, \tag{46}$$

$$\varphi_{2i} = \mathrm{tg}^{-1} \frac{Z_{Gi} - Z_{Bi}}{X_{Gi} - X_{Bi}}. \tag{47}$$

To solve the problem of the positions of the dyad II (3.4), we derive a vector *DEH* loop-closure equation:

$$l_{DE}\mathbf{e}_{3i} + l_{EH}\mathbf{e}_{4i} - l_{(DH)i}\mathbf{e}_{(DH)i} = 0, \tag{48}$$

where:

$$l_{(DH)i} = \left[\left(X_{Hi} - X_D \right)^2 + \left(Z_{Hi} - Z_D \right)^2 \right]^{\frac{1}{2}}, \tag{49}$$

$$\varphi_{(DH)i} = \mathrm{tg}^{-1} \frac{Z_{Hi} - Z_D}{X_{Hi} - X_D}, \tag{50}$$

$$\begin{bmatrix} X_{Hi} \\ Z_{Hi} \end{bmatrix} = \begin{bmatrix} X_I \\ Z_I \end{bmatrix} + \begin{bmatrix} \cos\varphi_{5i} & -\sin\varphi_{5i} \\ \sin\varphi_{5i} & \cos\varphi_{5i} \end{bmatrix} \cdot \begin{bmatrix} x_G^{(5)} \\ y_G^{(5)} \end{bmatrix}. \tag{51}$$

Transfer $l_{EH}\mathbf{e}_{4i}$ to the right side of Equation (48) and square both sides. As a result, we obtain:

$$\varphi_{3i} = \varphi_{(DH)i} + \cos^{-1}\frac{l_{DE}^2 + l_{(DH)i}^2 - l_{EH}^2}{2l_{DE}\cdot l_{(DH)i}}. \tag{52}$$

Next, we define:

$$\begin{bmatrix} X_{Ei} \\ Z_{Ei} \end{bmatrix} = \begin{bmatrix} X_D \\ Z_D \end{bmatrix} + l_{DE}\begin{bmatrix} \cos\varphi_{3i} \\ \sin\varphi_{3i} \end{bmatrix}, \tag{53}$$

$$\varphi_{4i} = \mathrm{tg}^{-1}\frac{Z_{Hi} - Z_{Ei}}{X_{Hi} - X_{Ei}}. \tag{54}$$

Coordinates of the output points *C* and *F* in the absolute coordinate system *OXYZ* are determined by the equations:

$$\begin{bmatrix} X_{Ci} \\ Z_{Ci} \end{bmatrix} = \begin{bmatrix} X_{Bi} \\ Z_{Bi} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{2i} & -\sin\varphi_{2i} \\ \sin\varphi_{2i} & \cos\varphi_{2i} \end{bmatrix}\cdot\begin{bmatrix} x_C^{(2)} \\ y_C^{(2)} \end{bmatrix}, \tag{55}$$

$$\begin{bmatrix} X_{Fi} \\ Z_{Fi} \end{bmatrix} = \begin{bmatrix} X_{Ei} \\ Z_{Ei} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{4i} & -\sin\varphi_{4i} \\ \sin\varphi_{4i} & \cos\varphi_{4i} \end{bmatrix}\cdot\begin{bmatrix} x_F^{(4)} \\ y_F^{(4)} \end{bmatrix}. \tag{56}$$

3.2. Analogues of Velocities and Accelerations

To determine the analogues of the angular velocities of the PM with two end effectors, we derive the vector *ABGI* and *IHED* loop-closure equations:

$$l_{AB}\mathbf{e}_{1i} + l_{BG}\mathbf{e}_{2i} - l_{IG}\mathbf{e}_{5i} - l_{AI}\mathbf{e}_{AI} = 0 \tag{57}$$

and

$$l_{IH}\mathbf{e}_{(IH)i} - l_{EH}\mathbf{e}_{4i} - l_{DE}\mathbf{e}_{3i} - l_{ID}\mathbf{e}_{ID} = 0 \tag{58}$$

