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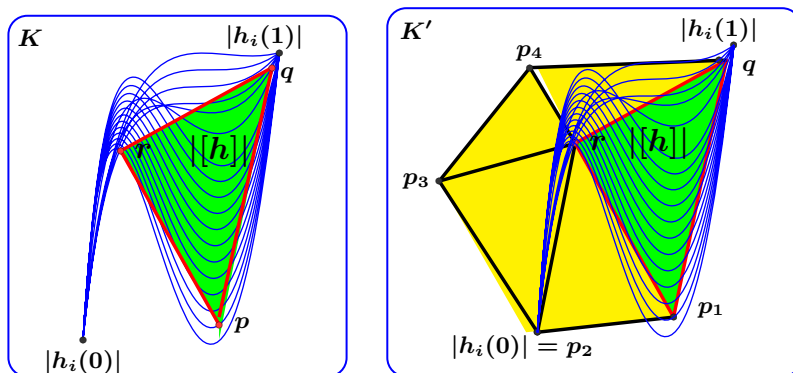
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**HOMOTOPIC CLASSES AND TRIANGLE CLUSTERS IN
MINIMAL PLANAR REGION COVERINGS.
SEVERAL FORMS OF THE GEOMETRIC
LUSTERNIK-SCHNIREL'MANN CATEGORY**

JANES F. PETERS AND FARIHA N. PEU & JUWAIIRIAH ZIA



1.1: $\Delta pqr \subseteq |[h]|$ homotopy class covering Δpqr

1.2:
 $TC = \{\Delta T \cap r\} \subseteq \min \{|[h]|\},$
 $\min \{\text{homotopic classes}\}$ covering

FIGURE 1. $\exists gcat(X) : X = \min \{|[h]| \in 2^{\mathbb{R}^2}\} \supset \Delta pqr$

from Theorem 1 & $\exists gcat(E) : E = \min \{\{|[h]| \in 2^{\mathbb{R}^2}\}\} \supset \{\Delta pqr\} \cap r$
from Corollary 1.

This paper introduces results for several forms of the geometric Lusternik-Schirel'mann categories (LS gcat), namely, geometric realization of one or more homotopy classes containing multiple sinusoidal paths that minimally cover planar triangles and clusters of Delaunay triangles with a common vertex that provide an application of gcat in a minimal covering of a foreground object in a video frame.

1. GEOMETRIC LUSTERNIK-SCHNIREL'MANN CATEGORY

Let $h : I \rightarrow K$ be a continuous map called **homotopy** (briefly, **path** in a space K). A **homotopic class** for different maps h (denoted by $[h] = \{h_0, \dots, h_i, \dots, h_{n-1[n]}\}$ with $[n] = \text{mod } n \in \mathbb{Z}^+$) is a collection of $h_{i[n]}$ homotopic maps that have the same endpoints, namely, $h_i(0)$ and $h_i(1)$. The geometric realization of $[n]$ (denoted by $|[h]|$) is a collection of sinusoidal curves, each being the geometric realization of a path h .

Definition 1. [Geometric LS Category] [2].

For a topological space X , the geometric category of X is the minimal covering of X with contractible open subsets of X . ■

Lemma 1. Let \mathbf{h} be a homotopic sinusoidal path. The geometric realization $|\mathbf{h}|$ is a planar sinusoidal curve.

Lemma 2. Let $[\mathbf{h}]$ be a collection of homotopic paths with common endpoints. The geometric realization $||[\mathbf{h}]||$ is a collection space filling planar curves.

Lemma 3. Let $\Delta\mathbf{pqr}$ be a planar filled triangle in a space K , geometric realization $||[\mathbf{h}]||$ such that each path h has endpoints $h(0), h(1) \in \mathbf{K} \setminus \Delta\mathbf{pqr}$. Then $\lim_{i \rightarrow \infty} h_i \in [h] \supseteq \Delta\mathbf{pqr}$.

Lemma 4. Let h be a homotopic path in space K .

1° Every path \mathbf{h} is contractible.

2° There exists a minimal $||[\mathbf{h}]||$ in space K with $h_i \in [h]$ with the same boundary endpoints $\mathbf{h}(0), \mathbf{h}(1) \in \partial\Delta\mathbf{pqr}$ such that $||[\mathbf{h}]||$ covers triangle $\Delta\mathbf{pqr} \subset K$.

3° Every planar triangle in K has a minimal covering $||[\mathbf{h}]||$.

Theorem 1. There exists $\mathbf{gcat}(|[\mathbf{h}]| \in \mathbf{2}^{\mathbf{K}})$ such that $\mathbf{min} ||[\mathbf{h}]|| \supseteq \Delta\mathbf{pqr} \in \mathbf{2}^{\mathbf{K}}$.

Example 1. From Theorem 1, the triangle T in Fig. 1.1 has a $||[\mathbf{h}]||$ minimal covering, which is a $\mathbf{gcat}(T)$. ■

A cluster of triangles $\{\Delta\mathbf{pqr}\}$ in a Euclidean space K is a collection of triangles attached to a common vertex. From Theorem 1, we have

Corollary 1. Let $|\{[\mathbf{h}]\}| \subset \mathbf{K}$ such that each $\mathbf{h} \in \{[\mathbf{h}]\} \subset \mathbf{K} \setminus \{\Delta\mathbf{pqr}\}$ has the same endpoints. There exists $\mathbf{gcat}(|\{[\mathbf{h}]\}|)$ such that $\mathbf{min} |\{[\mathbf{h}]\}| \supseteq \{\Delta\mathbf{pqr}\} \in \mathbf{2}^{\mathbf{K}}$.

Example 2. From Corollary 1, there is a $\mathbf{gcat}(|\{[\mathbf{h}]\}|)$ such that $\mathbf{gcat}(|\{[\mathbf{h}]\}|)$ is a minimal covering of the triangle cluster $\{\Delta\mathbf{pqr}\} \cap \mathbf{p}$, which is a bounded region in Fig. 1.2. ■

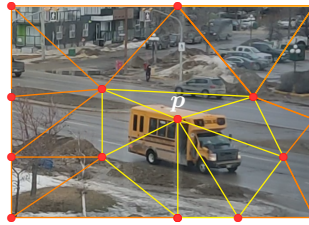


FIGURE 2. Delaunay Triangle cluster minimally covering bus in video frame.

2. DELAUNAY TRIANGLE CLUSTER MINIMAL COVERING OF VIDEO FRAME FOREGROUND OBJECT

Delaunay triangulations, introduced by B.N Delone [Delaunay] [1], represent pieces of a continuous space in form of triangles with edges attached to selected vertices [3].

Theorem 2. *Let \mathcal{M} be a Delaunay triangle cluster with k triangles minimally covering a planar bounded region $E \in 2^{\mathbb{R}^2}$. Then $\mathbf{gcat}(\mathcal{M}) = k$.*

Example 3. *With restrictions on the selection of vertices (e.g., centroids), we obtain a minimal cluster T of k triangles covering the bus, which is a bounded region in a Delaunay triangulation of the video frame in Fig. 2. Hence, from Theorem 2, $\mathbf{gcat}(T) = k$. ■*

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