and project them on the axes *OX* and *OZ* of the absolute coordinate system *OXYZ*

$$\left.\begin{array}{c} l_{AB}\cos\varphi_{1i} + l_{BG}\cos\varphi_{2i} - l_{IG}\cos\varphi_{5i} - l_{AI}\cos\varphi_{AI} = 0 \\ l_{AB}\sin\varphi_{1i} + l_{BG}\sin\varphi_{2i} - l_{IG}\sin\varphi_{5i} - l_{AI}\sin\varphi_{AI} = 0 \end{array}\right\} \tag{59}$$

and

$$\left.\begin{array}{c} l_{IH}\cos(\varphi_{5i} + \alpha_5) - l_{EH}\cos\varphi_{4i} - l_{DE}\cos\varphi_{3i} - l_{ID}\cos\varphi_{ID} = 0 \\ l_{IH}\sin(\varphi_{5i} + \alpha_5) - l_{EH}\sin\varphi_{4i} - l_{DE}\sin\varphi_{3i} - l_{ID}\sin\varphi_{ID} = 0 \end{array}\right\} \tag{60}$$

differentiate the systems of Equations (59) and (60) with respect to the generalized coordinate φ_{1i}

$$\left.\begin{array}{c} -l_{AB}\sin\varphi_{1i} - l_{BG}\sin\varphi_{2i}\cdot\varphi'_{2i} + l_{IG}\sin\varphi_{5i}\cdot\varphi'_{5i} = 0 \\ l_{AB}\cos\varphi_{1i} + l_{BG}\cos\varphi_{2i}\cdot\varphi'_{2i} - l_{IG}\cos\varphi_{5i}\cdot\varphi'_{5i} = 0 \end{array}\right\} \tag{61}$$

and

$$\left.\begin{array}{c} -l_{IH}\sin(\varphi_{5i} + \alpha_5)\cdot\varphi'_{5i} + l_{EH}\sin\varphi_{4i}\cdot\varphi'_{4i} + l_{DE}\sin\varphi_{3i}\cdot\varphi'_{3i} = 0 \\ l_{IH}\cos(\varphi_{5i} + \alpha_5)\cdot\varphi'_{5i} - l_{EH}\cos\varphi_{4i}\cdot\varphi'_{4i} - l_{DE}\cos\varphi_{3i}\cdot\varphi'_{3i} = 0 \end{array}\right\} \tag{62}$$

From the system of Equation (61), we determine the analogues of the angular velocities φ'_{2i} and φ'_{5i}

$$\mathbf{A}_1^{-1}\cdot\mathbf{u}_1 = \mathbf{b}_1 \tag{63}$$

where:

$$\mathbf{A}_1 = \begin{bmatrix} Z_{Bi} - Z_{Gi} & Z_{Gi} - Z_I \\ X_{Gi} - X_{Bi} & X_I - X_{Gi} \end{bmatrix},$$

$$\mathbf{u}_1 = \begin{bmatrix} \varphi'_{2i} \\ \varphi'_{5i} \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} Z_{Bi} - Z_A \\ X_A - X_{Bi} \end{bmatrix}.$$

Substituting the obtained values of the angular velocity analogue φ'_{5i} into the system of Equation (62), from this system, we determine the angular velocities analogues φ'_{3i} and φ'_{4i}

$$\mathbf{A}_2^{-1} \cdot \mathbf{u}_2 = \mathbf{b}_2, \tag{64}$$

where:

$$\mathbf{A}_2 = \begin{bmatrix} Z_{Ei} - Z_D & Z_{Hi} - Z_{Ei} \\ X_D - X_{Ei} & X_{Ei} - X_{Hi} \end{bmatrix},$$

$$\mathbf{u}_2 = \begin{bmatrix} \varphi'_{3i} \\ \varphi'_{4i} \end{bmatrix}, \qquad \mathbf{b}_2 = \begin{bmatrix} (Z_{Hi} - Z_I) \cdot \varphi'_{5i} \\ (X_I - X_{Hi}) \cdot \varphi'_{5i} \end{bmatrix}.$$

Projections of the linear velocities analogues of the output points C and F on the axis of the absolute coordinate system $OXYZ$ are determined by differentiating Equations (55) and (56) with respect to the generalized coordinate φ_{1i}

$$\begin{bmatrix} u^X_{Ci} \\ u^Z_{Ci} \end{bmatrix} = \begin{bmatrix} u^X_{Bi} \\ u^Z_{Bi} \end{bmatrix} + \begin{bmatrix} -\sin\varphi_{2i} & -\cos\varphi_{2i} \\ \cos\varphi_{2i} & -\sin\varphi_{2i} \end{bmatrix} \cdot \begin{bmatrix} x^{(2)}_C \\ y^{(2)}_C \end{bmatrix} \cdot \varphi'_{2i}, \tag{65}$$

$$\begin{bmatrix} u^X_{Fi} \\ u^Z_{Fi} \end{bmatrix} = \begin{bmatrix} u^X_{Ei} \\ u^Z_{Ei} \end{bmatrix} + \begin{bmatrix} -\sin\varphi_{4i} & -\cos\varphi_{4i} \\ \cos\varphi_{4i} & -\sin\varphi_{4i} \end{bmatrix} \cdot \begin{bmatrix} x^{(4)}_F \\ y^{(4)}_F \end{bmatrix} \cdot \varphi'_{4i}, \tag{66}$$

where the projections of the linear velocity analogues of the joints B and E are determined by differentiating Equations (44) and (53) with respect to the generalized coordinate φ_{1i}

$$\begin{bmatrix} u^X_{Bi} \\ u^Z_{Bi} \end{bmatrix} = l_{AB} \begin{bmatrix} -\sin\varphi_{1i} \\ \cos\varphi_{1i} \end{bmatrix}, \tag{67}$$

$$\begin{bmatrix} u^X_{Ei} \\ u^Z_{Ei} \end{bmatrix} = l_{DE} \begin{bmatrix} -\sin\varphi_{3i} \\ \cos\varphi_{3i} \end{bmatrix} \cdot \varphi'_{3i}. \tag{68}$$

To determine the angular acceleration analogues of the links, we differentiate the systems of Equations (61) and (62) with respect to the generalized coordinate φ_{1i}

$$\left.\begin{array}{c} -l_{AB}\cos\varphi_{1i} - l_{BG}\cos\varphi_{2i} \cdot \varphi'^2_{2i} - l_{BG}\sin\varphi_{2i} \cdot \varphi''_{2i} + \\ l_{IG}\cos\varphi_{5i} \cdot \varphi'^2_{5i} + l_{IG}\sin\varphi_{5i} \cdot \varphi''_{5i} = 0 \\ -l_{AB}\sin\varphi_{1i} - l_{BG}\sin\varphi_{2i} \cdot \varphi'^2_{2i} + l_{BG}\cos\varphi_{2i} \cdot \varphi''_{2i} + \\ l_{IG}\sin\varphi_{5i} \cdot \varphi'^2_{5i} - l_{IG}\cos\varphi_{5i} \cdot \varphi''_{5i} = 0 \end{array}\right\} \tag{69}$$

and

$$\left.\begin{array}{l} -l_{IH}\cos(\varphi_{5i} + \alpha_5) \cdot \varphi'^2_{5i} - l_{IH}\sin(\varphi_{5i} + \alpha_5) \cdot \varphi''_{5i} + l_{EH}\cos\varphi_{4i} \cdot \varphi'^2_{4i} \\ +l_{EH}\sin\varphi_{4i} \cdot \varphi''_{4i} + l_{DE}\cos\varphi_{3i} \cdot \varphi'^2_{3i} + l_{DE}\sin\varphi_{3i} \cdot \varphi''_{3i} = 0 \\ -l_{IH}\sin(\varphi_{5i} + \alpha_5) \cdot \varphi'^2_{5i} + l_{IH}\cos(\varphi_{5i} + \alpha_5) \cdot \varphi''_{5i} + l_{EH}\sin\varphi_{4i} \cdot \varphi'^2_{4i} \\ -l_{EH}\cos\varphi_{4i} \cdot \varphi''_{4i} + l_{DE}\sin\varphi_{3i} \cdot \varphi'^2_{3i} - l_{DE}\cos\varphi_{3i} \cdot \varphi''_{3i} = 0 \end{array}\right\} \tag{70}$$

From the systems of Equation (69), we determine the angular accelerations analogues φ''_{2i} and φ''_{5i}

$$\mathbf{A}_1^{-1} \cdot \mathbf{w}_1 = \mathbf{b}_3, \tag{71}$$

where:

$$\mathbf{w}_1 = \begin{bmatrix} \varphi''_{2i} \\ \varphi''_{5i} \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} (X_{Bi} - X_A) + (X_{Gi} - X_{Bi}) \cdot \varphi'^2_{2i} + (X_I - X_{Gi}) \cdot \varphi'^2_{5i} \\ (Z_{Bi} - Z_A) + (Z_{Gi} - Z_A) \cdot \varphi'^2_{2i} + (Z_I - Z_{Gi}) \cdot \varphi'^2_{5i} \end{bmatrix}.$$

Substituting the obtained values of the angular velocity analogues φ''_{2i} and φ''_{5i} into the system of Equation (70), from this system, we determine the angular velocities analogues φ''_{3i} and φ''_{4i}

$$\mathbf{A}_2^{-1} \cdot \mathbf{w}_2 = \mathbf{b}_4, \tag{72}$$

where:

$$\mathbf{w}_2 = \begin{bmatrix} \varphi''_{3i} \\ \varphi''_{4i} \end{bmatrix}, \mathbf{b}_4 = \begin{bmatrix} (X_{Hi} - X_I) \cdot \varphi'^2_{5i} + (X_{Ei} - X_{Hi}) \cdot \varphi'^2_{4i} + (X_D - X_{Ei}) \cdot \varphi'^2_{3i} \\ (Z_{Hi} - Z_I) \cdot \varphi'^2_{5i} + (Z_{Ei} - Z_{Hi}) \cdot \varphi'^2_{4i} + (Z_D - Z_{Ei}) \cdot \varphi'^2_{3i} \end{bmatrix}. \tag{73}$$

Projections of the linear velocities analogues of the output points C and F on the axis of the absolute coordinate system $OXYZ$ are determined by differentiating Equations (65) and (66) with respect to the generalized coordinate φ_{1i}

$$\begin{bmatrix} w^X_{Ci} \\ w^Z_{Ci} \end{bmatrix} = \begin{bmatrix} w^X_{Bi} \\ w^Z_{Bi} \end{bmatrix} + \begin{bmatrix} -\cos\varphi_{2i} & \sin\varphi_{2i} \\ -\sin\varphi_{2i} & -\cos\varphi_{2i} \end{bmatrix} \cdot \begin{bmatrix} x^{(2)}_C \\ z^{(2)}_C \end{bmatrix} \cdot \varphi'^2_{2i} + \begin{bmatrix} -\sin\varphi_{2i} & -\cos\varphi_{2i} \\ \cos\varphi_{2i} & -\sin\varphi_{2i} \end{bmatrix} \cdot \begin{bmatrix} x^{(2)}_C \\ z^{(2)}_C \end{bmatrix} \cdot \varphi''_{2i}, \tag{74}$$

$$\begin{bmatrix} w^X_{Fi} \\ w^Z_{Fi} \end{bmatrix} = \begin{bmatrix} w^X_{Ei} \\ w^Z_{Ei} \end{bmatrix} + \begin{bmatrix} -\cos\varphi_{4i} & \sin\varphi_{4i} \\ -\sin\varphi_{4i} & -\cos\varphi_{4i} \end{bmatrix} \cdot \begin{bmatrix} x^{(4)}_F \\ z^{(4)}_F \end{bmatrix} \cdot \varphi'^2_{4i} + \begin{bmatrix} -\sin\varphi_{4i} & -\cos\varphi_{4i} \\ \cos\varphi_{4i} & -\sin\varphi_{4i} \end{bmatrix} \cdot \begin{bmatrix} x^{(2)}_F \\ z^{(2)}_F \end{bmatrix} \cdot \varphi''_{4i}, \tag{75}$$

where the projections of the linear velocity analogues of the joints B and E are determined by differentiating Equations (67) and (68) with respect to the generalized coordinate φ_{1i}

$$\begin{bmatrix} w^X_{Bi} \\ w^Z_{Bi} \end{bmatrix} = l_{AB} \begin{bmatrix} -\cos\varphi_{1i} \\ -\sin\varphi_{1i} \end{bmatrix}, \tag{76}$$

$$\begin{bmatrix} w^X_{Ei} \\ w^Z_{Ei} \end{bmatrix} = l_{DE} \begin{bmatrix} -\cos\varphi_{3i} \\ -\sin\varphi_{3i} \end{bmatrix} \cdot \varphi'^2_{3i} + \begin{bmatrix} -\sin\varphi_{3i} \\ \cos\varphi_{3i} \end{bmatrix} \cdot \varphi''_{3i}. \tag{77}$$

4. Numerical Results

Table 1 shows N = 11 positions of the grippers P_1 and P_2 of the PM with two end effectors.

Table 1. Positions of the grippers P_1 and P_2.

i	1	2	3	4	5	6	7	8	9	10	11
$X_{P_{1,i}}$, mm	0	4.6	9.2	13.8	18.40	23.0	27.60	32.20	36.80	41.40	46.3882
$Y_{P_{1,i}}$, mm	0	−6.2512	−11.0478	−14.8398	−17.8092	−20.1433	−21.8922	−23.1166	−23.8528	−24.1065	−23.8384
$X_{P_{2,i}}$, mm	46.3882	53.5938	59.8359	65.3367	70.4344	75.0223	79.1896	82.9453	86.4738	89.6476	92.7766
$Y_{P_{2,i}}$, mm	−23.8384	−24.0439	−23.2688	−21.8398	−19.8480	−17.4431	−14.6785	−11.6242	−8.1590	−4.4228	0

Tables 2–4 show the obtained values of the synthesis parameters of the two passive CKCs *ABC*, *DEF* and negative CKC *GHI*, respectively.

Table 2. Synthesis parameters of the CKC *ABC*.

X_A, mm	Z_A, mm	l_{AB}, mm	l_{BC}, mm
14.04	−57.11	46.5309	43.0

Table 3. Synthesis parameters of the CKC *DEF*.

X_D, mm	Z_D, mm	l_{DE}, mm	l_{EF}, mm
78.74	−57.11	46.5309	43.0

Table 4. Synthesis parameters of the CKC *GHI*.

$x_G^{(2)}$, mm	$y_G^{(2)}$, mm	$x_H^{(4)}$, mm	$y_H^{(4)}$, mm	l_{GH}, mm	X_I, mm	Z_I, mm	l_{GI}, mm	l_{HI}, mm
6.6072	9.9607	−6.6072	−9.9607	45.0	46.39	57.11	50.0	50.0

3D CAD model of the synthesized PM with two grippers is shown in Figure 5.

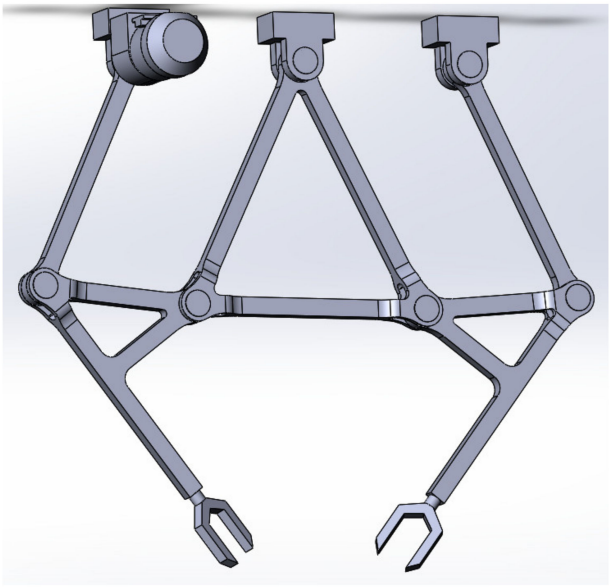

Figure 5. 3D CAD model of the PM with two grippers.

Table 5 shows the obtained values of the coordinates $X_{P_{1,i}}, Y_{P_{1,i}}, X_{P_{2,i}}, Y_{P_{2,i}}$ and projections of analogues of linear velocities $u^X_{P_{1,i}}, u^Z_{P_{1,i}}, u^X_{P_{2,i}}, u^Z_{P_{2,i}}$ and linear accelerations $w^X_{P_{1,i}}, w^Z_{P_{1,i}}$, $w^X_{P_{2,i}}, w^Z_{P_{2,i}}$ of the grippers of the PM with two end effectors.

Table 5. Positions and analogues of linear velocities and accelerations of the grippers.

i	1	2	3	4	5	6	7	8	9	10	11
$X_{P_{1,i}}$, mm	0	4.6	9.2	13.8	18.40	23.0	27.60	32.20	36.80	41.40	46.3882
$Z_{P_{1,i}}$, mm	0	−6.2512	−11.0478	−14.8398	−17.8092	−20.1433	−21.8922	−23.1166	−23.8528	−24.1065	−23.8384
$X_{P_{2,i}}$, mm	46.3882	53.5938	59.8359	65.3367	70.4344	75.0223	79.1896	82.9453	86.4738	89.6476	92.7766
$Z_{P_{2,i}}$, mm	−23.8384	−24.0439	−23.2688	−21.8398	−19.8480	−17.4431	−14.6785	−11.6242	−8.1590	−4.4228	0
$u^X_{P_{1,i}}$, mm	0.8440	0.5204	0.2432	−0.026	−0.276	−0.495	−0.693	−0.8729	−1.036	−1.184	−1.325
$u^Z_{P_{1,i}}$, mm	0.9830	0.9627	0.9489	0.9362	0.9236	0.9110	0.8977	0.8829	0.8662	0.8465	0.8211
$u^X_{P_{2,i}}$, mm	1.0733	1.0380	1.0173	0.9998	0.9845	0.9710	0.9583	0.9459	0.9335	0.9212	0.9094
$u^Z_{P_{2,i}}$, mm	−0.1071	−0.0733	−0.0650	−0.0515	−0.0399	−0.0294	−0.0189	−0.0075	0.0062	0.0240	0.0515
$w^X_{P_{1,i}}$, mm	−0.1096	−0.1066	−0.0868	−0.1056	−0.1964	−0.3570	−0.5768	−0.8396	−1.1265	−1.4157	−1.6956
$w^Z_{P_{1,i}}$, mm	−0.8688	−0.2504	−0.0283	0.0221	−0.0380	−0.1620	−0.3271	−0.5223	−0.7421	−0.9843	−1.2642
$w^X_{P_{2,i}}$, mm	−0.1116	−0.0885	−0.0968	−0.1112	−0.1353	−0.1676	−0.2100	−0.2661	−0.3420	−0.4486	−0.6182
$w^Z_{P_{2,i}}$, mm	0.6401	0.4710	0.3803	0.3069	0.2416	0.1771	0.1038	0.01319	−0.1059	−0.2713	−0.5320

Positions and modules of the velocities and acceleration analogues of the synthesized PM grippers P_1 and P_2 are also presented with the graphical plots in Figures 6–8.

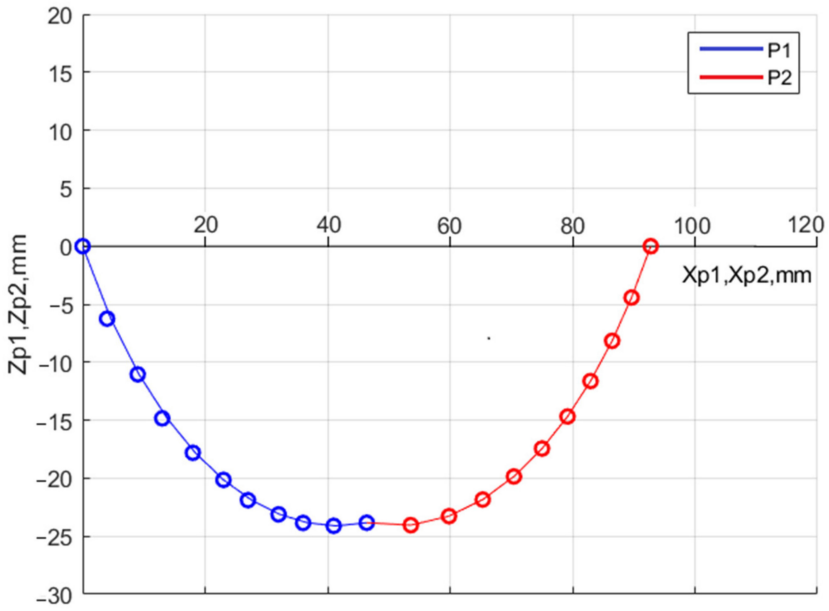

Figure 6. Graphics of the grippers P_1 and P_2 positions.

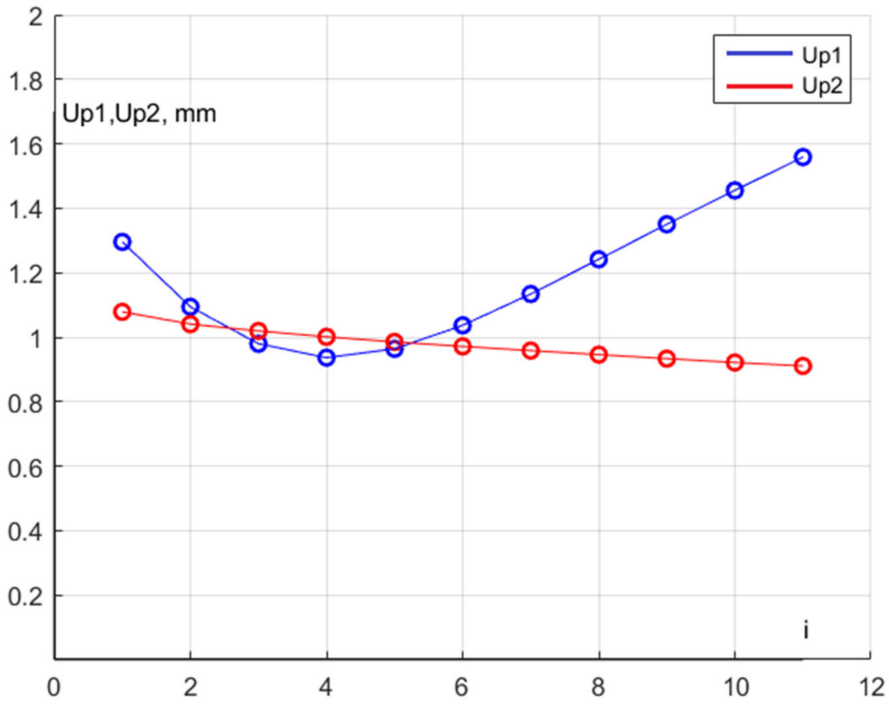

Figure 7. Graphics of the grippers P_1 and P_2 linear velocities analogues.

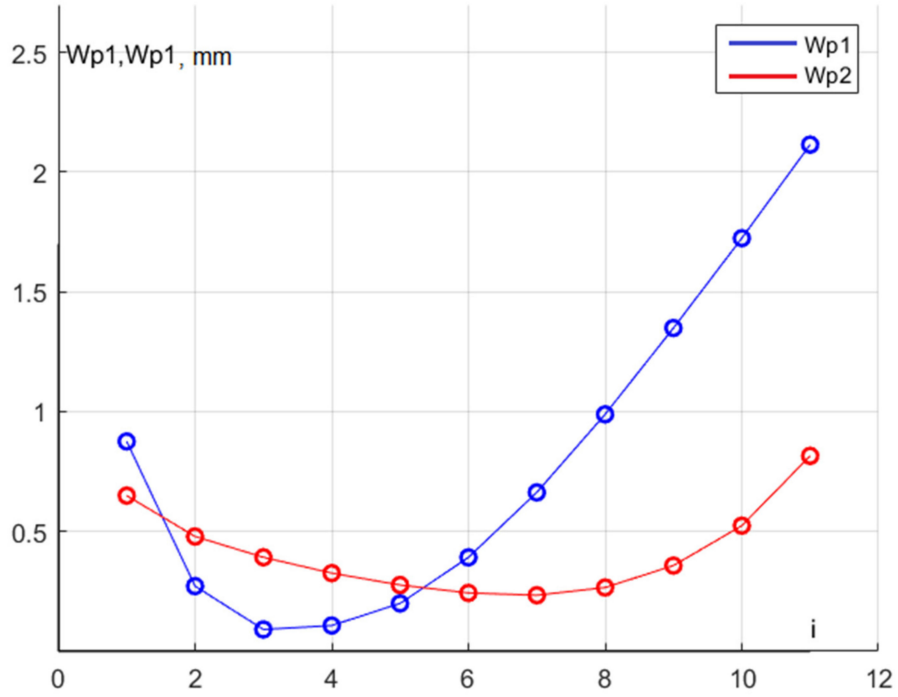

Figure 8. Graphics of the grippers P_1 and P_2 linear accelerations analogues.

5. Conclusions

Kinematic synthesis and analysis of the PM with two end effectors have been carried out. In the kinematic synthesis according to the given laws of motions (or positions) of two end effectors, the structural scheme, and geometric parameters of links of the synthesized PM are determined. The structural scheme of this PM is formed by connecting two output objects (end effectors) and a base using three CKCs: two passive and one negative CKC. Passive and negative CKCs are structural modules from which the PM is formed. Passive CKCs are two movable serial manipulators, and the negative CKCs is a three-jointed link. Serial manipulators (passive CKCs) do not impose geometric constraints on the movement of the output objects, and the three-jointed link (negative CKC) imposes three geometric constraints. Therefore, the geometric parameters of the links of the negative CKCs are determined, and the geometric parameters of the links of the passive CKCs are varied depending on the imposed geometric constraints of the negative CKC. Kinematic synthesis of the negative CKC was carried out on the basis of the Chebyshev and least-square approximations. Since the structure of the synthesized PM consists of two dyads, the position analysis is solved analytically. Analogues of angular velocities and accelerations are determined from two systems of linear equations obtained by differentiating the loop-closure equations with respect to the generalized coordinate.

Author Contributions: Z.B. established the methods of structural and kinematic synthesis of the RoboMech class parallel manipulator (PM) with two grippers. M.A.L. established the methods of kinematic analysis of this PM. A.M. created the programs in the MatLab for kinematic synthesis of the PM and obtained numerical results. A.K. created the programs in the MatLab for kinematic analysis of the PM and obtained numerical results. All authors contributed equally to the evaluation of the results and writing of this paper. All authors have read and agreed to the published version of the manuscript.

Funding: This research is founded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP08857522).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

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